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NUMERICAL PROCEDURES FOR THE CALCULATION OF THE STRESSES IN MONOCOQUES

IV - INFLUENCE COEFFICIENTS OF CURVED BARS FOR DISTORTIONS IN THEIR OWN PLANE

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IV - INFLUENCE COEFFICIENTS OF CURVED BARS FOR DISTORTIONS IN THEIR OWN PLANE

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SUMMARY

Formulas for the calculation of influence coefficients of curved beams of constant radius of curvature are presented corresponding to displacements in the plane of curvature. Moreover, numerical values are given in tabular and graphic form for a limited number of values of the parameters involved.

INTRODUCTION

In part III of these investigations (reference 1) the bending moment distribution in closed rings was calculated by numerical methods. In the course of these calculations it was observed that the determination of the influence coefficients of ring segments is rather laborious. Influence coefficients are the force and moment reactions at the ends of ring segments that correspond to prescribed unit displacements. Their values must be known when ring problems are solved by the numerical methods.

Formulas for the computation of the influence coefficients are presented here in order to reduce the work of the stress analyst availing himself of the numerical methods. Moreover, numerical values of the coefficients are given in tabular and graphic form for a limited number of values of the parameters involved.

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SYMBOLS

A	cross-sectional area of bar
A^*	effective shear area (based on tension)
A_{fl}	area of flange of built-up cross section
b	width of rectangular cross section of bar
C-C	cross section of bar
d	dimension of cross section of bar
E	Young's modulus of elasticity
f_{nn1}, f_{nn2}, etc.	functions of β
f_s	shearing stress
F	"fixed" end of bar
F_{nn1}, F_{nn2}, etc.	functions of β
G	shear modulus
h	height of rectangular cross section of bar
I	moment of inertia
L	developed length of median line of bar
M	"movable" end of bar
N	end moment reacting on bar

O	center of curvature of circular median line of bar
P	tension force acting on a cross section
q	shear flow acting along bar
Q	static moment about neutral axis
r	radius of circular median line of bar
R	end radial reaction acting on bar
t	thickness of cross section
t_{fl}	thickness of flange of section
t_w	thickness of web of section
T	end tangential reaction acting on bar
U	total strain energy
$U_{bending}$	bending strain energy
U_{shear}	shear strain energy
$U_{tension}$	tension strain energy
V	shear force acting on a cross section
x	rectangular coordinate
y	rectangular coordinate
β	angle subtended by circular median line of bar
γ	section-length parameter (AL^2/I)
$\delta_N, \delta_R, \delta_T$	unit movements of movable end of bar
Δ	determinant
θ	angular coordinate

$\kappa = 1 + (\beta^2/\gamma)\left[(1/\xi) + 1\right]$

$\lambda = 1 - (\beta^2/\gamma)\left[(1/\xi) - 1\right]$

$\nu = (G/E)$

$\xi = (A^*/A)$

φ angular coordinate

The symbols used to denote influence coefficients are defined in the following manner:

The symbol $(\widehat{ab})$ stands for the force or moment a caused by a unit movement in the direction of b (which direction is that of the force R or T, or of the moment N). Thus $(\widehat{nn})$ is the moment due to a unit rotation, while $(\widehat{tr})$ is the tangential force arising from a unit radial displacement. Further, to distinguish the reactions at the fixed end from those at the movable end, the subscripts F and M are employed. Consequently, $(\widehat{nt})_F$ is the moment arising at the fixed end of the curved bar as a result of a unit tangential displacement of the movable end, while $(\widehat{tt})_M$ stands for the tangential force at the movable end due to a unit tangential displacement of that end.

The moment, radial force, and tangential force caused by a constant shear flow are denoted by the symbols n_q, r_q, and t_q, respectively. The sign convention is that shown in figure 1, and the values given are valid for the end of the bar toward which the shear flows. At the opposite end the reactions are equal in magnitude and opposite in sign because of the antisymmetry of the system. It should be noted that all the reactions considered here are acting from the support upon the curved bar.

