NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1043

CHARTS FOR THE RAPID CALCULATION OF THE WORK REQUIRED TO COMPRESS DRY AIR

By William A. Benser, Ward W. Wilcox
and Charles H. Voit

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SUMMARY

Accurate values of the specific heat for dry air were applied to the calculation of the work required to compress dry air. Two work charts are presented from which the work of compression can be quickly and accurately determined from the initial temperature, the over-all pressure ratio, and either the compression exponent or the adiabatic efficiency of the compressor. The first chart presents the work of compression at pressure ratios up to 100, initial temperatures of 400° to 600° R, and compression exponents of 1.40 to 2.00. The second chart presents the same values for pressure ratios up to 25, initial temperatures of 400° to 1000° R, and compression exponents of 1.40 to 2.00. The accuracy of the results thus obtained is of the same order as the accuracy obtained when temperature-enthalpy tables are used for determining the work of compression.

INTRODUCTION

In view of recent developments in the field of compressor-turbine combinations, increasing emphasis has been placed on the use of such units for aircraft propulsion. In order to facilitate the design of such units, a simple accurate method of predicting the performance is essential. One of the most important problems encountered in the performance calculations is the determination of the work required by the compressor. For conventional single-stage centrifugal compressors, the calculation of the work of compression is simplified by the assumption that the state of the gas is defined by the perfect gas law and that the value of the specific heat at constant pressure is constant. When multistage centrifugal compressors or axial-flow compressors of high over-all pressure ratio are considered, however, these assumptions may lead to an appreciable error in the calculation of the work required to compress the air.

Although the work needed for compression can be accurately determined from temperature-enthalpy tables, a method of calculation based directly on compressor operating variables would be advantageous. A study of available data was made at the NACA Cleveland laboratory to determine the applicability of the perfect gas law to express the state condition of the dry air and to determine the variation of the specific heat at constant pressure with pressure and temperature. An empirical expression was fitted to recently obtained data (reference 1) that are more accurate than previously available data for the variation of the specific heat of air at constant pressure to permit the calculation of the work of compression from the usual thermodynamic equations.

Two work charts were prepared from which it is possible to determine graphically the work necessary for the compression of dry air when the pressure ratio, the initial temperature, and a measure of the compressor efficiency are known. The first of these charts covers pressure ratios up to 100, the range of initial temperatures for altitudes up to the isothermal region, and a wide range of compressor efficiencies. The second chart is an enlarged version of the first for pressure ratios up to 25, initial temperatures from 400° to 1000° R, and the same range of efficiencies.

ANALYSIS

Properties of Dry Air

Equations of state. - In the determination of the work necessary for compression of any gas, it is first necessary to establish the relation between the pressure, the specific volume, and the temperature of the gas. The common engineering practice is to consider that dry air can be treated as a perfect gas and thus

$$pV = RT \tag{1}$$

where

p pressure, pounds per square foot

V specific volume, cubic feet per pound

T temperature, °R

R universal gas constant, 53.35 foot-pounds per pound per °R for dry air

A closer approximation to the actual state conditions for dry air, particularly at low temperatures and high pressures, may be

obtained from the equation of state proposed by Beattie and Bridgeman in reference 2. In reference 1, the maximum deviation of this equation from measurements of the actual gas is given as 0.5 percent and the authors state that the average error with pressures up to 177 atmospheres and for the temperature range of 230° to 851° R is only 0.198 percent.

The deviation of the perfect gas law from the Beattie and Bridgeman equation presented in references 1 and 2 for pressures up to 25 atmospheres and temperatures up to 3000° R is given in figure 1. The high positive values of the variation shown in figure 1 represent a condition of temperature and pressure that is not encountered in continuous-flow compressors.

According to Keenan and Kaye (reference 3), air at a temperature of 32° F obeys the perfect gas law with a deviation of 1 percent at 300 pounds per square inch and 0.1 percent at atmospheric pressure. The gain in accuracy resulting from the use of the Beattie and Bridgeman equation over that obtained with the perfect gas law is not great enough to justify the additional complexity of the equations thus introduced nor has the validity of the Beattie and Bridgeman equation been established beyond 851° R. Consequently, dry air has been treated as a perfect gas for the purposes of this report.

Variation of specific heat. - Although the assumption that air acts as a perfect gas implies a change in the specific heat c_p only with temperature, for air a slight variation with pressure also exists, which may be neglected in this instance. References 4, 5, 6, and 7 show that considerable disagreement existed among early investigators of the exact variation of c_p with T. In recent years, however, a method of calculating specific heats from spectrographic data has been developed and the method is now recognized as being very accurate over a large range of temperatures. A great deal of work has been done by H. L. Johnston and his associates, some of which is presented in reference 8, in applying this technique to most of the common monatomic and diatomic gases. The absence of limitations due to strength considerations permits calculation of specific heats for temperatures up to 5500° R.

The values of the specific heat of dry air at zero constant pressure presented herein were obtained from reference 1, the authors of which attribute them to H. L. Johnston who calculated the values from spectrographic data. These values of specific heat are given as data points in figure 2; the solid line consists of arcs of two parabolas, which are matched empirically to these data. This empirical curve was also found to agree with values determined from spectrographic data (reference 9) with a maximum deviation of 1.07 percent, which occurred at 1300° R.

The empirical mathematical expressions fitted to the data of figure 2 to form two parabolic arcs give the variation of specific heat at constant pressure with temperature as:

$$c_p = 0.2445 - 2.206 \times 10^{-5}\ T + 2.758 \times 10^{-8}\ T^2 \qquad (2)$$

for temperatures from 360° to 1140° R and

$$c_p = 0.2413 + 1.0886 \times 10^{-3} \sqrt{T - 976} \qquad (3)$$

for temperatures from 1140° to 5500° R.

Temperature-Pressure Relation for Compression Process

Polytropic relation. - In continuous-flow compressor work, it is customary to base the efficiency of the compression process upon the reversible adiabatic (isentropic) case. (See reference 10.) Actually, the internal losses normally resulting from friction, throttling, and mixing cause part of the energy added to be rendered unavailable and the process is no longer isentropic, although adiabatic. A convenient means of expressing the actual relation between the initial and the final conditions is afforded by the use of an exponent n in the expression

$$p_1\ V_1^{\ n} = p_2\ V_2^{\ n} \qquad (4)$$

where the subscripts 1 and 2 denote the initial and final states, respectively. The value of n is a measure of the compression efficiency, which may be converted to adiabatic efficiency, polytropic efficiency, or any desired efficiency factor. When the perfect gas law is combined with this expression, a convenient expression may be had, from which the polytropic relations of temperatures and pressures may be found:

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \frac{T_2}{T_1} \qquad (5)$$

The numerical value of the exponent n for several continuous-flow compressors lies between 1.45 and 1.80 for all practical operating conditions.

Isentropic relation. - The work required for adiabatic compression is a minimum for an isentropic process and, therefore, the adiabatic efficiency, or ratio of isentropic to actual work, is

commonly used as a measure of compressor efficiency. If c_p is assumed to be constant, the temperature and pressure relation may be obtained from equation (5) by substituting the ratio of specific heats γ for n. For the present case, however, γ may not be considered as constant. From common thermodynamic relations, however,

$$\int_1^2 \frac{c_p dT}{T} = \frac{R}{J} \int_1^2 \frac{dp}{p} \tag{6}$$

where J is the mechanical equivalent of heat, 778.3 foot-pounds per Btu. When the empirical expressions of equations (2) and (3) are substituted in equation (6) and integrated, the following equations for the relation between pressure and temperature for isentropic compression are obtained:

$$\log \frac{p_2}{p_1} = 3.55690 \log \frac{T_2}{T_1} - 1.39374 \times 10^{-4}\, T_1 \left[\left(\frac{T_2}{T_1}\right) - 1\right] + 8.71247 \times 10^{-8}\, T_1^{\,2} \left[\left(\frac{T_2}{T_1}\right)^2 - 1\right] \tag{7}$$

for final temperatures below 1140° R and

$$\begin{aligned}\log \frac{p_2}{p_1} = {} & 3.55690 \log \frac{1140}{T_1} - 1.39374 \times 10^{-4}\, T_1 \left[\left(\frac{1140}{T_1}\right) - 1\right] \\ & + 8.71247 \times 10^{-8}\, T_1^{\,2} \left[\left(\frac{1140}{T_1}\right)^2 - 1\right] \\ & + 3.51064 \log \frac{T_2}{1140} + 1.37555 \times 10^{-2} \sqrt{T_2 - 976} \\ & - 0.429734 \arctan \sqrt{\frac{T_2 - 976}{976}} - 0.0089776\end{aligned} \tag{8}$$

for initial temperatures below and final temperatures above 1140° R. The angle in equation (8) must, of course, be in radians and the logarithms are to the base 10.

Work of Compression

By means of the foregoing relations, it is possible to find the final temperature T_2 from known values of pressure ratio, inlet temperature, and polytropic exponent for both polytropic and isentropic compression. When the expressions for c_p in equations (2) and (3) are substituted into the conventional work equation and the resulting expressions are integrated between the initial and final conditions, the following equations are obtained:

$$\text{work (hp)/(lb)(sec)} = 0.3460\ (T_2 - T_1) - 1.56084 \times 10^{-5}\left(T_2^2 - T_1^2\right) + 1.3009 \times 10^{-8}\left(T_2^3 - T_1^3\right) \tag{9}$$

for temperatures between 360° and 1140° R and

$$\text{work (hp)/(lb)(sec)} = 0.3460\ (1140 - T_1) - 1.56084 \times 10^{-5}\left(1140^2 - T_1^2\right) + 1.3009 \times 10^{-8}\left(1140^3 - T_1^3\right) + 0.34149\ (T_2 - 1140) + 1.0270 \times 10^{-3}\ (T_2 - 976)^{3/2} - 2.157 \tag{10}$$

when the initial temperature is below and the final temperature is above 1140° R. The work value thus obtained does not include any mechanical losses of the compressor.

Thus, for a process of the type under consideration, the work of compression between two temperatures remains at one fixed value and may be found by determining the end temperature from the pressure-temperature relations previously discussed.

DISCUSSION

Error resulting from assumption of constant c_p. - The percentage difference between the work of compression found by using a constant value for c_p of 0.243 (Btu)/(lb)(°F) (the value commonly used for normal air in engineering work) and the work of compression found from equations (9) and (10) is plotted in figure 3 for pressure ratios up to 100 and for various values of the exponent n. At a pressure ratio of 25 and a value of n of 1.80, the percentage error due to the assumption of constant c_p is 6.5 percent. For

pressure ratios below 5, however, this error will be less than 1 percent for the entire range of values of n considered. The use of variable values of c_p is therefore necessary for compressors operating in the range of pressure ratios above 5.

