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A RELAXATION PROCEDURE FOR THE STRESS ANALYSIS OF A CONTINUOUS BEAM-COLUMN ELASTICALLY RESTRAINED AGAINST DEFLECTION AND ROTATION AT THE SUPPORTS

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SUMMARY

A method of stress analysis is presented for a continuous beam-column supported by deflectional and rotational springs. The principal feature of the method is the use of a relaxation procedure to determine the deflections and rotations of the supports. The shears and moments at the supports and between the supports can then be calculated with the aid of simple equations and graphs. An example is presented to illustrate the use of the method.

INTRODUCTION

The usual simplifying assumptions made in the stress analysis of a continuous beam-column are that the supports are unyielding and that they provide no resistance to rotation. In actual structures the supports are always somewhat flexible. A simple way of taking the flexible resistance of the supports into account is to assume that each support consists of two independent elastic springs - one, a deflectional spring and the other, a rotational spring. This idealization is shown in figure 1. The values of the spring stiffnesses depend upon the nature of the supports and must be calculated or estimated by the designer.

Even after this simplified representation of the supports has been made, the usual methods of analysis are quite inadequate except in special cases. The unknown yielding of the supports makes the usual type of moment distribution described in references 1 and 2 not readily applicable, and solution by the differential-equation method becomes exceedingly involved as the number of spans increases.

The method of the present paper is proposed as a practicable procedure for the stress analysis of an elastically supported continuous beam-column having supports that may be represented by deflectional and rotational springs. A relaxation process is used to determine the deflections and rotations of the supports and the deflection curve, shears, and bending moments are then calculated with the aid of simple equations and graphs. The method is applicable only when the stresses are below the proportional limit. In all other respects it is quite general. However, tables and graphs to facilitate the use of the method have been prepared only for the more important cases involving axial compression and spans with uniform flexural stiffness.

A simple problem is solved to illustrate the use of the method.

SIGN CONVENTIONS

In the present paper the sign conventions are as follows: Deflection is positive downward. Rotation is positive clockwise. Lateral loads and external forces, except the resisting force of a deflectional spring, are positive downward. The resisting force of a deflectional spring is positive upward. Externally applied moments, except the resisting moment of a rotational spring, are positive clockwise. The resisting moment of a rotational spring is positive counterclockwise. The internal shears and moments acting upon the ends of a single span (spring supports excluded) are positive downward and clockwise, respectively. In the figures of the present paper deflections, rotations, forces, and moments are always shown in their positive directions.

SYMBOLS

General.- The following symbols apply throughout the present paper:

P axial load (parallel to undeflected axis of beam-column)

L length of span

E Young's modulus of elasticity

I moment of inertia of cross-sectional area about neutral bending axis

$j = \sqrt{EI/P}$

L/j	nondimensional measure of axial load in span $\left(L\sqrt{P/EI}\right)$
C	deflectional stiffness of support, force per unit deflection
K	rotational stiffness of support, moment per radian
x	distance of point on span from left end of span
y	deflection of point on loaded beam-column from its position before loading
Δ	deflection of point on span with respect to left end of span; positive downward
S	shear
M	moment

Single span.- The following symbols apply to a single axially loaded span free at one end and clamped at the other end. (For examples, see figs. 5 and 6.) In general, a symbol C stands for a force per unit deflection or rotation, and a symbol K stands for a moment per unit deflection or rotation. The subscript δ indicates that an effect (force or moment) associated with deflection of the free end is being considered. The subscript θ indicates that an effect associated with rotation of the free end is being considered. The subscript F indicates that the effect denoted by the symbol occurs at the free end, and the subscript C indicates that the effect is at the clamped end. Those symbols with the subscript L apply to a span extending to the left from the free end (for example, see fig. 5), and those with the subscript R apply to a span extending to the right from the free end. (For example, see fig. 6.)

δ	deflection at free end
θ	rotation at free end
$C_{F\theta L(\text{or } R)}$, $K_{F\theta L(\text{or } R)}$	force and moment required at free end to produce unit rotation $(\theta = 1)$ and zero deflection $(\delta = 0)$ at free end

$C_{C\theta_{L(\text{or } R)}}$, $K_{C\theta_{L(\text{or } R)}}$ — force and moment produced at clamped end by unit rotation ($\theta = 1$) and zero deflection ($\delta = 0$) of free end

$C_{F\delta_{L(\text{or } R)}}$, $K_{F\delta_{L(\text{or } R)}}$ — force and moment required at free end to produce unit deflection ($\delta = 1$) and zero rotation ($\theta = 0$) at free end

$C_{C\delta_{L(\text{or } R)}}$, $K_{C\delta_{L(\text{or } R)}}$ — force and moment produced at clamped end by unit deflection ($\delta = 1$) and zero rotation ($\theta = 0$) of free end

Two-span beam-column.- The following symbols apply to a two-span continuous beam-column with the ends clamped and the center joint supported by a deflectional and a rotational spring. (For example, see fig. 4.)

δ — deflection of center joint

θ — rotation of center joint

C_θ, K_θ — external force and moment required at center joint to produce unit rotation ($\theta = 1$) and zero deflection ($\delta = 0$) of joint

$$\left(C_\theta = C_{F\theta_L} + C_{F\theta_R} \text{ and } K_\theta = K + K_{F\theta_L} + K_{F\theta_R}\right)$$

C_δ, K_δ — external force and moment required at center joint to produce unit deflection ($\delta = 1$) and zero rotation ($\theta = 0$) of joint

$$\left(C_\delta = C + C_{F\delta_L} + C_{F\delta_R} \text{ and } K_\delta = K_{F\delta_L} + K_{F\delta_R}\right)$$

Other symbols are defined throughout the text where they are first used.

CALCULATION OF DEFLECTIONS AND ROTATIONS OF SUPPORTS BY THE PROCEDURE OF FORCE AND MOMENT DISTRIBUTION

When an elastically supported continuous beam-column, such as that shown diagrammatically in figure 1, is gradually loaded all the supports deflect and rotate until a condition of stable equilibrium

is reached. To solve for these deflections and rotations directly by means of the beam-flexure differential equation may be feasible for a beam-column supported at only two or three points. A direct solution for a beam-column supported at many points, however, usually involves a great, if not a prohibitive, amount of algebraic and numerical work. In order to effect a solution for this case, a relaxation procedure somewhat similar to moment distribution may be used.

At the outset all the joints are assumed to be "locked" or "frozen" against deflection and rotation, and the loads are applied. The fixed-end shears and moments produced at the span ends are then calculated. The locking force required at any joint is equal to the algebraic sum of the fixed-end shears at the joint; similarly, the locking moment is equal to the algebraic sum of the fixed-end moments at the joint. (Figs. 2 and 3 in the present paper and graphs III to X of reference 1 or figs. 14:14 to 14:21 of reference 2 can be used to calculate the fixed-end moments produced by several common types of loading in a span having uniform EI and constant axial compression. The fixed-end shears for any span can be calculated by applying the equations of statics to the span after the fixed-end moments have been determined.)

Then, at any joint a force F (hereinafter called the "balancing force") equal but of opposite sign to the locking force and a moment M (hereinafter called the "balancing moment") equal but of opposite sign to the locking moment are applied. The effect of applying these quantities F and M is to "unlock" or to release the joint. As a result of this unlocking the joint undergoes a vertical deflection δ and a rotation θ (see fig. 4) which are given by the following formulas identical to equations (A17) and (A18) in appendix A:

$$\delta = -\frac{C_\theta}{D} M + \frac{K_\theta}{D} F \tag{1}$$

$$\theta = \frac{C_\delta}{D} M - \frac{K_\delta}{D} F \tag{2}$$

The symbol K_θ stands for the total rotational stiffness of the joint, as used in ordinary moment distribution. The quantities C_θ, C_δ, and K_δ are other kinds of stiffness which have special

significance for a joint with two degrees of freedom. These four stiffnesses are defined physically in the symbols and are given by the equations

$$K_\theta = K + K_{F\theta_L} + K_{F\theta_R} \tag{3}$$

$$C_\theta = C_{F\theta_L} + C_{F\theta_R} \tag{4}$$

$$C_\delta = C + C_{F\delta_L} + C_{F\delta_R} \tag{5}$$

$$K_\delta = K_{F\delta_L} + K_{F\delta_R} = C_\theta \tag{6}$$

where C is the deflectional stiffness of the support at the joint in units of force per unit deflection and K is the rotational stiffness of the support at the joint in units of moment per radian. The quantities $K_{F\theta_L}$, $K_{F\theta_R}$, $C_{F\theta_L}$, $C_{F\theta_R}$, $C_{F\delta_L}$, $C_{F\delta_R}$, $K_{F\delta_L}$, and $K_{F\delta_R}$ are component stiffnesses contributed to the joint by the two members entering the joint. They are defined in the symbols and can be readily evaluated with the aid of tables 1 and 2 for the special case of a span having uniform EI and constant axial compression. The symbol D represents a particular combination of the stiffnesses K_θ, C_θ, and C_δ and is defined by the equation

$$D = K_\theta C_\delta - C_\theta^2 \tag{7}$$

For the special case in which the rotational spring constant K is infinite, no rotation of the joint occurs, and the deflection is given by the equation

$$\delta = \frac{F}{C_\delta} \tag{8}$$

For the case in which the deflectional spring constant C is infinite, no deflection of the joint occurs, and the rotation is given by the equation

$$\theta = \frac{M}{K_\theta} \tag{9}$$

After deflection and rotation, the unlocked joint is in equilibrium under the balancing force F and the balancing moment M. The joint is therefore balanced and is again locked, this time in its new equilibrium position.

In the course of the balancing, shears and moments are induced at the clamped ends of the two spans entering the joint. These shears and moments are shown on the free-body diagrams in figures 5 and 6. They may be calculated from the formulas

$$S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta \tag{10}$$

$$S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \tag{11}$$

$$M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta \tag{12}$$

$$M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \tag{13}$$

where

S_{C_L} shear induced at clamped end of left-hand span (fig. 5)

M_{C_L} moment induced at clamped end of left-hand span (fig. 5)

S_{C_R} shear induced at clamped end of right-hand span (fig. 6)

M_{C_R} moment induced at clamped end of right-hand span (fig. 6)

δ deflection of balanced joint; from equation (1) or (8)

θ rotation of the balanced joint; from equation (2) or (9)

The quantities $C_{C\theta_L}$, $C_{C\delta_L}$, $C_{C\theta_R}$, and $C_{C\delta_R}$ and $K_{C\theta_L}$, $K_{C\delta_L}$, $K_{C\theta_R}$, and $K_{C\delta_R}$ which appear in equations (10) to (13) can be calculated with the aid of tables 1 and 2 for the special case of a span having uniform EI and constant axial compression. The shears and moments given by equations (10) to (13) represent additional locking forces and moments required at the neighboring joints to keep them in a locked condition and must be considered in computing the balancing forces and moments for the neighboring joints.

The balancing procedure just described, including the evaluation of the additional locking forces and moments at the neighboring joints, is successively repeated at all the joints except those which are locked in the actual structure. If the structure is stable under the given axial loading, a stage will be reached at which any additional locking forces and moments will be small enough to be neglected for the degree of accuracy desired. At this point the balancings may stop, and the final deflection and rotation of any support may be obtained by summing the deflections and rotations produced by all the individual balancings of that support or joint.

The essential operations will now be restated briefly as follows:

(1) The calculation of the deflection δ and rotation θ by use of equations (1) or (8) and equations (2) or (9) when a given joint is balanced

(2) The calculation of the shears and moments induced at the neighboring joints by use of equations (10) to (13).

The execution of step (1) requires that the balancing force F and the balancing moment M first be calculated. In calculating F and M for a joint that has not previously been balanced, consideration must be given to the fixed-end shears and moments at the joint as well as to the shears and moments that were induced by the balancing of neighboring joints. In calculating F and M for a joint that has previously been balanced, only those shears and moments that were induced at the joint since its last balancing need be considered.

The derivation of equations (1) to (13) is given in appendix A. The equations used to evaluate the various coefficients for tables 1 and 2 are also given in appendix A.

DETERMINATION OF SHEARS AND MOMENTS AT THE SUPPORTS

After the deflections and rotations of the supports have been determined, the state of the beam-column is uniquely defined, and the shears and moments at the joints can be obtained from simple slope-deflection equations. Each equation expresses the final shear or moment at the end of any span as the sum of the following five parts:

(1) The fixed-end shear S_F or fixed-end moment M_F

(2) Shear or moment produced by the deflection without rotation of the left end of the span

(3) Shear or moment produced by the rotation without deflection of the left end of the span

(4) Shear or moment produced by the deflection without rotation of the right end of the span

(5) Shear or moment produced by the rotation without deflection of the right end of the span

For a typical span jk, the final shear S_{jk} and final moment M_{jk} at the left end j can be written as

$$S_{jk} = (S_F)_{jk} + C_{F\delta_R}\delta_j + C_{F\theta_R}\theta_j + C_{C\delta_L}\delta_k + C_{C\theta_L}\theta_k \tag{14}$$

$$M_{jk} = (M_F)_{jk} + K_{F\delta_R}\delta_j + K_{F\theta_R}\theta_j + K_{C\delta_L}\delta_k + K_{C\theta_L}\theta_k \tag{15}$$

The shear S_{kj} and the moment M_{kj} at the right end k of the typical span jk are given by the equations

$$S_{kj} = (S_F)_{kj} + C_{C\delta_R}\delta_j + C_{C\theta_R}\theta_j + C_{F\delta_L}\delta_k + C_{F\theta_L}\theta_k \tag{16}$$

$$M_{kj} = (M_F)_{kj} + K_{C\delta_R}\delta_j + K_{C\theta_R}\theta_j + K_{F\delta_L}\delta_k + K_{F\theta_L}\theta_k \tag{17}$$

The coefficients of the δ's and θ's in equations (14) to (17) are readily calculable with the aid of tables 1 and 2.

An over-all check on the δ's and θ's and the span-end shears and moments can be made by applying the equations of static equilibrium to the joints as free bodies. If some of the joints are found to be not in equilibrium, a mistake in either the relaxation procedure or in the use of the slope-deflection equations (14) to (17) is indicated. The correct use of the slope-deflection equations can be checked by an investigation of the static equilibrium of the spans as free bodies.

DEFLECTIONS AND BENDING MOMENTS BETWEEN SUPPORTS

In order to obtain the bending moments at points between supports, the deflection curve of the beam-column must first be determined. The deflection curve for any span, say jk, can be obtained by superimposing the five following individual deflection curves:

(1) Deflection curve produced by lateral loading when both ends of the span are clamped

(2) Deflection curve produced by total deflection δ_j of left end when left end is restrained against rotation and right end is clamped

(3) Deflection curve produced by total rotation θ_j of left end when left end is restrained against deflection and right end is clamped

(4) Deflection curve produced by total deflection δ_k of right end when right end is restrained against rotation and left end is clamped

(5) Deflection curve produced by total rotation θ_k of right end when right end is restrained against deflection and left end is clamped

These five deflection curves can be obtained by use of figures 7 to 10 for spans having uniform EI and constant axial compression. The figures provided for the calculation of deflection curve (1) for both ends clamped consider only two types of lateral loading: a uniform load along the entire span (fig. 7) and a single concentrated load at the successive tenth points (figs. 8(a) to 8(e)). Most lateral loadings can be approximated by suitable combinations of these concentrated loads. The equations of the deflection curves, if desired, are given in appendix B.

After the deflection curve for a span has been obtained, the bending moment at any point on the span can be calculated. For a span in which the axial compression is constant, the bending moment at a point a distance x from the left end of the span is given by the expression

$$P\Delta + M_{jk} - S_{jk}x + M_{L.L.} \tag{18}$$

where

P axial compression (parallel to undeflected axis of beam-column)

Δ deflection of point with respect to left end of span (positive downward)

$M_{L.L.}$ total moment about point under consideration of any lateral loads between point and left end of span (moment tending to cause compression on top fiber is considered positive)

and M_{jk} is obtained from equation (15) and S_{jk} is obtained from equation (14).

The sign of the bending moment, as determined from expression (18), is consistent with the convention that bending moment tending to cause compression on the top fiber is positive.

SHEAR DIAGRAM

The shear diagram for any span can be drawn in the conventional manner, the shear at any point being simply the algebraic sum of the left-end shear and all the vertical loads between the left end and the point under consideration. It should be noticed, however, that because of the presence of axial load the usual relationship between the shear and moment, namely $S = \frac{dM}{dx}$ with no distinction made between the vertical shear and the shear normal to the elastic curve, needs to be modified. This relationship is still correct if S represents the shear normal to the elastic curve of the beam-column. The quantity S, as used in the present paper, however, represents the vertical shear, and the appropriate equation for it is

$$S = \frac{dM}{dx} - P\frac{dy}{dx} \tag{19}$$

where S is positive if it acts upward on the right-hand part of the span, $\frac{dM}{dx}$ is the slope of the bending-moment curve at the point,

and $\frac{dy}{dx}$ is the slope of the deflection curve at the point. Equation (19) can be used to check the mutual consistency of the deflection curve and the bending-moment diagram.

ILLUSTRATIVE EXAMPLE

Figure 11 shows a continuous beam-column free at the left end A and built in at the right end D with two intermediate supports B and C. The support at B is an unyielding hinge, and the support at C is a deflectional spring having a stiffness of 686.7 pounds per inch of deflection. Neither support includes a rotational spring. The cross-sectional moment of inertia I of the beam-column is 0.2 inch^4, and the modulus of elasticity E is 29,000,000 psi. Span AB has no axial load. Spans BC and CD are axially loaded with a compressive force of 8156.25 pounds, which was chosen to give these two spans a value of L/j equal to 3. The lateral loading and dimensions are shown in figure 11.

In applying the method of force and moment distribution, the cantilever span AB could be thought of as an ordinary span supported at the left end by springs with zero stiffness. Labor will be saved however by regarding span AB as simply a loading device to provide a constant lateral force and moment just to the left of support B.

Deflections and rotations of the supports.- The tabular scheme for recording the force and moment distribution computations is shown in table 3. Each support is represented in the tabulation by a vertical line. Above each vertical line are written the necessary balancing equations for the deflection δ and rotation θ of the joint and the equations for shears and moments induced at the adjacent joints: equation (1) or (8) for δ, equation (2) or (9) for θ, and equations (10) to (13) for the induced shears and moments. Across the first horizontal line of the tabulation are written the fixed-end shears and fixed-end moments. One fixed-end moment is written on each side of the vertical line representing the joint. The fixed-end shears are written alongside the fixed-end moments but farther from the vertical line. The rest of the table is used to record the computations involved in the joint balancings. Each balancing of a joint requires the calculation of the following three sets of quantities which are recorded on separate horizontal lines of the table:

(1) The balancing force F and balancing moment M at joint

(2) The deflection δ and rotation θ of joint

(3) The shears and moments induced at neighboring joints

The induced shears are recorded in the same vertical lines as the fixed-end shears; similarly, the induced moments are recorded in the same columns as the fixed-end moments. After the calculated values of δ and θ have been recorded (item (2)), a short horizontal line is drawn under them. Item (3) is omitted in the final balancing when the induced shears and moments are assumed to be negligible.

In table 3, for purposes of illustration, a complete set of balancing, induced-shear, and induced-moment equations is written above joint C even though three of them S_{C_L}, S_{C_R}, and M_{C_R} are not required in this particular problem. The equation for δ is omitted above joint B since the support at E is unyielding. Since span AB is considered to be merely a loading device, the quantities S_{C_L} and M_{C_L} do not exist for joint B and therefore their equations are omitted. The arithmetic involved in setting up the equations is given in detail in appendix C.

The fixed-end moments shown in table 3 were taken from figure 12 of reference 1 in which a moment-distribution analysis was made of the same beam-column but on unyielding supports. The fixed-end shears were calculated by applying the equations of statics to the spans.

Joint C is balanced first. The locking force (the algebraic sum of the fixed-end shears) is -173.06 - 119.04 or -292.10 pounds. The balancing force F is recorded as the negative of the locking force, or 292.10 pounds. Similarly, the locking moment is the algebraic sum of the fixed-end moments, or 2229.2 inch-pounds, and the balancing moment M is the negative of the locking moment, or -2229.2 inch-pounds. From the formulas for δ and θ given for joint C in table 3, the movement of the joint for the first balancing is calculated and recorded as $\delta = 0.410847$ inch and $\theta = -5.85845 \times 10^{-3}$ radian. From the formula for M_{C_L} above joint C the moment induced at joint B is calculated as -2899.15 inch-pounds. Because the formula for the rotation of joint B involves only the balancing moment M, the shear S_{C_L} induced at joint B is not calculated; and since joint D will never be permitted to deflect or rotate, neither the shear S_{C_R} nor the moment M_{C_R} induced at joint D is calculated.

Joint B is balanced next. The locking moment (the algebraic sum of the fixed-end moments and the moment induced during the balancing of joint C) is 5000 - 6989.8 - 2899.15 or -4888.95 inch-pounds. The balancing moment M is therefore 4888.95. From the formula for θ given above joint B in table 3, the rotation of the joint is calculated as $\theta = 25.6968 \times 10^{-3}$ radian. From the formulas for S_{C_R} and M_{C_R} given for joint B the shear and moment induced at joint C during the balancing of joint B are calculated as -117.269 pounds and 4492.57 inch-pounds, respectively, and are recorded in their appropriate places near the vertical line representing joint C.

At this point one cycle of the procedure of force and moment distribution has been completed. Another cycle is begun by balancing joint C again. The locking force at joint C (the algebraic sum of the shears induced there since the last balancing of the joint) is -117.269 pounds. Similarly, the locking moment is 4492.57 inch-pounds. The balancing force F and the balancing moment M are then equal to 117.269 pounds and -4492.57 inch-pounds, respectively. By use of the formulas for δ and θ above joint C, the additional movement caused by this second balancing of the joint is calculated as $\delta = 0.164942$ inch and $\theta = 11.8067 \times 10^{-3}$ radian. The moment induced at joint B is calculated as -2816.89 inch-pounds.

The successive balancings of joints C and B are continued until the shears and moments induced by the balancings are small enough to be neglected. At this point the process stops, and the final deflections and rotations are obtained by summing the deflections and rotations produced by the individual balancings. Four cycles of balancing are shown in table 3. The results obtained after 10 and 20 cycles are also given and compared with the exact results. The error after 10 cycles is seen to be only about 0.6 percent.

For this particular problem it was possible to calculate the exact results by means of the formula for the sum of an infinite geometric series, since a point was reached early in the process where the value δ or θ corresponding to any balancing was simply a constant factor (2816.89/4888.95) times the value of δ or θ for the preceding balancing. (In a more general problem, this fortuitous circumstance would not arise.) The calculation of the exact results is given at the end of appendix C.

The illustrative example just explained was adapted from a problem solved in reference 1. In reference 1 a moment-distribution analysis was made of the same structure, with support C assumed to

be 0.8 inch below supports B and D. In order to make a problem that was suitable to the method of the present paper, the support at C was replaced by a deflectional spring. Furthermore, in order to provide a check on the calculations, the spring stiffness was so chosen as to give 0.8-inch deflection at joint C and was obtained by dividing the reaction at C, as calculated from the data given in the solution of reference 1, page 27, by the desired deflection of 0.8 inch. As shown in table 3, the exact value obtained for the deflection at C is 0.8 inch, which was the answer to be expected if the computations were correct.

Shears and moments at span ends.- The preceding computations have yielded the deflections and rotations necessary for calculating the shears and moments at the ends of spans BC and CD from equations (14) to (17). These shears and moments are found to be:

$$S_{BC} = -213.82 \text{ pounds}$$

$$M_{BC} = -5000.0 \text{ inch-pounds}$$

$$S_{CB} = -286.18 \text{ pounds}$$

$$M_{CB} = 5369.1 \text{ inch-pounds}$$

$$S_{CD} = -263.20 \text{ pounds}$$

$$M_{CD} = -5369.1 \text{ inch-pounds}$$

$$S_{DC} = -136.80 \text{ pounds}$$

$$M_{DC} = 1505.2 \text{ inch-pounds}$$

The final shear and moment at the right end of the cantilever AB are, from static considerations,

$$S_{BA} = -100 \text{ pounds}$$

$$M_{BA} = 5000 \text{ inch-pounds}$$

Bending-moment diagram for span BC.- The span BC is chosen for purposes of illustration. The deflection curve for the span is first obtained in the manner explained in the section of the present paper entitled "Deflections and Bending Moments between Supports." The computations for this deflection curve are given

in table 4 and the deflection curve obtained is shown in figure 12. The bending-moment diagram for the span is then obtained by use of expression (18). The computations involved are summarized in table 5. The bending-moment diagram is shown in figure 13.

DISCUSSION

Scope of Method

The method of force and moment distribution is quite general in its applicability. In order to apply it to cases other than those considered in the present paper - such as axial tension, nonuniform EI, or nonuniform axial load within a span - it is only necessary to prepare tables and figures for these cases. The special case in which there is a sudden change in EI or axial load within a span can be handled by assuming the beam-column to be supported by springs of zero stiffness at the point of discontinuity. The span can then be regarded as two spans, each of which has uniform EI and constant axial load.

Thus far, application of the method has been restricted to beam-columns, the supports of which are such that the restraining force and restraining moment of a support are directly proportional to the deflection and rotation, respectively; that is,

$$\text{Restraining force} = C\delta$$

$$\text{Restraining moment} = K\theta$$

It can be shown, however, that the method is also applicable to a more general, mathematically possible case in which the restraining force and moment are linear functions of both the deflection and rotation; that is

$$\left.\begin{aligned} \text{Restraining force} &= C_1\delta + C_2\theta \\ \text{Restraining moment} &= K_1\delta + K_2\theta \end{aligned}\right\} \tag{20}$$

In order for the method to be applicable to this case, it is only necessary to revise equations (3) to (7) to read

$$K_\theta = K_2 + K_{F\theta_L} + K_{F\theta_R} \tag{21}$$

$$C_\theta = C_2 + C_{F\theta_L} + C_{F\theta_R} \tag{22}$$

$$C_\delta = C_1 + C_{F\delta_L} + C_{F\delta_R} \tag{23}$$

$$K_\delta = K_1 + K_{F\delta_L} + K_{F\delta_R} \tag{24}$$

$$D = K_\zeta C_\delta - K_\delta C_\theta \tag{25}$$

The support consisting of two independent elastic springs is a special case of the more general type of support and occurs when C_2 and K_1 are both equal to zero.

The method of force and moment distribution is inapplicable if the stresses are above the proportional limit. In making a force and moment distribution analysis, the stresses are assumed to be below the proportional limit. The results obtained are correct only if the final stresses bear out this assumption.

Procedure of Force and Moment Distribution

There is no definite order in which the joints of the beam-column must be balanced. In order to facilitate checking, however, and to minimize the possibilities for error, a definite order of balancing should be maintained. Not all the joints need be balanced the same number of times; small induced effects may be allowed to accumulate at a joint until the locking force and locking moment there are appreciable.

The problem of slow convergence or nonconvergence will sometimes arise. Nonconvergence is an indication that the structure is unstable under the axial loading. Slow convergence, accompanied by large induced shears and moments, is an indication that the structure is close to instability. When the convergence is slow, an alternate method of solution, based upon the slope-deflection equations (14) to (17) may be adopted. The internal shears and moments as given by equations (14) to (17) are combined with the external forces and moments and spring forces and moments at the joints in writing two equations of static equilibrium for each joint. The system of static-equilibrium equations is then solved simultaneously for the deflections and rotations at the joints.

Principle of Superposition

The modified principle of superposition is of basic importance to the method of force and moment distribution, and in the explanation presented it was assumed to hold for a continuous beam-column

supported by elastic deflectional and rotational springs. The fact that this principle does apply can be proved in a manner similar to that used in reference 3 for a single-span beam-column with hinged ends.