STATEMENT OF THE PROBLEM

Figure 1 represents a bar the median line of which is an arc of a circle subtending an angle β. The forces acting upon the bar are the radial and tangential components R and T, respectively, of the reaction forces at the end points F and M, the reaction moments N at the end points, and the uniformly distributed shear flow q, all taken positive as shown.

The first step in the calculations is the determination of the bending moment, the tensile force, and the shear force

acting in an arbitrary section of the curved bar. Next, the total strain energy stored in the bar is calculated.

The unit problem proper consists of the calculation of the six reaction forces and moments caused by a generalized unit displacement of end point M (the "movable end point") when there is no shear flow acting on the bar. It is convenient to choose the generalized unit displacement as a tangential displacement, a radial displacement, or a rotation. The calculations are carried out with the aid of Castigliano's theorem, according to which the partial differential coefficient of the strain energy with respect to any unknown generalized reaction is equal to the generalized displacement of the point of attack of the generalized reaction in its own direction. The curved bar is considered a cantilever fixed at point F, and the strain energy is expressed as a function of the unknown reactions at point M. By setting the partial differential coefficient with respect to one of the reactions at M equal to unity and the other two equal to zero, the resulting three simultaneous linear equations can be solved for the unknown reactions at the movable end. The three reactions at the fixed end F of the bar can then be determined from conditions of equilibrium.

If in turn the three generalized displacements are set equal to unity, the reactions corresponding to three unit problems are obtained.

In the last problem discussed in this report all the three displacements are equal to zero, but the shear flow q has a finite constant value. In this case again three simultaneous linear equations are obtained which can be solved for the unknown reactions at one end point. The reactions at the other end are equal in magnitude and opposite in sense because of antisymmetry.

CALCULATION OF THE STRAIN ENERGY

Bending Strain Energy

The bending moment at point C (fig. 1) caused by the uniform shear flow q is

$$N_q = \int_0^{\varphi} qr^2 d\theta \, [1 - \cos(\varphi - \theta)] = qr^2 (\varphi - \sin \varphi) \quad (1)$$

The radial reaction R contributes a moment of $Rr \sin \varphi$, and the tangential unknown force T gives $Tr(1 - \cos \varphi)$. For convenience the subscript M is omitted. The omission cannot cause any misunderstanding, since the reactions at point F do not appear in the expressions for the strain energy.

The total bending moment at point C due to all forces up to that point is the sum of the above-mentioned terms and the unknown end moment N:

$$N_\varphi = qr^2 (\varphi - \sin \varphi) + N + Rr \sin \varphi + Tr (1 - \cos\varphi) \qquad (2)$$

The strain energy stored in the structure due to bending is:

$$U_{bending} = (1/2EI) \int_0^\beta N_\varphi{}^2 r d\varphi \qquad (3)$$

Tension Strain Energy

Upon each radial cross section such as $C-C$ there acts a resultant tension force equal to the sum of the components of all forces up to that point taken in the tangential direction at $C-C$. The force due to the shear flow is

$$P_q = \int_0^\varphi qrd\theta \cos (\varphi - \theta) = qr \sin \varphi \qquad (4)$$

If the appropriate components of the reaction forces are added, the resultant becomes

$$P = T \cos \varphi - R \sin \varphi + qr \sin \varphi \qquad (5)$$

The expression for the tension strain energy is

$$U_{tension} = (1/2EA) \int_0^\beta P^2 rd\varphi \qquad (6)$$

A being the cross-sectional area of the bar.

Shear Strain Energy

Upon each radial cross section there acts a resultant shear force V. The strain energy due to shear cannot be determined quite so easily as that due to tension, because the shear stresses are not uniformly distributed over the cross section. The strain energy stored in an element of the bar of an infinitesimal length dL can be calculated from the formula:

$$dU_{shear} = (dL/2G)\int_A f_s^2 \, dA \tag{7}$$

where

f_s shearing stress at any point of the cross section due to V

G shear modulus of the material

dA elementary area of section

After integration this can be written in the form,

$$dU_{shear} = (V^2/2EA^*)dL \tag{8}$$

which is equivalent to the strain energy caused by a uniform tension force equal to the shear force V, acting over an effective shear area A^*. In general, A^* will be less than A, the total cross-sectional area, and the ratio $A^*/A = \xi$ will depend on the type of cross section. This value will now be calculated for four sections that may be taken to represent the probable types and cover the usual range in aircraft structures.