Effect of varying initial-temperature conditions. - The effect on the work of compression of varying the initial-temperature conditions is shown in figure 4, where the work of compression is plotted against pressure ratio for a number of values of T_1 and for values of n of 1.40, 1.60, and 2.00. The values of T_1 used herein correspond to the standard atmospheric temperature at altitudes from sea level to the isothermal region. The decrease in the work of compression as the initial temperature is decreased is clearly shown, although the effect is less pronounced at low values of n.

From this figure, it may also be seen that the over-all work of compression decreases rapidly as the value of the exponent n decreases. At high pressure ratios a given decrease in n results in a greater percentage decrease in the work required than at lower pressure ratios.

GRAPHICAL WORK CHARTS

In figures 5 and 6, charts are presented from which the actual and the isentropic work of compression may be determined for the entire range of practical operating conditions. The use of these charts enables the final temperature, the actual work of compression, and the isentropic work of compression to be found when the pressure ratio, the compression exponent or the adiabatic efficiency, and the initial temperature are known. For the convenience of the reader, figures 5 and 6 have been prepared in original size to permit more accurate readings and are attached.

In figure 5, pressure ratios up to 100 were used and values of n were chosen in close increments from 1.40 to 2.00. The final temperature was limited to 2000° R, which exceeds the present practical limit imposed by strength considerations of metal compressors. Initial temperatures were selected to allow interpolation and to cover inlet conditions for all altitudes from sea level to the isothermal region.

The chart presented in figure 6 covers a low range of pressure ratios. A large range of initial-temperature conditions was presented to permit the use of the chart when compressors are installed in series. The use of these charts is fully illustrated by the sample calculations in the appendix.

Only dry air has been considered in all calculations. Any corrections for humidity effects will increase the value of c_p slightly and thus increase the work of compression.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, July 7, 1945.

APPENDIX - USE OF GRAPHICAL CHARTS

The charts given in figures 5 and 6 give a step-by-step graphical plot of the calculation of the actual work of compression from known initial conditions. The actual work of compression may be determined directly from the chart by proceeding around the chart in a counterclockwise direction from the upper right quadrant. In the first quadrant a plot of pressure ratio against temperature ratio for various values of the polytropic exponent n is given, which fixes the pressure-temperature relation. The second quadrant contains the transition from temperature ratio to final temperature by the multiplication of temperature ratio by the various values of the initial temperature. As previously stated, for each initial temperature there is a fixed relation between final temperature and the work of compression. This relation is given in the third quadrant. In the fourth quadrant, two sets of curves are superimposed. The isentropic work of compression is given for the complete range of pressure ratios at a number of initial temperatures. In addition, curves of constant values of adiabatic efficiency are plotted to show the relation of actual work to isentropic work.

It should be noted that if the charts are used in the reverse sense, the values of n may be found when the over-all adiabatic efficiency is known. Further explanation of the use of these charts is given by the following sample calculations.

Sample Calculations

Determination of work of compression and adiabatic efficiency. - Assume the following conditions for the sample calculation: pressure ratio, 15; n, 1.50; initial temperature, 500° R. The dashed line in figure 6 shows the path taken for this example.

Enter the first quadrant at a pressure ratio of 15 and go up to the n curve of 1.50 and over to the temperature-ratio scale; the temperature ratio obtained is 2.47. Continue across at this temperature ratio to the 500° R initial-temperature line and down to the final-temperature scale; the final temperature obtained is 1235° R. Continue down to the 500° R initial-temperature line in the third quadrant and across to the scale for the actual work of compression in the fourth quadrant; the value obtained is 256 horsepower per pound per second.

The isentropic work of compression is found by entering the fourth quadrant at a value of 15 on the vertical pressure-ratio scale at the right and proceeding across to the 500° R initial-temperature curve and down to the isentropic-work scale; the value obtained is 198 horsepower per pound per second.

The adiabatic efficiency may be found from the intersection of the previously determined values of actual and isentropic work by interpolation between the constant-efficiency curves. The adiabatic efficiency obtained is approximately 77 percent.

Determination of the polytropic exponent n and work of compression. - Assume the following conditions for the sample calculation: pressure ratio, 15; adiabatic efficiency, 77 percent; initial temperature, 500° R. The path followed in this example is the reverse of that in the previous example.

Enter the fourth quadrant at a value of 15 on the vertical pressure-ratio scale and go across to the 500° R initial-temperature curve and down to the isentropic work of compression scale; the value obtained is 198 horsepower per pound per second. Proceed from this value of the isentropic work of compression up to an adiabatic efficiency of 77 percent, which is found by interpolating between the constant-efficiency curves. Continue across from this point to the actual work of compression, which is 256 horsepower per pound per second. Continue across to the 500° R initial-temperature curve in the third quadrant and up to the final-temperature scale, which results in a final temperature of 1235° R. Continue up to the 500° R initial-temperature curve in the second quadrant and across into the first quadrant to a pressure ratio of 15. This point is seen to fall on the 1.50 polytropic-exponent curve.

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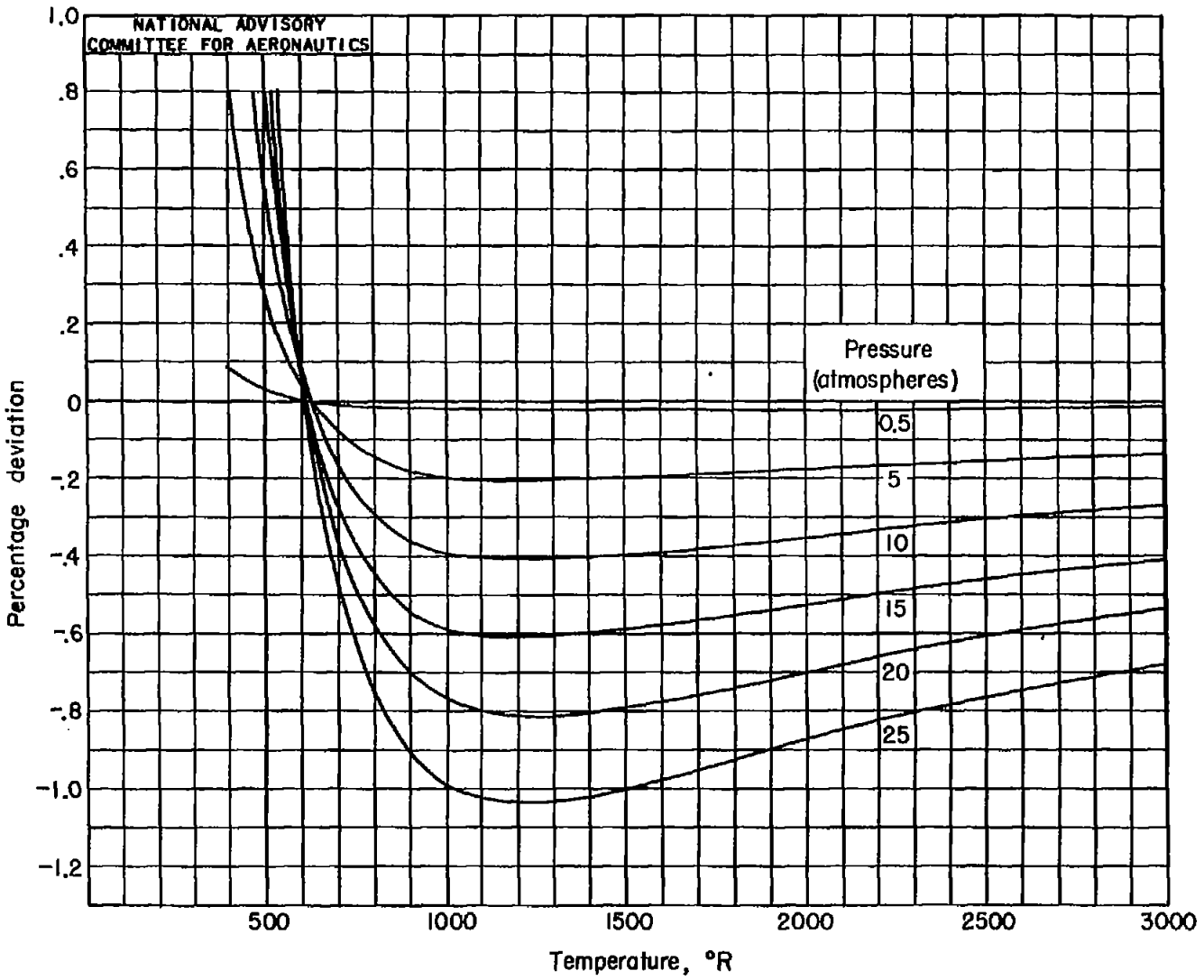


Figure 1.— Percentage deviation of the perfect gas law from the equation presented in references 1 and 2 for a range of state conditions.