The modified principle of superposition allows the original lateral loading to be resolved into any number of component loadings, each of which is applied separately in conjunction with the axial loads. The final deflections, rotations, shears, and bending moments in the beam-column may then be obtained by adding algebraically the effects of all the component loadings.

This principle is employed throughout the procedure of force and moment distribution. The locking of the joints at the start of the process is equivalent to applying to the beam-column a component loading which consists of the original lateral loads plus the locking forces and moments at the joints. The fixed-end shears and moments are calculated. This component loading is then removed, and the balancing force and moment applied at any joint together with the required locking forces and moments at the neighboring joints represent the second component loading. The effects of the second component loading - the deflection and rotation of the joint being balanced - are calculated. When this second component loading is added algebraically to the first, the original loading condition at the balanced joint is restored since the locking force and moment are canceled. Similarly, each balancing represents the restoration of the original lateral-loading conditions at a joint and the application of the required locking forces and moments at the neighboring joints. If the process is stopped when the locking forces and moments are negligibly small and all component lateral loadings are added algebraically, the resultant lateral loading will consist of the original lateral loads plus negligibly small locking forces and moments at some of the joints. By the principle of superposition, then, algebraic addition of the deflections and rotations produced by the individual balancings (each of which represents the application of a component loading) will yield the deflections and rotations produced by the original lateral loads plus the negligible locking forces and moments.

The principle of superposition is also important for stress analysis. It justifies the determination of the final deflection curve of any span by means of the algebraic addition of five individual deflection curves, as is done in the section entitled

"Deflections and Bending Moments between Supports." It was also employed in writing the slope-deflection equations (14) to (17).

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va. June 25, 1946

APPENDIX A

DERIVATION OF THE BALANCING, INDUCED-SHEAR, AND INDUCED-MOMENT EQUATIONS

Equations will be derived only for spans of uniform EI and constant axial compression.

Derivation of the equations for induced shear and moment.- In figure 4 a balancing force F and a balancing moment M are shown producing a deflection δ and a rotation θ at the center support of a two-span continuous beam-column clamped at the ends. The left-hand span and right-hand span of the beam-column are shown in figures 5 and 6, respectively.

By use of the ordinary beam-flexure differential equation, the equilibrium of internal and external moments about any section in the left-hand span (fig. 5) may be expressed mathematically as

$$EI \frac{d^2y}{dx^2} = -M_{C_L} - Py + S_{C_L}x \tag{A1}$$

The boundary conditions at $x = 0$ are $y = 0$ and $\frac{dy}{dx} = 0$ and at $x = L$ are $y = \delta$ and $\frac{dy}{dx} = \theta$.

By solving the differential equation and satisfying the boundary conditions and the equations of statics for the span as a whole, the following expressions are obtained for the end shears and moments in the left-hand span:

$$S_{F_L} = C_{F\theta_L}\theta + C_{F\delta_L}\delta \tag{A2}$$

$$M_{F_L} = K_{F\theta_L}\theta + K_{F\delta_L}\delta \tag{A3}$$

$$S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta \tag{A4}$$

$$M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta \tag{A5}$$

where

$$\left.\begin{aligned}
K_{F\theta_L}\left(\frac{L}{EI}\right) &= \frac{-\frac{L}{j}\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{F\delta_L}\left(\frac{L^2}{EI}\right) &= \frac{\left(\frac{L}{j}\right)^2\left(1 - \cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
C_{F\theta_L}\left(\frac{L^2}{EI}\right) &= K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{F\delta_L}\left(\frac{L^3}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^3\sin\frac{L}{j}}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{C\theta_L}\left(\frac{L}{EI}\right) &= -K_{F\delta_L}\left(\frac{L^2}{EI}\right) - K_{F\theta_L}\left(\frac{L}{EI}\right) \\
K_{C\delta_L}\left(\frac{L^2}{EI}\right) &= K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{C\theta_L}\left(\frac{L^2}{EI}\right) &= -K_{F\delta_L}\left(\frac{L^2}{EI}\right) \\
C_{C\delta_L}\left(\frac{L^3}{EI}\right) &= -C_{F\delta_L}\left(\frac{L^3}{EI}\right)
\end{aligned}\right\} \qquad \text{(A6)}$$

and

$$j = \sqrt{EI/P}$$

For the right-hand span (fig. 6) the differential equation of equilibrium may be written as

$$EI \frac{d^2y}{dx^2} = -M_{T_R} - P\left(y - \delta\right) + S_{F_R}x \tag{A7}$$

and the boundary conditions to be satisfied at $x = 0$ are $y = \delta$ and $\frac{dy}{dx} = \theta$ and at $x = L$ are $y = 0$ and $\frac{dy}{dx} = 0$.

By solving the differential equation and satisfying the boundary conditions and the equations of statics for the span as a whole, the following expressions for the end shears and moments in the right-hand span are obtained:

$$S_{F_R} = C_{F\theta_R}\theta + C_{F\delta_R}\delta \tag{A8}$$

$$M_{T_R} = K_{F\theta_R}\theta + K_{F\delta_R}\delta \tag{A9}$$

$$S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \tag{A10}$$

$$M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \tag{A11}$$

where

$$\left.\begin{aligned}
K_{F\theta_R}\left(\frac{L}{EI}\right) &= \frac{-\frac{L}{j}\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{F\delta_R}\left(\frac{L^2}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^2\left(1 - \cos\frac{L}{j}\right)}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
C_{F\theta_R}\left(\frac{L^2}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{F\delta_R}\left(\frac{L^3}{EI}\right) &= \frac{-\left(\frac{L}{j}\right)^3\sin\frac{L}{j}}{\frac{L}{j}\sin\frac{L}{j} - 2\left(1 - \cos\frac{L}{j}\right)} \\
K_{C\theta_R}\left(\frac{L}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) - K_{F\theta_R}\left(\frac{L}{EI}\right) \\
K_{C\delta_R}\left(\frac{L^2}{EI}\right) &= K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{C\theta_R}\left(\frac{L^2}{EI}\right) &= -K_{F\delta_R}\left(\frac{L^2}{EI}\right) \\
C_{C\delta_R}\left(\frac{L^3}{EI}\right) &= -C_{F\delta_R}\left(\frac{L^3}{EI}\right)
\end{aligned}\right\} \qquad \text{(A12)}$$

and

$$j = \sqrt{EI/P}$$

Equations (A4), (A5), (A10), and (A11) are identical to equations (10), (12), (11), and (13), respectively. The terms in equations (A6) and (A12) are defined physically in the symbols.

Derivation of balancing equations.- The static equilibrium of the center joint of the beam-column in figure 4 requires that

$$F = C\delta + S_{F_L} + S_{F_R} \tag{A13}$$

and

$$M = K\theta + M_{F_L} + M_{F_R} \tag{A14}$$

Substitution of S_{F_L} from equation (A2), S_{F_R} from equation (A8), M_{F_L} from equation (A3), and M_{F_R} from equation (A9) in equations (A13) and (A14) results in

$$F = C_\theta\theta + C_\delta\delta \tag{A15}$$

$$M = K_\theta\theta + K_\delta\delta \tag{A16}$$

where

$$C_\theta = C_{F\theta_L} + C_{F\theta_R}$$

$$C_\delta = C + C_{F\delta_L} + C_{F\delta_R}$$

$$K_\theta = K + K_{F\theta_L} + K_{F\theta_R}$$

$$K_\delta = K_{F\delta_L} + K_{F\delta_R} = C_\theta$$

Equations (A15) and (A16) can be solved for δ and θ with the result that

$$\delta = -\frac{C_\theta}{D} M + \frac{K_\theta}{D} F \tag{A17}$$

$$\theta = \frac{C_\delta}{D} M - \frac{K_\delta}{D} F \tag{A18}$$

where

$$D = K_\theta C_\delta - C_\theta^{\ 2}$$

For the special case in which the spring constant K is infinite, $\theta = 0$, and, from equation (A15),

$$\delta = \frac{F}{C_\delta} \tag{A19}$$

If C is infinite, $\delta = 0$, and, from equation (A16),

$$\theta = \frac{M}{K_\theta} \tag{A20}$$

Equations (A17), (A18), (A19), and (A20) are identical to equations (1), (2), (8), and (9), respectively.

APPENDIX B

DEFLECTION EQUATIONS

Graphs for calculating the five types of deflection discussed in the section entitled "Deflections and Bending Moments between Supports" are presented in figures 7 to 10 for a span having uniform EI and constant axial compression and the corresponding deflection equations will now be given. In these equations x represents the distance of a point on the elastic curve from the left end of the span and y represents the downward deflection of the point. Detailed derivations are omitted. The equations are obtained by the solution of the ordinary beam-flexure differential equation $M = -EI \frac{d^2y}{dx^2}$ and the satisfaction of simple boundary conditions.

Deflections due to lateral loading.- The following two types of loading will be considered:

(1) Uniform load: The deflections y produced by a total uniformly distributed load W (fig. 7) are defined by the equation

$$\frac{y}{\frac{WL^3}{EI}} = \frac{\sin\frac{L}{j}\frac{x}{L} + \sin\left[\frac{L}{j}\left(1 - \frac{x}{L}\right)\right] - \sin\frac{L}{j}}{2\left(\frac{L}{j}\right)^3\left(1 - \cos\frac{L}{j}\right)} - \frac{1}{2\left(\frac{L}{j}\right)^2}\frac{x}{L}\left(1 - \frac{x}{L}\right) \qquad \text{(B1)}$$

(2) Concentrated load: The deflections y produced by a lateral load Q acting a distance c from the right end (figs. 8(a) to 8(e)) are given by the equations

$$\frac{y}{\frac{QL^3}{EI}} = \frac{1}{\left(\frac{L}{j}\right)^2} \left\{ \frac{\sin \frac{L}{j}\frac{c}{L} \sin \frac{L}{j}\frac{x}{L}}{\frac{L}{j} \sin \frac{L}{j}} - \frac{c}{L}\frac{x}{L} + H_1\left(\frac{\sin \frac{L}{j}\frac{x}{L}}{\sin \frac{L}{j}} - \frac{x}{L}\right) + H_2\left[\frac{\sin \frac{L}{j}\left(1 - \frac{x}{L}\right)}{\sin \frac{L}{j}} - 1 + \frac{x}{L}\right] \right\}$$

for $x \leqq L - c$ and

$$\frac{y}{\frac{QL^3}{EI}} = \frac{1}{\left(\frac{L}{j}\right)^2} \left\{ \frac{\sin \frac{L}{j}\left(1 - \frac{c}{L}\right) \sin \frac{L}{j}\left(1 - \frac{x}{L}\right)}{\frac{L}{j} \sin \frac{L}{j}} - \left(1 - \frac{c}{L}\right)\left(1 - \frac{x}{L}\right) + H_1\left(\frac{\sin \frac{L}{j}\frac{x}{L}}{\sin \frac{L}{j}} - \frac{x}{L}\right) + H_2\left[\frac{\sin \frac{L}{j}\left(1 - \frac{x}{L}\right)}{\sin \frac{L}{j}} - 1 + \frac{x}{L}\right] \right\} \tag{B2}$$

for $x \geqq L - c$ where

$$H_1 = \frac{\left[\left(1 - \frac{c}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{c}{L}\right)\right]\left[\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right] - \left[\frac{c}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{c}{L}\right]\left[\frac{L}{j} - \sin\frac{L}{j}\right]}{\frac{L}{j}\left(1 - \cos\frac{L}{j}\right)\left(2\sin\frac{L}{j} - \frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}$$

$$H_2 = \frac{\left(\frac{c}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{c}{L}\right)\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left[\left(1 - \frac{c}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{c}{L}\right)\right]\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\frac{L}{j}\left(1 - \cos\frac{L}{j}\right)\left(2\sin\frac{L}{j} - \frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right)}$$

Deflections due to displacement at left end.- The deflections y due to a displacement at the left end δ_L (fig. 9) are defined by the equation

$$\frac{y}{\delta_L} = \frac{\sin\frac{L}{j}\frac{x}{L}\sin\frac{L}{j} - \frac{x}{L}\frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} + \left(\cos\frac{L}{j} - 1\right)\left[\cos\frac{L}{j}\left(1 - \frac{x}{L}\right) - \cos\frac{L}{j}\right]}{\sin^2\frac{L}{j} - \frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} - \left(1 - \cos\frac{L}{j}\right)^2} \tag{B3}$$

Deflections due to rotation at left end.- The deflections y due to a rotation at the left end θ_L (fig. 10) are defined by the equation

$$\frac{y}{\theta_L L} = \frac{\left[\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\right]\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left(\frac{x}{L}\sin\frac{L}{j} - \sin\frac{L}{j}\frac{x}{L}\right)\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\left[-2\frac{L}{j}\left(1 - \cos\frac{L}{j}\right) + \left(\frac{L}{j}\right)^2 \sin\frac{L}{j}\right]\sin\frac{L}{j}} \tag{B4}$$

Deflections due to displacement at right end.- The deflections y due to a displacement at the the right end δ_R (fig. 9) are defined by the equation

$$\frac{y}{\delta_R} = \frac{\sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \left(1 - \frac{x}{L}\right)\frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} + \left(\cos\frac{L}{j} - 1\right)\left(\cos\frac{L}{j}\frac{x}{L} - \cos\frac{L}{j}\right)}{\sin^2\frac{L}{j} - \frac{L}{j}\sin\frac{L}{j}\cos\frac{L}{j} - \left(1 - \cos\frac{L}{j}\right)^2} \tag{B5}$$

Deflections due to rotation at right end.- The deflections y due to a rotation at the right end θ_R (fig. 10) are defined by the equation

$$-\frac{y}{\theta_R L} = \frac{\left(\frac{x}{L}\sin\frac{L}{j} - \sin\frac{x}{L}\frac{L}{j}\right)\left(\sin\frac{L}{j} - \frac{L}{j}\cos\frac{L}{j}\right) - \left[\left(1 - \frac{x}{L}\right)\sin\frac{L}{j} - \sin\frac{L}{j}\left(1 - \frac{x}{L}\right)\right]\left(\frac{L}{j} - \sin\frac{L}{j}\right)}{\left[-2\frac{L}{j}\left(1 - \cos\frac{L}{j}\right) + \left(\frac{L}{j}\right)^2 \sin\frac{L}{j}\right]\sin\frac{L}{j}} \tag{B6}$$

APPENDIX C

PRELIMINARY COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE

Computations at joint B.- Setting up the balancing equation (9) for joint B requires the preliminary evaluation of K_θ from equation (3), which in turn requires the calculation of $K_{F\theta_R}$. Setting up equations (11) and (13) for induced effects necessitates the evaluation of only $C_{C\theta_R}$ and $K_{C\theta_R}$ since δ at joint B equals zero. These quantities can be obtained by entering table 2 with the value of $\frac{L}{j}$ for span BC, namely $\frac{L}{j} = 3$, and substituting numerical values of $\frac{EI}{L}$, $\frac{EI}{L^2}$, and $\frac{EI}{L^3}$. These numerical values are

$$\left.\begin{aligned} \frac{EI}{L} &= \frac{29{,}000{,}000 \times 0.2}{80} = 72{,}500 \\ \frac{EI}{L^2} &= 906.25 \\ \frac{EI}{L^3} &= 11.328 \end{aligned}\right\} \tag{C1}$$

In this manner the following data are obtained:

$$K_{F\theta_R} = 2.62420\,\frac{EI}{L} = 190{,}255 \tag{C2}$$

$$C_{C\theta_R} = -\,5.03565\,\frac{EI}{L^2} = -\,4563.56 \tag{C3}$$

$$K_{C\theta_R} = 2.41145\,\frac{EI}{L} = 174{,}830 \tag{C4}$$

From equation (3)

$$K_\theta = 0 + 0 + 190,255 = 190,255 \tag{C5}$$

The balancing equation (9) can now be written

$$\theta = \frac{M}{190,255} = 5.2561 \times 10^{-6}\ M \tag{C6}$$

Equation (11) for induced shear and equation (13) for induced moment can be written as

$$S_{C_R} = -4563.56\theta + 0 = -4563.56\theta \tag{C7}$$

$$M_{C_R} = 174,830\theta + 0 = 174,830\theta \tag{C8}$$

Computations at joint C.- The computations at joint C are similar to, but more extensive than, those at joint B because joint C can deflect as well as rotate and has two spans effective in resisting its motion instead of one.

The following data are obtained for joint C by entering table 1 with the value of $\frac{L}{j} = 3$ for span CB:

$$\left.\begin{aligned}
K_{F\theta_L} &= 2.62420\,\frac{EI}{L} = 190{,}255 \\
K_{F\delta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{F\theta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{F\delta_L} &= 1.07131\,\frac{EI}{L^3} = 12.1358 \\
K_{C\theta_L} &= 2.41145\,\frac{EI}{L} = 174{,}830 \\
K_{C\delta_L} &= -5.03565\,\frac{EI}{L^2} = -4563.56 \\
C_{C\theta_L} &= 5.03565\,\frac{EI}{L^2} = 4563.56 \\
C_{C\delta_L} &= -1.07131\,\frac{EI}{L^3} = -12.1358
\end{aligned}\right\} \qquad (C9)$$

The following information is obtained from table 2 by entering it with $\frac{L}{j} = 3$ for span CD:

$$\left.\begin{aligned}
K_{F\theta_R} &= 2.62420 \frac{EI}{L} = 190,255 \\
K_{F\delta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{F\theta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{F\delta_R} &= 1.07131 \frac{EI}{L^3} = 12.1358 \\
K_{C\theta_R} &= 2.41145 \frac{EI}{L} = 174,830 \\
K_{C\delta_R} &= 5.03565 \frac{EI}{L^2} = 4563.56 \\
C_{C\theta_R} &= -5.03565 \frac{EI}{L^2} = -4563.56 \\
C_{C\delta_R} &= -1.07131 \frac{EI}{L^3} = -12.1358
\end{aligned}\right\} \qquad \text{(C10)}$$

Expressions (3), (4), (5), and (7) can now be evaluated as

$$K_\theta = 0 + 190,255 + 190,255 = 380,510 \tag{C11}$$

$$C_\theta = -4563.56 + 4563.56 = 0 \tag{C12}$$

$$C_\delta = 686.7 + 12.1358 + 12.1358 = 710.972 \tag{C13}$$

$$D = 380,510 \times 710.972 - 0 = 270,532,000 \tag{C14}$$

The balancing equations (1) and (2) can now be set up as follows:

$$\delta = 0 + \frac{380,510}{270,532,000} F = 0.00140653 F \tag{C15}$$

$$\theta = \frac{710.972}{270,532,000} M - 0 = 2.62805 \times 10^{-6} M \tag{C16}$$

Equations (10) to (13) for induced effects at neighboring joints can be written

$$S_{C_L} = 4563.56\theta - 12.13588 \tag{C17}$$

$$S_{C_R} = -4563.56\theta - 12.13588 \tag{C18}$$

$$M_{C_L} = 174,830\theta - 4563.568 \tag{C19}$$

$$M_{C_R} = 174,830\theta + 4563.568 \tag{C20}$$

Calculation of exact results shown in table 3.- From table 3 it is clear that all the δ's and θ's that were calculated after the second balancing of joint C are simply 2816.89/4888.95 times the corresponding preceding entries. The formula for the sum of an infinite geometric series being $S = \frac{a}{1 - r}$, where S is the sum, a the first term of the series, and r the ratio of any term to the preceding term, the total rotation of joint B can be calculated as

$$\theta_B = \frac{25.6968 \times 10^{-3}}{1 - \frac{2816.89}{4888.95}} = 60.6307 \times 10^{-3} \tag{C21}$$

Similarly, the deflection of joint C is

$$\delta_C = 0.410847 + \frac{0.164942}{1 - \frac{2816.89}{4888.95}} = 0.800022 \text{ inch} \tag{C22}$$

and the rotation of joint C is

$$\theta_C = -5.85845 \times 10^{-3} - \frac{11.8067 \times 10^{-3}}{1 - \frac{2816.89}{4888.95}}$$

$$= -33.7160 \times 10^{-3} \tag{C23}$$

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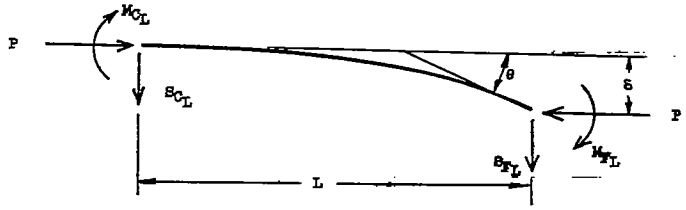
TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED

$$\left[\begin{array}{ll} S_{C_L} = C_{C\theta_L}\theta + C_{C\delta_L}\delta & S_{F_L} = C_{F\theta_L}\theta + C_{F\delta_L}\delta \\ M_{C_L} = K_{C\theta_L}\theta + K_{C\delta_L}\delta & M_{F_L} = K_{F\theta_L}\theta + K_{F\delta_L}\delta \end{array}\right]$$



$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\delta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_L}\left(\frac{L^3}{EI}\right)$
0	4.00000	-6.00000	12.0000	2.00000	-6.00000	6.00000	-12.0000
.1	3.99865	-5.99901	11.9880	2.00036	-5.99901	5.99901	-11.9880
.2	3.99465	-5.99597	11.9519	2.00132	-5.99597	5.99597	-11.9519
.3	3.98798	-5.99099	11.8920	2.00300	-5.99099	5.99099	-11.8920
.4	3.97862	-5.98398	11.8080	2.00536	-5.98398	5.98398	-11.8080
.5	3.96656	-5.97495	11.6999	2.00840	-5.97495	5.97495	-11.6999
.6	3.95177	-5.96391	11.5678	2.01214	-5.96391	5.96391	-11.5678
.7	3.93424	-5.95083	11.4117	2.01658	-5.95083	5.95083	-11.4117
.8	3.91394	-5.93571	11.2314	2.02176	-5.93571	5.93571	-11.2314
.9	3.89083	-5.91853	11.0271	2.02769	-5.91853	5.91853	-11.0271
1.0	3.86488	-5.89928	10.7986	2.03440	-5.89928	5.89928	-10.7986
1.01	3.86213	-5.89724	10.7744	2.03511	-5.89724	5.89724	-10.7744
1.02	3.85935	-5.89518	10.7500	2.03583	-5.89518	5.89518	-10.7500
1.03	3.85654	-5.89310	10.7253	2.03656	-5.89310	5.89310	-10.7253
1.04	3.85370	-5.89099	10.7004	2.03730	-5.89099	5.89099	-10.7004
1.05	3.85083	-5.88887	10.6752	2.03804	-5.88887	5.88887	-10.6752
1.06	3.84793	-5.88673	10.6499	2.03880	-5.88673	5.88673	-10.6499
1.07	3.84500	-5.88456	10.6242	2.03956	-5.88456	5.88456	-10.6242
1.08	3.84204	-5.88238	10.5984	2.04033	-5.88238	5.88238	-10.5984
1.09	3.83906	-5.88017	10.5722	2.04111	-5.88017	5.88017	-10.5722
1.10	3.83604	-5.87794	10.5459	2.04190	-5.87794	5.87794	-10.5459
1.11	3.83300	-5.87569	10.5193	2.04269	-5.87569	5.87569	-10.5193
1.12	3.82992	-5.87342	10.4924	2.04350	-5.87342	5.87342	-10.4924
1.13	3.82682	-5.87113	10.4654	2.04431	-5.87113	5.87113	-10.4654
1.14	3.82369	-5.86882	10.4380	2.04513	-5.86882	5.86882	-10.4380
1.15	3.82052	-5.86648	10.4105	2.04596	-5.86648	5.86648	-10.4105
1.16	3.81733	-5.86413	10.3827	2.04679	-5.86413	5.86413	-10.3827
1.17	3.81411	-5.86175	10.3546	2.04764	-5.86175	5.86175	-10.3546
1.18	3.81086	-5.85935	10.3263	2.04850	-5.85935	5.85935	-10.3263
1.19	3.80758	-5.85693	10.2978	2.04936	-5.85693	5.85693	-10.2978
1.20	3.80426	-5.85449	10.2690	2.05023	-5.85449	5.85449	-10.2690
1.21	3.80092	-5.85203	10.2400	2.05111	-5.85203	5.85203	-10.2400
1.22	3.79755	-5.84955	10.2107	2.05200	-5.84955	5.84955	-10.2107
1.23	3.79415	-5.84705	10.1812	2.05290	-5.84705	5.84705	-10.1812
1.24	3.79072	-5.84452	10.1514	2.05380	-5.84452	5.84452	-10.1514
1.25	3.78726	-5.84197	10.1215	2.05472	-5.84197	5.84197	-10.1215
1.26	3.78377	-5.83941	10.0912	2.05564	-5.83941	5.83941	-10.0912
1.27	3.78024	-5.83682	10.0607	2.05658	-5.83682	5.83682	-10.0607
1.28	3.77669	-5.83421	10.0300	2.05752	-5.83421	5.83421	-10.0300
1.29	3.77311	-5.83157	9.99905	2.05847	-5.83157	5.83157	-9.99905
1.30	3.76949	-5.82892	9.96784	2.05943	-5.82892	5.82892	-9.96784
1.31	3.76585	-5.82625	9.93639	2.06040	-5.82625	5.82625	-9.93639
1.32	3.76217	-5.82355	9.90470	2.06137	-5.82355	5.82355	-9.90470
1.33	3.75847	-5.82083	9.87276	2.06236	-5.82083	5.82083	-9.87276
1.34	3.75473	-5.81809	9.84058	2.06336	-5.81809	5.81809	-9.84058