1. <u>Thin-walled I-section</u>.- For a thin-walled I-section such as shown in figure 2(a), the shearing stress at any point is:

$$f_s = VQ/I_x t \tag{9}$$

where

Q static moment about the neutral axis of the portion of the section extending from the free edge to the point in question

I_x moment of inertia of the section about the neutral axis

t thickness of wall

For the right-hand half of the flange, with the notation of figure 2 (a),

$$f_s = (V/I_x t_f)(d - x)t_f h = (V/I_x)(d - x)h \qquad (10a)$$

$$\text{when} \quad d > x \geq 0$$

For a point on the web:

$$f_s = (2Vht_f d/I_x t_w) + (V/2I_x)(h^2 - y^2) \qquad (10b)$$

$$\text{when} \quad x = 0, \; h \geq y \geq -h$$

The total shear strain energy of the infinitesimal element will be:

$$dU_{shear} = dL \int_A (1/2G) f_s^2 dA = (V/I_x)^2 (dL/G) \left\{ (2h^2 d^3 t_f/3) \right.$$

$$\left. + 2t_w h^2 \left[2hd^2 (t_f/t_w)^2 + (2h^2 d t_f/3t_w) + (h^3/15) \right] \right\} \qquad (10c)$$

But

$$I_x = 4h^2 \left[t_f d + (ht_w/6) \right]$$

and

$$A = 2(2t_f d + ht_w)$$

Thus

$$\xi = (A^*/A) = \frac{(V^2 dL/2E)}{AdL \int_A (1/2G) f_s^2 dA}$$

$$= \left(\frac{2}{6}\right) \frac{\left[6 \dfrac{t_f d}{t_w h} + 1\right]^2}{\left[2 \dfrac{t_f d}{t_w h} + 1\right]\left[\left(\dfrac{d}{h}\right)^2 \dfrac{t_f d}{t_w h} + 6\left(\dfrac{t_f d}{t_w h}\right)^2 + 2\dfrac{t_f d}{t_w h} + \dfrac{1}{5}\right]} \qquad (10)$$

If d/h is taken as 1 and t_f/t_w as 1, ξ is found to be 0.114. If $d/h = 1/2$ and $t_f/t_w = 1$, then $\xi = 0.182$.

2. Thin-walled channel section.- With the notation of figure 2(b) the value of ξ for the channel section is

$$\xi = (A^*/A) = \left(\frac{\nu}{3}\right) \frac{\left[3\left(\frac{d}{h}\right) + 1\right]^2}{\left[\frac{d}{h} + 1\right]\left[\left(\frac{d}{h}\right)^3 + 3\left(\frac{d}{h}\right)^2 + 2\left(\frac{d}{h}\right) + \left(\frac{2}{5}\right)\right]} \tag{11}$$

This result is approximately valid for a Z-section also.

In a typical case $d/h = 1$ and ξ is found to be 0.16. With $d/h = 1/2$ the value 0.236 is obtained.

3. Solid rectangular section.- In the case of the solid rectangular section

$$\xi = \frac{A^*}{A} = \frac{(V^2/2E)\ dL}{A(9V^2 dL/15AG)} = (15\ \nu/18) = 0.32 \tag{12}$$

It is instructive to note that the result is independent of the ratio b/h. Moreover, equations (10) and (11) reduce to equation (12) when the ratio d/h is equal to zero.

4. Built-up section.- For a built-up section like that of figure 2(d), ξ is merely the ratio of the area of the web to the total area of the section, times the ratio $\nu = G/E$. Therefore

$$\xi = A^*/A = \nu\ ht/(ht + 2A_{fl}) = \nu/[1 + (2A_{fl}/ht)] \tag{13}$$

As far as numerical values are concerned, ht will always be smaller than $2A_{fl}$. Consequently, ξ cannot exceed the value 0.192. For the other extreme, that of a very light web, ξ may become 0.01.