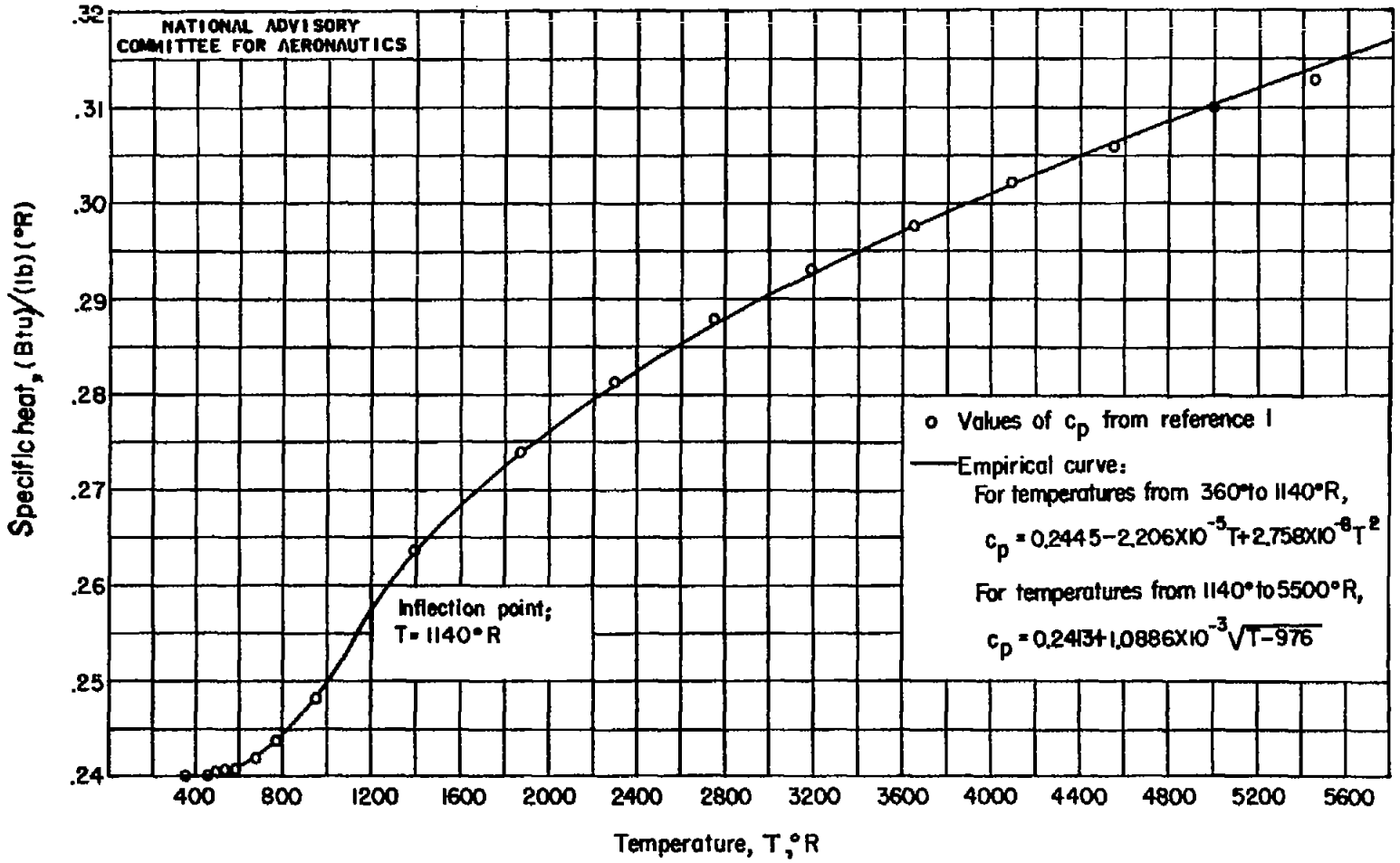


Figure 2.– The variation of specific heat with temperature for dry air at zero constant pressure.

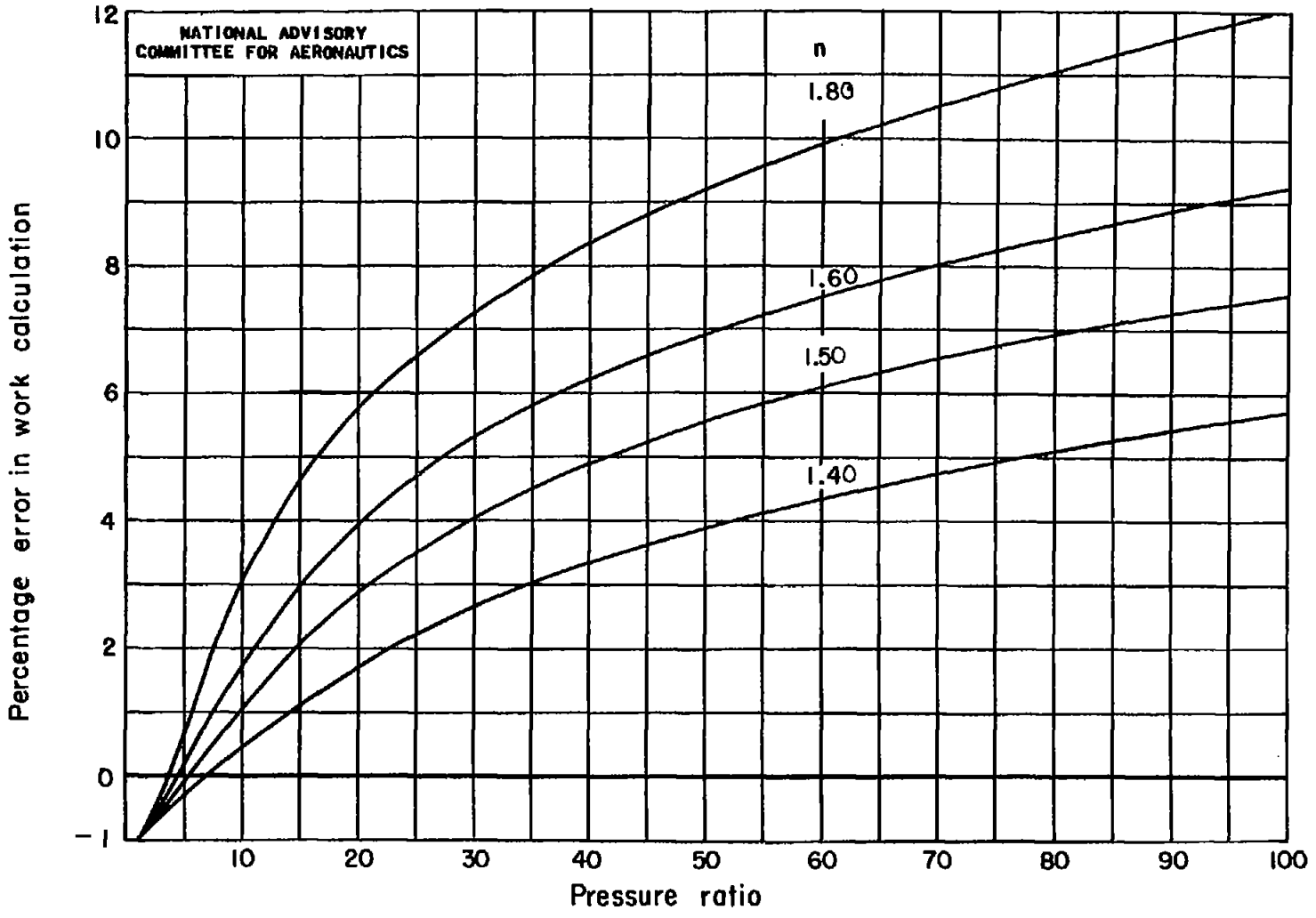


Figure 3.-The percentage error introduced in determining the work of compression for dry air by assuming a fixed value of c_p of 0.243 (Btu)/(lb)(°F). Sea-level altitude temperature at inlet, 518.4 °R.

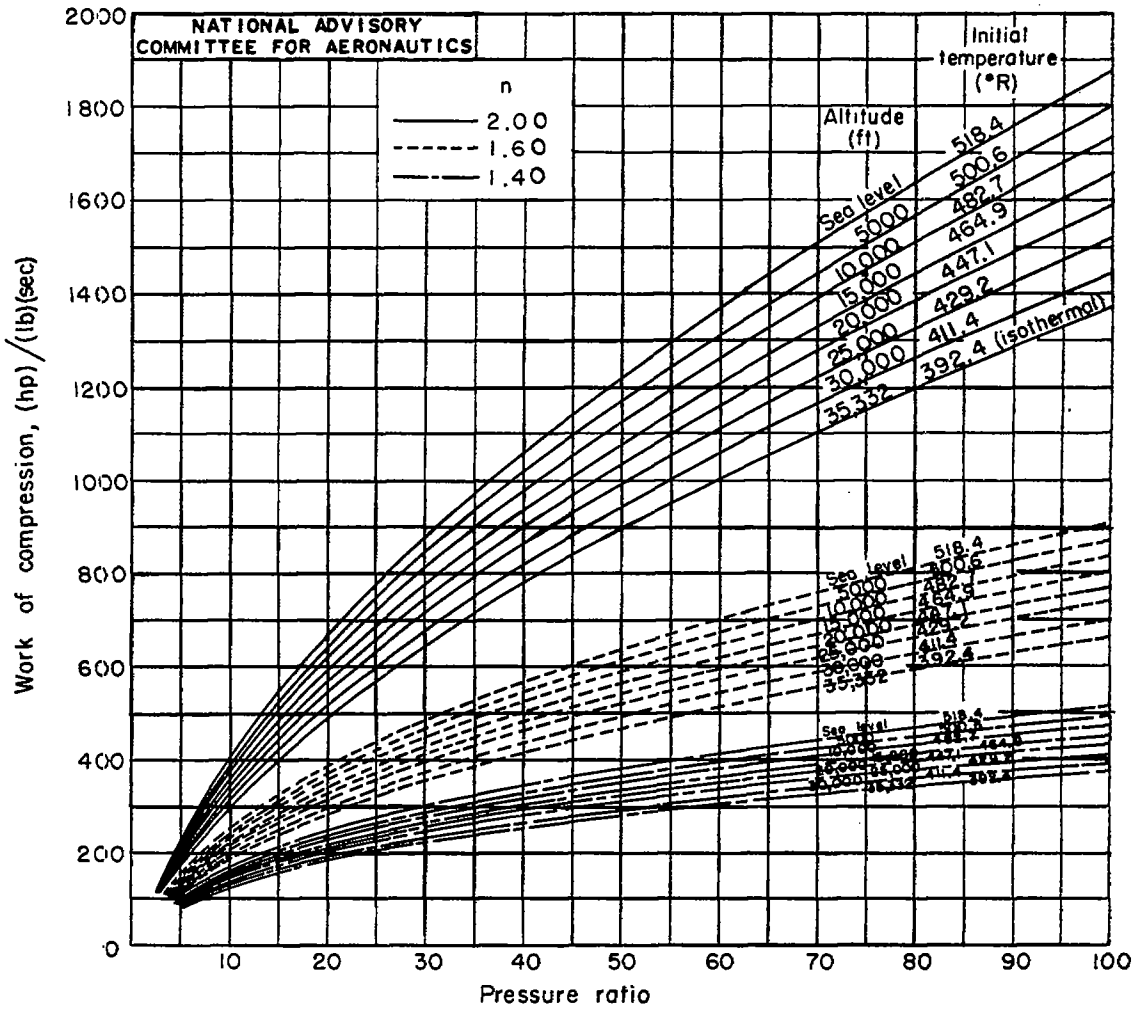


Figure 4.— The work required to compress dry air at several initial temperatures to various pressure ratios for three values of the polytropic exponent n.