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\delta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_L}\left(\frac{L^3}{EI}\right)$
1.35	3.75097	-5.81533	9.80816	2.06436	-5.81533	5.81533	-9.80816
1.36	3.74717	-5.81255	9.77549	2.06538	-5.81255	5.81255	-9.77549
1.37	3.74334	-5.80974	9.74258	2.06640	-5.80974	5.80974	-9.74258
1.38	3.73948	-5.80691	9.70943	2.06743	-5.80691	5.80691	-9.70943
1.39	3.73559	-5.80407	9.67603	2.06848	-5.80407	5.80407	-9.67603
1.40	3.73167	-5.80119	9.64239	2.06953	-5.80119	5.80119	-9.64239
1.41	3.72771	-5.79830	9.60851	2.07059	-5.79830	5.79830	-9.60851
1.42	3.72373	-5.79539	9.57438	2.07166	-5.79539	5.79539	-9.57438
1.43	3.71971	-5.79245	9.54001	2.07274	-5.79245	5.79245	-9.54001
1.44	3.71567	-5.78950	9.50539	2.07383	-5.78950	5.78950	-9.50539
1.45	3.71159	-5.78652	9.47053	2.07493	-5.78652	5.78652	-9.47053
1.46	3.70748	-5.78352	9.43543	2.07604	-5.78352	5.78352	-9.43543
1.47	3.70333	-5.78049	9.40009	2.07716	-5.78049	5.78049	-9.40009
1.48	3.69916	-5.77745	9.36450	2.07829	-5.77745	5.77745	-9.36450
1.49	3.69495	-5.77438	9.32866	2.07943	-5.77438	5.77438	-9.32866
1.50	3.69072	-5.77129	9.29259	2.08058	-5.77129	5.77129	-9.29259
1.51	3.68645	-5.76818	9.25626	2.08173	-5.76818	5.76818	-9.25626
1.52	3.68214	-5.76505	9.21969	2.08290	-5.76505	5.76505	-9.21969
1.53	3.67781	-5.76189	9.18288	2.08408	-5.76189	5.76189	-9.18288
1.54	3.67344	-5.75871	9.14583	2.08527	-5.75871	5.75871	-9.14583
1.55	3.66904	-5.75551	9.10853	2.08647	-5.75551	5.75551	-9.10853
1.56	3.66461	-5.75229	9.07098	2.08768	-5.75229	5.75229	-9.07098
1.57	3.66015	-5.74905	9.03319	2.08890	-5.74905	5.74905	-9.03319
1.58	3.65565	-5.74578	8.99516	2.09013	-5.74578	5.74578	-8.99516
1.59	3.65112	-5.74249	8.95689	2.0913	-5.74249	5.74249	-8.95689
1.60	3.64656	-5.73918	8.91836	2.09262	-5.73918	5.73918	-8.91836
1.61	3.64197	-5.73585	8.87960	2.09388	-5.73585	5.73585	-8.87960
1.62	3.63734	-5.73249	8.84059	2.09515	-5.73249	5.73249	-8.84059
1.63	3.63268	-5.72911	8.80133	2.09643	-5.72911	5.72911	-8.80133
1.64	3.62799	-5.72571	8.76183	2.09773	-5.72571	5.72571	-8.76183
1.65	3.62326	-5.72229	8.72208	2.09903	-5.72229	5.72229	-8.72208
1.66	3.61850	-5.71884	8.68209	2.10035	-5.71884	5.71884	-8.68209
1.67	3.61371	-5.71538	8.64185	2.10167	-5.71538	5.71538	-8.64185
1.68	3.60888	-5.71189	8.60137	2.10301	-5.71189	5.71189	-8.60137
1.69	3.60402	-5.70837	8.56064	2.10435	-5.70837	5.70837	-8.56064
1.70	3.59912	-5.70484	8.51967	2.10571	-5.70484	5.70484	-8.51967
1.71	3.59420	-5.70128	8.47845	2.10708	-5.70128	5.70128	-8.47845
1.72	3.58923	-5.69770	8.43699	2.10846	-5.69770	5.69770	-8.43699
1.73	3.58424	-5.69409	8.39528	2.10985	-5.69409	5.69409	-8.39528
1.74	3.57921	-5.69046	8.35333	2.11126	-5.69046	5.69046	-8.35333
1.75	3.57414	-5.68681	8.31113	2.11267	-5.68681	5.68681	-8.31113
1.76	3.56905	-5.68314	8.26868	2.11410	-5.68314	5.68314	-8.26868
1.77	3.56391	-5.67945	8.22599	2.11553	-5.67945	5.67945	-8.22599
1.78	3.55875	-5.67573	8.18305	2.11698	-5.67573	5.67573	-8.18305
1.79	3.55354	-5.67199	8.13987	2.11844	-5.67199	5.67199	-8.13987
1.80	3.54831	-5.66822	8.09644	2.11991	-5.66822	5.66822	-8.09644
1.81	3.54304	-5.66443	8.05277	2.12140	-5.66443	5.66443	-8.05277
1.82	3.53773	-5.66062	8.00884	2.12289	-5.66062	5.66062	-8.00884
1.83	3.53239	-5.65679	7.96468	2.12440	-5.65679	5.65679	-7.96468
1.84	3.52701	-5.65293	7.92026	2.12592	-5.65293	5.65293	-7.92026
1.85	3.52160	-5.64905	7.87560	2.12745	-5.64905	5.64905	-7.87560
1.86	3.51615	-5.64515	7.83070	2.12900	-5.64515	5.64515	-7.83070
1.87	3.51067	-5.64122	7.78554	2.13055	-5.64122	5.64122	-7.78554
1.88	3.50515	-5.63727	7.74014	2.13212	-5.63727	5.63727	-7.74014
1.89	3.49960	-5.63330	7.69450	2.13370	-5.63330	5.63330	-7.69450
1.90	3.49401	-5.62930	7.64860	2.13529	-5.62930	5.62930	-7.64860
1.91	3.48838	-5.62528	7.60246	2.13690	-5.62528	5.62528	-7.60246
1.92	3.48272	-5.62124	7.55607	2.13852	-5.62124	5.62124	-7.55607
1.93	3.47702	-5.61717	7.50944	2.14015	-5.61717	5.61717	-7.50944
1.94	3.47129	-5.61308	7.46256	2.14179	-5.61308	5.61308	-7.46256

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\theta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\theta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^3}{EI}\right)$
1.95	3.46552	-5.60896	7.41543	2.14345	-5.60896	5.60896	-7.41543
1.96	3.45971	-5.60483	7.36805	2.14512	-5.60483	5.60483	-7.36805
1.97	3.45386	-5.60067	7.32043	2.14680	-5.60067	5.60067	-7.32043
1.98	3.44798	-5.59648	7.27256	2.14850	-5.59648	5.59648	-7.27256
1.99	3.44207	-5.59227	7.22444	2.15020	-5.59227	5.59227	-7.22444
2.00	3.43611	-5.58804	7.17608	2.15193	-5.58804	5.58804	-7.17608
2.01	3.43012	-5.58378	7.12746	2.15366	-5.58378	5.58378	-7.12746
2.02	3.42409	-5.57950	7.07860	2.15541	-5.57950	5.57950	-7.07860
2.03	3.41802	-5.57520	7.02949	2.15717	-5.57520	5.57520	-7.02949
2.04	3.41192	-5.57087	6.98013	2.15895	-5.57087	5.57087	-6.98013
2.05	3.40578	-5.56651	6.93053	2.16074	-5.56651	5.56651	-6.93053
2.06	3.39960	-5.56214	6.88068	2.16254	-5.56214	5.56214	-6.88068
2.07	3.39338	-5.55774	6.83057	2.16436	-5.55774	5.55774	-6.83057
2.08	3.38712	-5.55331	6.78022	2.16619	-5.55331	5.55331	-6.78022
2.09	3.38083	-5.54886	6.72963	2.16803	-5.54886	5.54886	-6.72963
2.10	3.37450	-5.54439	6.67878	2.16989	-5.54439	5.54439	-6.67878
2.11	3.36812	-5.53989	6.62768	2.17177	-5.53989	5.53989	-6.62768
2.12	3.36171	-5.53537	6.57634	2.17366	-5.53537	5.53537	-6.57634
2.13	3.35526	-5.53082	6.52475	2.17556	-5.53082	5.53082	-6.52475
2.14	3.34878	-5.52625	6.47291	2.17748	-5.52625	5.52625	-6.47291
2.15	3.34225	-5.52166	6.42081	2.17941	-5.52166	5.52166	-6.42081
2.16	3.33568	-5.51704	6.36848	2.18135	-5.51704	5.51704	-6.36848
2.17	3.32908	-5.51239	6.31589	2.18332	-5.51239	5.51239	-6.31589
2.18	3.32243	-5.50772	6.26305	2.18529	-5.50772	5.50772	-6.26305
2.19	3.31575	-5.50303	6.20996	2.18728	-5.50303	5.50303	-6.20996
2.20	3.30902	-5.49831	6.15662	2.18929	-5.49831	5.49831	-6.15662
2.21	3.30226	-5.49357	6.10304	2.19131	-5.49357	5.49357	-6.10304
2.22	3.29545	-5.48880	6.04920	2.19335	-5.48880	5.48880	-6.04920
2.23	2.28860	-5.48401	5.99512	2.19540	-5.48401	5.48401	-5.99512
2.24	3.28172	-5.47919	5.94078	2.19747	-5.47919	5.47919	-5.94078
2.25	3.27479	-5.47435	5.88619	2.19956	-5.47435	5.47435	-5.88619
2.26	3.26782	-5.46948	5.83136	2.20166	-5.46948	5.46948	-5.83136
2.27	3.26081	-5.46459	5.77627	2.20377	-5.46459	5.46459	-5.77627
2.28	3.25376	-5.45967	5.72094	2.20591	-5.45967	5.45967	-5.72094
2.29	3.24667	-5.45473	5.66535	2.20806	-5.45473	5.45473	-5.66535
2.30	3.23954	-5.44976	5.60951	2.21022	-5.44976	5.44976	-5.60951
2.31	3.23236	-5.44476	5.55343	2.21240	-5.44476	5.44476	-5.55343
2.32	3.22514	-5.43974	5.49709	2.21460	-5.43974	5.43974	-5.49709
2.33	3.21788	-5.43470	5.44050	2.21682	-5.43470	5.43470	-5.44050
2.34	3.21058	-5.42963	5.38366	2.21905	-5.42963	5.42963	-5.38366
2.35	3.20324	-5.42453	5.32657	2.22130	-5.42453	5.42453	-5.32657
2.36	3.19585	-5.41941	5.26923	2.22356	-5.41941	5.41941	-5.26923
2.37	3.18842	-5.41427	5.31163	2.22585	-5.41427	5.41427	-5.31163
2.38	3.18095	-5.40909	5.15379	2.22815	-5.40909	5.40909	-5.15379
2.39	3.17343	-5.40390	5.09569	2.23047	-5.40390	5.40390	-5.09569
2.40	3.16587	-5.39867	5.03734	2.23280	-5.39867	5.39867	-5.03734
2.41	3.15827	-5.39342	4.97875	2.23515	-5.39342	5.39342	-4.97875
2.42	3.15062	-5.38815	4.91989	2.23753	-5.38815	5.38815	-4.91989
2.43	3.14293	-5.38285	4.86079	2.23992	-5.38285	5.38285	-4.86079
2.44	3.13519	-5.37752	4.80144	2.24232	-5.37752	5.37752	-4.80144
2.45	3.12742	-5.37216	4.74183	2.24475	-5.37216	5.37216	-4.74183
2.46	3.11959	-5.36679	4.68197	2.24719	-5.36679	5.36679	-4.68197
2.47	3.11172	-5.36138	4.62186	2.24966	-5.36138	5.36138	-4.62186
2.48	3.10381	-5.35595	4.56149	2.25214	-5.35595	5.35595	-4.56149
2.49	3.09585	-5.35049	4.50088	2.25464	-5.35049	5.35049	-4.50088
2.50	3.08784	-5.34500	4.44001	2.25716	-5.34500	5.34500	-4.44001
2.51	3.07979	-5.33949	4.37888	2.25970	-5.33949	5.33949	-4.37888
2.52	3.07170	-5.33395	4.31751	2.26226	-5.33395	5.33395	-4.31751
2.53	3.06355	-5.32839	4.25588	2.26484	-5.32839	5.32839	-4.25588
2.54	3.05536	-5.32280	4.19400	2.26743	-5.32280	5.32280	-4.19400

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TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\xi_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\xi_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\xi_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\xi_L}\left(\frac{L^3}{EI}\right)$
2.55	3.04713	-5.31718	4.13186	2.27005	-5.31718	5.31718	-4.13186
2.56	3.03885	-5.31154	4.06947	2.27269	-5.31154	5.31154	-4.06947
2.57	3.03052	-5.30587	4.00683	2.27535	-5.30587	5.30587	-4.00683
2.58	3.02214	-5.30017	3.94393	2.27803	-5.30017	5.30017	-3.94393
2.59	3.01372	-5.29444	3.88079	2.28073	-5.29444	5.29444	-3.88079
2.60	3.00524	-5.28869	3.81738	2.28344	-5.28869	5.28869	-3.81738
2.61	2.99673	-5.28291	3.75372	2.28619	-5.28291	5.28291	-3.75372
2.62	2.98816	-5.27711	3.68981	2.28895	-5.27711	5.27711	-3.68981
2.63	2.97954	-5.27127	3.62564	2.29173	-5.27127	5.27127	-3.62564
2.64	2.97088	-5.26541	3.56122	2.29453	-5.26541	5.26541	-3.56122
2.65	2.96216	-5.25952	3.49654	2.29736	-5.25952	5.25952	-3.49654
2.66	2.95340	-5.25361	3.43161	2.30021	-5.25361	5.25361	-3.43161
2.67	2.94459	-5.24766	3.36643	2.30308	-5.24766	5.24766	-3.36643
2.68	2.93572	-5.24169	3.30099	2.30597	-5.24169	5.24169	-3.30099
2.69	2.92681	-5.23570	3.23529	2.30888	-5.23570	5.23570	-3.23529
2.70	2.91785	-5.22967	3.16934	2.31182	-5.22967	5.22967	-3.16934
2.71	2.90884	-5.22362	3.10313	2.31478	-5.22362	5.22362	-3.10313
2.72	2.89977	-5.21753	3.03667	2.31776	-5.21753	5.21753	-3.03667
2.73	2.89066	-5.21143	2.96995	2.32077	-5.21143	5.21143	-2.96995
2.74	2.88149	-5.20529	2.90298	2.32380	-5.20529	5.20529	-2.90298
2.75	2.87228	-5.19912	2.83575	2.32685	-5.19912	5.19912	-2.83575
2.76	2.86301	-5.19293	2.76826	2.32992	-5.19293	5.19293	-2.76826
2.77	2.85368	-5.18671	2.70052	2.33302	-5.18671	5.18671	-2.70052
2.78	2.84431	-5.18046	2.63252	2.33615	-5.18046	5.18046	-2.63252
2.79	2.83488	-5.17418	2.56426	2.33930	-5.17418	5.17418	-2.56426
2.80	2.82540	-5.16787	2.49575	2.34247	-5.16787	5.16787	-2.49575
2.81	2.81587	-5.16154	2.42698	2.34567	-5.16154	5.16154	-2.42698
2.82	2.80628	-5.15517	2.35795	2.34889	-5.15517	5.15517	-2.35795
2.83	2.79664	-5.14878	2.28866	2.35214	-5.14878	5.14878	-2.28866
2.84	2.78695	-5.14236	2.21912	2.35541	-5.14236	5.14236	-2.21912
2.85	2.77720	-5.13591	2.14932	2.35871	-5.13591	5.13591	-2.14932
2.86	2.76740	-5.12943	2.07926	2.36204	-5.12943	5.12943	-2.07926
2.87	2.75754	-5.12292	2.00895	2.36539	-5.12292	5.12292	-2.00895
2.88	2.74762	-5.11639	1.93838	2.36877	-5.11639	5.11639	-1.93838
2.89	2.73765	-5.10982	1.86754	2.37217	-5.10982	5.10982	-1.86754
2.90	2.72763	-5.10323	1.79645	2.37560	-5.10323	5.10323	-1.79645
2.91	2.71754	-5.09660	1.72511	2.37906	-5.09660	5.09660	-1.72511
2.92	2.70740	-5.08995	1.65350	2.38255	-5.08995	5.08995	-1.65350
2.93	2.69721	-5.08326	1.58163	2.38606	-5.08326	5.08326	-1.58163
2.94	2.68695	-5.07656	1.50951	2.38960	-5.07656	5.07656	-1.50951
2.95	2.67664	-5.06981	1.43712	2.39317	-5.06981	5.06981	-1.43712
2.96	2.66627	-5.06304	1.36448	2.39677	-5.06304	5.06304	-1.36448
2.97	2.65584	-5.05624	1.29158	2.40040	-5.05624	5.05624	-1.29158
2.98	2.64535	-5.04941	1.21841	2.40406	-5.04941	5.04941	-1.21841
2.99	2.63481	-5.04255	1.14499	2.40774	-5.04255	5.04255	-1.14499
3.00	2.62420	-5.03565	1.07131	2.41145	-5.03565	5.03565	-1.07131
3.01	2.61353	-5.02873	.997365	2.41520	-5.02873	5.02873	-.997365
3.02	2.60280	-5.02178	.923161	2.41897	-5.02178	5.02178	-.923161
3.03	2.59202	-5.01480	.848697	2.42278	-5.01480	5.01480	-.848697
3.04	2.58117	-5.00778	.773971	2.42661	-5.00778	5.00778	-.773971
3.05	2.57026	-5.00074	.698986	2.43048	-5.00074	5.00074	-.698986
3.06	2.55929	-4.99367	.623740	2.43438	-4.99367	4.99367	-.623740
3.07	2.54825	-4.98656	.548231	2.43831	-4.98656	4.98656	-.548231
3.08	2.53715	-4.97943	.472461	2.44227	-4.97943	4.97943	-.472461
3.09	2.52600	-4.97226	.396430	2.44627	-4.97226	4.97226	-.396430
3.10	2.51478	-4.96507	.320138	2.45030	-4.96507	4.96507	-.320138
3.11	2.50349	-4.95784	.243582	2.45436	-4.95784	4.95784	-.243582
3.12	2.49214	-4.95059	.168765	2.45845	-4.95059	4.95059	-.168765
3.13	2.48073	-4.94331	.0896851	2.46259	-4.94331	4.94331	-.0896851
3.14	2.46897	-4.93543	.0123409	2.46647	-4.93543	4.93543	-.0123409

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TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\theta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\theta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^3}{EI}\right)$
∞	2.46740	-4.93480	0	2.46740	-4.93480	4.93480	0
3.15	2.45772	-4.92867	-.0652638	2.47096	-4.92867	4.92867	.0652638
3.16	2.44608	-4.92125	-.143132	2.47517	-4.92125	4.92125	.143132
3.17	2.43439	-4.91381	-.221263	2.47942	-4.91381	4.91381	.221263
3.18	2.42265	-4.90638	-.299658	2.48373	-4.90638	4.90638	.299658
3.19	2.41083	-4.89889	-.378316	2.48806	-4.89889	4.89889	.378316
3.20	2.39895	-4.89138	-.457239	2.49243	-4.89138	4.89138	.457239
3.21	2.38700	-4.88384	-.536424	2.49684	-4.88384	4.88384	.536424
3.22	2.37498	-4.87626	-.615875	2.50128	-4.87626	4.87626	.615875
3.23	2.36289	-4.86866	-.695590	2.50576	-4.86866	4.86866	.695590
3.24	2.35073	-4.86101	-.775568	2.51028	-4.86101	4.86101	.775568
3.25	2.33850	-4.85334	-.855811	2.51484	-4.85334	4.85334	.855811
3.26	2.32620	-4.84564	-.936321	2.51943	-4.84564	4.84564	.936321
3.27	2.31383	-4.83790	-1.01710	2.52407	-4.83790	4.83790	1.01710
3.28	2.30139	-4.83013	-1.09814	2.52874	-4.83013	4.83013	1.09814
3.29	2.28887	-4.82233	-1.17944	2.53345	-4.82233	4.82233	1.17944
3.30	2.27629	-4.81449	-1.26101	2.53821	-4.81449	4.81449	1.26101
3.31	2.26362	-4.80662	-1.34285	2.54300	-4.80662	4.80662	1.34285
3.32	2.25089	-4.79873	-1.42495	2.54783	-4.79873	4.79873	1.42495
3.33	2.23808	-4.79079	-1.50732	2.55271	-4.79079	4.79079	1.50732
3.34	2.22520	-4.78282	-1.58996	2.55763	-4.78282	4.78282	1.58996
3.35	2.21223	-4.77482	-1.67286	2.56258	-4.77482	4.77482	1.67286
3.36	2.19920	-4.76678	-1.75603	2.56759	-4.76678	4.76678	1.75603
3.37	2.18608	-4.75871	-1.83947	2.57263	-4.75871	4.75871	1.83947
3.38	2.17289	-4.75061	-1.92318	2.57772	-4.75061	4.75061	1.92318
3.39	2.15962	-4.74247	-2.00715	2.58285	-4.74247	4.74247	2.00715
3.40	2.14627	-4.73430	-2.09139	2.58803	-4.73430	4.73430	2.09139
3.41	2.13285	-4.72610	-2.17590	2.59325	-4.72610	4.72610	2.17590
3.42	2.11934	-4.71786	-2.26068	2.59852	-4.71786	4.71786	2.26068
3.43	2.10575	-4.70958	-2.34573	2.60383	-4.70958	4.70958	2.34573
3.44	2.09209	-4.70127	-2.43105	2.60919	-4.70127	4.70127	2.43105
3.45	2.07834	-4.69293	-2.51663	2.61460	-4.69293	4.69293	2.51663
3.46	2.06450	-4.68455	-2.60249	2.62005	-4.68455	4.68455	2.60249
3.47	2.05059	-4.67614	-2.68861	2.62555	-4.67614	4.67614	2.68861
3.48	2.03659	-4.66770	-2.77501	2.63111	-4.66770	4.66770	2.77501
3.49	2.02251	-4.65921	-2.86167	2.63671	-4.65921	4.65921	2.86167
3.50	2.00834	-4.65070	-2.94861	2.64236	-4.65070	4.65070	2.94861
3.51	1.99409	-4.64214	-3.03581	2.64806	-4.64214	4.64214	3.03581
3.52	1.97975	-4.63355	-3.12329	2.65381	-4.63355	4.63355	3.12329
3.53	1.96532	-4.62493	-3.21104	2.65961	-4.62493	4.62493	3.21104
3.54	1.95081	-4.61627	-3.29906	2.66546	-4.61627	4.61627	3.29906
3.55	1.93620	-4.60758	-3.38735	2.67137	-4.60758	4.60758	3.38735
3.56	1.92151	-4.59884	-3.47591	2.67733	-4.59884	4.59884	3.47591
3.57	1.90673	-4.59007	-3.56475	2.68335	-4.59007	4.59007	3.56475
3.58	1.89185	-4.58127	-3.65386	2.68942	-4.58127	4.58127	3.65386
3.59	1.87689	-4.57243	-3.74324	2.69554	-4.57243	4.57243	3.74324
3.60	1.86183	-4.56355	-3.83289	2.70172	-4.56355	4.56355	3.83289
3.61	1.84668	-4.55464	-3.92282	2.70796	-4.55464	4.55464	3.92282
3.62	1.83143	-4.54569	-4.01302	2.71425	-4.54569	4.54569	4.01302
3.63	1.81609	-4.53670	-4.10350	2.72061	-4.53670	4.53670	4.10350
3.64	1.80066	-4.52768	-4.19425	2.72702	-4.52768	4.52768	4.19425
3.65	1.78513	-4.51862	-4.28527	2.73349	-4.51862	4.51862	4.28527
3.66	1.76950	-4.50952	-4.37657	2.74002	-4.50952	4.50952	4.37657
3.67	1.75377	-4.50038	-4.46814	2.74661	-4.50038	4.50038	4.46814
3.68	1.73794	-4.49120	-4.55999	2.75326	-4.49120	4.49120	4.55999
3.69	1.72202	-4.48199	-4.65211	2.75997	-4.48199	4.48199	4.65211
3.70	1.70599	-4.47274	-4.74451	2.76675	-4.47274	4.47274	4.74451
3.71	1.68986	-4.46345	-4.83719	2.77359	-4.46345	4.46345	4.83719
3.72	1.67363	-4.45413	-4.93014	2.78050	-4.45413	4.45413	4.93014
3.73	1.65729	-4.44764	-5.02337	2.78747	-4.44764	4.44764	5.02337

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
$\frac{L}{j}$	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\delta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_L}\left(\frac{L^3}{EI}\right)$
3.74	1.64085	-4.43536	-5.11688	2.79451	-4.43536	4.43536	5.11688
3.75	1.62431	-4.42592	-5.21066	2.80161	-4.42592	4.42592	5.21066
3.76	1.60765	-4.41644	-5.30472	2.80879	-4.41644	4.41644	5.30472
3.77	1.59089	-4.40692	-5.39906	2.81603	-4.40692	4.40692	5.39906
3.78	1.57402	-4.39736	-5.49368	2.82334	-4.39736	4.39736	5.49368
3.79	1.55705	-4.38776	-5.58857	2.83072	-4.38776	4.38776	5.58857
3.80	1.53996	-4.37813	-5.68374	2.83817	-4.37813	4.37813	5.68374
3.81	1.52275	-4.36845	-5.77920	2.84570	-4.36845	4.36845	5.77920
3.82	1.50544	-4.35873	-5.87493	2.85329	-4.35873	4.35873	5.87493
3.83	1.48802	-4.34898	-5.97095	2.86097	-4.34898	4.34898	5.97095
3.84	1.47047	-4.33918	-6.06724	2.86871	-4.33918	4.33918	6.06724
3.85	1.45281	-4.32934	-6.16381	2.87654	-4.32934	4.32934	6.16381
3.86	1.43503	-4.31947	-6.26067	2.88444	-4.31947	4.31947	6.26067
3.87	1.41713	-4.30955	-6.35780	2.89242	-4.30955	4.30955	6.35780
3.88	1.39912	-4.29959	-6.45522	2.90047	-4.29959	4.29959	6.45522
3.89	1.38098	-4.28959	-6.55292	2.90861	-4.28959	4.28959	6.55292
3.90	1.36272	-4.27955	-6.65089	2.91683	-4.27955	4.27955	6.65089
3.91	1.34434	-4.26947	-6.74916	2.92513	-4.26947	4.26947	6.74916
3.92	1.32583	-4.25935	-6.84770	2.93352	-4.25935	4.25935	6.84770
3.93	1.30720	-4.24919	-6.94653	2.94199	-4.24919	4.24919	6.94653
3.94	1.28844	-4.23898	-7.04564	2.95054	-4.23898	4.23898	7.04564
3.95	1.26955	-4.22873	-7.14504	2.95919	-4.22873	4.22873	7.14504
3.96	1.25052	-4.21844	-7.24472	2.96792	-4.21844	4.21844	7.24472
3.97	1.23137	-4.20811	-7.34468	2.97674	-4.20811	4.20811	7.34468
3.98	1.21209	-4.19774	-7.44493	2.98565	-4.19774	4.19774	7.44493
3.99	1.19267	-4.18732	-7.54546	2.99465	-4.18732	4.18732	7.54546
4.00	1.17311	-4.17686	-7.64628	3.00374	-4.17686	4.17686	7.64628
4.01	1.15342	-4.16636	-7.74739	3.01293	-4.16636	4.16636	7.74739
4.02	1.13359	-4.15581	-7.84878	3.02222	-4.15581	4.15581	7.84878
4.03	1.11362	-4.14522	-7.95046	3.03160	-4.14522	4.14522	7.95046
4.04	1.09351	-4.13459	-8.05242	3.04108	-4.13459	4.13459	8.05242
4.05	1.07325	-4.12391	-8.15468	3.05066	-4.12391	4.12391	8.15468
4.06	1.05285	-4.11319	-8.25722	3.06034	-4.11319	4.11319	8.25722
4.07	1.03230	-4.10243	-8.36005	3.07013	-4.10243	4.10243	8.36005
4.08	1.01160	-4.09162	-8.46316	3.08002	-4.09162	4.09162	8.46316
4.09	.990754	-4.08076	-8.56657	3.09001	-4.08076	4.08076	8.56657
4.10	.969755	-4.06987	-8.67027	3.10011	-4.06987	4.06987	8.67027
4.11	.948603	-4.05892	-8.77425	3.11032	-4.05892	4.05892	8.77425
4.12	.927295	-4.04794	-8.87853	3.12064	-4.04794	4.04794	8.87853
4.13	.905830	-4.03690	-8.98309	3.13107	-4.03690	4.03690	8.98309
4.14	.884206	-4.02582	-9.08795	3.14162	-4.02582	4.02582	9.08795
4.15	.862420	-4.01470	-9.19310	3.15228	-4.01470	4.01470	9.19310
4.16	.840473	-4.00353	-9.29854	3.16306	-4.00353	4.00353	9.29854
4.17	.818360	-3.99231	-9.40427	3.17395	-3.99231	3.99231	9.40427
4.18	.796080	-3.98105	-9.51030	3.18497	-3.98105	3.98105	9.51030
4.19	.773632	-3.96974	-9.61661	3.19611	-3.96974	3.96974	9.61661
4.20	.751013	-3.95839	-9.72323	3.20737	-3.95839	3.95839	9.72323
4.21	.728221	-3.94698	-9.83013	3.21876	-3.94698	3.94698	9.83013
4.22	.705254	-3.93553	-9.93733	3.23028	-3.93553	3.93553	9.93733
4.23	.682109	-3.92404	-10.0448	3.24193	-3.92404	3.92404	10.0448
4.24	.658785	-3.91249	-10.1526	3.25371	-3.91249	3.91249	10.1526
4.25	.635279	-3.90090	-10.2607	3.26562	-3.90090	3.90090	10.2607
4.26	.611589	-3.88926	-10.3691	3.27767	-3.88926	3.88926	10.3691
4.27	.587712	-3.87757	-10.4778	3.28986	-3.87757	3.87757	10.4778
4.28	.563647	-3.86583	-10.5867	3.30219	-3.86583	3.86583	10.5867
4.29	.539390	-3.85405	-10.6960	3.31466	-3.85405	3.85405	10.6960
4.30	.514939	-3.84221	-10.8056	3.32727	-3.84221	3.84221	10.8056
4.31	.490292	-3.83033	-10.9154	3.34004	-3.83033	3.83033	10.9154
4.32	.465446	-3.81839	-11.0256	3.35295	-3.81839	3.81839	11.0256
4.33	.440399	-3.80641	-11.1361	3.36601	-3.80641	3.80641	11.1361