It is seen that the parameter ξ may thus vary from about 0.01 to 0.32. Because of the work involved, it was not found practical to compute the influence coefficients for many values of ξ. Moreover, the effect of ξ is not

very important. Hence only three values of ξ, namely, 0.01, 0.1, and 0.25 were chosen when the tables and graphs were set up. In an actual problem the value should be used which is closest to that of the actual section. If more accuracy is desired, the actual value of ξ may be used, but then the influence coefficients must be calculated from formulas developed later in this report, and no advantage can be gained from the tabulated values.

On returning now to the calculation of the shear strain energy, it is necessary to determine the shear force V acting at any section subtending an angle φ. The part contributed by the shear flow is

$$\int_0^{\varphi} qrd\theta \sin(\varphi - \theta) = qr(1 - \cos\varphi) \tag{f4}$$

With the components of R and T the shear force becomes

$$V = T \sin\varphi + R \cos\varphi + qr(1 - \cos\varphi) \tag{15}$$

The Total Strain Energy

The total strain energy of the system can now be written down.

$$U = U_{bending} + U_{tension} + U_{shear}$$

$$= \int_0^{\beta} (M_{\varphi}^2/2EI)\, rd\varphi + \int_0^{\beta} (P^2/2EA)\, rd\varphi + \int_0^{\beta} (V^2/2EA^*)\, rd\varphi \tag{16}$$

where the expressions for M, P, and V are given by equations (2), (5), and (15) respectively.

DETERMINATION OF THE DIFFERENTIAL COEFFICIENTS

In the solution of the unit problem with the aid of Castigliano's theorem the partial differential coefficients

of the total strain energy (equation (16)) are needed with respect to the reactions N, R, and T at the movable end.

These partial differential coefficients can be written in the form

$$(EI/r)(\partial U/\partial N) = N\beta + Rr(1 - \cos\beta) + Tr(\beta - \sin\beta)$$

$$+ qr^2[(\beta^2/2) + \cos\beta - 1] = (EI/r)\delta_N \qquad (17a)$$

$$(EI/r^2)(\partial U/\partial R) = N(1 - \cos\beta) + Rr\Big\{(\beta/2)[1 + (\beta^2/\gamma)$$

$$+ (\beta^2/\gamma\xi)] - (1/4)(\sin 2\beta)[1 + (\beta^2/\gamma) - (\beta^2/\gamma\xi)]\Big\}$$

$$+ Tr\Big\{1 - \cos\beta - (1/2)(\sin^2\beta)[1 + (\beta^2/\gamma) - (\beta^2/\gamma\xi)]\Big\}$$

$$+ qr^2\Big\{-\beta\cos\beta + (\sin\beta)[1 + (\beta^2/\gamma\xi)] - (\beta/2)[1 + (\beta^2/\gamma)$$

$$+ (\beta^2/\gamma\xi)] + (1/4)(\sin 2\beta)[1 + (\beta^2/\gamma) - (\beta^2/\gamma\xi)]\Big\}$$

$$= (EI/r^2)\delta_R \qquad (17b)$$

$$(EI/r^2)(\partial U/\partial T) = N(\beta - \sin\beta) + Rr\Big\{1 - \cos\beta$$

$$-(1/2)(\sin^2\beta)[1 + (\beta^2/\gamma) - (\beta^2/\gamma\xi)]\Big\} + Tr\Big\{\beta - 2\sin\beta$$

$$+ (\beta/2)[1 + (\beta^2/\gamma) + (\beta^2/\gamma\xi)] + (1/4)(\sin 2\beta)[1 + (\beta^2/\gamma)$$

$$- (\beta^2/\gamma\xi)]\Big\} + qr^2\Big\{(\beta^2/2) - \beta\sin\beta + (1/2)(\sin^2\beta)[1 + (\beta^2/\gamma)$$

$$- (\beta^2/\gamma\xi)] + (1 - \cos\beta)(\beta^2/\gamma\xi)\Big\} = (EI/r^2)\delta_T \qquad (17c)$$