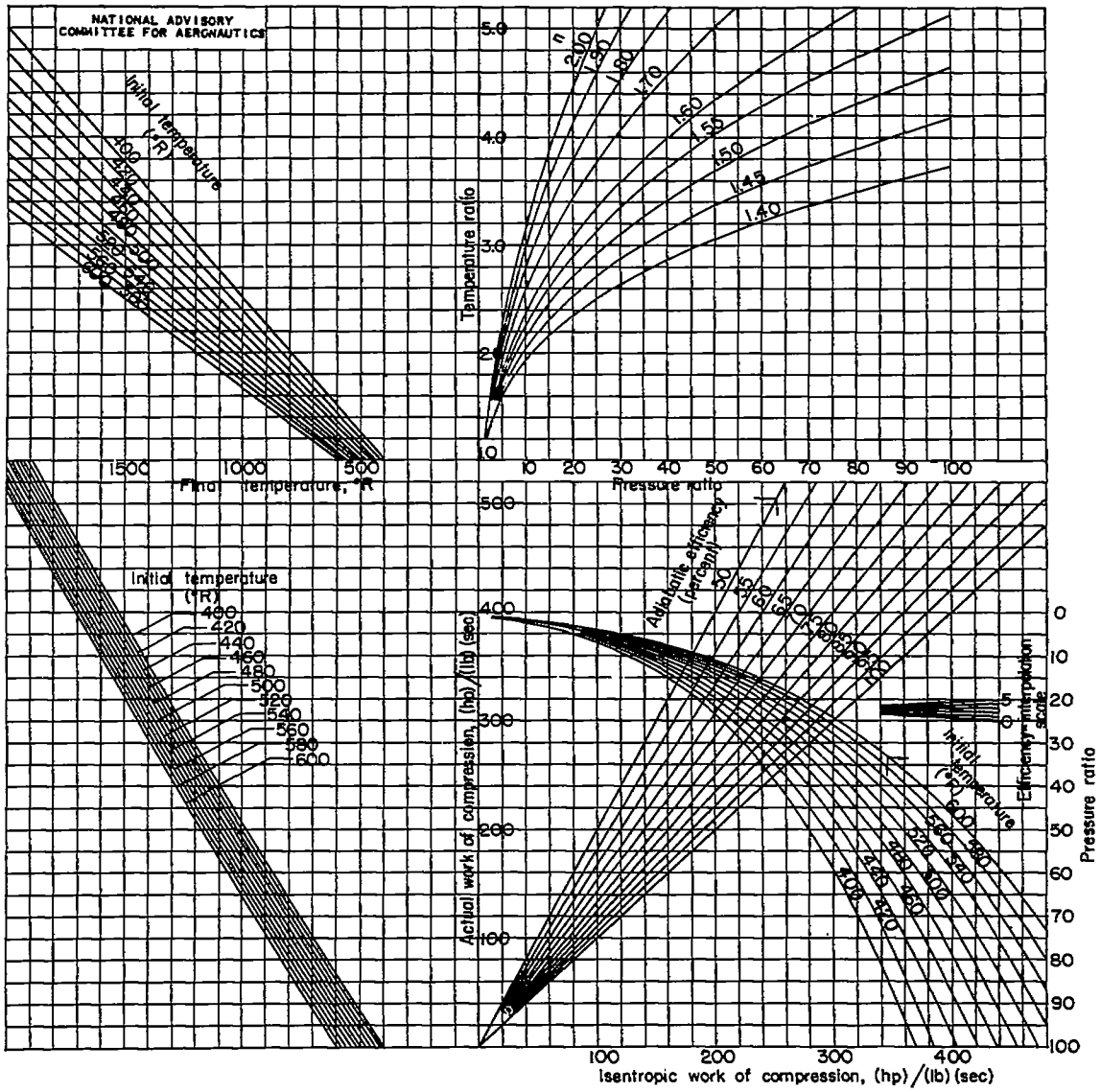


Figure 5.—Graphical work chart for determining the actual and isentropic work of compression and adiabatic efficiency for pressure ratios to 100 : 1 and initial temperatures from 400° to 600°R.

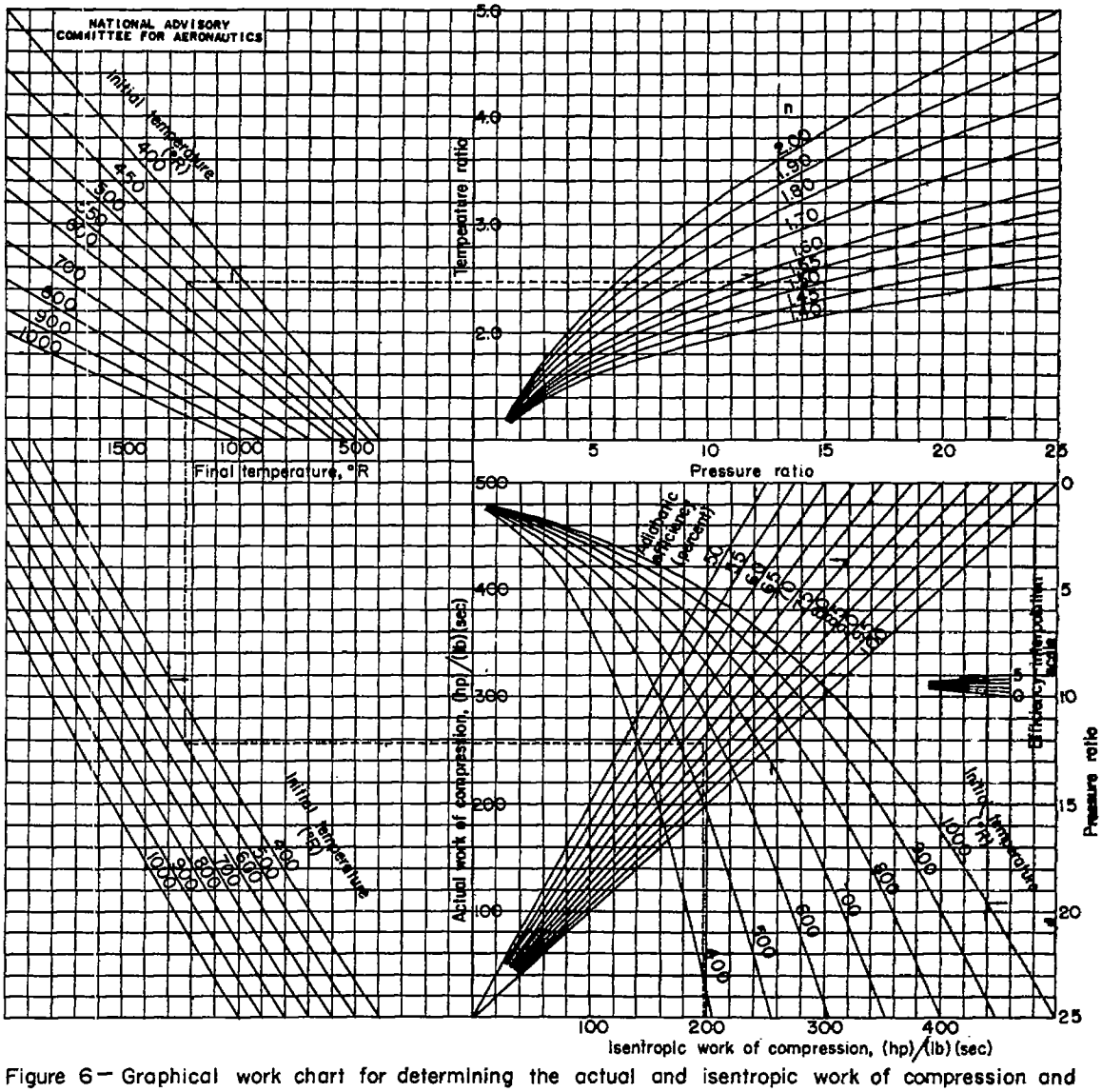
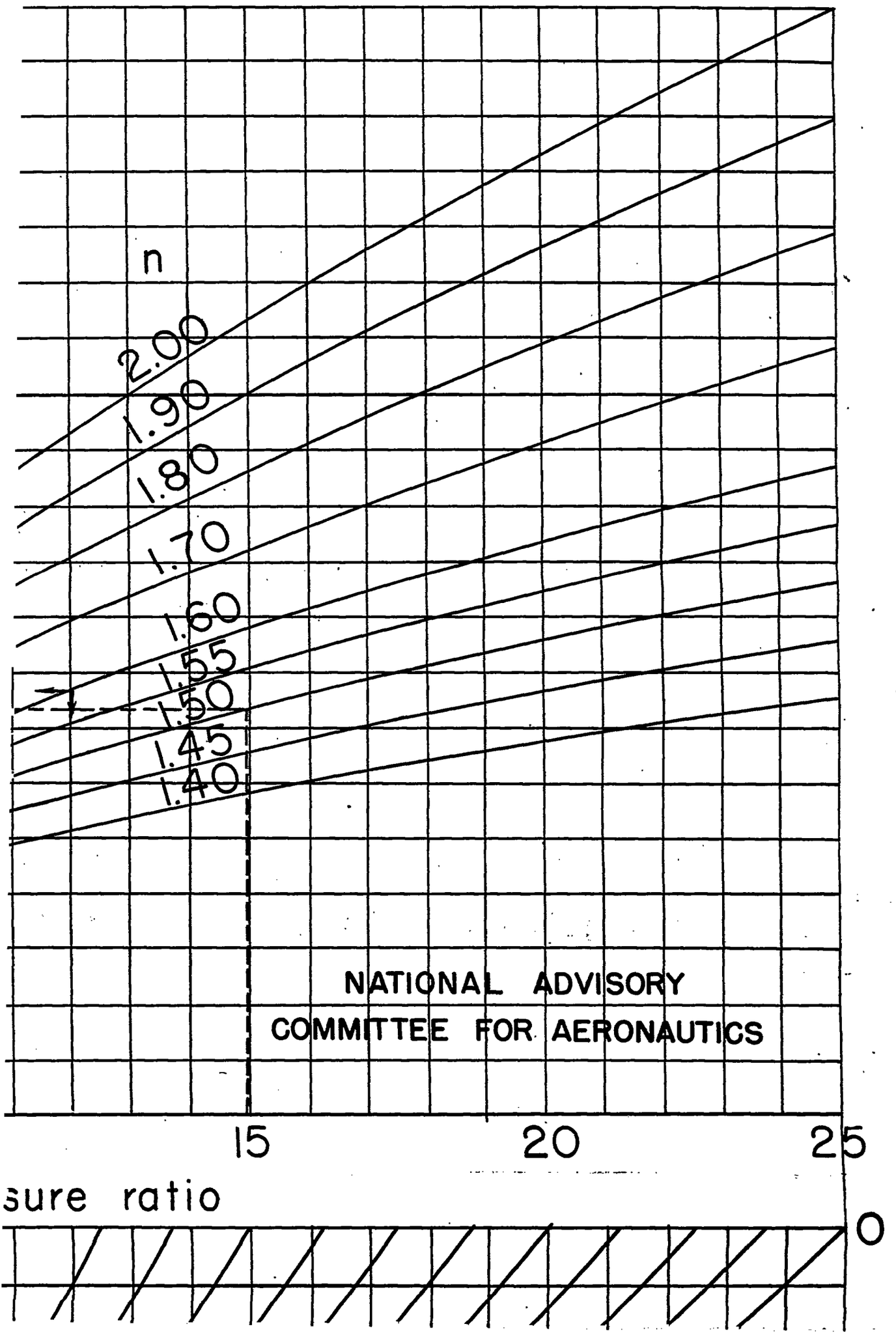


Figure 6 — Graphical work chart for determining the actual and isentropic work of compression and adiabatic efficiency for pressure ratios to 25 : 1 and initial temperatures from 400° to 1000° R.