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TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\delta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_L}\left(\frac{L^3}{EI}\right)$
4.34	0.415147	-3.79438	-11.2468	3.37923	-3.79438	3.79438	11.2468
4.35	.389688	-3.78229	-11.3579	3.39261	-3.78229	3.78229	11.3579
4.36	.364019	-3.77016	-11.4693	3.40614	-3.77016	3.77016	11.4693
4.37	.338138	-3.75797	-11.5809	3.41984	-3.75797	3.75797	11.5809
4.38	.312040	-3.74574	-11.6929	3.43370	-3.74574	3.74574	11.6929
4.39	.285724	-3.73345	-11.8052	3.44773	-3.73345	3.73345	11.8052
4.40	.259187	-3.72111	-11.9178	3.46193	-3.72111	3.72111	11.9178
4.41	.232424	-3.70873	-12.0306	3.47630	-3.70873	3.70873	12.0306
4.42	.205434	-3.69628	-12.1438	3.49085	-3.69628	3.69628	12.1438
4.43	.178213	-3.68379	-12.2573	3.50558	-3.68379	3.68379	12.2573
4.44	.150756	-3.67125	-12.3711	3.52049	-3.67125	3.67125	12.3711
4.45	.123062	-3.65865	-12.4852	3.53558	-3.65865	3.65865	12.4852
4.46	.0951270	-3.64599	-12.5996	3.55087	-3.64599	3.64599	12.5996
4.47	.0669467	-3.63329	-12.7143	3.56634	-3.63329	3.63329	12.7143
4.48	.0385179	-3.62053	-12.8293	3.58201	-3.62053	3.62053	12.8293
4.49	.00983696	-3.60772	-12.9447	3.59788	-3.60772	3.60772	12.9447
4.50	-.0190999	-3.59485	-13.0603	3.61395	-3.59485	3.59485	13.0603
4.51	-.0482966	-3.58193	-13.1762	3.63023	-3.58193	3.58193	13.1762
4.52	-.0777570	-3.56896	-13.2925	3.64672	-3.56896	3.56896	13.2925
4.53	-.107485	-3.55593	-13.4090	3.66341	-3.55593	3.55593	13.4090
4.54	-.137485	-3.54284	-13.5259	3.68033	-3.54284	3.54284	13.5259
4.55	-.167761	-3.52970	-13.6431	3.69746	-3.52970	3.52970	13.6431
4.56	-.198317	-3.51651	-13.7606	3.71482	-3.51651	3.51651	13.7606
4.57	-.229158	-3.50325	-13.8784	3.73241	-3.50325	3.50325	13.8784
4.58	-.260288	-3.48995	-13.9965	3.75023	-3.48995	3.48995	13.9965
4.59	-.291712	-3.47658	-14.1149	3.76829	-3.47658	3.47658	14.1149
4.60	-.323435	-3.46316	-14.2337	3.78659	-3.46316	3.46316	14.2337
4.61	-.355461	-3.44968	-14.3527	3.80514	-3.44968	3.44968	14.3527
4.62	-.387795	-3.43614	-14.4721	3.82394	-3.43614	3.43614	14.4721
4.63	-.420442	-3.42255	-14.5918	3.84299	-3.42255	3.42255	14.5918
4.64	-.453408	-3.40890	-14.7118	3.86230	-3.40890	3.40890	14.7118
4.65	-.486697	-3.39518	-14.8321	3.88188	-3.39518	3.39518	14.8321
4.66	-.520316	-3.38141	-14.9528	3.90173	-3.38141	3.38141	14.9528
4.67	-.554269	-3.36759	-15.0737	3.92185	-3.36759	3.36759	15.0737
4.68	-.588563	-3.35370	-15.1950	3.94226	-3.35370	3.35370	15.1950
4.69	-.623202	-3.33975	-15.3166	3.96295	-3.33975	3.33975	15.3166
4.70	-.658194	-3.32574	-15.4385	3.98394	-3.32574	3.32574	15.4385
4.71	-.693545	-3.31167	-15.5608	4.00522	-3.31167	3.31167	15.5608
4.72	-.729259	-3.29754	-15.6833	4.02680	-3.29754	3.29754	15.6833
4.73	-.765345	-3.28335	-15.8062	4.04870	-3.28335	3.28335	15.8062
4.74	-.801808	-3.26910	-15.9294	4.07091	-3.26910	3.26910	15.9294
4.75	-.838656	-3.25479	-16.0529	4.09344	-3.25479	3.25479	16.0529
4.76	-.875895	-3.24041	-16.1768	4.11631	-3.24041	3.24041	16.1768
4.77	-.913532	-3.22597	-16.3010	4.13950	-3.22597	3.22597	16.3010
4.78	-.951574	-3.21147	-16.4255	4.16305	-3.21147	3.21147	16.4255
4.79	-.990030	-3.19691	-16.5503	4.18694	-3.19691	3.19691	16.5503
4.80	-1.02891	-3.18228	-16.6754	4.21119	-3.18228	3.18228	16.6754
4.81	-1.06821	-3.16759	-16.8009	4.23580	-3.16759	3.16759	16.8009
4.82	-1.10795	-3.15283	-16.9267	4.26079	-3.15283	3.15283	16.9267
4.83	-1.14814	-3.13801	-17.0529	4.28615	-3.13801	3.13801	17.0529
4.84	-1.18878	-3.12313	-17.1793	4.31191	-3.12313	3.12313	17.1793
4.85	-1.22989	-3.10818	-17.3061	4.33806	-3.10818	3.10818	17.3061
4.86	-1.27146	-3.09316	-17.4333	4.36462	-3.09316	3.09316	17.4333
4.87	-1.31352	-3.07808	-17.5607	4.39159	-3.07808	3.07808	17.5607
4.88	-1.35606	-3.06293	-17.6885	4.41899	-3.06293	3.06293	17.6885
4.89	-1.39911	-3.04771	-17.8167	4.44682	-3.04771	3.04771	17.8167
4.90	-1.44266	-3.03243	-17.9451	4.47509	-3.03243	3.03243	17.9451
4.91	-1.48674	-3.01708	-18.0740	4.50381	-3.01708	3.01708	18.0740
4.92	-1.53135	-3.00166	-18.2031	4.53300	-3.00166	3.00166	18.2031
4.93	-1.57650	-2.98617	-18.3326	4.56266	-2.98617	2.98617	18.3326

TABLE 1 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_L}\left(\frac{L}{EI}\right)$	$K_{F\delta_L}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_L}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_L}\left(\frac{L}{EI}\right)$	$K_{C\delta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_L}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_L}\left(\frac{L^3}{EI}\right)$
4.94	-1.62220	-2.97061	-18.4624	4.59281	-2.97061	2.97061	18.4624
4.95	-1.66847	-2.95498	-18.5925	4.62345	-2.95498	2.95498	18.5925
4.96	-1.71531	-2.93929	-18.7230	4.65460	-2.93929	2.93929	18.7230
4.97	-1.76275	-2.92352	-18.8539	4.68627	-2.92352	2.92352	18.8539
4.98	-1.81079	-2.90769	-18.9850	4.71847	-2.90769	2.90769	18.9850
4.99	-1.85944	-2.89178	-19.1165	4.75122	-2.89178	2.89178	19.1165
5.00	-1.90872	-2.87580	-19.2484	4.78452	-2.87580	2.87580	19.2484
5.01	-1.95865	-2.85975	-19.3806	4.81839	-2.85975	2.85975	19.3806
5.02	-2.00923	-2.84362	-19.5132	4.85285	-2.84362	2.84362	19.5132
5.03	-2.06048	-2.82743	-19.6461	4.88791	-2.82743	2.82743	19.6461
5.04	-2.11242	-2.81116	-19.7793	4.92358	-2.81116	2.81116	19.7793
5.05	-2.16507	-2.79481	-19.9129	4.95988	-2.79481	2.79481	19.9129
5.06	-2.21843	-2.77839	-20.0468	4.99682	-2.77839	2.77839	20.0468
5.07	-2.27253	-2.76190	-20.1811	5.03443	-2.76190	2.76190	20.1811
5.08	-2.32738	-2.74533	-20.3157	5.07272	-2.74533	2.74533	20.3157
5.09	-2.38301	-2.72869	-20.4507	5.11170	-2.72869	2.72869	20.4507
5.10	-2.43942	-2.71197	-20.5861	5.15139	-2.71197	2.71197	20.5861
5.11	-2.49665	-2.69517	-20.7218	5.19182	-2.69517	2.69517	20.7218
5.12	-2.55470	-2.67830	-20.8578	5.23300	-2.67830	2.67830	20.8578
5.13	-2.61360	-2.66135	-20.9942	5.27495	-2.66135	2.66135	20.9942
5.14	-2.67338	-2.64432	-21.1310	5.32336	-2.64432	2.64432	21.1310
5.15	-2.73404	-2.62721	-21.2681	5.36126	-2.62721	2.62721	21.2681
5.16	-2.79562	-2.61003	-21.4055	5.40565	-2.61003	2.61003	21.4055
5.17	-2.85815	-2.59276	-21.5434	5.45091	-2.59276	2.59276	21.5434
5.18	-2.92163	-2.57541	-21.6816	5.49704	-2.57541	2.57541	21.6816
5.19	-2.98611	-2.55799	-21.8201	5.54409	-2.55799	2.55799	21.8201
5.20	-3.05159	-2.54048	-21.9590	5.59207	-2.54048	2.54048	21.9590
5.21	-3.11812	-2.52289	-22.0983	5.64101	-2.52289	2.52289	22.0983
5.22	-3.18573	-2.50522	-22.2380	5.69094	-2.50522	2.50522	22.2380
5.23	-3.25443	-2.48746	-22.3780	5.74189	-2.48746	2.48746	22.3780
5.24	-3.32426	-2.46962	-22.5184	5.79388	-2.46962	2.46962	22.5184
5.25	-3.39525	-2.45170	-22.6591	5.84695	-2.45170	2.45170	22.6591
5.26	-3.46744	-2.43369	-22.8002	5.90113	-2.43369	2.43369	22.8002
5.27	-3.54086	-2.41560	-22.9417	5.95646	-2.41560	2.41560	22.9417
5.28	-3.61554	-2.39742	-23.0836	6.01296	-2.39742	2.39742	23.0836
5.29	-3.69152	-2.37916	-23.2258	6.07067	-2.37916	2.37916	23.2258
5.30	-3.76884	-2.36081	-23.3684	6.12964	-2.36081	2.36081	23.3684
5.31	-3.84754	-2.34237	-23.5114	6.18990	-2.34237	2.34237	23.5114
5.32	-3.92766	-2.32384	-23.6547	6.25150	-2.32384	2.32384	23.6547
5.33	-4.00924	-2.30522	-23.7984	6.31446	-2.30522	2.30522	23.7984
5.34	-4.09233	-2.28652	-23.9426	6.37885	-2.28652	2.28652	23.9426
5.35	-4.17698	-2.26772	-24.0870	6.44470	-2.26772	2.26772	24.0870
5.36	-4.26323	-2.24884	-24.2319	6.51207	-2.24884	2.24884	24.2319
5.37	-4.35114	-2.22986	-24.3772	6.58100	-2.22986	2.22986	24.3772
5.38	-4.44076	-2.21079	-24.5228	6.65155	-2.21079	2.21079	24.5228
5.39	-4.53214	-2.19163	-24.6688	6.72377	-2.19163	2.19163	24.6688
5.40	-4.62534	-2.17238	-24.8152	6.79773	-2.17238	2.17238	24.8152
5.41	-4.72043	-2.15303	-24.9620	6.87347	-2.15303	2.15303	24.9620
5.42	-4.81747	-2.13359	-25.1092	6.95106	-2.13359	2.13359	25.1092
5.43	-4.91653	-2.11405	-25.2568	7.03058	-2.11405	2.11405	25.2568
5.44	-5.01766	-2.09442	-25.4048	7.11208	-2.09442	2.09442	25.4048
5.45	-5.12096	-2.07469	-25.5531	7.19565	-2.07469	2.07469	25.5531
5.46	-5.22649	-2.05486	-25.7019	7.28135	-2.05486	2.05486	25.7019
5.47	-5.33434	-2.03493	-25.8510	7.36927	-2.03493	2.03493	25.8510
5.48	-5.44458	-2.01491	-26.0006	7.45950	-2.01491	2.01491	26.0006
5.49	-5.55732	-1.99479	-26.1505	7.55211	-1.99479	1.99479	26.1505
5.50	-5.67264	-1.97456	-26.3009	7.64720	-1.97456	1.97456	26.3009
5.51	-5.79063	-1.95424	-26.4516	7.74487	-1.95424	1.95424	26.4516
5.52	-5.91142	-1.93381	-26.6028	7.84523	-1.93381	1.93381	26.6028
5.53	-6.03509	-1.91328	-26.7543	7.94837	-1.91328	1.91328	26.7543

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

TABLE I - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH LEFT END CLAMPED - Concluded

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{Fe_L}\left(\frac{L}{EI}\right)$	$K_{Fe_L}\left(\frac{L^2}{EI}\right)$ or $C_{Fe_L}\left(\frac{L^2}{EI}\right)$	$C_{Fe_L}\left(\frac{L^3}{EI}\right)$	$K_{Cs_L}\left(\frac{L}{EI}\right)$	$K_{Cs_L}\left(\frac{L^2}{EI}\right)$	$C_{Cs_L}\left(\frac{L^2}{EI}\right)$	$C_{Cs_L}\left(\frac{L^3}{EI}\right)$
5.54	-6.16177	-1.89269	-26.9063	8.05442	-1.89269	1.89269	26.9063
5.55	-6.29157	-1.87191	-27.0587	8.16349	-1.87191	1.87191	27.0587
5.56	-6.42464	-1.85108	-27.2115	8.27571	-1.85108	1.85108	27.2115
5.57	-6.56109	-1.83013	-27.3646	8.39122	-1.83013	1.83013	27.3646
5.58	-6.70107	-1.80908	-27.5183	8.51015	-1.80908	1.80908	27.5183
5.59	-6.84474	-1.78792	-27.6722	8.63266	-1.78792	1.78792	27.6722
5.60	-6.99225	-1.76664	-27.8267	8.75889	-1.76664	1.76664	27.8267
5.61	-7.14376	-1.74527	-27.9816	8.88903	-1.74527	1.74527	27.9816
5.62	-7.29947	-1.72378	-28.1368	9.02326	-1.72378	1.72378	28.1368
5.63	-7.45955	-1.70219	-28.2925	9.16174	-1.70219	1.70219	28.2925
5.64	-7.62422	-1.68048	-28.4486	9.30469	-1.68048	1.68048	28.4486
5.65	-7.79367	-1.65866	-28.6052	9.45233	-1.65866	1.65866	28.6052
5.66	-7.96815	-1.63672	-28.7622	9.60487	-1.63672	1.63672	28.7622
5.67	-8.14790	-1.61467	-28.9196	9.76257	-1.61467	1.61467	28.9196
5.68	-8.33166	-1.59251	-29.0774	9.92568	-1.59251	1.59251	29.0774
5.69	-8.52423	-1.57023	-29.2356	10.0945	-1.57023	1.57023	29.2356
5.70	-8.72139	-1.54783	-29.3943	10.2692	-1.54783	1.54783	29.3943
5.71	-8.92497	-1.52532	-29.5535	10.4503	-1.52532	1.52532	29.5535
5.72	-9.13528	-1.50269	-29.7130	10.6380	-1.50269	1.50269	29.7130
5.73	-9.35272	-1.47993	-29.8730	10.8327	-1.47993	1.47993	29.8730
5.74	-9.57765	-1.45706	-30.0335	11.0347	-1.45706	1.45706	30.0335
5.75	-9.81051	-1.43407	-30.1944	11.2446	-1.43407	1.43407	30.1944
5.76	-10.0517	-1.41095	-30.3557	11.4627	-1.41095	1.41095	30.3557
5.77	-10.3018	-1.38771	-30.5175	11.6895	-1.38771	1.38771	30.5175
5.78	-10.5613	-1.36434	-30.6797	11.9256	-1.36434	1.36434	30.6797
5.79	-10.8307	-1.34085	-30.8424	12.1715	-1.34085	1.34085	30.8424
5.80	-11.1106	-1.31723	-31.0055	12.4278	-1.31723	1.31723	31.0055
5.81	-11.4018	-1.29348	-31.1691	12.6953	-1.29348	1.29348	31.1691
5.82	-11.7049	-1.26961	-31.3332	12.9745	-1.26961	1.26961	31.3332
5.83	-12.0208	-1.24560	-31.4977	13.2664	-1.24560	1.24560	31.4977
5.84	-12.3502	-1.22147	-31.6627	13.5717	-1.22147	1.22147	31.6627
5.85	-12.6942	-1.19720	-31.8281	13.8914	-1.19720	1.19720	31.8281
5.86	-13.0537	-1.17279	-31.9940	14.2265	-1.17279	1.17279	31.9940
5.87	-13.4299	-1.14826	-32.1604	14.5781	-1.14826	1.14826	32.1604
5.88	-13.8240	-1.12358	-32.3272	14.9476	-1.12358	1.12358	32.3272
5.89	-14.2373	-1.09877	-32.4945	15.3361	-1.09877	1.09877	32.4945
5.90	-14.6719	-1.07383	-32.6623	15.7453	-1.07383	1.07383	32.6623
5.91	-15.1280	-1.04874	-32.8306	16.1768	-1.04874	1.04874	32.8306
5.92	-15.6089	-1.02351	-32.9994	16.6324	-1.02351	1.02351	32.9994
5.93	-16.1161	-.998143	-33.1686	17.1142	-.998143	.998143	33.1686
5.94	-16.6519	-.972631	-33.3383	17.6245	-.972631	.972631	33.3383
5.95	-17.2189	-.946975	-33.5086	18.1659	-.946975	.946975	33.5086
5.96	-17.8201	-.921174	-33.6793	18.7412	-.921174	.921174	33.6793
5.97	-18.4585	-.895225	-33.8505	19.3538	-.895225	.895225	33.8505
5.98	-19.1380	-.869126	-34.0221	20.0071	-.869126	.869126	34.0221
5.99	-19.8628	-.842881	-34.1944	20.7057	-.842881	.842881	34.1944
6.00	-20.6375	-.816483	-34.3670	21.4540	-.816483	.816483	34.3670
6.01	-21.4678	-.789934	-34.5402	22.2577	-.789934	.789934	34.5402
6.02	-22.3598	-.763230	-34.7139	23.1230	-.763230	.763230	34.7139
6.03	-23.3210	-.736374	-34.8882	24.0574	-.736374	.736374	34.8882
6.04	-25.3598	-.709359	-35.0629	25.0692	-.709359	.709359	35.0629
6.05	-25.4863	-.682186	-35.2381	26.1685	-.682186	.682186	35.2381
6.06	-26.7121	-.654853	-35.4138	27.3669	-.654853	.654853	35.4138
6.07	-28.0513	-.627362	-35.5902	28.6787	-.627362	.627362	35.5902
6.08	-29.5205	-.599706	-35.7670	30.1202	-.599706	.599706	35.7670
6.09	-31.1401	-.571888	-35.9443	31.7120	-.571888	.571888	35.9443
6.10	-32.9343	-.543900	-36.1220	33.4782	-.543900	.543900	36.1220
6.11	-34.9340	-.515752	-36.3006	35.4398	-.515752	.515752	36.3006
6.12	-37.1765	-.487433	-36.4796	37.6639	-.487433	.487433	36.4796
6.13	-39.7095	-.458946	-36.6593	40.1684	-.458946	.458946	36.6593
6.14	-42.5931	-.430281	-36.8391	43.0234	-.430281	.430281	36.8391
6.15	-45.9073	-.401445	-37.0196	46.3087	-.401445	.401445	37.0196
6.16	-49.7565	-.372434	-37.2007	50.1289	-.372434	.372434	37.2007
6.17	-54.2825	-.343246	-37.3824	54.6258	-.343246	.343246	37.3824
6.18	-59.6822	-.313880	-37.5646	59.9960	-.313880	.313880	37.5646
6.19	-66.2363	-.284334	-37.7474	66.5209	-.284334	.284334	37.7474
6.20	-74.3621	-.254605	-37.9308	74.6167	-.254605	.254605	37.9308
6.21	-84.7028	-.224693	-38.1147	84.9275	-.224693	.224693	38.1147
6.22	-98.3097	-.194593	-38.2989	98.5043	-.194593	.194593	38.2989
6.23	-117.028	-.164309	-38.4843	117.192	-.164309	.164309	38.4843
6.24	-144.404	-.133833	-38.6700	144.538	-.133833	.133833	38.6700
6.25	-188.268	-.103166	-38.8562	188.371	-.103166	.103166	38.8562
6.26	-269.950	-.072306	-39.0430	270.023	-.072306	.072306	39.0430
6.27	-475.501	-.041247	-39.2303	475.542	-.041247	.041247	39.2303
6.28	-1971.76	-.00999181	-39.4227	1971.77	-.00999181	.00999181	39.4227
2π	$-\infty$	0	-39.4784	∞	0	0	39.4784

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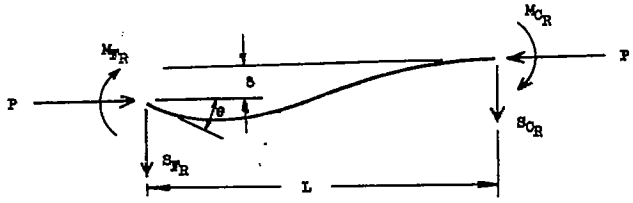
TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED

$$\left[\begin{array}{ll} S_{T_R} = C_{T\theta_R}\theta + C_{T\delta_R}\delta & S_{C_R} = C_{C\theta_R}\theta + C_{C\delta_R}\delta \\ M_{T_R} = K_{T\theta_R}\theta + K_{T\delta_R}\delta & M_{C_R} = K_{C\theta_R}\theta + K_{C\delta_R}\delta \end{array}\right]$$



$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{T\theta_R}\left(\frac{L}{EI}\right)$	$K_{T\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{T\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{T\delta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\delta_R}\left(\frac{L^2}{EI}\right)$	$C_{T\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_R}\left(\frac{L^3}{EI}\right)$
0	4.00000	6.00000	12.0000	2.00000	6.00000	-6.00000	-12.0000
.1	3.99865	5.99901	11.9880	2.00036	5.99901	-5.99901	-11.9880
.2	3.99465	5.99597	11.9519	2.00132	5.99597	-5.99597	-11.9519
.3	3.98798	5.99099	11.8920	2.00300	5.99099	-5.99099	-11.8920
.4	3.97862	5.98398	11.8080	2.00536	5.98398	-5.98398	-11.8080
.5	3.96656	5.97495	11.6999	2.00840	5.97495	-5.97495	-11.6999
.6	3.95177	5.96391	11.5678	2.01214	5.96391	-5.96391	-11.5678
.7	3.93424	5.95083	11.4117	2.01658	5.95083	-5.95083	-11.4117
.8	3.91394	5.93571	11.2314	2.02176	5.93571	-5.93571	-11.2314
.9	3.89083	5.91853	11.0271	2.02769	5.91853	-5.91853	-11.0271
1.0	3.86488	5.89928	10.7986	2.03440	5.89928	-5.89928	-10.7986
1.01	3.86213	5.89724	10.7744	2.03511	5.89724	-5.89724	-10.7744
1.02	3.85935	5.89518	10.7500	2.03583	5.89518	-5.89518	-10.7500
1.03	3.85654	5.89310	10.7253	2.03656	5.89310	-5.89310	-10.7253
1.04	3.85370	5.89099	10.7004	2.03730	5.89099	-5.89099	-10.7004
1.05	3.85083	5.88887	10.6752	2.03804	5.88887	-5.88887	-10.6752
1.06	3.84793	5.88673	10.6499	2.03880	5.88673	-5.88673	-10.6499
1.07	3.84500	5.88456	10.6242	2.03956	5.88456	-5.88456	-10.6242
1.08	3.84204	5.88238	10.5984	2.04033	5.88238	-5.88238	-10.5984
1.09	3.83906	5.88017	10.5722	2.04111	5.88017	-5.88017	-10.5722
1.10	3.83604	5.87794	10.5459	2.04190	5.87794	-5.87794	-10.5459
1.11	3.83300	5.87569	10.5193	2.04269	5.87569	-5.87569	-10.5193
1.12	3.82992	5.87342	10.4924	2.04350	5.87342	-5.87342	-10.4924
1.13	3.82682	5.87113	10.4654	2.04431	5.87113	-5.87113	-10.4654
1.14	3.82369	5.86882	10.4380	2.04513	5.86882	-5.86882	-10.4380
1.15	3.82052	5.86648	10.4105	2.04596	5.86648	-5.86648	-10.4105
1.16	3.81733	5.86413	10.3827	2.04679	5.86413	-5.86413	-10.3827
1.17	3.81411	5.86175	10.3546	2.04764	5.86175	-5.86175	-10.3546
1.18	3.81086	5.85935	10.3263	2.04850	5.85935	-5.85935	-10.3263
1.19	3.80758	5.85693	10.2978	2.04936	5.85693	-5.85693	-10.2978
1.20	3.80426	5.85449	10.2690	2.05023	5.85449	-5.85449	-10.2690
1.21	3.80092	5.85203	10.2400	2.05111	5.85203	-5.85203	-10.2400
1.22	3.79755	5.84955	10.2107	2.05200	5.84955	-5.84955	-10.2107
1.23	3.79415	5.84705	10.1812	2.05290	5.84705	-5.84705	-10.1812
1.24	3.79072	5.84452	10.1514	2.05380	5.84452	-5.84452	-10.1514
1.25	3.78726	5.84197	10.1215	2.05472	5.84197	-5.84197	-10.1215
1.26	3.78377	5.83941	10.0912	2.05564	5.83941	-5.83941	-10.0912
1.27	3.78024	5.83682	10.0607	2.05658	5.83682	-5.83682	-10.0607
1.28	3.77669	5.83421	10.0300	2.05752	5.83421	-5.83421	-10.0300
1.29	3.77311	5.83157	9.99905	2.05847	5.83157	-5.83157	-9.99905
1.30	3.76949	5.82892	9.96784	2.05943	5.82892	-5.82892	-9.96784
1.31	3.76585	5.82625	9.93639	2.06040	5.82625	-5.82625	-9.93639
1.32	3.76217	5.82355	9.90470	2.06137	5.82355	-5.82355	-9.90470
1.33	3.75847	5.82083	9.87276	2.06236	5.82083	-5.82083	-9.87276
1.34	3.75473	5.81809	9.84058	2.06336	5.81809	-5.81809	-9.84058