where

$$\gamma = AL^2/I = Ar^2\beta^2/I \tag{18}$$

Some simplification can be gained by the substitutions:

$$1 + (\beta^2/\gamma) + (\beta^2/\gamma\xi) = \kappa \tag{19a}$$

$$1 + (\beta^2/\gamma) - (\beta^2/\gamma\xi) = \lambda \tag{19b}$$

CALCULATION OF THE UNIT PROBLEM WITHOUT SHEAR FLOW

Influence Coefficients

If in equations (17) q and two of the three generalized displacements δ_N, δ_R, and δ_T are set equal to zero, while the third generalized displacement is assumed to be equal to unity, the resulting three simultaneous equations represent one of the three unit problems without shear flow. By letting each of the displacements in turn take on the value 1, the equations corresponding to the three unit problems are obtained. Since they are linear in the unknown reactions, they can best be solved by the method of determinants. The following notation is used throughout this paper:

The symbol $(\widehat{ab})$ stands for the force or moment a caused by a unit movement in the direction of b (which direction is that of the force R or T, or of the moment N). Thus $(\widehat{nn})$ is the moment due to a unit rotation, while $(\widehat{tr})$ is the tangential force arising from a unit radial displacement. Further, to distinguish the reactions at the fixed end from those at the movable end, the subscripts F and M are employed. Consequently, $(\widehat{nt})_F$ is the moment arising at the fixed end of the curved bar as a result of a unit tangential displacement of the movable end, while $(\widehat{tt})_M$ stands for the tangential force at the movable end due to a unit tangential displacement of that end.

It would appear then that there were $2 \times 3 \times 3 = 18$ distinct reactions to be determined. In reality there are only 12, since $(\widehat{ab}) = (\widehat{ba})$ for either the movable or the fixed end in accordance with Maxwell's theorem of reciprocity.

The six independent influence coefficients for the movable end can be calculated from the following equations:

$$(\widehat{nn})_M \frac{L}{EI} = \beta \frac{\Delta_{\widehat{nn}}}{\Delta} \left(\frac{r}{EI}\right) \tag{20a}$$

$$-(\widehat{nr})_M \frac{L^2}{EI} = \beta^2 \frac{\Delta_{\widehat{nr}}}{\Delta} \left(\frac{r^2}{EI}\right) \tag{20b}$$

$$(\widehat{nt})_M \frac{L^2}{EI} = \beta^2 \frac{\Delta_{\widehat{nt}}}{\Delta} \left(\frac{r^2}{EI}\right) \tag{20c}$$

$$(\widehat{rr})_M \frac{L^3}{EI} = \beta^3 \frac{\Delta_{\widehat{rr}}}{\Delta} \left(\frac{r^3}{EI}\right) \tag{20d}$$

$$-(\widehat{rt})_M \frac{L^3}{EI} = \beta^3 \frac{\Delta_{\widehat{rt}}}{\Delta} \left(\frac{r^3}{EI}\right) \tag{20e}$$

$$(\widehat{tt})_M \frac{L^3}{EI} = \beta^3 \frac{\Delta_{\widehat{tt}}}{\Delta} \left(\frac{r^3}{EI}\right) \tag{20f}$$

where

$$(\Delta/r^2) = F_{\Delta 1} k^2 + F_{\Delta 2} k + F_{\Delta 3} \lambda^2 + F_{\Delta 4} \lambda \tag{21a}$$

$$(\Delta_{\widehat{nn}}/EIr)\ \beta = F_{nn1} k^2 + F_{nn2} k + F_{nn3} \lambda^2 + F_{nn4} \lambda + F_{nn5} \tag{21b}$$

$$-(\Delta_{\widehat{rn}}/EI)\ \beta^2 = F_{rn1} k + F_{rn2} \lambda + F_{rn3} \tag{21c}$$

$$(\Delta_{\widehat{tn}}/EI)\ \beta^2 = +F_{tn1} k + F_{tn2} \lambda + F_{tn3} \tag{21d}$$

$$(\Delta_{\widehat{rr}}/EI)\ r\beta^3 = F_{rr1} k + F_{rr2} \lambda + F_{rr3} \tag{21e}$$

$$-(\Delta_{\widehat{tr}}/EI)\ r\beta^3 = F_{tr1} \lambda + F_{tr2} \tag{21f}$$

$$(\Delta_{\widehat{tt}}/EI)\ r\beta^3 = F_{tt1} k + F_{tt2} \lambda + F_{tt3} \tag{21g}$$