Fig. 6


n
2.00
1.90
1.80
1.70
1.60
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1.50
1.45
1.40
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25
sure ratio
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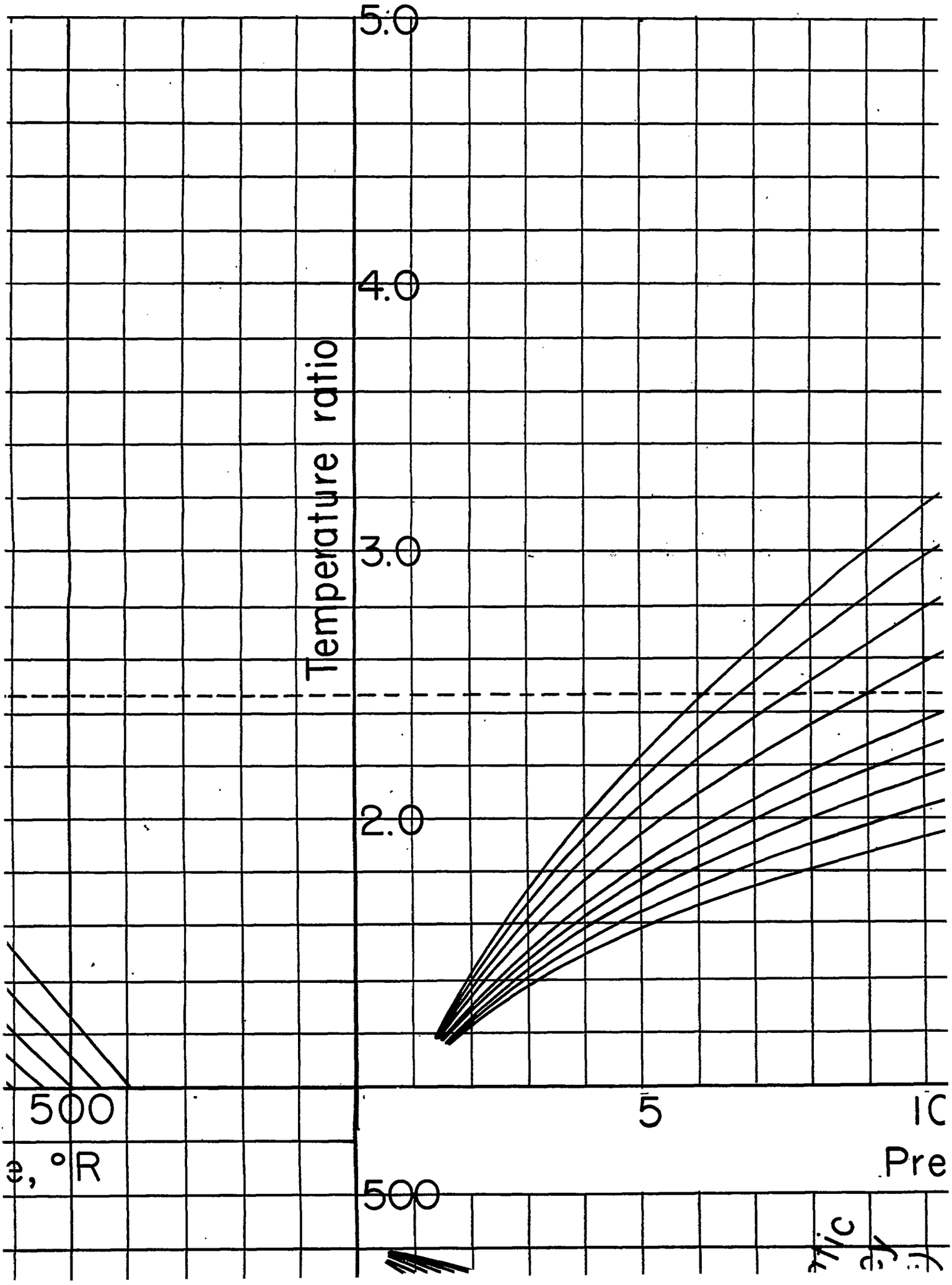
5.0
4.0
3.0
2.0
Temperature ratio
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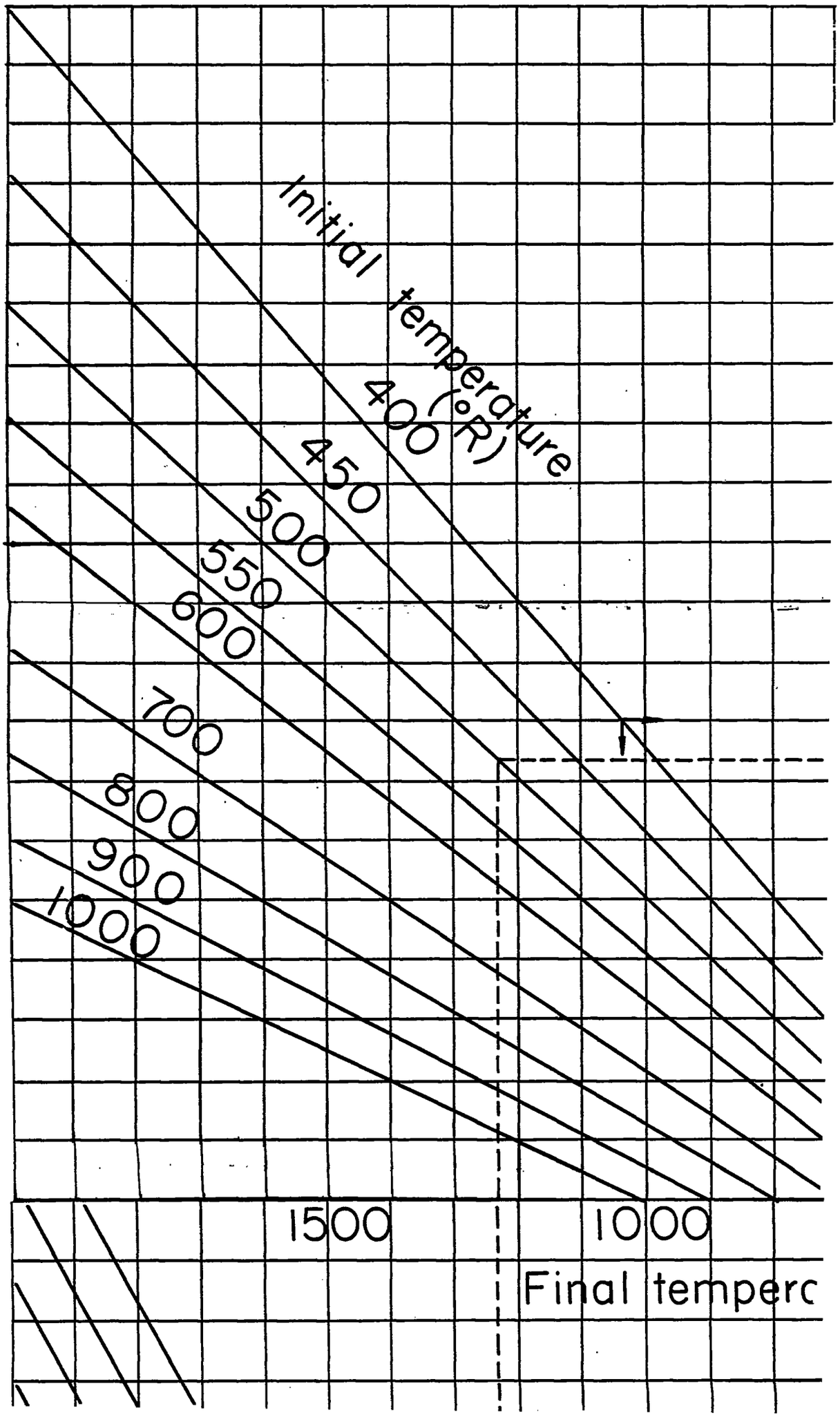
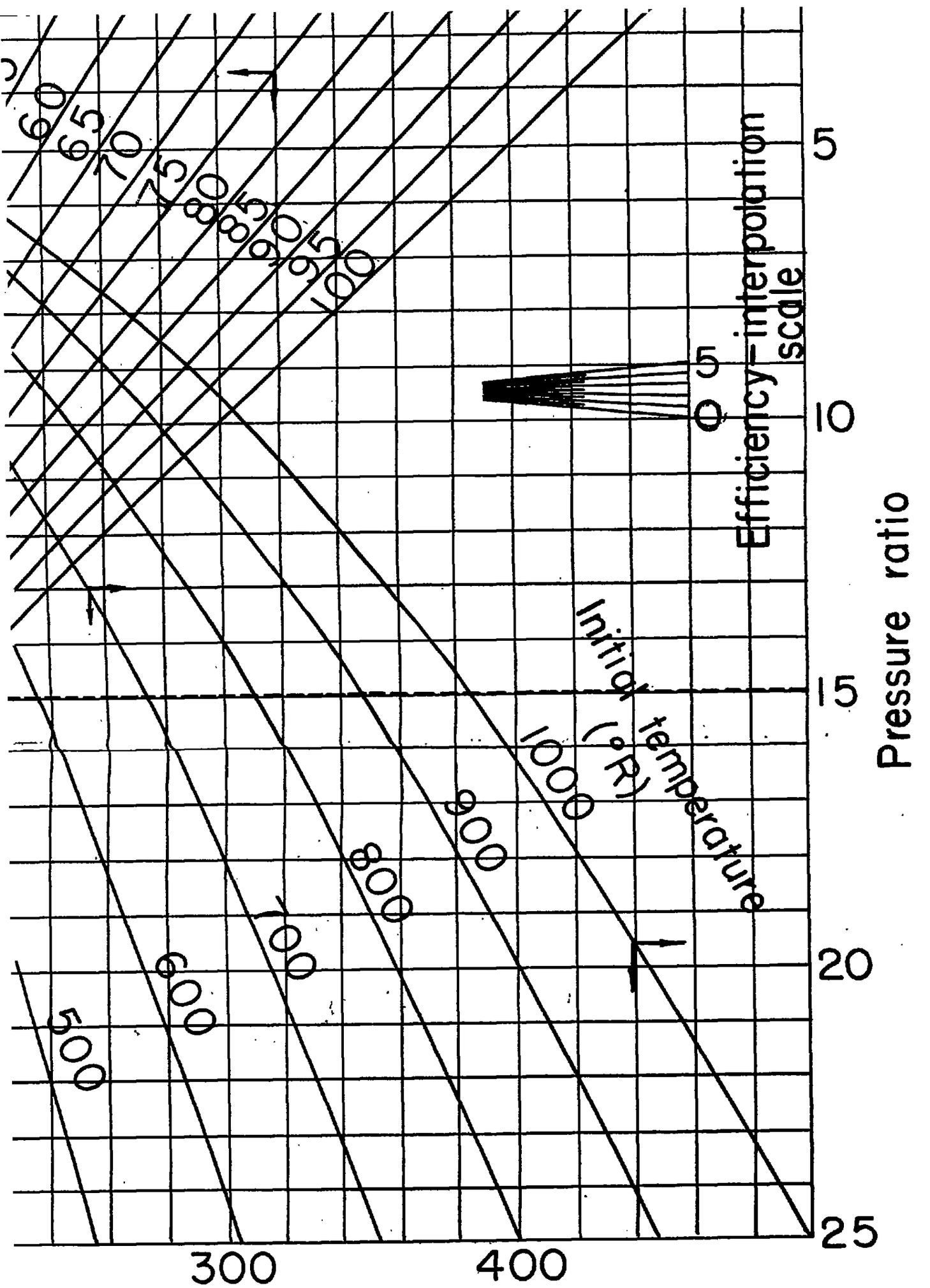
5
10
e, °R
Pre
500