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TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{d}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{Fe_R}\left(\frac{L}{EI}\right)$	$K_{Fe_R}\left(\frac{L^2}{EI}\right)$ or $C_{Fe_R}\left(\frac{L^2}{EI}\right)$	$C_{Fe_R}\left(\frac{L^3}{EI}\right)$	$K_{Ce_R}\left(\frac{L}{EI}\right)$	$K_{Ce_R}\left(\frac{L^2}{EI}\right)$	$C_{Ce_R}\left(\frac{L^2}{EI}\right)$	$C_{Ce_R}\left(\frac{L^3}{EI}\right)$
1.35	3.75097	5.81533	9.80816	2.06436	5.81533	-5.81533	-9.80816
1.36	3.74717	5.81255	9.77549	2.06538	5.81255	-5.81255	-9.77549
1.37	3.74334	5.80974	9.74258	2.06640	5.80974	-5.80974	-9.74258
1.38	3.73948	5.80691	9.70943	2.06743	5.80691	-5.80691	-9.70943
1.39	3.73559	5.80407	9.67603	2.06848	5.80407	-5.80407	-9.67603
1.40	3.73167	5.80119	9.64239	2.06953	5.80119	-5.80119	-9.64239
1.41	3.72771	5.79830	9.60851	2.07059	5.79830	-5.79830	-9.60851
1.42	3.72373	5.79539	9.57438	2.07166	5.79539	-5.79539	-9.57438
1.43	3.71971	5.79245	9.54001	2.07274	5.79245	-5.79245	-9.54001
1.44	3.71567	5.78950	9.50539	2.07383	5.78950	-5.78950	-9.50539
1.45	3.71159	5.78652	9.47053	2.07493	5.78652	-5.78652	-9.47053
1.46	3.70748	5.78352	9.43543	2.07604	5.78352	-5.78352	-9.43543
1.47	3.70333	5.78049	9.40009	2.07716	5.78049	-5.78049	-9.40009
1.48	3.69916	5.77745	9.36450	2.07829	5.77745	-5.77745	-9.36450
1.49	3.69495	5.77438	9.32866	2.07943	5.77438	-5.77438	-9.32866
1.50	3.69072	5.77129	9.29259	2.08058	5.77129	-5.77129	-9.29259
1.51	3.68645	5.76818	9.25626	2.08173	5.76818	-5.76818	-9.25626
1.52	3.68214	5.76505	9.21969	2.08290	5.76505	-5.76505	-9.21969
1.53	3.67781	5.76189	9.18288	2.08408	5.76189	-5.76189	-9.18288
1.54	3.67344	5.75871	9.14583	2.08527	5.75871	-5.75871	-9.14583
1.55	3.66904	5.75551	9.10853	2.08647	5.75551	-5.75551	-9.10853
1.56	3.66461	5.75229	9.07098	2.08768	5.75229	-5.75229	-9.07098
1.57	3.66015	5.74905	9.03319	2.08890	5.74905	-5.74905	-9.03319
1.58	3.65565	5.74578	8.99516	2.09013	5.74578	-5.74578	-8.99516
1.59	3.65112	5.74249	8.95689	2.09137	5.74249	-5.74249	-8.95689
1.60	3.64656	5.73918	8.91836	2.09262	5.73918	-5.73918	-8.91836
1.61	3.64197	5.73585	8.87960	2.09388	5.73585	-5.73585	-8.87960
1.62	3.63734	5.73249	8.84059	2.09515	5.73249	-5.73249	-8.84059
1.63	3.63268	5.72911	8.80133	2.09643	5.72911	-5.72911	-8.80133
1.64	3.62799	5.72571	8.76183	2.09773	5.72571	-5.72571	-8.76183
1.65	3.62326	5.72229	8.72208	2.09903	5.72229	-5.72229	-8.72208
1.66	3.61850	5.71884	8.68209	2.10035	5.71884	-5.71884	-8.68209
1.67	3.61371	5.71538	8.64185	2.10167	5.71538	-5.71538	-8.64185
1.68	3.60888	5.71189	8.60137	2.10301	5.71189	-5.71189	-8.60137
1.69	3.60402	5.70837	8.56064	2.10435	5.70837	-5.70837	-8.56064
1.70	3.59912	5.70484	8.51967	2.10571	5.70484	-5.70484	-8.51967
1.71	3.59420	5.70128	8.47845	2.10708	5.70128	-5.70128	-8.47845
1.72	3.58923	5.69770	8.43699	2.10846	5.69770	-5.69770	-8.43699
1.73	3.58424	5.69409	8.39528	2.10985	5.69409	-5.69409	-8.39528
1.74	3.57921	5.69046	8.35333	2.11126	5.69046	-5.69046	-8.35333
1.75	3.57414	5.68681	8.31113	2.11267	5.68681	-5.68681	-8.31113
1.76	3.56905	5.68314	8.26868	2.11410	5.68314	-5.68314	-8.26868
1.77	3.56391	5.67945	8.22599	2.11553	5.67945	-5.67945	-8.22599
1.78	3.55875	5.67573	8.18305	2.11698	5.67573	-5.67573	-8.18305
1.79	3.55354	5.67199	8.13987	2.11844	5.67199	-5.67199	-8.13987
1.80	3.54831	5.66822	8.09644	2.11991	5.66822	-5.66822	-8.09644
1.81	3.54304	5.66443	8.05277	2.12140	5.66443	-5.66443	-8.05277
1.82	3.53773	5.66062	8.00884	2.12289	5.66062	-5.66062	-8.00884
1.83	3.53239	5.65679	7.96468	2.12440	5.65679	-5.65679	-7.96468
1.84	3.52701	5.65293	7.92026	2.12592	5.65293	-5.65293	-7.92026
1.85	3.52160	5.64905	7.87560	2.12745	5.64905	-5.64905	-7.87560
1.86	3.51615	5.64515	7.83070	2.12900	5.64515	-5.64515	-7.83070
1.87	3.51067	5.64122	7.78554	2.13055	5.64122	-5.64122	-7.78554
1.88	3.50515	5.63727	7.74014	2.13212	5.63727	-5.63727	-7.74014
1.89	3.49960	5.63330	7.69450	2.13370	5.63330	-5.63330	-7.69450
1.90	3.49401	5.62930	7.64860	2.13529	5.62930	-5.62930	-7.64860
1.91	3.48838	5.62528	7.60246	2.13690	5.62528	-5.62528	-7.60246
1.92	3.48272	5.62124	7.55607	2.13852	5.62124	-5.62124	-7.55607
1.93	3.47702	5.61717	7.50944	2.14015	5.61717	-5.61717	-7.50944
1.94	3.47129	5.61308	7.46256	2.14179	5.61308	-5.61308	-7.46256

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TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\delta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_R}\left(\frac{L^3}{EI}\right)$
1.95	3.46552	5.60896	7.41543	2.14345	5.60896	-5.60896	-7.41543
1.96	3.45971	5.60483	7.36805	2.14512	5.60483	-5.60483	-7.36805
1.97	3.45386	5.60067	7.32043	2.14680	5.60067	-5.60067	-7.32043
1.98	3.44798	5.59648	7.27256	2.14850	5.59648	-5.59648	-7.27256
1.99	3.44207	5.59227	7.22444	2.15020	5.59227	-5.59227	-7.22444
2.00	3.43611	5.58804	7.17608	2.15193	5.58804	-5.58804	-7.17608
2.01	3.43012	5.58378	7.12746	2.15366	5.58378	-5.58378	-7.12746
2.02	3.42409	5.57950	7.07860	2.15541	5.57950	-5.57950	-7.07860
2.03	3.41802	5.57520	7.02949	2.15717	5.57520	-5.57520	-7.02949
2.04	3.41192	5.57087	6.98013	2.15895	5.57087	-5.57087	-6.98013
2.05	3.40578	5.56651	6.93053	2.16074	5.56651	-5.56651	-6.93053
2.06	3.39960	5.56214	6.88068	2.16254	5.56214	-5.56214	-6.88068
2.07	3.39338	5.55774	6.83057	2.16436	5.55774	-5.55774	-6.83057
2.08	3.38712	5.55331	6.78022	2.16619	5.55331	-5.55331	-6.78022
2.09	3.38083	5.54886	6.72963	2.16803	5.54886	-5.54886	-6.72963
2.10	3.37450	5.54439	6.67878	2.16989	5.54439	-5.54439	-6.67878
2.11	3.36812	5.53989	6.62768	2.17177	5.53989	-5.53989	-6.62768
2.12	3.36171	5.53537	6.57634	2.17366	5.53537	-5.53537	-6.57634
2.13	3.35526	5.53082	6.52475	2.17556	5.53082	-5.53082	-6.52475
2.14	3.34878	5.52625	6.47291	2.17748	5.52625	-5.52625	-6.47291
2.15	3.34225	5.52166	6.42081	2.17941	5.52166	-5.52166	-6.42081
2.16	3.33568	5.51704	6.36848	2.18135	5.51704	-5.51704	-6.36848
2.17	3.32908	5.51239	6.31589	2.18332	5.51239	-5.51239	-6.31589
2.18	3.32243	5.50772	6.26305	2.18529	5.50772	-5.50772	-6.26305
2.19	3.31575	5.50303	6.20996	2.18728	5.50303	-5.50303	-6.20996
2.20	3.30902	5.49831	6.15662	2.18929	5.49831	-5.49831	-6.15662
2.21	3.30226	5.49357	6.10304	2.19131	5.49357	-5.49357	-6.10304
2.22	3.29545	5.48880	6.04920	2.19335	5.48880	-5.48880	-6.04920
2.23	3.28860	5.48401	5.99512	2.19540	5.48401	-5.48401	-5.99512
2.24	3.28172	5.47919	5.94078	2.19747	5.47919	-5.47919	-5.94078
2.25	3.27479	5.47435	5.88619	2.19956	5.47435	-5.47435	-5.88619
2.26	3.26782	5.46948	5.83136	2.20166	5.46948	-5.46948	-5.83136
2.27	3.26081	5.46459	5.77627	2.20377	5.46459	-5.46459	-5.77627
2.28	3.25376	5.45967	5.72094	2.20591	5.45967	-5.45967	-5.72094
2.29	3.24667	5.45473	5.66535	2.20806	5.45473	-5.45473	-5.66535
2.30	3.23954	5.44976	5.60951	2.21022	5.44976	-5.44976	-5.60951
2.31	3.23236	5.44476	5.55343	2.21240	5.44476	-5.44476	-5.55343
2.32	3.22514	5.43974	5.49709	2.21460	5.43974	-5.43974	-5.49709
2.33	3.21788	5.43470	5.44050	2.21682	5.43470	-5.43470	-5.44050
2.34	3.21058	5.42963	5.38366	2.21905	5.42963	-5.42963	-5.38366
2.35	3.20324	5.42453	5.32657	2.22130	5.42453	-5.42453	-5.32657
2.36	3.19585	5.41941	5.26923	2.22356	5.41941	-5.41941	-5.26923
2.37	3.18842	5.41427	5.31163	2.22585	5.41427	-5.41427	-5.31163
2.38	3.18095	5.40909	5.15379	2.22815	5.40909	-5.40909	-5.15379
2.39	3.17343	5.40390	5.09569	2.23047	5.40390	-5.40390	-5.09569
2.40	3.16587	5.39867	5.03734	2.23280	5.39867	-5.39867	-5.03734
2.41	3.15827	5.39342	4.97875	2.23515	5.39342	-5.39342	-4.97875
2.42	3.15062	5.38815	4.91989	2.23753	5.38815	-5.38815	-4.91989
2.43	3.14293	5.38285	4.86079	2.23992	5.38285	-5.38285	-4.86079
2.44	3.13519	5.37752	4.80144	2.24232	5.37752	-5.37752	-4.80144
2.45	3.12742	5.37216	4.74183	2.24475	5.37216	-5.37216	-4.74183
2.46	3.11959	5.36679	4.68197	2.24719	5.36679	-5.36679	-4.68197
2.47	3.11172	5.36138	4.62186	2.24966	5.36138	-5.36138	-4.62186
2.48	3.10381	5.35595	4.56149	2.25214	5.35595	-5.35595	-4.56149
2.49	3.09585	5.35049	4.50088	2.25464	5.35049	-5.35049	-4.50088
2.50	3.08784	5.34500	4.44001	2.25716	5.34500	-5.34500	-4.44001
2.51	3.07979	5.33949	4.37888	2.25970	5.33949	-5.33949	-4.37888
2.52	3.07170	5.33395	4.31751	2.26226	5.33395	-5.33395	-4.31751
2.53	3.06355	5.32839	4.25588	2.26484	5.32839	-5.32839	-4.25588
2.54	3.05536	5.32280	4.19400	2.26743	5.32280	-5.32280	-4.19400

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TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{J}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\theta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\theta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^3}{EI}\right)$
2.55	3.04713	5.31718	4.13186	2.27005	5.31718	-5.31718	-4.13186
2.56	3.03885	5.31154	4.06947	2.27269	5.31154	-5.31154	-4.06947
2.57	3.03052	5.30587	4.00683	2.27535	5.30587	-5.30587	-4.00683
2.58	3.02214	5.30017	3.94393	2.27803	5.30017	-5.30017	-3.94393
2.59	3.01372	5.29444	3.88079	2.28073	5.29444	-5.29444	-3.88079
2.60	3.00524	5.28869	3.81738	2.28344	5.28869	-5.28869	-3.81738
2.61	2.99673	5.28291	3.75372	2.28619	5.28291	-5.28291	-3.75372
2.62	2.98816	5.27711	3.68981	2.28895	5.27711	-5.27711	-3.68981
2.63	2.97954	5.27127	3.62564	2.29173	5.27127	-5.27127	-3.62564
2.64	2.97088	5.26541	3.56122	2.29453	5.26541	-5.26541	-3.56122
2.65	2.96216	5.25952	3.49654	2.29736	5.25952	-5.25952	-3.49654
2.66	2.95340	5.25361	3.43161	2.30021	5.25361	-5.25361	-3.43161
2.67	2.94459	5.24766	3.36643	2.30308	5.24766	-5.24766	-3.36643
2.68	2.93572	5.24169	3.30099	2.30597	5.24169	-5.24169	-3.30099
2.69	2.92681	5.23570	3.23529	2.30888	5.23570	-5.23570	-3.23529
2.70	2.91785	5.22967	3.16934	2.31182	5.22967	-5.22967	-3.16934
2.71	2.90884	5.22362	3.10313	2.31478	5.22362	-5.22362	-3.10313
2.72	2.89977	5.21753	3.03667	2.31776	5.21753	-5.21753	-3.03667
2.73	2.89066	5.21143	2.96995	2.32077	5.21143	-5.21143	-2.96995
2.74	2.88149	5.20529	2.90298	2.32380	5.20529	-5.20529	-2.90298
2.75	2.87228	5.19912	2.83575	2.32685	5.19912	-5.19912	-2.83575
2.76	2.86301	5.19293	2.76826	2.32992	5.19293	-5.19293	-2.76826
2.77	2.85368	5.18671	2.70052	2.33302	5.18671	-5.18671	-2.70052
2.78	2.84431	5.18046	2.63252	2.33615	5.18046	-5.18046	-2.63252
2.79	2.83488	5.17418	2.56426	2.33930	5.17418	-5.17418	-2.56426
2.80	2.82540	5.16787	2.49575	2.34247	5.16787	-5.16787	-2.49575
2.81	2.81587	5.16154	2.42698	2.34567	5.16154	-5.16154	-2.42698
2.82	2.80628	5.15517	2.35795	2.34889	5.15517	-5.15517	-2.35795
2.83	2.79664	5.14878	2.28866	2.35214	5.14878	-5.14878	-2.28866
2.84	2.78695	5.14236	2.21912	2.35541	5.14236	-5.14236	-2.21912
2.85	2.77720	5.13591	2.14932	2.35871	5.13591	-5.13591	-2.14932
2.86	2.76740	5.12943	2.07926	2.36204	5.12943	-5.12943	-2.07926
2.87	2.75754	5.12292	2.00895	2.36539	5.12292	-5.12292	-2.00895
2.88	2.74762	5.11639	1.93838	2.36877	5.11639	-5.11639	-1.93838
2.89	2.73765	5.10982	1.86754	2.37217	5.10982	-5.10982	-1.86754
2.90	2.72763	5.10323	1.79645	2.37560	5.10323	-5.10323	-1.79645
2.91	2.71754	5.09660	1.72511	2.37906	5.09660	-5.09660	-1.72511
2.92	2.70740	5.08995	1.65350	2.38255	5.08995	-5.08995	-1.65350
2.93	2.69721	5.08326	1.58163	2.38606	5.08326	-5.08326	-1.58163
2.94	2.68695	5.07656	1.50951	2.38960	5.07656	-5.07656	-1.50951
2.95	2.67664	5.06981	1.43712	2.39317	5.06981	-5.06981	-1.43712
2.96	2.66627	5.06304	1.36448	2.39677	5.06304	-5.06304	-1.36448
2.97	2.65584	5.05624	1.29158	2.40040	5.05624	-5.05624	-1.29158
2.98	2.64535	5.04941	1.21841	2.40406	5.04941	-5.04941	-1.21841
2.99	2.63481	5.04255	1.14499	2.40774	5.04255	-5.04255	-1.14499
3.00	2.62420	5.03565	1.07131	2.41145	5.03565	-5.03565	-1.07131
3.01	2.61353	5.02873	.997365	2.41520	5.02873	-5.02873	-.997365
3.02	2.60280	5.02178	.923161	2.41897	5.02178	-5.02178	-.923161
3.03	2.59202	5.01480	.848697	2.42278	5.01480	-5.01480	-.848697
3.04	2.58117	5.00778	.773971	2.42661	5.00778	-5.00778	-.773971
3.05	2.57026	5.00074	.698986	2.43048	5.00074	-5.00074	-.698986
3.06	2.55929	4.99367	.623740	2.43438	4.99367	-4.99367	-.623740
3.07	2.54825	4.98656	.548231	2.43831	4.98656	-4.98656	-.548231
3.08	2.53715	4.97943	.472461	2.44227	4.97943	-4.97943	-.472461
3.09	2.52600	4.97226	.396430	2.44627	4.97226	-4.97226	-.396430
3.10	2.51478	4.96507	.320138	2.45030	4.96507	-4.96507	-.320138
3.11	2.50349	4.95784	.243582	2.45436	4.95784	-4.95784	-.243582
3.12	2.49214	4.95059	.166765	2.45845	4.95059	-4.95059	-.166765
3.13	2.48073	4.94331	.0896851	2.46259	4.94331	-4.94331	-.0896851
3.14	2.46897	4.93543	.0123409	2.46647	4.93543	-4.93543	-.0123409

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED.- Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\delta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_R}\left(\frac{L^3}{EI}\right)$
∞	2.46740	4.93480	0	2.46740	4.93480	-4.93480	0
3.15	2.45772	4.92867	-.0652638	2.47096	4.92867	-4.92867	.0652638
3.16	2.44608	4.92125	-.143132	2.47517	4.92125	-4.92125	.143132
3.17	2.43439	4.91381	-.221263	2.47942	4.91381	-4.91381	.221263
3.18	2.42265	4.90638	-.299658	2.48373	4.90638	-4.90638	.299658
3.19	2.41083	4.89889	-.378316	2.48806	4.89889	-4.89889	.378316
3.20	2.39895	4.89138	-.457239	2.49243	4.89138	-4.89138	.457239
3.21	2.38700	4.88384	-.536424	2.49684	4.88384	-4.88384	.536424
3.22	2.37498	4.87626	-.615875	2.50128	4.87626	-4.87626	.615875
3.23	2.36289	4.86866	-.695590	2.50576	4.86866	-4.86866	.695590
3.24	2.35073	4.86101	-.775568	2.51028	4.86101	-4.86101	.775568
3.25	2.33850	4.85334	-.855811	2.51484	4.85334	-4.85334	.855811
3.26	2.32620	4.84564	-.936321	2.51943	4.84564	-4.84564	.936321
3.27	2.31383	4.83790	-1.01710	2.52407	4.83790	-4.83790	1.01710
3.28	2.30139	4.83013	-1.09814	2.52874	4.83013	-4.83013	1.09814
3.29	2.28887	4.82233	-1.17944	2.53345	4.82233	-4.82233	1.17944
3.30	2.27629	4.81449	-1.26101	2.53821	4.81449	-4.81449	1.26101
3.31	2.26362	4.80662	-1.34285	2.54300	4.80662	-4.80662	1.34285
3.32	2.25089	4.79873	-1.42495	2.54783	4.79873	-4.79873	1.42495
3.33	2.23808	4.79079	-1.50732	2.55271	4.79079	-4.79079	1.50732
3.34	2.22520	4.78282	-1.58996	2.55763	4.78282	-4.78282	1.58996
3.35	2.21223	4.77482	-1.67286	2.56258	4.77482	-4.77482	1.67286
3.36	2.19920	4.76678	-1.75603	2.56759	4.76678	-4.76678	1.75603
3.37	2.18608	4.75871	-1.83947	2.57263	4.75871	-4.75871	1.83947
3.38	2.17289	4.75061	-1.92318	2.57772	4.75061	-4.75061	1.92318
3.39	2.15962	4.74247	-2.00715	2.58285	4.74247	-4.74247	2.00715
3.40	2.14627	4.73430	-2.09139	2.58803	4.73430	-4.73430	2.09139
3.41	2.13285	4.72610	-2.17590	2.59325	4.72610	-4.72610	2.17590
3.42	2.11934	4.71786	-2.26068	2.59852	4.71786	-4.71786	2.26068
3.43	2.10575	4.70958	-2.34573	2.60383	4.70958	-4.70958	2.34573
3.44	2.09209	4.70127	-2.43105	2.60919	4.70127	-4.70127	2.43105
3.45	2.07834	4.69293	-2.51663	2.61460	4.69293	-4.69293	2.51663
3.46	2.06450	4.68455	-2.60249	2.62005	4.68455	-4.68455	2.60249
3.47	2.05059	4.67614	-2.68861	2.62555	4.67614	-4.67614	2.68861
3.48	2.03659	4.66770	-2.77501	2.63111	4.66770	-4.66770	2.77501
3.49	2.02251	4.65921	-2.86167	2.63671	4.65921	-4.65921	2.86167
3.50	2.00834	4.65070	-2.94861	2.64236	4.65070	-4.65070	2.94861
3.51	1.99409	4.64214	-3.03581	2.64806	4.64214	-4.64214	3.03581
3.52	1.97975	4.63355	-3.12329	2.65381	4.63355	-4.63355	3.12329
3.53	1.96532	4.62493	-3.21104	2.65961	4.62493	-4.62493	3.21104
3.54	1.95081	4.61627	-3.29906	2.66546	4.61627	-4.61627	3.29906
3.55	1.93620	4.60758	-3.38735	2.67137	4.60758	-4.60758	3.38735
3.56	1.92151	4.59884	-3.47591	2.67733	4.59884	-4.59884	3.47591
3.57	1.90673	4.59007	-3.56475	2.68335	4.59007	-4.59007	3.56475
3.58	1.89185	4.58127	-3.65386	2.68942	4.58127	-4.58127	3.65386
3.59	1.87689	4.57243	-3.74324	2.69554	4.57243	-4.57243	3.74324
3.60	1.86183	4.56355	-3.83289	2.70172	4.56355	-4.56355	3.83289
3.61	1.84668	4.55464	-3.92282	2.70796	4.55464	-4.55464	3.92282
3.62	1.83143	4.54569	-4.01302	2.71425	4.54569	-4.54569	4.01302
3.63	1.81609	4.53670	-4.10350	2.72061	4.53670	-4.53670	4.10350
3.64	1.80066	4.52768	-4.19425	2.72702	4.52768	-4.52768	4.19425
3.65	1.78513	4.51862	-4.28527	2.73349	4.51862	-4.51862	4.28527
3.66	1.76950	4.50952	-4.37657	2.74002	4.50952	-4.50952	4.37657
3.67	1.75377	4.50038	-4.46814	2.74661	4.50038	-4.50038	4.46814
3.68	1.73794	4.49120	-4.55999	2.75326	4.49120	-4.49120	4.55999
3.69	1.72202	4.48199	-4.65211	2.75997	4.48199	-4.48199	4.65211
3.70	1.70599	4.47274	-4.74451	2.76675	4.47274	-4.47274	4.74451
3.71	1.68986	4.46345	-4.83719	2.77359	4.46345	-4.46345	4.83719
3.72	1.67363	4.45413	-4.93014	2.78050	4.45413	-4.45413	4.93014
3.73	1.65729	4.44764	-5.02337	2.78747	4.44764	-4.44764	5.02337

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TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\delta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_R}\left(\frac{L^3}{EI}\right)$
3.74	1.64085	4.43536	-5.11688	2.79451	4.43536	-4.43536	5.11688
3.75	1.62431	4.42592	-5.21066	2.80161	4.42592	-4.42592	5.21066
3.76	1.60765	4.41644	-5.30472	2.80879	4.41644	-4.41644	5.30472
3.77	1.59089	4.40692	-5.39906	2.81603	4.40692	-4.40692	5.39906
3.78	1.57402	4.39736	-5.49368	2.82334	4.39736	-4.39736	5.49368
3.79	1.55705	4.38776	-5.58857	2.83072	4.38776	-4.38776	5.58857
3.80	1.53996	4.37813	-5.68374	2.83817	4.37813	-4.37813	5.68374
3.81	1.52275	4.36845	-5.77920	2.84570	4.36845	-4.36845	5.77920
3.82	1.50544	4.35873	-5.87493	2.85329	4.35873	-4.35873	5.87493
3.83	1.48802	4.34898	-5.97095	2.86097	4.34898	-4.34898	5.97095
3.84	1.47047	4.33918	-6.06724	2.86871	4.33918	-4.33918	6.06724
3.85	1.45281	4.32934	-6.16381	2.87654	4.32934	-4.32934	6.16381
3.86	1.43503	4.31947	-6.26067	2.88444	4.31947	-4.31947	6.26067
3.87	1.41713	4.30955	-6.35780	2.89242	4.30955	-4.30955	6.35780
3.88	1.39912	4.29959	-6.45522	2.90047	4.29959	-4.29959	6.45522
3.89	1.38098	4.28959	-6.55292	2.90861	4.28959	-4.28959	6.55292
3.90	1.36272	4.27955	-6.65089	2.91683	4.27955	-4.27955	6.65089
3.91	1.34434	4.26947	-6.74916	2.92513	4.26947	-4.26947	6.74916
3.92	1.32583	4.25935	-6.84770	2.93352	4.25935	-4.25935	6.84770
3.93	1.30720	4.24919	-6.94653	2.94199	4.24919	-4.24919	6.94653
3.94	1.28844	4.23898	-7.04564	2.95054	4.23898	-4.23898	7.04564
3.95	1.26955	4.22873	-7.14504	2.95919	4.22873	-4.22873	7.14504
3.96	1.25052	4.21844	-7.24472	2.96792	4.21844	-4.21844	7.24472
3.97	1.23137	4.20811	-7.34468	2.97674	4.20811	-4.20811	7.34468
3.98	1.21209	4.19774	-7.44493	2.98565	4.19774	-4.19774	7.44493
3.99	1.19267	4.18732	-7.54546	2.99465	4.18732	-4.18732	7.54546
4.00	1.17311	4.17686	-7.64628	3.00374	4.17686	-4.17686	7.64628
4.01	1.15342	4.16636	-7.74739	3.01293	4.16636	-4.16636	7.74739
4.02	1.13359	4.15581	-7.84878	3.02222	4.15581	-4.15581	7.84878
4.03	1.11362	4.14522	-7.95046	3.03160	4.14522	-4.14522	7.95046
4.04	1.09351	4.13459	-8.05242	3.04108	4.13459	-4.13459	8.05242
4.05	1.07325	4.12391	-8.15468	3.05066	4.12391	-4.12391	8.15468
4.06	1.05285	4.11319	-8.25722	3.06034	4.11319	-4.11319	8.25722
4.07	1.03230	4.10243	-8.36005	3.07013	4.10243	-4.10243	8.36005
4.08	1.01160	4.09162	-8.46316	3.08002	4.09162	-4.09162	8.46316
4.09	.990754	4.08076	-8.56657	3.09001	4.08076	-4.08076	8.56657
4.10	.969755	4.06987	-8.87027	3.10011	4.06987	-4.06987	8.87027
4.11	.948603	4.05892	-8.77425	3.11032	4.05892	-4.05892	8.77425
4.12	.927295	4.04794	-8.87853	3.12064	4.04794	-4.04794	8.87853
4.13	.905830	4.03690	-8.98309	3.13107	4.03690	-4.03690	8.98309
4.14	.884206	4.02582	-9.08795	3.14162	4.02582	-4.02582	9.08795
4.15	.862420	4.01470	-9.19310	3.15228	4.01470	-4.01470	9.19310
4.16	.840473	4.00353	-9.29854	3.16306	4.00353	-4.00353	9.29854
4.17	.818360	3.99231	-9.40427	3.17395	3.99231	-3.99231	9.40427
4.18	.796080	3.98105	-9.51030	3.18497	3.98105	-3.98105	9.51030
4.19	.773632	3.96974	-9.61661	3.19611	3.96974	-3.96974	9.61661
4.20	.751013	3.95839	-9.72323	3.20737	3.95839	-3.95839	9.72323
4.21	.728221	3.94698	-9.83013	3.21876	3.94698	-3.94698	9.83013
4.22	.705254	3.93553	-9.93733	3.23028	3.93553	-3.93553	9.93733
4.23	.682109	3.92404	-10.0448	3.24193	3.92404	-3.92404	10.0448
4.24	.658785	3.91249	-10.1526	3.25371	3.91249	-3.91249	10.1526
4.25	.635279	3.90090	-10.2607	3.26562	3.90090	-3.90090	10.2607
4.26	.611589	3.88926	-10.3691	3.27767	3.88926	-3.88926	10.3691
4.27	.587712	3.87757	-10.4778	3.28986	3.87757	-3.87757	10.4778
4.28	.563647	3.86583	-10.5867	3.30219	3.86583	-3.86583	10.5867
4.29	.539390	3.85405	-10.6960	3.31466	3.85405	-3.85405	10.6960
4.30	.514939	3.84221	-10.8056	3.32727	3.84221	-3.84221	10.8056
4.31	.490292	3.83033	-10.9154	3.34004	3.83033	-3.83033	10.9154
4.32	.465446	3.81839	-11.0256	3.35295	3.81839	-3.81839	11.0256
4.33	.440399	3.80641	-11.1361	3.36601	3.80641	-3.80641	11.1361