The functions F are defined as

$$\left.\begin{aligned} F_{\Delta 1} &= (1/4)\ \beta^3 \\ F_{\Delta 2} &= -2\beta \sin^2 (\beta/2) \\ F_{\Delta 3} &= -(1/4)\ \beta \sin^2 \beta \\ F_{\Delta 4} &= -(1/2) \sin 2\beta + \sin \beta \end{aligned}\right\} \quad (22a)$$

$$\left.\begin{aligned} F_{nn1} &= (1/4)\ \beta^3 \\ F_{nn2} &= (1/2)\ \beta^3 - \beta^2 \sin \beta \\ F_{nn3} &= -(1/4)\ \beta \sin^2 \beta \\ F_{nn4} &= \beta \sin^2 \beta - (1/4)\ \beta^2 \sin 2\beta \\ F_{nn5} &= \beta \sin^2 \beta - 4\beta \sin^2 (\beta/2) \end{aligned}\right\} \quad (22b)$$

$$\left.\begin{aligned} F_{rn1} &= (\beta^3/2)(1 - \cos \beta) \\ F_{rn2} &= (\beta^2/2)[-\sin \beta + (1/2) \sin 2\beta + \beta \sin^2 \beta] \\ F_{rn3} &= \beta^2\ [-\sin \beta + (1/2) \sin 2\beta] \end{aligned}\right\} \quad (22c)$$

$$\left.\begin{aligned} F_{tn1} &= (\beta^3/2)(\sin \beta - \beta) \\ F_{tn2} &= (\beta^2/2)[(\beta/2) \sin 2\beta - \sin^2 \beta] \\ F_{tn3} &= \beta^2\ [4 \sin^2 (\beta/2) - \sin^2 \beta] \end{aligned}\right\} \quad (22d)$$

$$\left.\begin{aligned} F_{rr1} &= (\beta^5/2) \\ F_{rr2} &= (\beta^4/4) \sin 2\beta \\ F_{rr3} &= -\beta^3 \sin^2 \beta \end{aligned}\right\} \quad (22e)$$

$$\left.\begin{aligned} F_{tr1} &= -(\beta^4/2)\sin^2\beta \\ F_{tr2} &= \beta^3\left[\sin\beta - (1/2)\sin 2\beta\right] \end{aligned}\right\} \quad (22f)$$

$$\left.\begin{aligned} F_{tt1} &= (\beta^5/2) \\ F_{tt2} &= -(\beta^4/4)\sin 2\beta \\ F_{tt3} &= \beta^3\left[\sin^2\beta - 4\sin^2(\beta/2)\right] \end{aligned}\right\} \quad (22g)$$

Numerical values of these functions were calculated for $\beta = 60^\circ$, 75°, 90°, 120°, 150°, and 180°. They are collected in table I.

Series Expansions

In the numerical calculation of the expressions given for the influence coefficients, it was found that in most cases the value of the determinants is the small difference of large numbers. For small angles β the difference is so small that even the use of calculating machines is insufficient. The influence coefficients themselves, however, are not negligibly small, since they are the ratios of small quantities.

The difficulty can be overcome by expanding the trigonometric functions into their power series in the angle. The final expressions obtained for the determinants are then power series which converge rapidly for small angles β. For values of β greater than 90° the convergence is so slow that the original (unexpanded) expressions are more suitable for numerical calculations.