Initial temperature (°R)
400
450
500
550
600
700
800
900
1000
1500
1000
Final temperc



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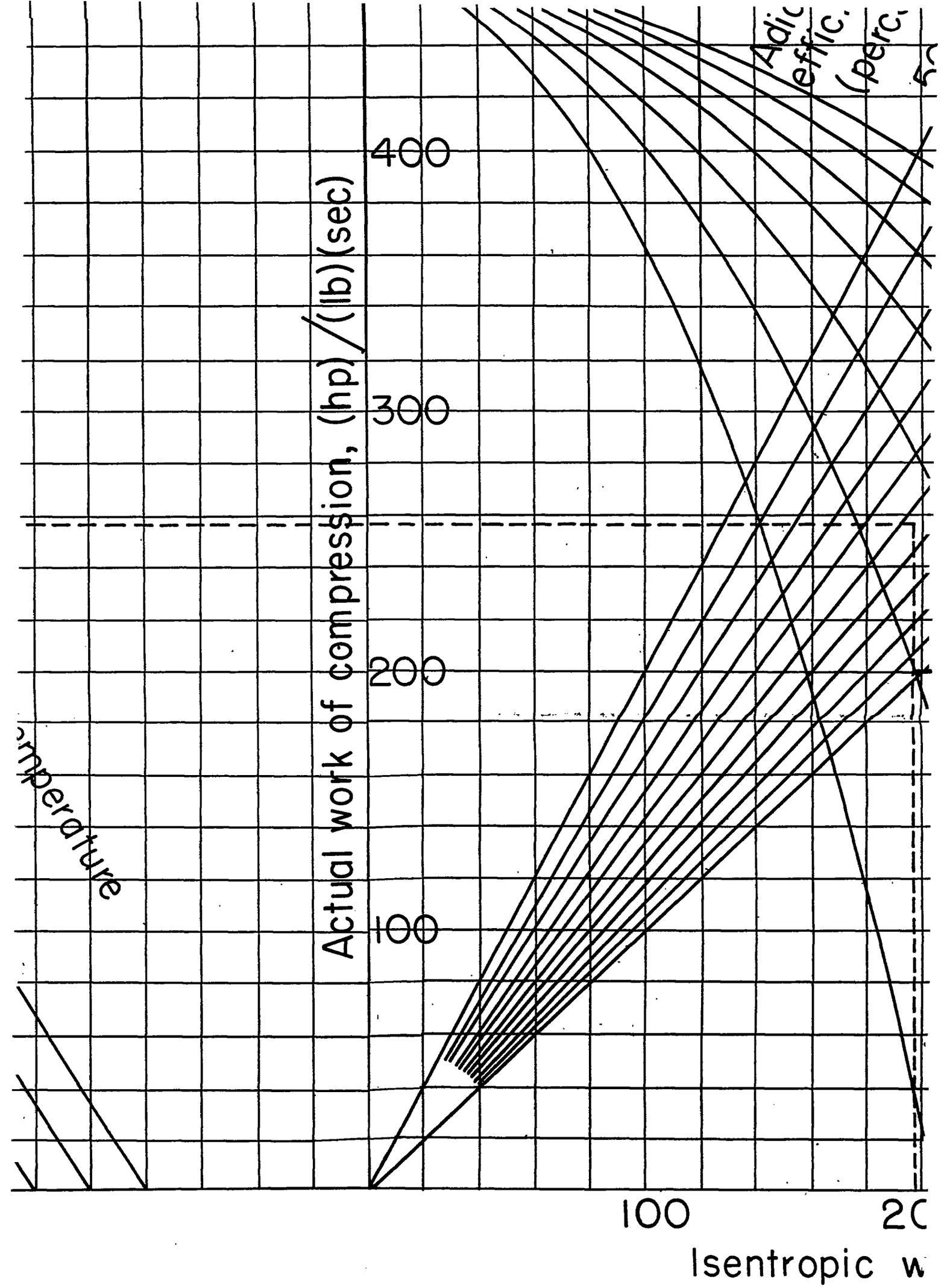
peratures from 400° to 1000° R.



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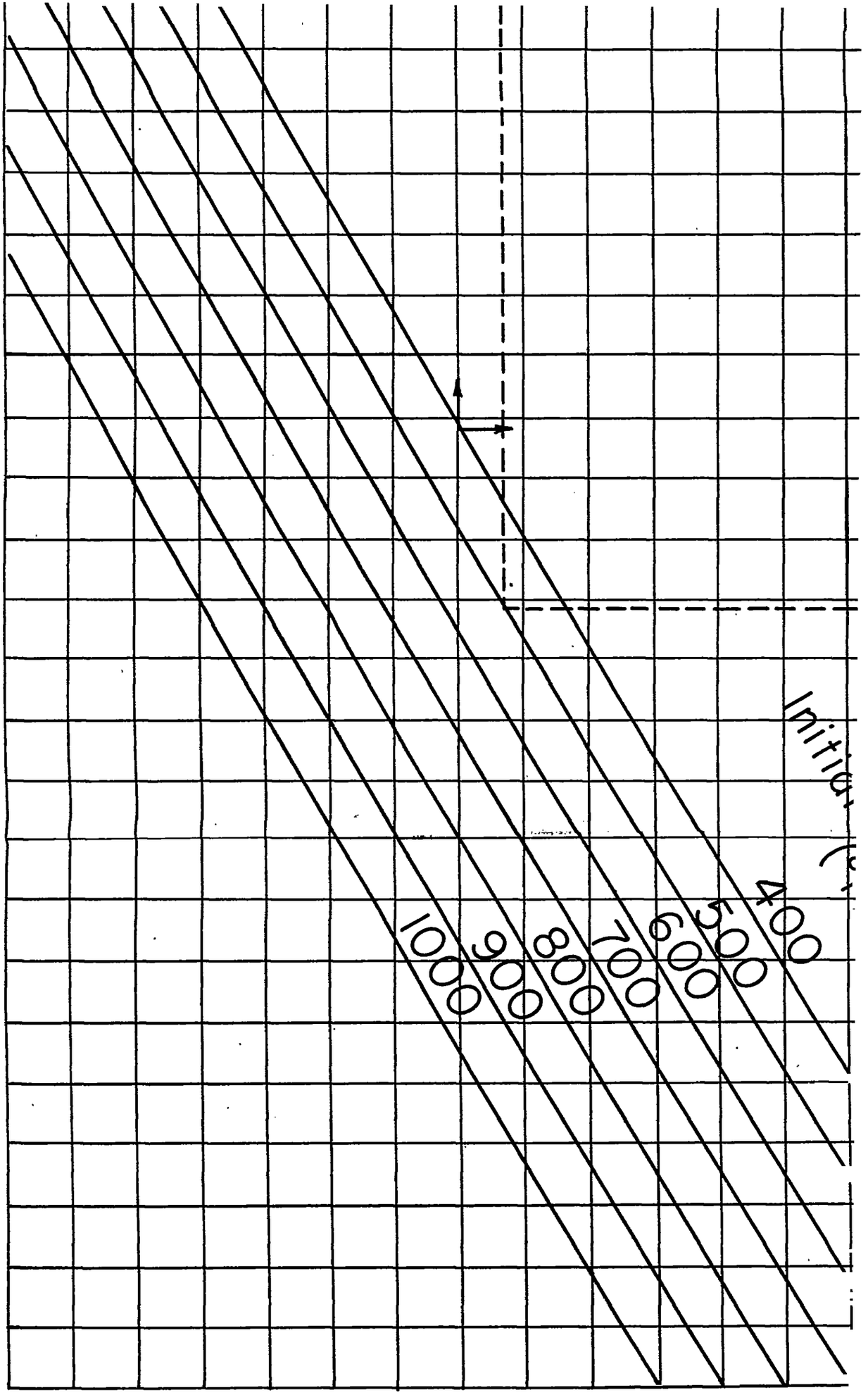
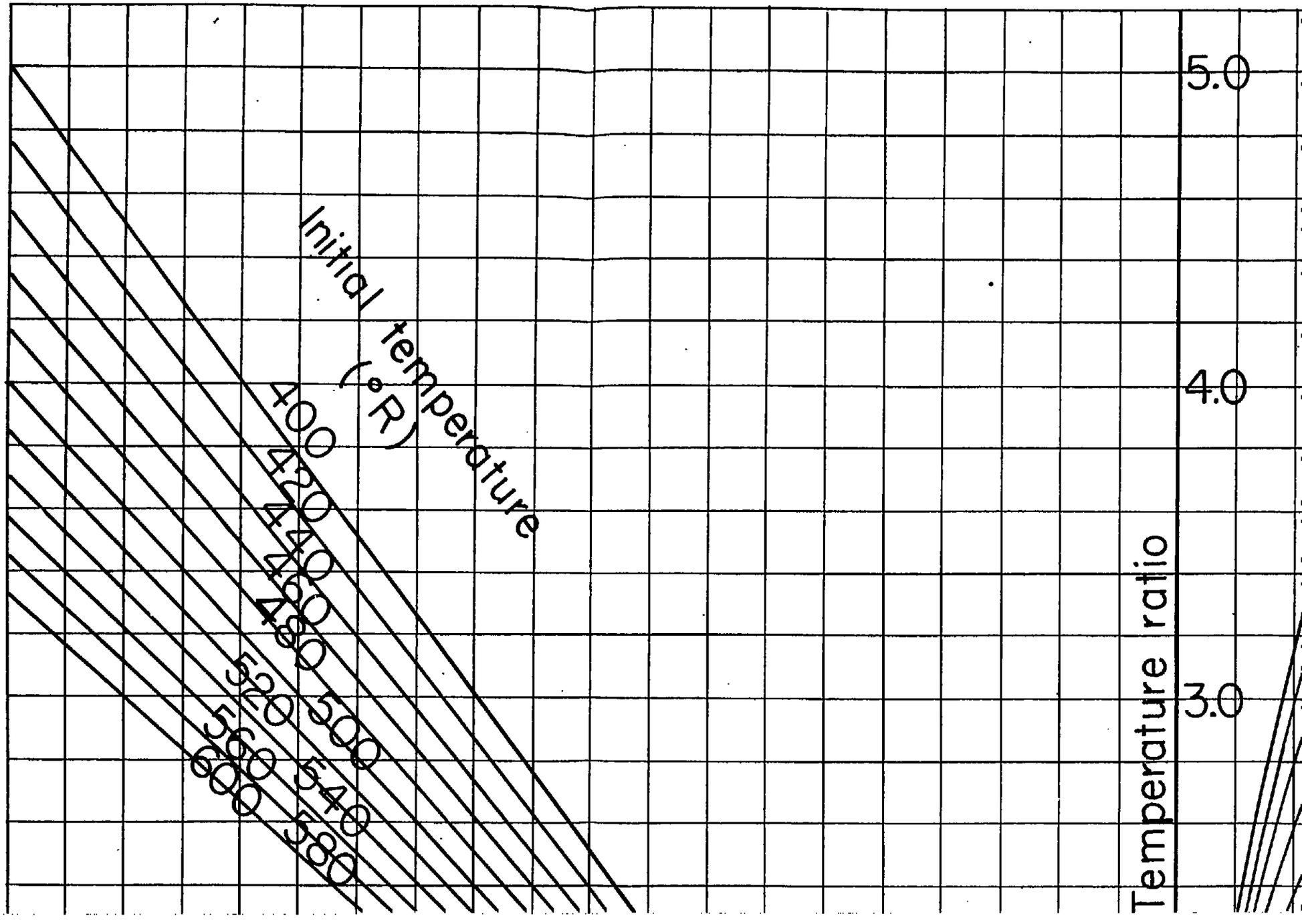