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\theta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\theta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^3}{EI}\right)$
4.34	.415147	3.79438	-11.2468	3.37923	3.79438	-3.79438	11.2468
4.35	.389688	3.78229	-11.3579	3.39261	3.78229	-3.78229	11.3579
4.36	.364019	3.77016	-11.4693	3.40614	3.77016	-3.77016	11.4693
4.37	.338138	3.75797	-11.5809	3.41984	3.75797	-3.75797	11.5809
4.38	.312040	3.74574	-11.6929	3.43370	3.74574	-3.74574	11.6929
4.39	.285724	3.73345	-11.8052	3.44773	3.73345	-3.73345	11.8052
4.40	.259187	3.72111	-11.9178	3.46193	3.72111	-3.72111	11.9178
4.41	.232424	3.70873	-12.0306	3.47630	3.70873	-3.70873	12.0306
4.42	.205434	3.69628	-12.1438	3.49085	3.69628	-3.69628	12.1438
4.43	.178213	3.68379	-12.2573	3.50558	3.68379	-3.68379	12.2573
4.44	.150756	3.67125	-12.3711	3.52049	3.67125	-3.67125	12.3711
4.45	.123062	3.65865	-12.4852	3.53558	3.65865	-3.65865	12.4852
4.46	.0951270	3.64599	-12.5996	3.55087	3.64599	-3.64599	12.5996
4.47	.0669467	3.63329	-12.7143	3.56634	3.63329	-3.63329	12.7143
4.48	.0385179	3.62053	-12.8293	3.58201	3.62053	-3.62053	12.8293
4.49	.00983696	3.60772	-12.9447	3.59788	3.60772	-3.60772	12.9447
4.50	-.0190999	3.59485	-13.0603	3.61395	3.59485	-3.59485	13.0603
4.51	-.0482966	3.58193	-13.1762	3.63023	3.58193	-3.58193	13.1762
4.52	-.0777570	3.56896	-13.2925	3.64672	3.56896	-3.56896	13.2925
4.53	-.107465	3.55593	-13.4090	3.66341	3.55593	-3.55593	13.4090
4.54	-.137485	3.54284	-13.5259	3.68033	3.54284	-3.54284	13.5259
4.55	-.167761	3.52970	-13.6431	3.69746	3.52970	-3.52970	13.6431
4.56	-.198317	3.51651	-13.7606	3.71482	3.51651	-3.51651	13.7606
4.57	-.229158	3.50325	-13.8784	3.73241	3.50325	-3.50325	13.8784
4.58	-.260288	3.48995	-13.9965	3.75023	3.48995	-3.48995	13.9965
4.59	-.291712	3.47658	-14.1149	3.76829	3.47658	-3.47658	14.1149
4.60	-.323435	3.46316	-14.2337	3.78659	3.46316	-3.46316	14.2337
4.61	-.355461	3.44968	-14.3527	3.80514	3.44968	-3.44968	14.3527
4.62	-.387795	3.43614	-14.4721	3.82394	3.43614	-3.43614	14.4721
4.63	-.420442	3.42255	-14.5918	3.84299	3.42255	-3.42255	14.5918
4.64	-.453408	3.40890	-14.7118	3.86230	3.40890	-3.40890	14.7118
4.65	-.486697	3.39518	-14.8321	3.88188	3.39518	-3.39518	14.8321
4.66	-.520316	3.38141	-14.9528	3.90173	3.38141	-3.38141	14.9528
4.67	-.554269	3.36759	-15.0737	3.92185	3.36759	-3.36759	15.0737
4.68	-.588563	3.35370	-15.1950	3.94226	3.35370	-3.35370	15.1950
4.69	-.623202	3.33975	-15.3166	3.96295	3.33975	-3.33975	15.3166
4.70	-.658194	3.32574	-15.4385	3.98394	3.32574	-3.32574	15.4385
4.71	-.693543	3.31167	-15.5608	4.00522	3.31167	-3.31167	15.5608
4.72	-.729259	3.29754	-15.6833	4.02680	3.29754	-3.29754	15.6833
4.73	-.765345	3.28335	-15.8062	4.04870	3.28335	-3.28335	15.8062
4.74	-.801808	3.26910	-15.9294	4.07091	3.26910	-3.26910	15.9294
4.75	-.838656	3.25479	-16.0529	4.09344	3.25479	-3.25479	16.0529
4.76	-.875895	3.24041	-16.1768	4.11631	3.24041	-3.24041	16.1768
4.77	-.913532	3.22597	-16.3010	4.13950	3.22597	-3.22597	16.3010
4.78	-.951574	3.21147	-16.4255	4.16305	3.21147	-3.21147	16.4255
4.79	-.990030	3.19691	-16.5503	4.18694	3.19691	-3.19691	16.5503
4.80	-1.02891	3.18228	-16.6754	4.21119	3.18228	-3.18228	16.6754
4.81	-1.06821	3.16759	-16.8009	4.23580	3.16759	-3.16759	16.8009
4.82	-1.10795	3.15283	-16.9267	4.26079	3.15283	-3.15283	16.9267
4.83	-1.14814	3.13801	-17.0529	4.28615	3.13801	-3.13801	17.0529
4.84	-1.18878	3.12313	-17.1793	4.31191	3.12313	-3.12313	17.1793
4.85	-1.22989	3.10818	-17.3061	4.33806	3.10818	-3.10818	17.3061
4.86	-1.27146	3.09316	-17.4333	4.36462	3.09316	-3.09316	17.4333
4.87	-1.31352	3.07808	-17.5607	4.39159	3.07808	-3.07808	17.5607
4.88	-1.35606	3.06293	-17.6885	4.41899	3.06293	-3.06293	17.6885
4.89	-1.39911	3.04771	-17.8167	4.44682	3.04771	-3.04771	17.8167
4.90	-1.44266	3.03243	-17.9451	4.47509	3.03243	-3.03243	17.9451
4.91	-1.48674	3.01708	-18.0740	4.50381	3.01708	-3.01708	18.0740
4.92	-1.53135	3.00166	-18.2031	4.53300	3.00166	-3.00166	18.2031
4.93	-1.57650	2.98617	-18.3326	4.56266	2.98617	-2.98617	18.3326

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED EFFECTS FOR COLUMN WITH RIGHT END CLAMPED - Continued

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{F\theta_R}\left(\frac{L}{EI}\right)$	$K_{F\delta_R}\left(\frac{L^2}{EI}\right)$ or $C_{F\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{F\delta_R}\left(\frac{L^3}{EI}\right)$	$K_{C\theta_R}\left(\frac{L}{EI}\right)$	$K_{C\delta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\theta_R}\left(\frac{L^2}{EI}\right)$	$C_{C\delta_R}\left(\frac{L^3}{EI}\right)$
4.94	-1.62220	2.97061	-18.4624	4.59281	2.97061	-2.97061	18.4624
4.95	-1.66847	2.95498	-18.5925	4.62345	2.95498	-2.95498	18.5925
4.96	-1.71531	2.93929	-18.7230	4.65460	2.93929	-2.93929	18.7230
4.97	-1.76275	2.92352	-18.8539	4.68627	2.92352	-2.92352	18.8539
4.98	-1.81079	2.90769	-18.9850	4.71847	2.90769	-2.90769	18.9850
4.99	-1.85944	2.89178	-19.1165	4.75122	2.89178	-2.89178	19.1165
5.00	-1.90872	2.87580	-19.2484	4.78452	2.87580	-2.87580	19.2484
5.01	-1.95865	2.85975	-19.3806	4.81839	2.85975	-2.85975	19.3806
5.02	-2.00923	2.84362	-19.5132	4.85285	2.84362	-2.84362	19.5132
5.03	-2.06048	2.82743	-19.6461	4.88791	2.82743	-2.82743	19.6461
5.04	-2.11242	2.81116	-19.7793	4.92358	2.81116	-2.81116	19.7793
5.05	-2.16507	2.79481	-19.9129	4.95988	2.79481	-2.79481	19.9129
5.06	-2.21843	2.77839	-20.0468	4.99682	2.77839	-2.77839	20.0468
5.07	-2.27253	2.76190	-20.1811	5.03443	2.76190	-2.76190	20.1811
5.08	-2.32738	2.74533	-20.3157	5.07272	2.74533	-2.74533	20.3157
5.09	-2.38301	2.72869	-20.4507	5.11170	2.72869	-2.72869	20.4507
5.10	-2.43942	2.71197	-20.5861	5.15139	2.71197	-2.71197	20.5861
5.11	-2.49665	2.69517	-20.7218	5.19182	2.69517	-2.69517	20.7218
5.12	-2.55470	2.67830	-20.8578	5.23300	2.67830	-2.67830	20.8578
5.13	-2.61360	2.66135	-20.9942	5.27495	2.66135	-2.66135	20.9942
5.14	-2.67338	2.64432	-21.1310	5.32336	2.64432	-2.64432	21.1310
5.15	-2.73404	2.62721	-21.2681	5.36126	2.62721	2.62721	21.2681
5.16	-2.79562	2.61003	-21.4055	5.40565	2.61003	-2.61003	21.4055
5.17	-2.85815	2.59276	-21.5434	5.45091	2.59276	-2.59276	21.5434
5.18	-2.92163	2.57541	-21.6816	5.49704	2.57541	-2.57541	21.6816
5.19	-2.98611	2.55799	-21.8201	5.54409	2.55799	-2.55799	21.8201
5.20	-3.05159	2.54048	-21.9590	5.59207	2.54048	-2.54048	21.9590
5.21	-3.11812	2.52289	-22.0983	5.64101	2.52289	-2.52289	22.0983
5.22	-3.18573	2.50522	-22.2380	5.69094	2.50522	-2.50522	22.2380
5.23	-3.25443	2.48746	-22.3780	5.74189	2.48746	-2.48746	22.3780
5.24	-3.32426	2.46962	-22.5184	5.79388	2.46962	-2.46962	22.5184
5.25	-3.39525	2.45170	-22.6591	5.84695	2.45170	-2.45170	22.6591
5.26	-3.46744	2.43369	-22.8002	5.90113	2.43369	-2.43369	22.8002
5.27	-3.54086	2.41560	-22.9417	5.95646	2.41560	-2.41560	22.9417
5.28	-3.61554	2.39742	-23.0836	6.01296	2.39742	-2.39742	23.0836
5.29	-3.69152	2.37916	-23.2258	6.07067	2.37916	-2.37916	23.2258
5.30	-3.76884	2.36081	-23.3684	6.12964	2.36081	-2.36081	23.3684
5.31	-3.84754	2.34237	-23.5114	6.18990	2.34237	-2.34237	23.5114
5.32	-3.92766	2.32384	-23.6547	6.25150	2.32384	-2.32384	23.6547
5.33	-4.00924	2.30522	-23.7984	6.31446	2.30522	-2.30522	23.7984
5.34	-4.09233	2.28652	-23.9426	6.37885	2.28652	-2.28652	23.9426
5.35	-4.17698	2.26772	-24.0870	6.44470	2.26772	-2.26772	24.0870
5.36	-4.26323	2.24884	-24.2319	6.51207	2.24884	-2.24884	24.2319
5.37	-4.35114	2.22986	-24.3772	6.58100	2.22986	-2.22986	24.3772
5.38	-4.44076	2.21079	-24.5228	6.65155	2.21079	-2.21079	24.5228
5.39	-4.53214	2.19163	-24.6688	6.72377	2.19163	-2.19163	24.6688
5.40	-4.62534	2.17238	-24.8152	6.79773	2.17238	-2.17238	24.8152
5.41	-4.72043	2.15303	-24.9620	6.87347	2.15303	-2.15303	24.9620
5.42	-4.81747	2.13359	-25.1092	6.95106	2.13359	-2.13359	25.1092
5.43	-4.91653	2.11405	-25.2568	7.03058	2.11405	-2.11405	25.2568
5.44	-5.01766	2.09442	-25.4048	7.11208	2.09442	-2.09442	25.4048
5.45	-5.12096	2.07469	-25.5531	7.19565	2.07469	-2.07469	25.5531
5.46	-5.22649	2.05486	-25.7019	7.28135	2.05486	-2.05486	25.7019
5.47	-5.33434	2.03493	-25.8510	7.36927	2.03493	-2.03493	25.8510
5.48	-5.44458	2.01491	-26.0006	7.45950	2.01491	-2.01491	26.0006
5.49	-5.55732	1.99479	-26.1505	7.55211	1.99479	-1.99479	26.1505
5.50	-5.67264	1.97456	-26.3009	7.64720	1.97456	-1.97456	26.3009
5.51	-5.79063	1.95424	-26.4516	7.74487	1.95424	-1.95424	26.4516
5.52	-5.91142	1.93381	-26.6028	7.84523	1.93381	-1.93381	26.6028
5.53	-6.03509	1.91328	-26.7543	7.94837	1.91328	-1.91328	26.7543

TABLE 2 - COEFFICIENTS FOR STIFFNESS AND INDUCED ERRORS FOR COLUMN WITH RIGHT END CLAMPED - Concluded

$\frac{L}{j}$	Stiffness coefficients at free end			Coefficients for induced shear and moment at clamped end			
	$K_{Fc_R}\left(\frac{L}{EI}\right)$	$K_{Fc_R}\left(\frac{L^2}{EI}\right)$ or $C_{Fc_R}\left(\frac{L^2}{EI}\right)$	$C_{Fc_R}\left(\frac{L^3}{EI}\right)$	$K_{Cs_R}\left(\frac{L}{EI}\right)$	$K_{Cs_R}\left(\frac{L^2}{EI}\right)$	$C_{Cs_R}\left(\frac{L^2}{EI}\right)$	$C_{Cs_R}\left(\frac{L^3}{EI}\right)$
5.54	-6.16177	1.89265	-26.9063	8.05442	1.89265	-1.89265	26.9063
5.55	-6.22157	1.87191	-27.0587	8.16349	1.87191	-1.87191	27.0587
5.56	-6.42464	1.85108	-27.2113	8.27571	1.85108	-1.85108	27.2113
5.57	-6.56109	1.83013	-27.3646	8.39122	1.83013	-1.83013	27.3646
5.58	-6.70107	1.80908	-27.5183	8.51015	1.80908	-1.80908	27.5183
5.59	-6.84474	1.78792	-27.6722	8.63266	1.78792	-1.78792	27.6722
5.60	-6.99225	1.76665	-27.8267	8.75889	1.76664	-1.76664	27.8267
5.61	-7.14376	1.74527	-27.9816	8.88903	1.74527	-1.74527	27.9816
5.62	-7.29947	1.72378	-28.1368	9.02326	1.72378	-1.72378	28.1368
5.63	-7.45955	1.70219	-28.2925	9.16174	1.70219	-1.70219	28.2925
5.64	-7.62422	1.68048	-28.4486	9.30469	1.68048	-1.68048	28.4486
5.65	-7.79387	1.65866	-28.6052	9.45233	1.65866	-1.65866	28.6052
5.66	-7.96815	1.63672	-28.7622	9.60487	1.63672	-1.63672	28.7622
5.67	-8.14790	1.61467	-28.9196	9.76257	1.61467	-1.61467	28.9196
5.68	-8.33166	1.59251	-29.0774	9.92568	1.59251	-1.59251	29.0774
5.69	-8.52423	1.57023	-29.2356	10.0945	1.57023	-1.57023	29.2356
5.70	-8.72139	1.54783	-29.3943	10.2692	1.54783	-1.54783	29.3943
5.71	-8.92457	1.52532	-29.5535	10.4503	1.52532	-1.52532	29.5535
5.72	-9.13288	1.50269	-29.7130	10.6380	1.50269	-1.50269	29.7130
5.73	-9.35272	1.47993	-29.8730	10.8327	1.47993	-1.47993	29.8730
5.74	-9.57765	1.45706	-30.0335	11.0347	1.45706	-1.45706	30.0335
5.75	-9.81051	1.43407	-30.1944	11.2446	1.43407	-1.43407	30.1944
5.76	-10.0517	1.41095	-30.3557	11.4627	1.41095	-1.41095	30.3557
5.77	-10.3018	1.38771	-30.5175	11.6895	1.38771	-1.38771	30.5175
5.78	-10.5613	1.36434	-30.6797	11.9256	1.36434	-1.36434	30.6797
5.79	-10.8307	1.34085	-30.8424	12.1715	1.34085	-1.34085	30.8424
5.80	-11.1106	1.31723	-31.0055	12.4278	1.31723	-1.31723	31.0055
5.81	-11.4018	1.29348	-31.1691	12.6953	1.29348	-1.29348	31.1691
5.82	-11.7049	1.26961	-31.3332	12.9745	1.26961	-1.26961	31.3332
5.83	-12.0208	1.24560	-31.4977	13.2664	1.24560	-1.24560	31.4977
5.84	-12.3502	1.22147	-31.6627	13.5717	1.22147	-1.22147	31.6627
5.85	-12.6942	1.19720	-31.8281	13.8914	1.19720	-1.19720	31.8281
5.86	-13.0537	1.17279	-31.9940	14.2265	1.17279	-1.17279	31.9940
5.87	-13.4299	1.14826	-32.1604	14.5781	1.14826	-1.14826	32.1604
5.88	-13.8240	1.12358	-32.3272	14.9476	1.12358	-1.12358	32.3272
5.89	-14.2373	1.09877	-32.4945	15.3361	1.09877	-1.09877	32.4945
5.90	-14.6715	1.07383	-32.6623	15.7453	1.07383	-1.07383	32.6623
5.91	-15.1280	1.04874	-32.8306	16.1768	1.04874	-1.04874	32.8306
5.92	-15.6089	1.02351	-32.9994	16.6324	1.02351	-1.02351	32.9994
5.93	-16.1161	.998143	-33.1686	17.1142	.998143	-.998143	33.1686
5.94	-16.6519	.972631	-33.3383	17.6245	.972631	-.972631	33.3383
5.95	-17.2189	.946975	-33.5086	18.1659	.946975	-.946975	33.5086
5.96	-17.8201	.921174	-33.6793	18.7412	.921174	-.921174	33.6793
5.97	-18.4585	.895225	-33.8505	19.3538	.895225	-.895225	33.8505
5.98	-19.1380	.869126	-34.0221	20.0071	.869126	-.869126	34.0221
5.99	-19.8628	.842881	-34.1944	20.7057	.842881	-.842881	34.1944
6.00	-20.6375	.816483	-34.3670	21.4540	.816483	-.816483	34.3670
6.01	-21.4678	.789934	-34.5402	22.2577	.789934	-.789934	34.5402
6.02	-22.3598	.763230	-34.7139	23.1230	.763230	-.763230	34.7139
6.03	-23.3210	.736374	-34.8882	24.0574	.736374	-.736374	34.8882
6.04	-25.3598	.709359	-35.0629	25.0692	.709359	-.709359	35.0629
6.05	-25.4863	.682186	-35.2381	26.1685	.682186	-.682186	35.2381
6.06	-26.7121	.654853	-35.4138	27.3669	.654853	-.654853	35.4138
6.07	-28.0513	.627362	-35.5902	28.6787	.627362	-.627362	35.5902
6.08	-29.5205	.599706	-35.7670	30.1202	.599706	-.599706	35.7670
6.09	-31.1401	.571888	-35.9443	31.7120	.571888	-.571888	35.9443
6.10	-32.9343	.543900	-36.1220	33.4782	.543900	-.543900	36.1220
6.11	-34.9340	.515752	-36.3006	35.4498	.515752	-.515752	36.3006
6.12	-37.1765	.487433	-36.4796	37.6639	.487433	-.487433	36.4796
6.13	-39.7095	.458946	-36.6593	40.1684	.458946	-.458946	36.6593
6.14	-42.5931	.430281	-36.8391	43.0234	.430281	-.430281	36.8391
6.15	-45.9073	.401445	-37.0196	46.3087	.401445	-.401445	37.0196
6.16	-49.7365	.372424	-37.2007	50.1289	.372424	-.372424	37.2007
6.17	-54.2825	.343246	-37.3824	54.6258	.343246	-.343246	37.3824
6.18	-59.6822	.313880	-37.5646	59.9960	.313880	-.313880	37.5646
6.19	-66.2365	.284334	-37.7474	66.5209	.284334	-.284334	37.7474
6.20	-74.3621	.254605	-37.9308	74.6167	.254605	-.254605	37.9308
6.21	-84.7028	.224693	-38.1147	84.9275	.224693	-.224693	38.1147
6.22	-98.3097	.194593	-38.2989	98.5043	.194593	-.194593	38.2989
6.23	-117.028	.164309	-38.4843	117.192	.164309	-.164309	38.4843
6.24	-144.404	.133833	-38.6700	144.538	.133833	-.133833	38.6700
6.25	-188.268	.103166	-38.8562	188.371	.103166	-.103166	38.8562
6.26	-269.930	.0723062	-39.0430	270.023	.0723062	-.0723062	39.0430
6.27	-475.501	.0412474	-39.2303	475.542	.0412474	-.0412474	39.2303
6.28	-1971.76	.00999181	-39.4227	1971.77	.00999181	-.00999181	39.4227
2π	-∞	0	-39.4784	∞	0	0	39.4784

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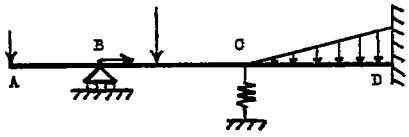
TABLE 3 - FORCE AND MOMENT DISTRIBUTION COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE



Joint B

$\theta = 5.25610 \times 10^{-6}$ M

$S_{C_R} = -4563.56\ \theta$

$M_{C_R} = 174{,}830\ \theta$

Joint C

$\delta = 0.00140653$ F

$\theta = 2.62805 \times 10^{-6}$ M

$M_{C_L} = 174{,}830\ \theta - 4563.56\ \delta$

Not used $\begin{cases} S_{C_L} = 4563.56\ \theta - 12.1358\ \delta \\ S_{C_R} = -4563.56\ \theta - 12.1358\ \delta \\ M_{C_R} = 174{,}830\ \theta + 4563.56\ \delta \end{cases}$

Shear	Moment	B Moment	Shear	Shear	Moment	C Moment	Shear	Shear	Moment	D
-100	5000	-6989.8	-326.94	-173.06	4834.6	-2605.4	-119.04	-280.96	3748.8	
				F = 292.10		M = -2229.2				
				δ = 0.410847 in.		$\theta = -5.85845 \times 10^{-3}$				
		-2899.15								
		M = 4888.95								
		$\theta = 25.6968 \times 10^{-3}$								
				-117.269	4492.57					
				F = 117.269		M = -4492.57				
				δ = 0.164942 in.		$\theta = -11.8067 \times 10^{-3}$				
		-2816.89								
		M = 2816.89								
		$\theta = 14.8059 \times 10^{-3}$								
				-87.5676	2588.52					
				F = 87.5676		M = -2588.52				
				δ = 0.0950359 in.		$\theta = -6.80276 \times 10^{-3}$				
		-1623.03								
		M = 1623.03								
		$\theta = 8.53081 \times 10^{-3}$								
				-38.9309	1491.44					
				F = 38.9309		M = -1491.44				
				δ = 0.0547575 in.		$\theta = -3.91958 \times 10^{-3}$				
		-935.149								
		M = 935.149								
		$\theta = 4.91524 \times 10^{-3}$								
Total deflections and rotations after 10 cycles		$\theta_B = 60.3862 \times 10^{-3}$		$\delta_C = 0.797298$ in.		$\theta_C = -33.5210 \times 10^{-3}$				
After 20 cycles		$\theta_B = 60.6297 \times 10^{-3}$		$\delta_C = 0.800011$ in.		$\theta_C = -33.7152 \times 10^{-3}$				
Exact results		$\theta_B = 60.6307 \times 10^{-3}$		$\delta_C = 0.800022$ in.		$\theta_C = -33.7160 \times 10^{-3}$				

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TABLE 4 - COMPUTATIONS FOR THE DEFLECTION CURVE

OF SPAN BC (ILLUSTRATIVE EXAMPLE)

$\left.\begin{array}{l}\theta_B = 0.06063 \\ \delta_C = 0.8000 \text{ inch} \\ \theta_C = -0.033716\end{array}\right\}$ from table 3

$Q = 500$ pounds

$L = 80$ inches

$\frac{EI}{L^3} = 11.328$ pounds per inch

$\frac{c}{L} = 0.6$

$\frac{L}{j} = 3$

$\frac{x}{L}$	$\frac{y_1}{\left(\frac{QL^3}{EI}\right)}$	$\frac{y_2}{\theta_B L}$	$\frac{y_3}{\theta_C L}$	$\frac{y_4}{\delta_C}$	y_1 (in.)	y_2 (in.)	y_3 (in.)	y_4 (in.)	Total deflection, y (in.) (a)
0	0	0	0	0	0	0	0	0	0
.1	.00078	.086	-.011	.025	.035	.419	.030	.020	.504
.2	.00258	.144	-.040	.096	.114	.698	.108	.077	.997
.3	.00458	.173	-.080	.207	.202	.837	.215	.166	1.420
.4	.00596	.175	-.121	.346	.263	.847	.326	.277	1.713
.5	.00612	.155	-.155	.500	.270	.753	.419	.400	1.842
.6	.00520	.121	-.175	.654	.230	.586	.471	.523	1.810
.7	.00366	.080	-.173	.793	.161	.386	.465	.634	1.646
.8	.00194	.040	-.144	.904	.086	.195	.388	.723	1.392
.9	.00057	.011	-.086	.975	.025	.054	.233	.780	1.092
1.0	0	0	0	1.000	0	0	0	.800	.800

a

$y = y_1 + y_2 + y_3 + y_4$

y_1 = deflection due to lateral load from figure 8(d)

y_2 = deflection due to rotation of joint B from figure 10

y_3 = deflection due to rotation of joint C from figure 10

y_4 = deflection due to deflection of joint C from figure 9

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TABLE 5 - COMPUTATIONS FOR BENDING-MOMENT DIAGRAM

FOR SPAN BC (ILLUSTRATIVE EXAMPLE)

$$P = 8156.25 \text{ pounds}$$

$$S_{BC} = -213.82 \text{ pounds}$$

$$M_{BC} = -5000 \text{ inch-pounds}$$

$$\Delta = y - (y)_{\frac{x}{L}=0}$$

Bending moment at $x \leq 32$ in. $= M_{BC} + P\Delta - S_{BC}x$

Bending moment at $x \geq 32$ in. $= M_{BC} + P\Delta - S_{BC}x - 500(x - 32)$

$\frac{x}{L}$	x (in.)	y from table 4 (in.)	Δ (in.)	$P\Delta$ (in.-lb)	$-S_{BC}x$ (in.-lb)	$-500(x - 32)$ for $x \geq 32$ (in.-lb)	Bending moment (in.-lb)
0	0	0	0	0	0	----------	-5000
.1	8	.504	.504	4100	1710	----------	810
.2	16	.997	.997	8130	3420	----------	6550
.3	24	1.420	1.420	11580	5130	----------	11710
.4	32	1.713	1.713	13970	6840	0	15810
.5	40	1.842	1.842	15020	8550	-4000	14570
.6	48	1.810	1.810	14760	10260	-8000	12020
.7	56	1.646	1.646	13430	11970	-12000	8400
.8	64	1.392	1.392	11350	13680	-16000	4030
.9	72	1.092	1.092	8910	15400	-20000	-690
1.0	80	.800	.800	6530	17110	-24000	-5360

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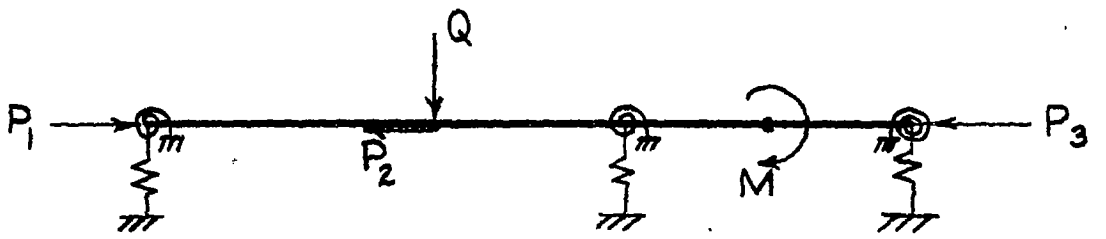

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Figure 1.- Continuous beam-column on elastic supports.