The influence coefficients can be calculated from the following expressions:

$$(\Delta/r^2\beta^7) = f_{\Delta 1} + (1/\gamma)\left[f_{\Delta 2} + (1/t)(f_{\Delta 3})\right] + (1/\gamma)^2\left\{(1/t) + \left[(1/t) - 1\right]^2 f_{\Delta 4}\right\} \tag{23a}$$

$$(\Delta_{\widehat{nn}}/EIr\beta^6) = f_{nn1} + (1/\gamma)\left[f_{nn2} + (1/t)\, f_{nn3}\right] + (1/\gamma)^2\left\{(1/t) + \left[(1/t) - 1\right]^2 f_{nn4}\right\} \tag{23b}$$

$$-(\Delta_{\widehat{rn}}/EI\beta^5) = f_{rn1} + (1/\gamma)\left[f_{rn2} + (1/t)\, f_{rn3}\right] \tag{23c}$$

$$(\Delta_{\widehat{tn}}/EI\beta^5) = f_{tn1} + (1/\gamma)\left[-f_{tn2} + (1/t)\, f_{tn3}\right] \tag{23d}$$

$$(\Delta_{\widehat{rr}}r/EI\beta^4) = f_{rr1} + (1/\gamma)\left\{1 + \left[(1/t) - 1\right] f_{rr2}\right\} \tag{23e}$$

$$-(\Delta_{\widehat{tr}}r/EI\beta^4) = f_{tr1} + (1/\gamma)\left[(1/t) - 1\right] f_{tr2} \tag{23f}$$

$$(\Delta_{\widehat{tt}}r/EI\beta^4) = f_{tt1} + (1/\gamma)\left\{(1/t) - \left[(1/t) - 1\right] f_{tt2}\right\} \tag{23g}$$

where:

$$\left.\begin{aligned}
f_{\Delta 1} &= (0.0001157407\beta^2 - 0.0000089206634\beta^4 + 0.0000004133361\beta^6 - 0.0000000010974\beta^8 + \ldots)\\
f_{\Delta 2} &= (0.08333\ldots - 0.01111\ldots\beta^2 + 0.0009093391\beta^4 - 0.0000451194\beta^6 + 0.000001482235\beta^8 - \ldots)\\
f_{\Delta 3} &= (0.008333\ldots\beta^2 - 0.0008597888\beta^4 + 0.0000446428\beta^6 - 0.000001478079\beta^8 + 0.0000000344\beta^{10} - \ldots)\\
f_{\Delta 4} &= (0.08333\ldots\beta^2 - 0.01111\ldots\beta^4 + 0.00079365\beta^6 - 0.0000352373\beta^8 + 0.000001088889\beta^{10} - \ldots)
\end{aligned}\right\} \tag{24a}$$

$$\left.\begin{aligned}
f_{nn1} &= (0.001041666\ldots\beta^2 - 0.00010912689\beta^4 + 0.000005373889\beta^6 - 0.000000164683\beta^8 + 0.0000000003524\beta^{10} - \ldots)\\
f_{nn2} &= (0.333\ldots - 0.0572777\ldots\beta^2 + 0.004960317\beta^4 - 0.000284942\beta^6 + 0.0000107140\beta^8 - 0.000000288\beta^{10} + \ldots)\\
f_{nn3} &= (0.038111\ldots\beta^2 - 0.00456349\beta^4 + 0.0002794318\beta^6 - 0.0000106638\beta^8 + 0.000000281\beta^{10} - \ldots)\\
f_{nn4} &= (0.08333\ldots\beta^2 - 0.0111\ldots\beta^4 + 0.00079365076\beta^6 - 0.000035273368\beta^8 + \ldots)
\end{aligned}\right\} \tag{24b}$$

$$\left.\begin{aligned}
f_{rn1} &= (0.0041888\ldots\beta^2 - 0.000545635\beta^4 + 0.00003224198\beta^6 - 0.0000011524\beta^8 + 0.000000018196\beta^{10} - \ldots)\\
f_{rn2} &= (0.5 - 0.125\beta^2 + 0.01666\ldots\beta^4 - 0.001248347\beta^6 + 0.000057870088\beta^8 - 0.000001181\beta^{10} + \ldots)\\
f_{rn3} &= (0.08333\ldots\beta^2 - 0.0152777\ldots\beta^4 + 0.0012235455\beta^6 - 0.000057594\beta^8 + 0.000001179\beta^{10} - \ldots)
\end{aligned}\right\} \tag{24c}$$