Figure 6.— Graphical work c
adiabatic efficiency for p


Initial temperature (°R)
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460
480
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520
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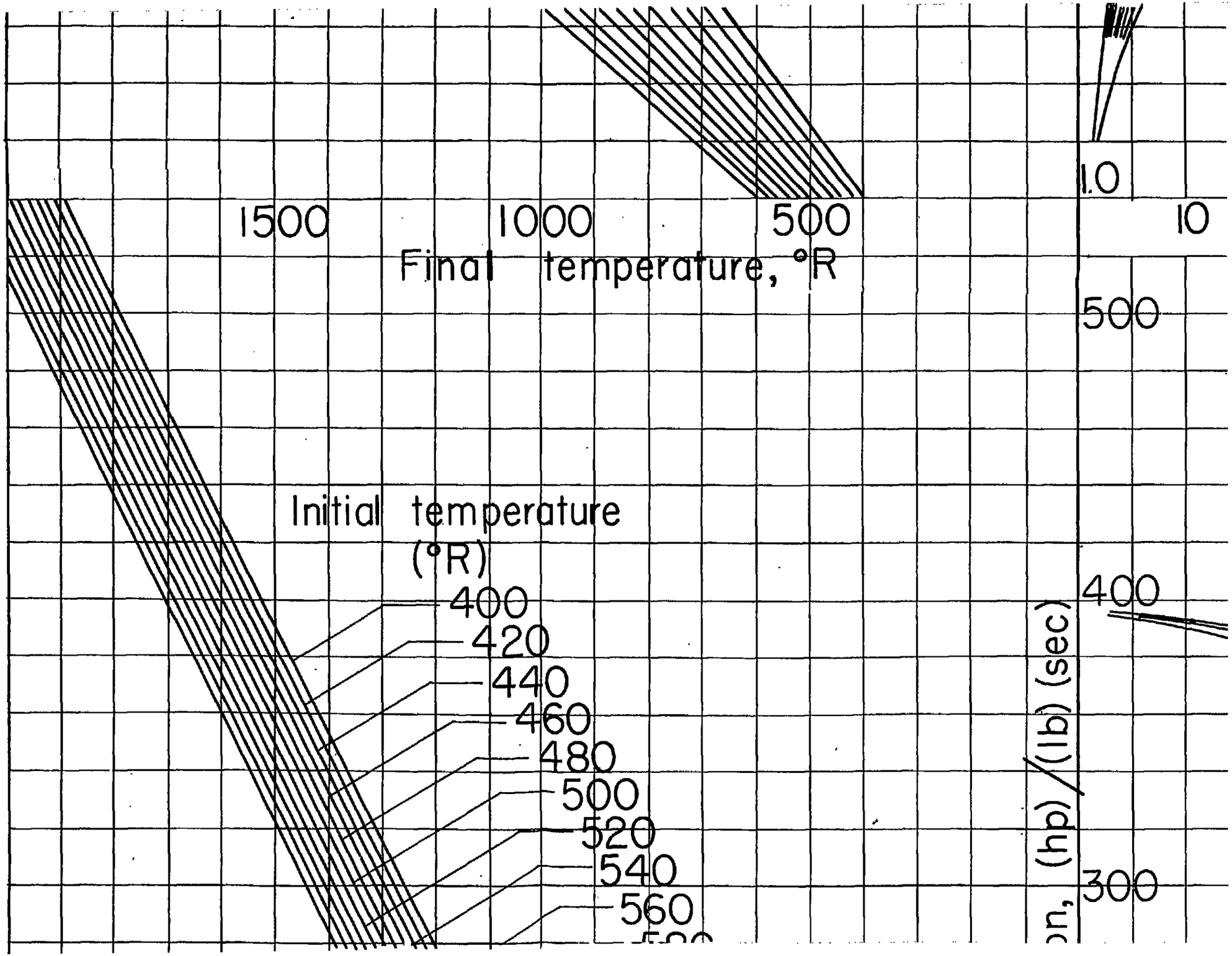
560
580
600
Temperature ratio
5.0
4.0
3.0


1500
1000
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Final temperature, °R
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(°R)
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420
440
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520
540
560
(hp)/(lb) (sec)

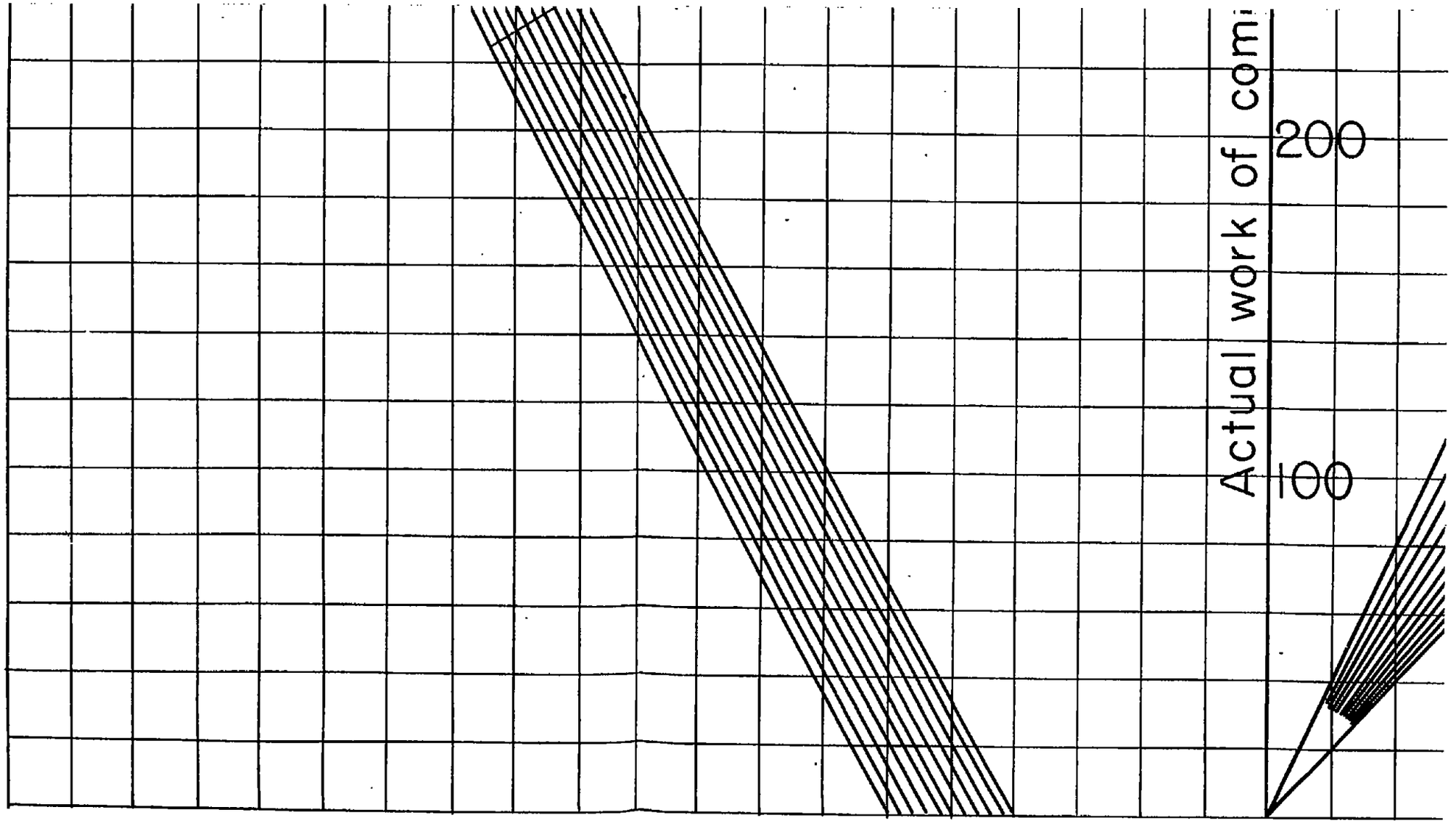
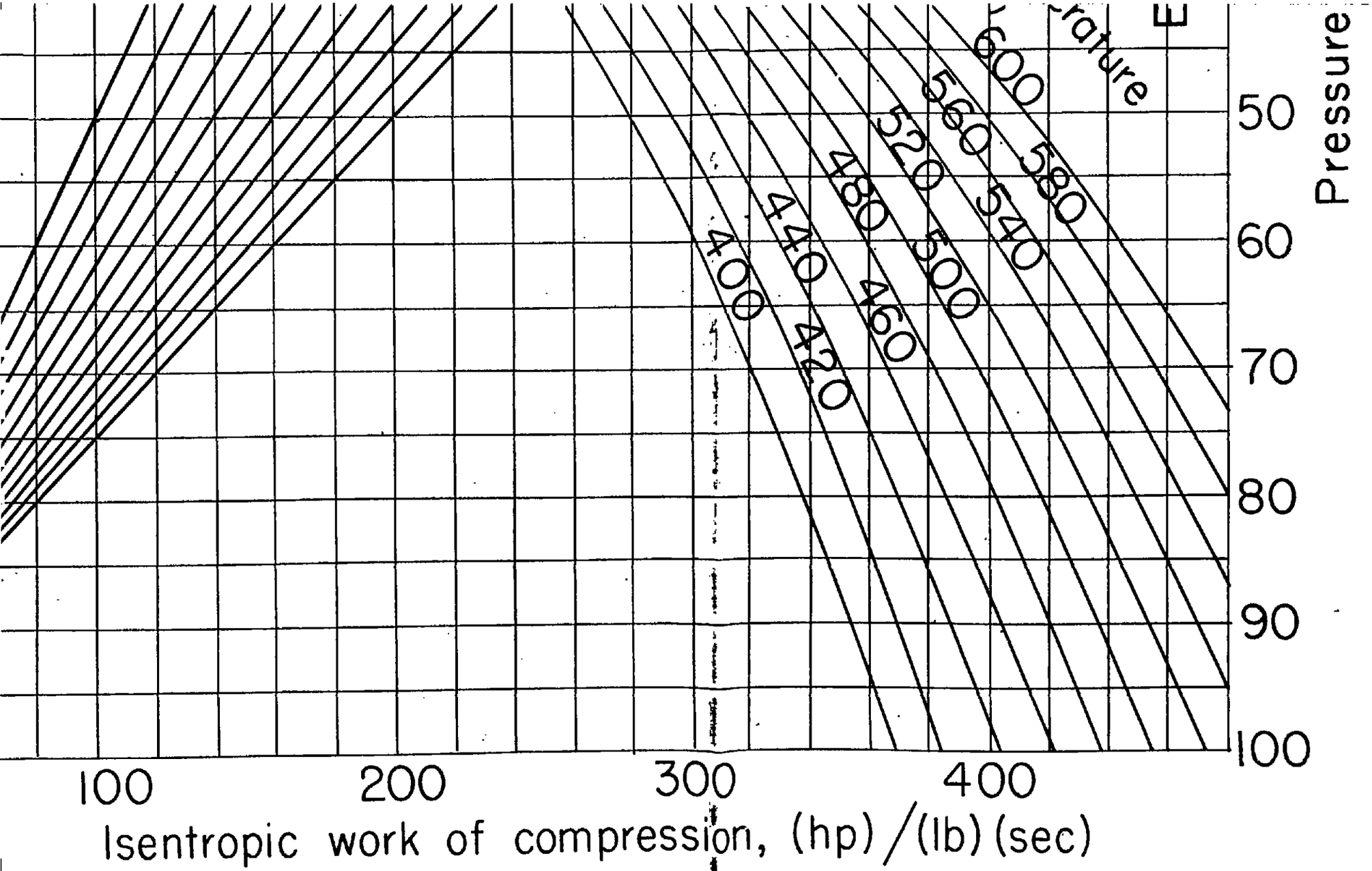