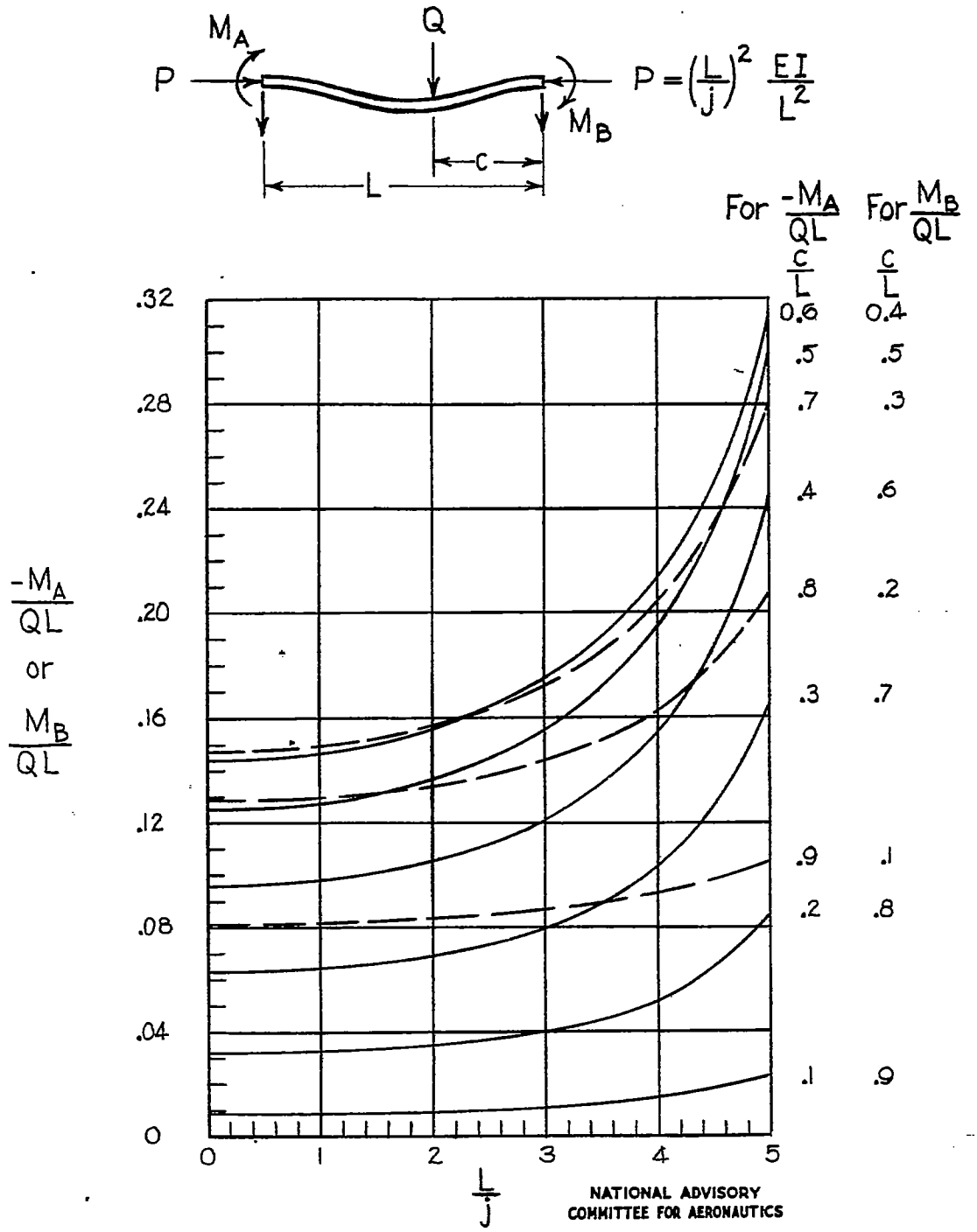

Figure 2.- Fixed-end moment coefficients for a beam-column subjected to a concentrated lateral load.

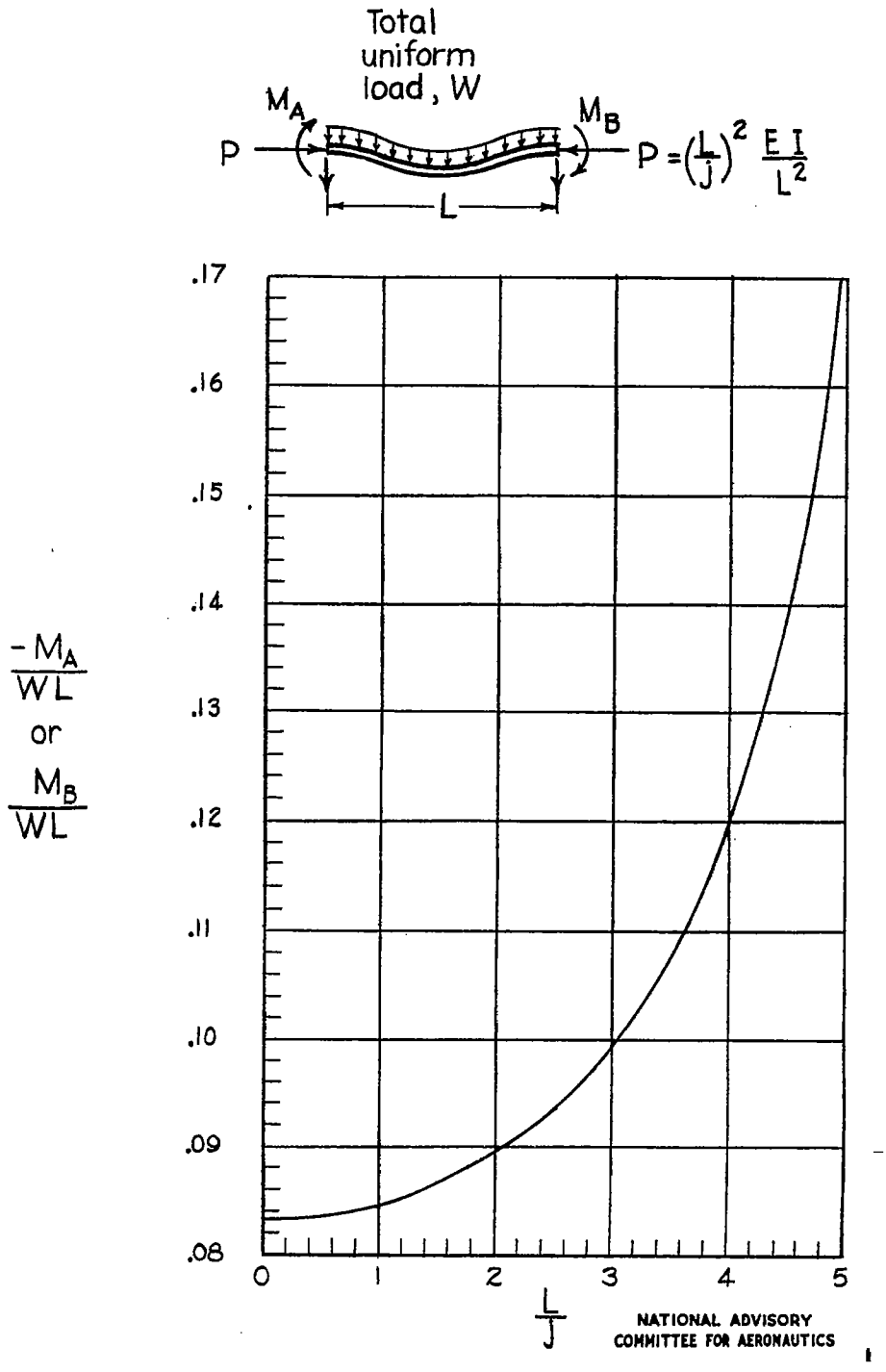

Figure 3.- Fixed-end moment coefficients for a beam-column subjected to a uniformly distributed lateral load.

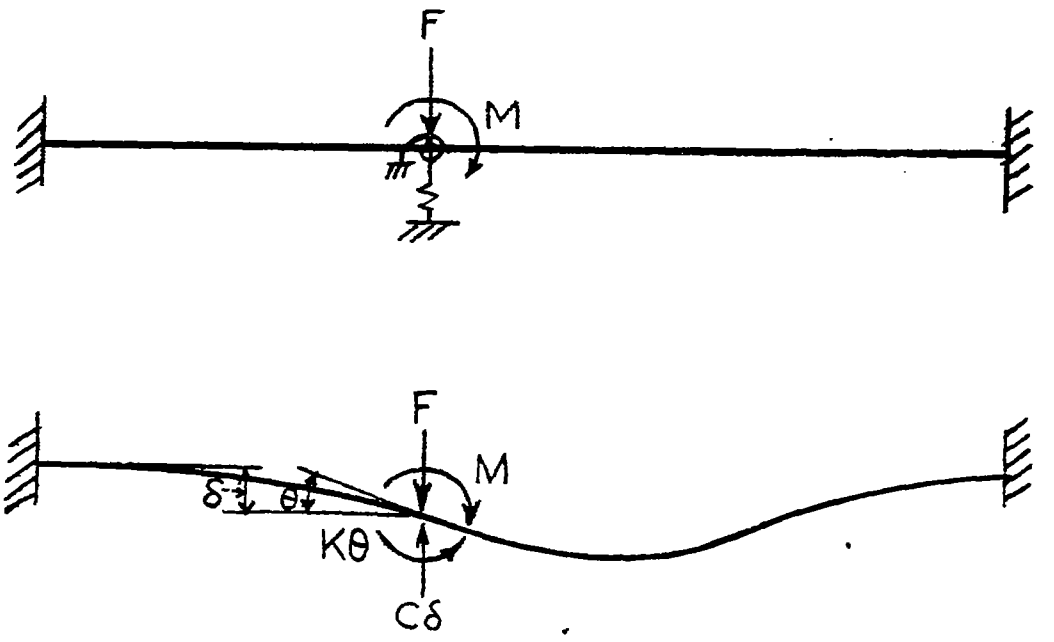

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Figure 4.- Two-span fundamental structural unit in force and moment distribution.

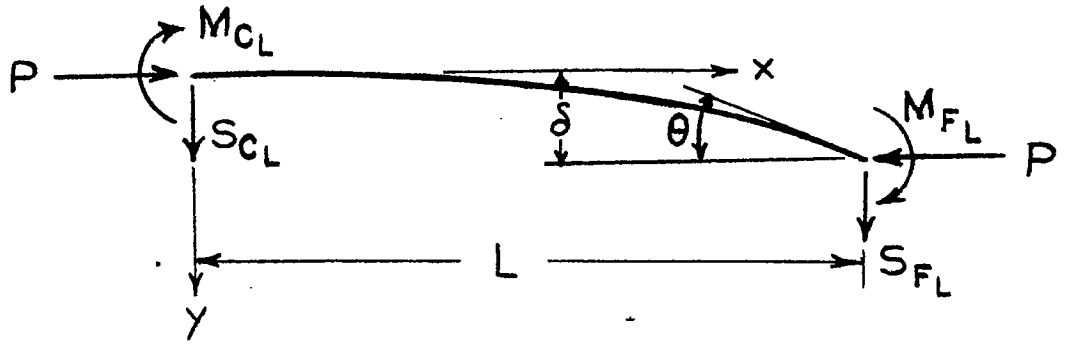

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Figure 5.- Free-body diagram of left-hand span.

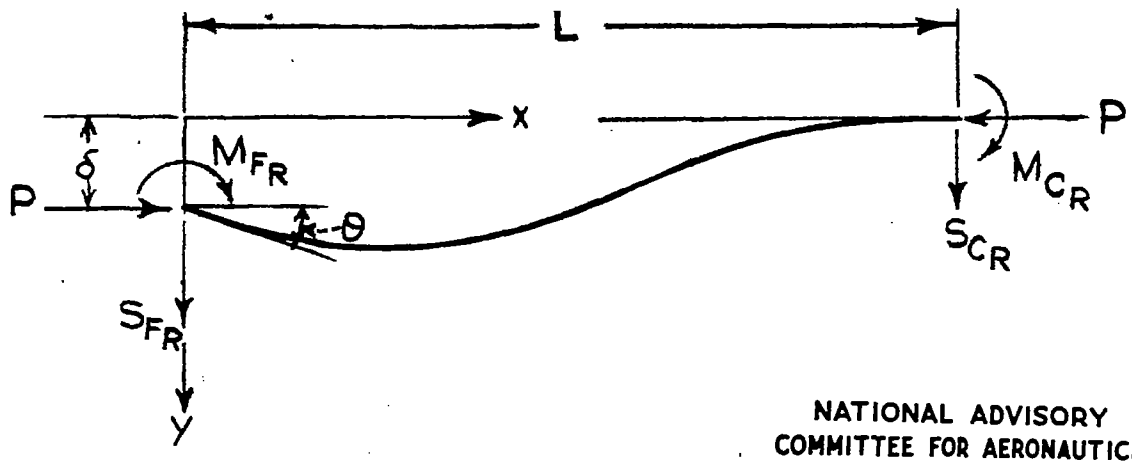

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Figure 6.- Free-body diagram of right-hand span.

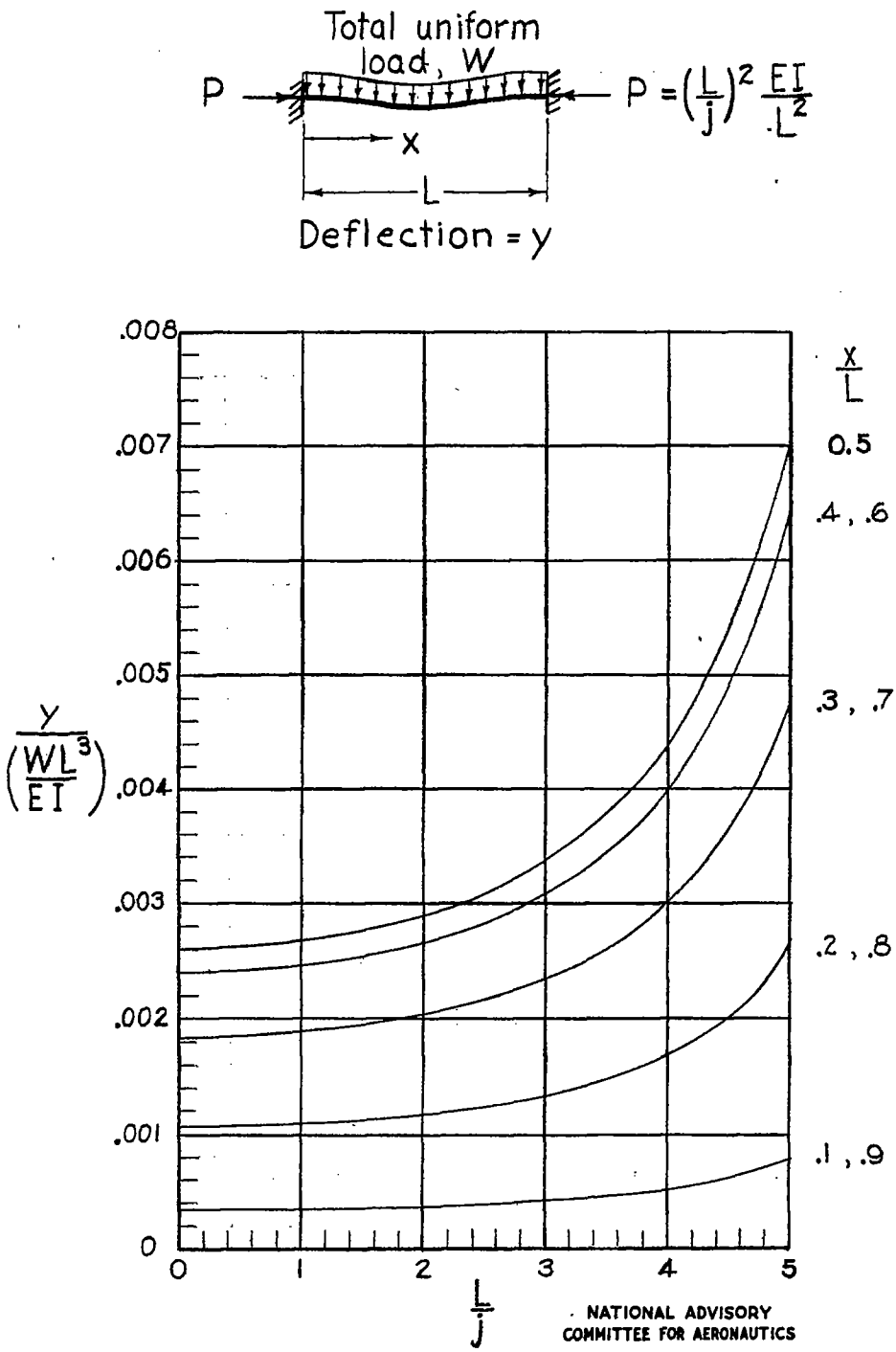
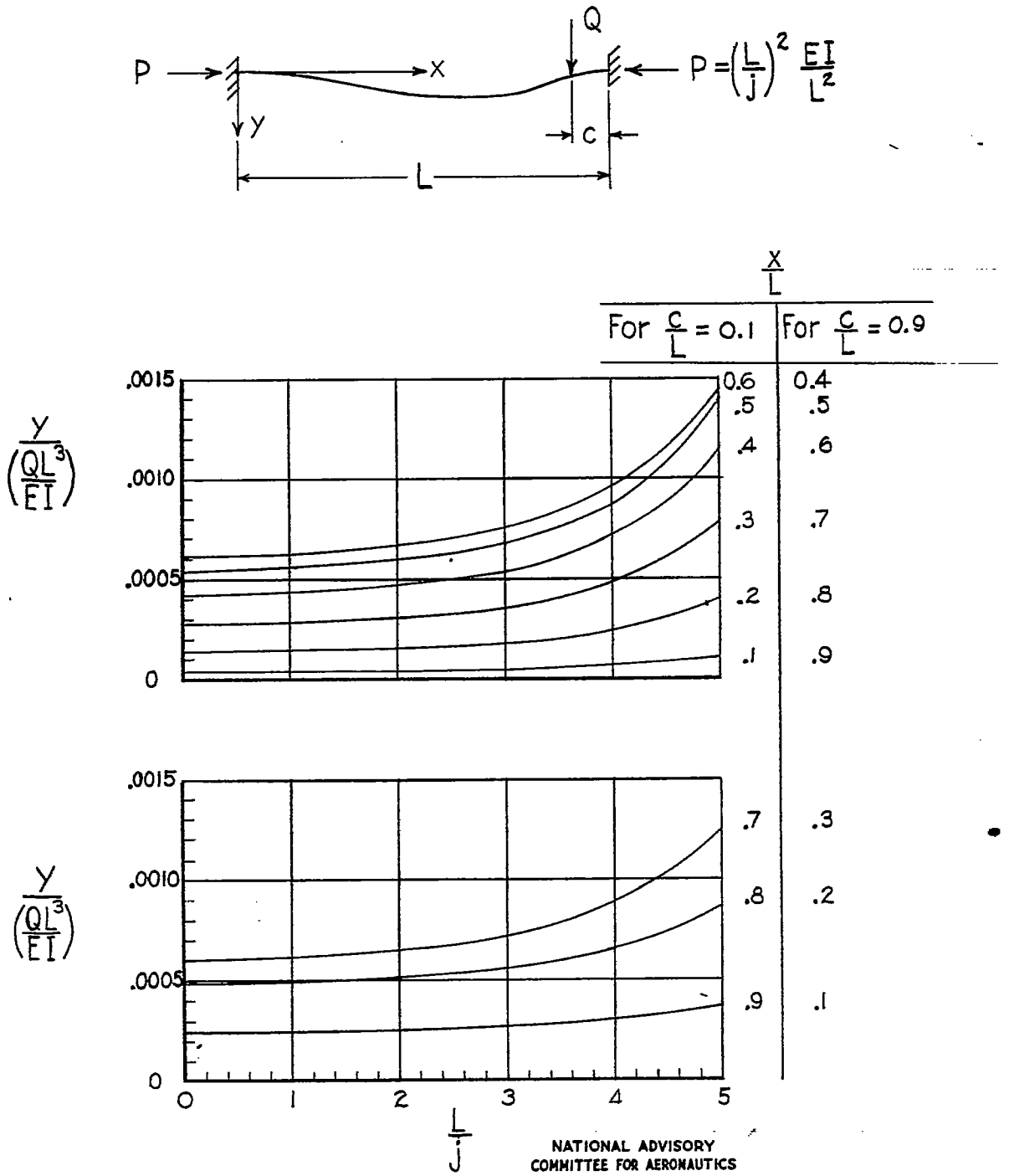
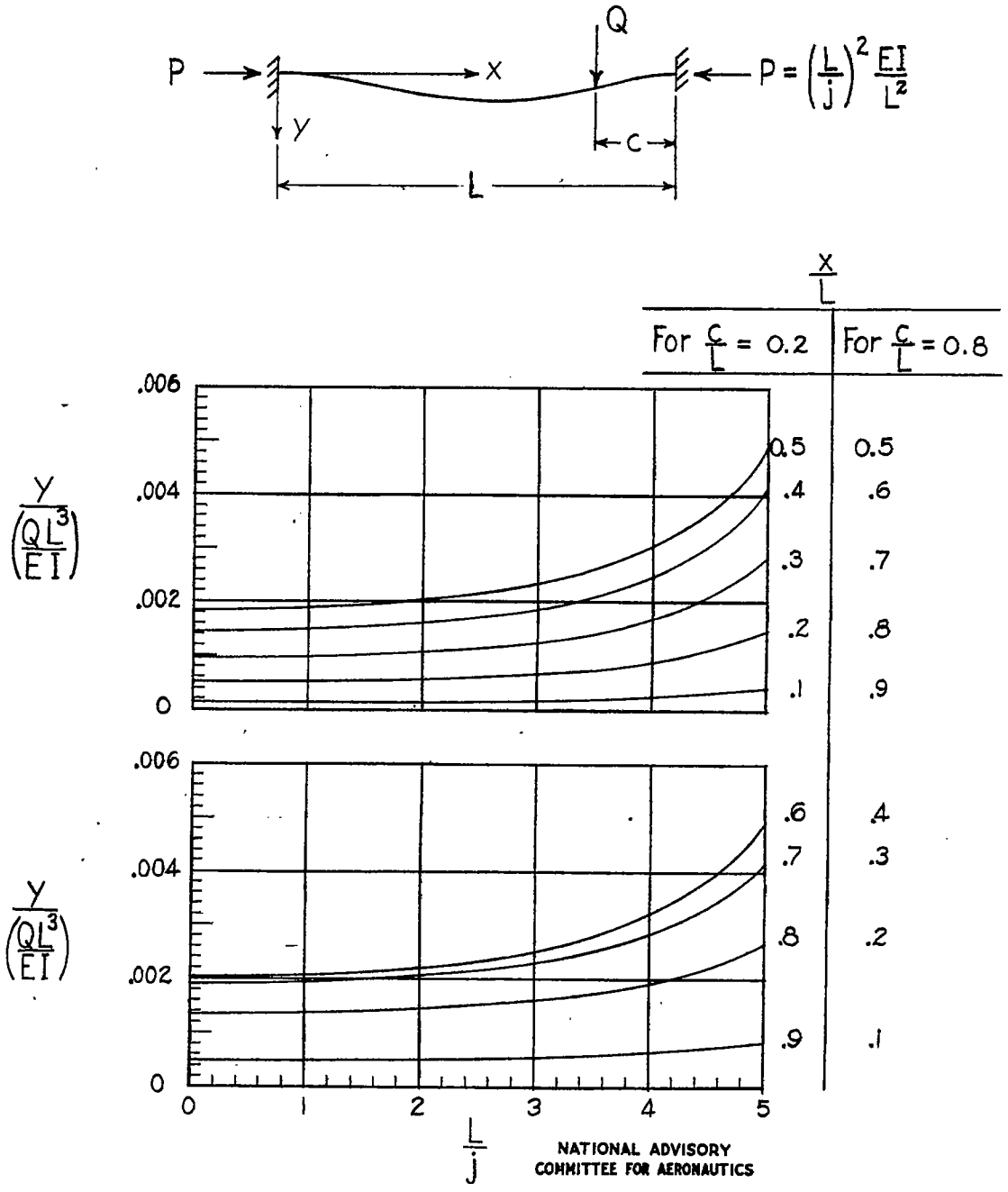


Figure 7.- Deflections of a clamped-end beam-column due to a uniformly distributed lateral load.