Figure 5.— Graphical work chart for determining the adiabatic efficiency for pressure ratios to 100 :



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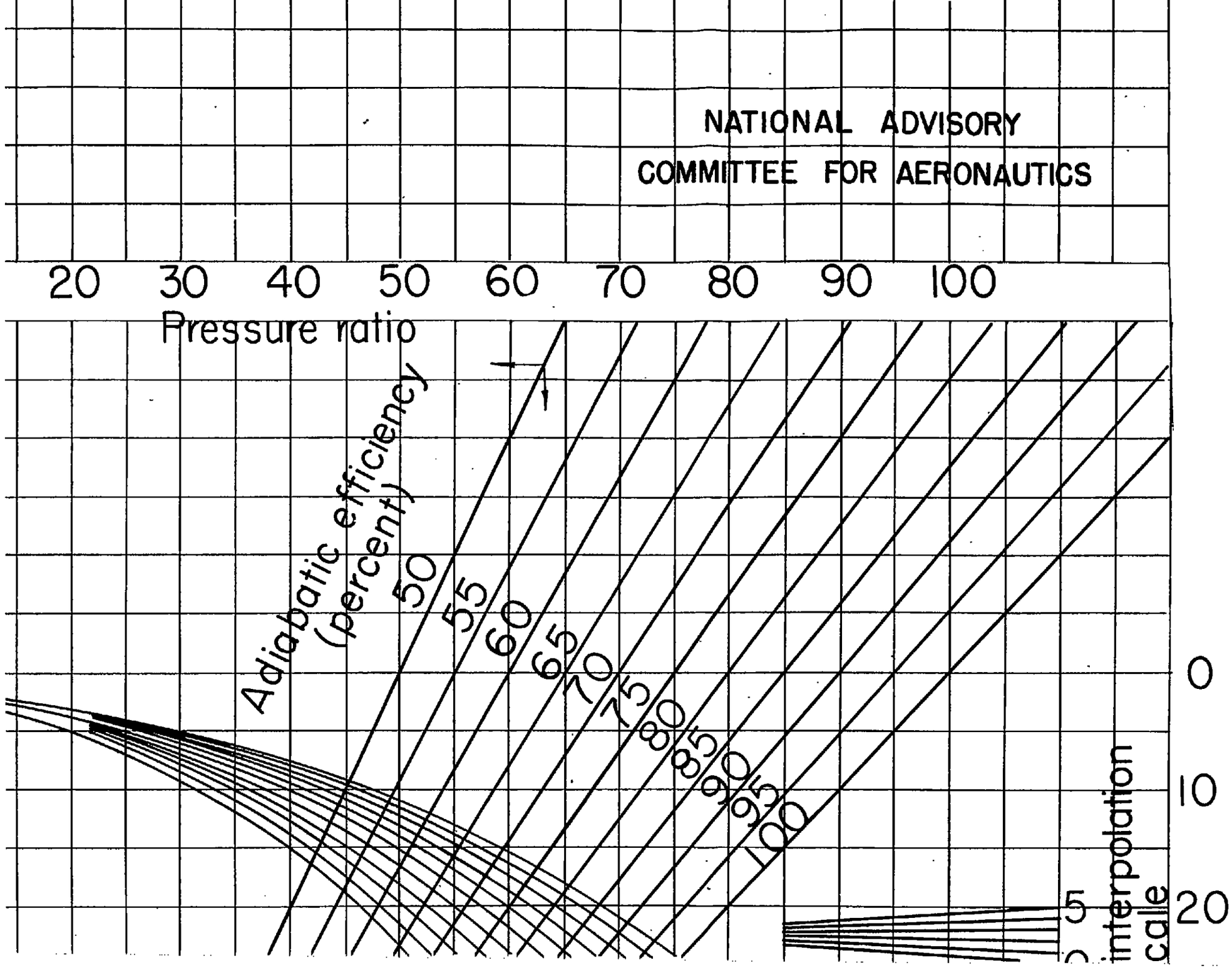
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80
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100
Pressure ratio
Adiabatic efficiency
(percent)
50
55
60
65
70
75
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90
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Interpolation
scale
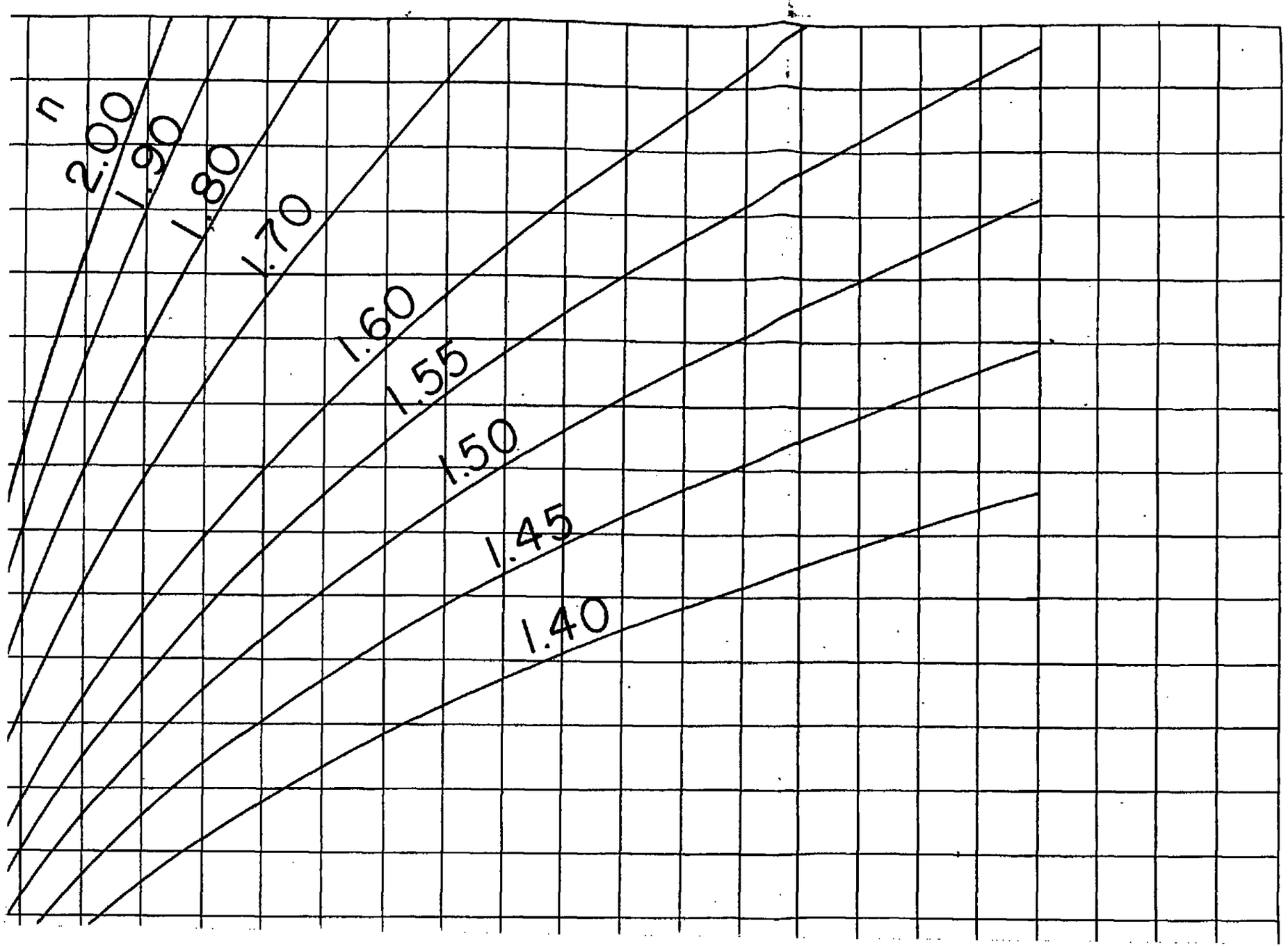
Fig. 5