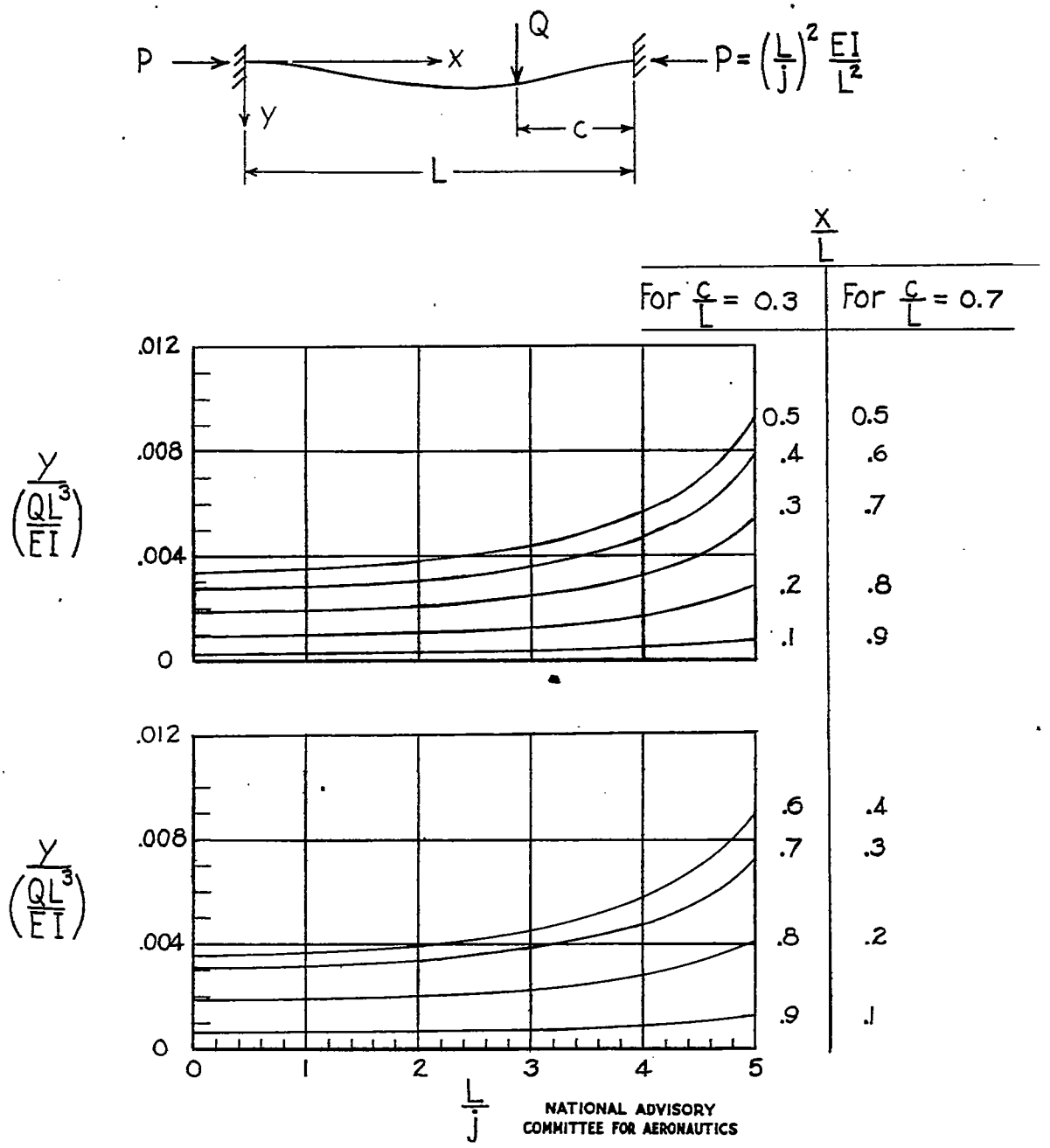
(a) $\frac{c}{L} = 0.1$ or 0.9.

Figure 8.- Deflections of a clamped-end beam-column due to a concentrated lateral load.



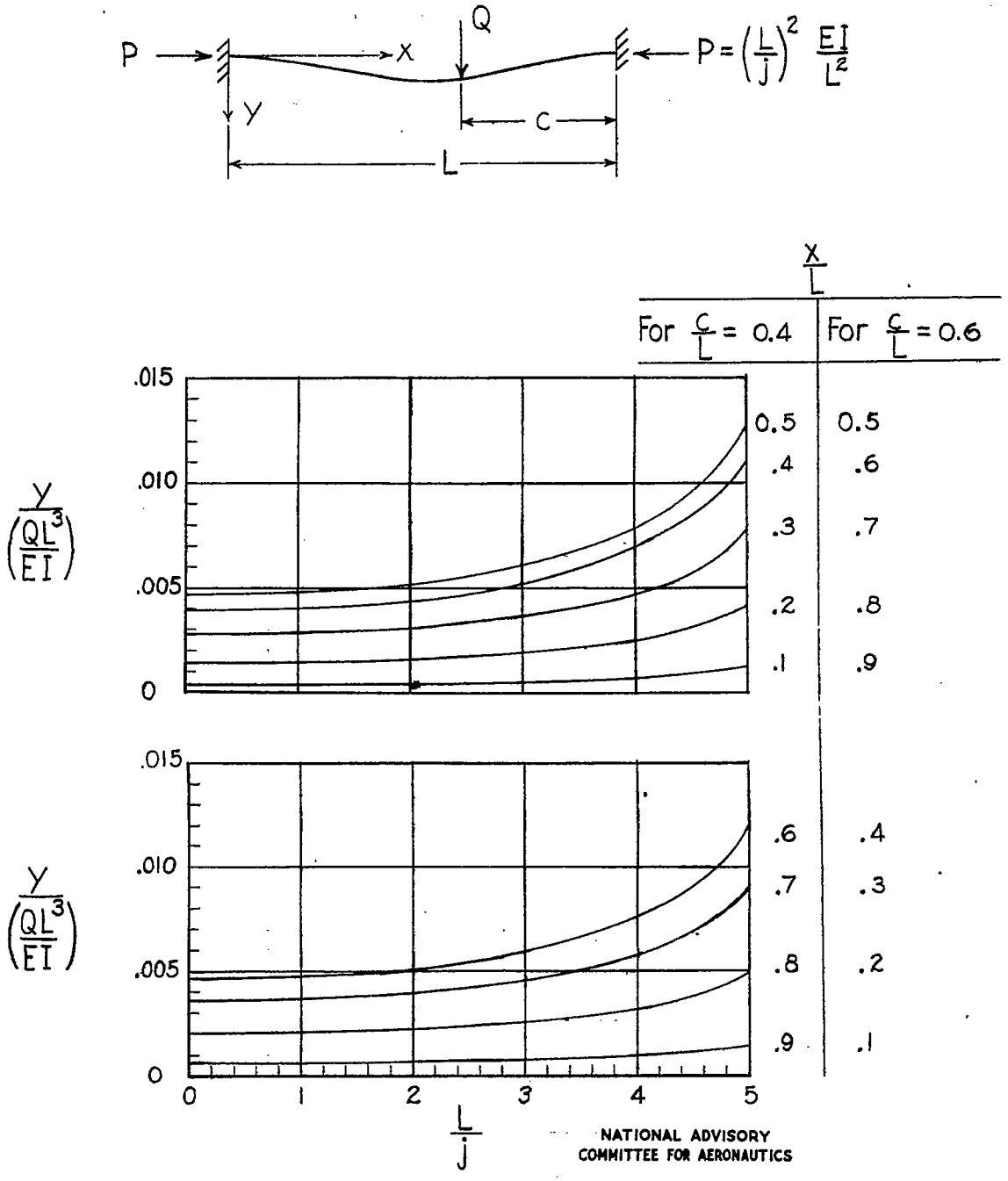
(b) $\frac{c}{L} = 0.2$ or 0.8.

Figure 8.- Continued.



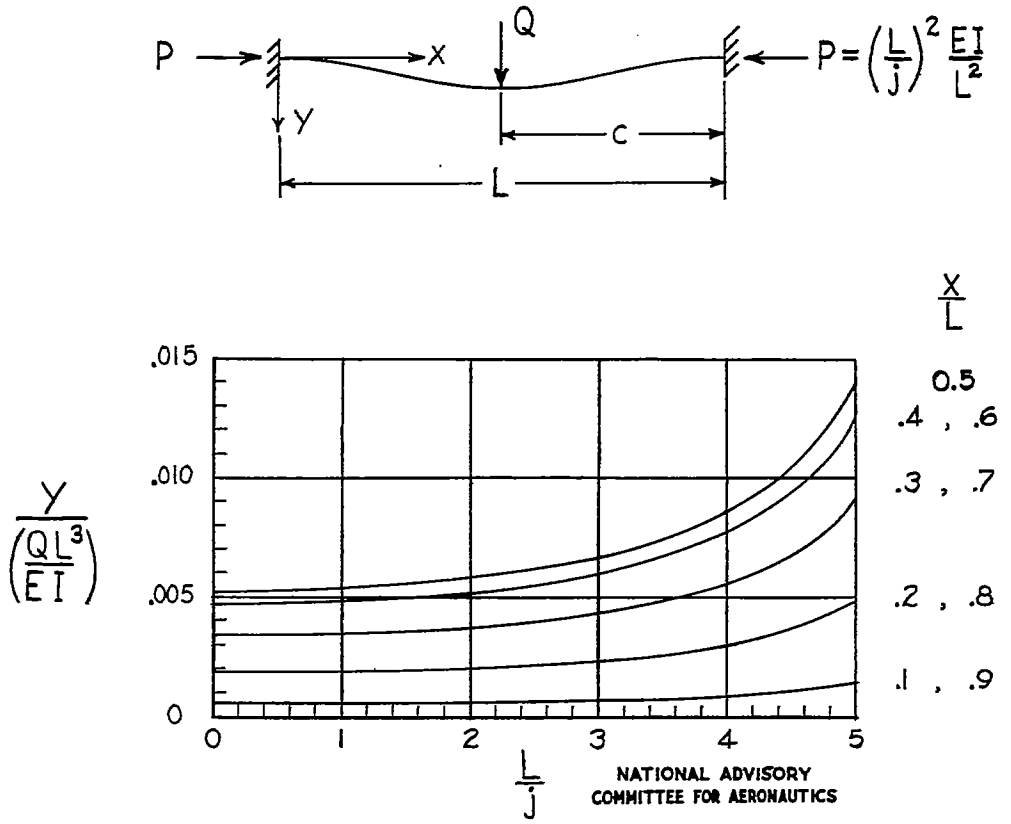
(c) $\frac{c}{L} = 0.3$ or 0.7.

Figure 8.- Continued.



(d) $\frac{c}{L} = 0.4$ or 0.6.

Figure 8.- Continued.



(e) $\frac{c}{L} = 0.5$.

Figure 8.- Concluded.

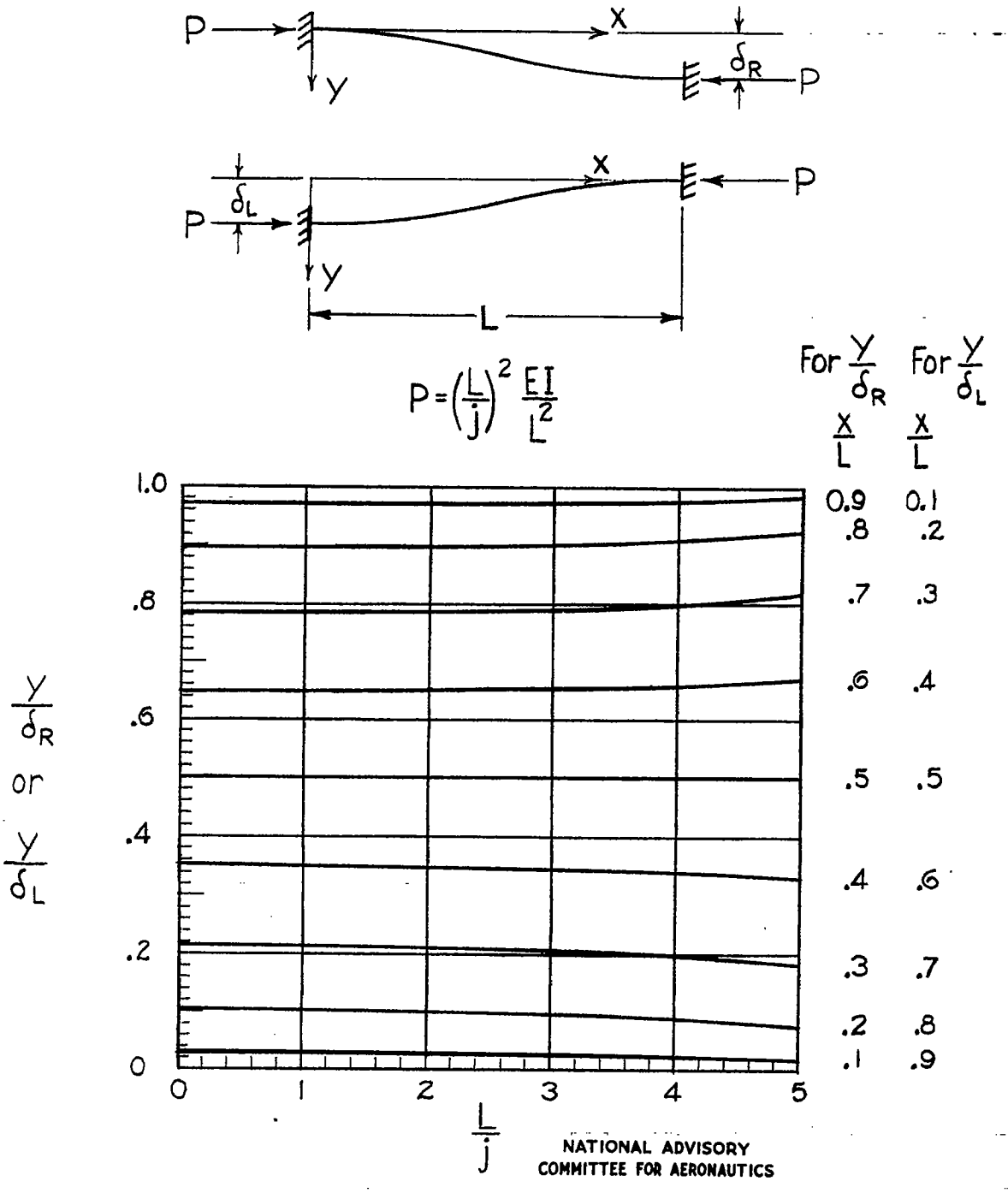


Figure 9.- Deflections of a clamped-end column due to lateral movement of one end.

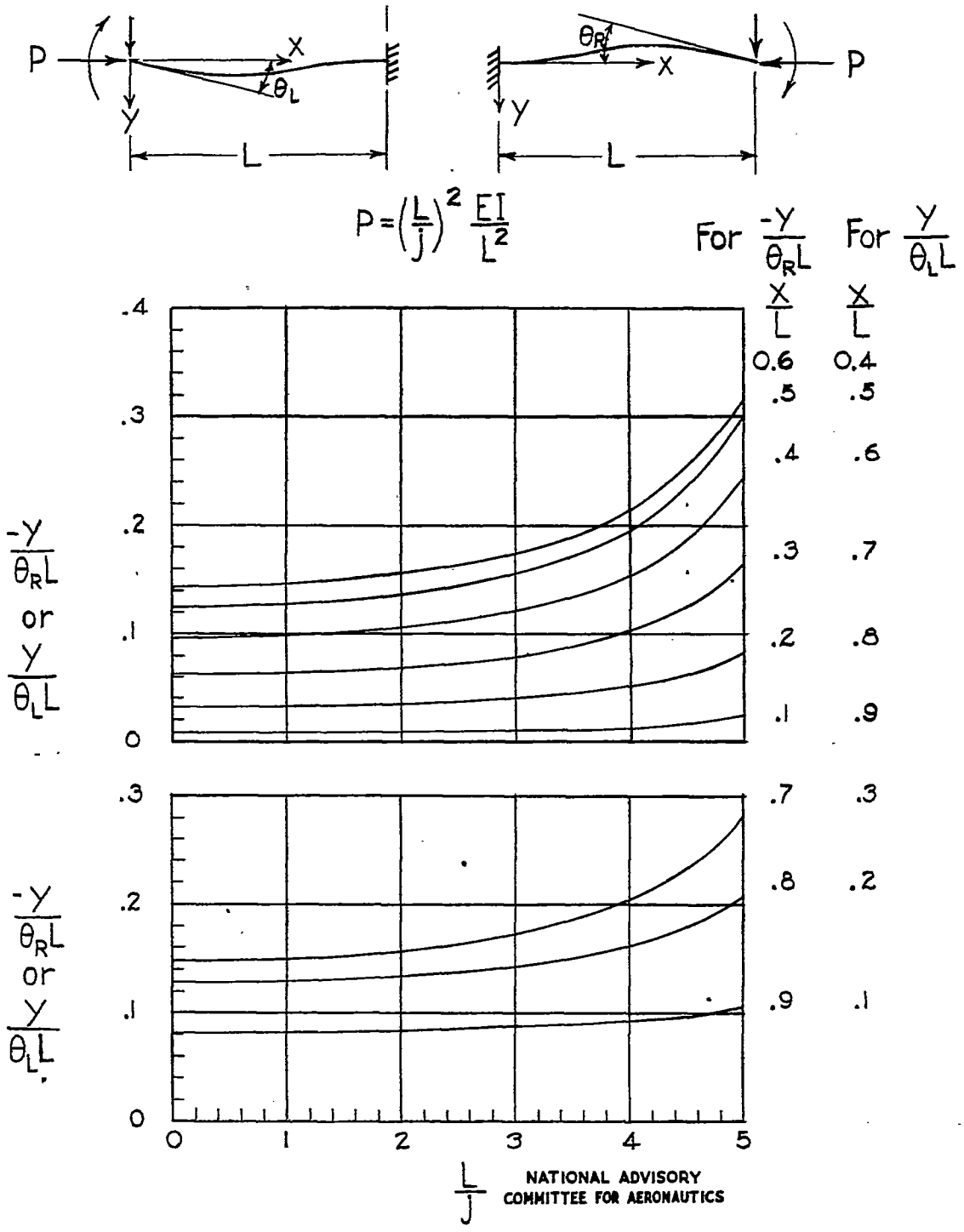
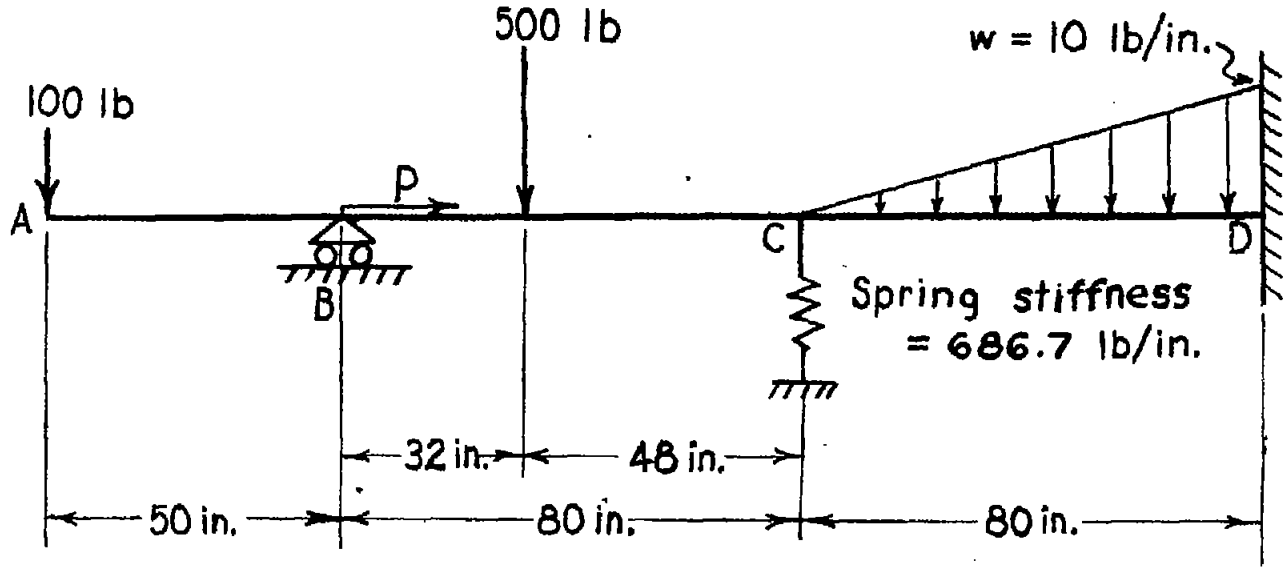


Figure 10.- Deflections of a clamped-end column due to rotation of one end.



$E = 29 \times 10^6$ psi

$I = 0.2$ inch^4

$\frac{L}{j} = 3$ for spans BC and CD

Span AB has no axial load.

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Figure 11.- Structure analyzed for illustrative example.

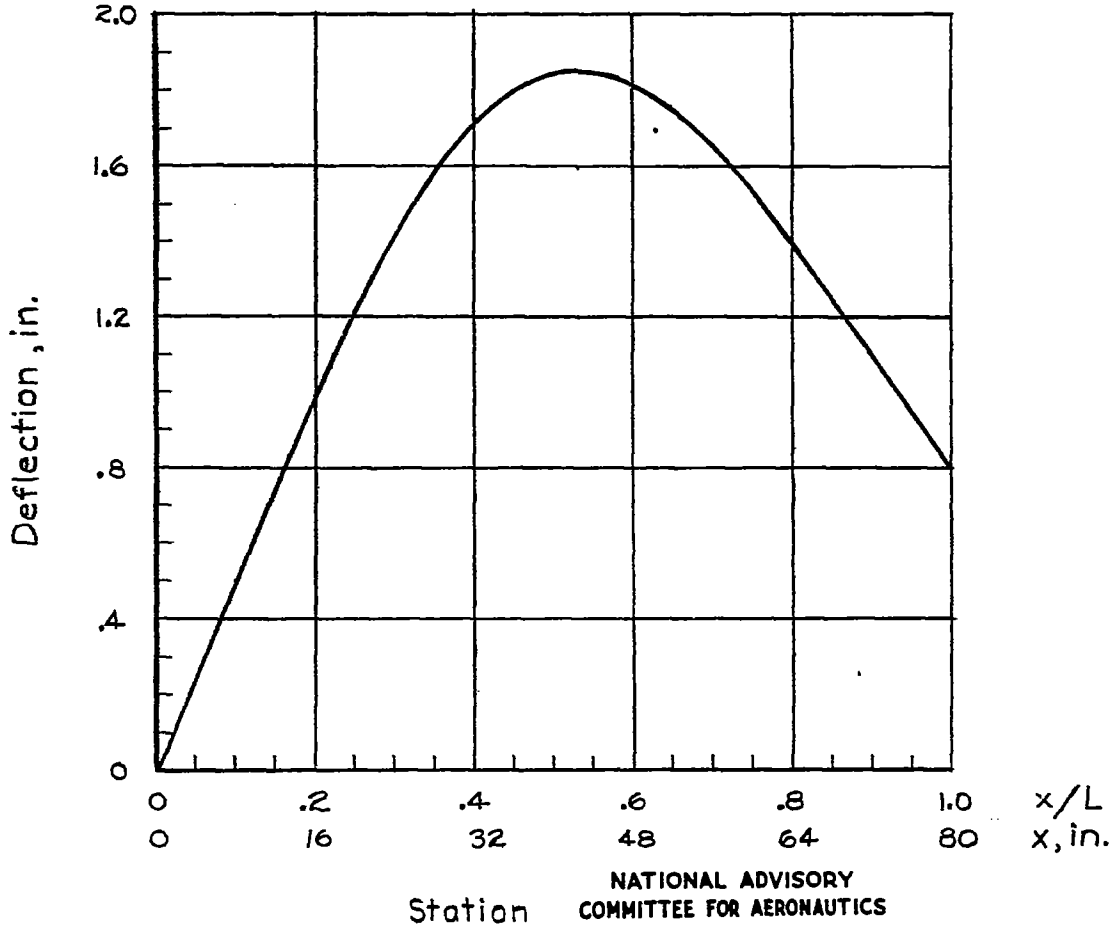

Figure 12.- Deflections of span BC in illustrative example.

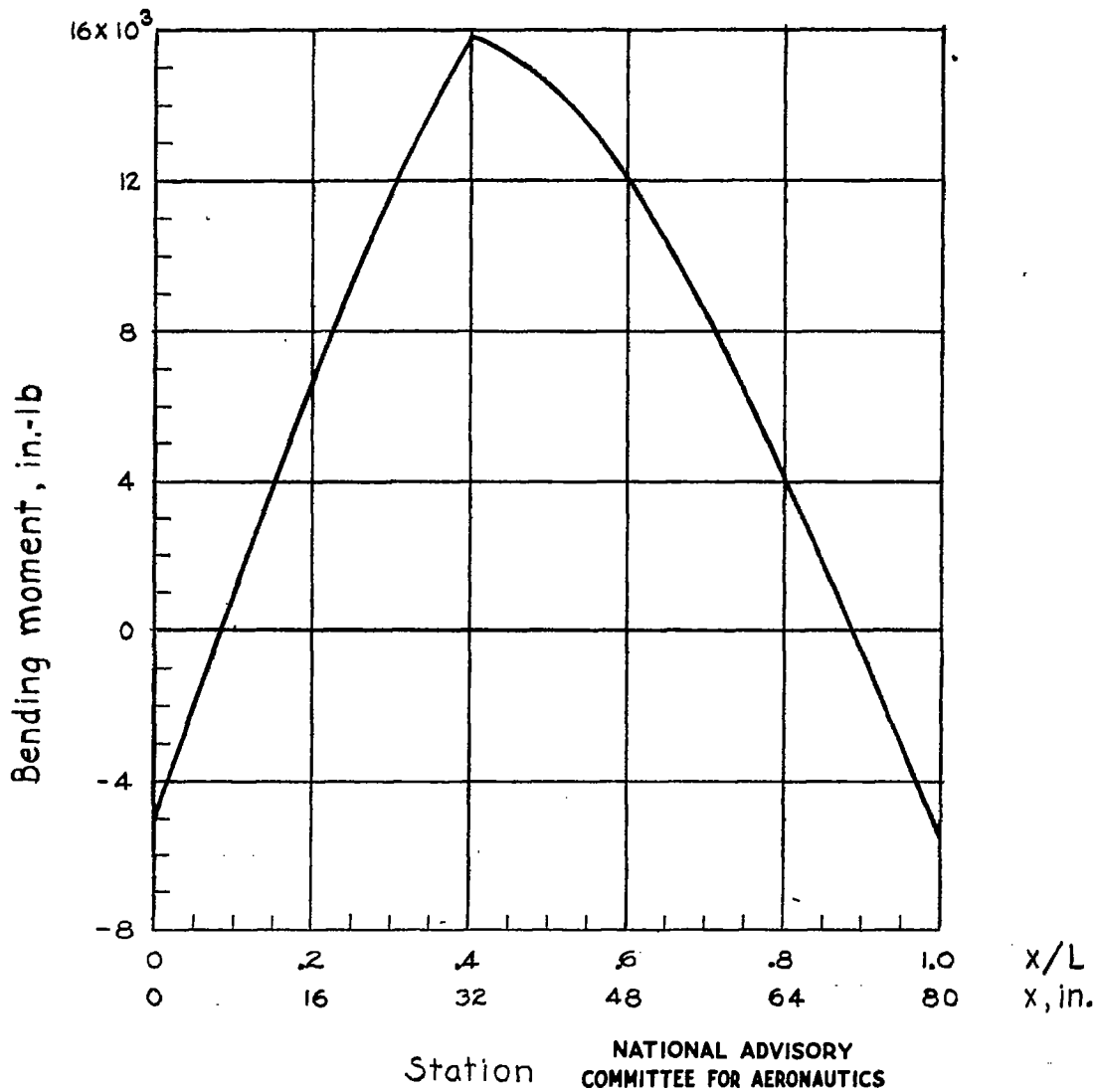

Figure 13.- Bending-moment diagram for span BC in illustrative example.