$$\left.\begin{aligned}
f_{tn1} &= (0.0989444\ldots - 0.001736111\ldots\beta^2 + 0.00014302248\beta^4 - 0.000006430068\beta^6 + 0.000000188048\beta^8\\
&\qquad - 0.000000006265\beta^{10} + \ldots)\\
f_{tn2} &= (0.25 - 0.0486111\ldots\beta^2 + 0.00486111\ldots\beta^4 - 0.000283564\beta^6 + 0.00001070\beta^8 - 0.000000282\beta^{10} + \ldots)\\
f_{tn3} &= (0.08333\ldots - 0.0403777\ldots\beta^2 + 0.004862869\beta^4 - 0.000280809\beta^6 + 0.0000108764\beta^8 - 0.000000281\beta^{10} + \ldots)
\end{aligned}\right\} \tag{24d}$$

$$\left.\begin{aligned}
f_{rr1} &= (0.0222\ldots\beta^2 - 0.0031746032\beta^4 + 0.000211640288\beta^6 - 0.000008551148\beta^8 + 0.000000234921\beta^{10}\\
&\qquad - 0.000000004698\beta^{12} + \ldots)\\
f_{rr2} &= (0.333\ldots\beta^2 - 0.0666\ldots\beta^4 + 0.006349206\beta^6 - 0.000352733\beta^8 + 0.000012886\beta^{10} - 0.000000328\beta^{12} + \ldots)
\end{aligned}\right\} \tag{24e}$$

$$\left.\begin{aligned}
f_{tr1} &= (0.041888 - 0.0097222\ldots\beta^2 + 0.000884589\beta^4 - 0.000044918288\beta^6 + 0.000001480187\beta^8 - 0.000000034468\beta^{10} + \ldots)\\
f_{tr2} &= (0.5 - 0.1666\ldots\beta^2 + 0.0222\ldots\beta^4 - 0.0015873015\beta^6 + 0.000070548787\beta^8 - 0.000002137788\beta^{10} + \ldots)
\end{aligned}\right\} \tag{24f}$$

$$\left.\begin{aligned}
f_{tt1} &= (0.08333\ldots - 0.025\beta^2 + 0.0032842\beta^4 - 0.000212128\beta^6 + 0.000008568\beta^8 - 0.000000235\beta^{10} + \ldots)\\
f_{tt2} &= (+0.333\ldots\beta^2 - 0.0666\ldots\beta^4 + 0.006349206\beta^6 - 0.000352738\beta^8 + 0.000012887\beta^{10} - \ldots)
\end{aligned}\right\} \tag{24g}$$

Numerical values of all these functions were computed for $\beta = 5^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$, and 90°. They are presented in table II. The functions also were plotted in the range 0° to 90° in figures 3 to 13.

Straight Bar

By taking the limit of the determinant ratios as β approaches zero and r approaches infinity, the influence coefficients for the straight bar case can be derived. The expressions so obtained are:

$$(\widehat{nn})_M \, (L/EI) = \frac{4[\gamma + (3/\xi)]}{[\gamma + (12/\xi)]} \tag{25a}$$

$$-(\widehat{rn})_M \, (L^2/EI) = 6\gamma/[\gamma + (12/\xi)] \tag{25b}$$

$$(\widehat{tn})_M = 0 \tag{25c}$$

$$(\widehat{rr})_M \, (L^3/EI) = 12\gamma/[\gamma + (12/\xi)] \tag{25d}$$

$$(\widehat{tr})_M = 0 \tag{25e}$$

and

$$(\widehat{tt})_M \, (L^3/EI) = \gamma \tag{25f}$$

This special case also could be solved directly by setting up the strain energy expressions for the straight bar. In this case again equations (25) are obtained if the tension and shear strain energies are considered. If they are neglected, the values 4, 6, 0, 12, and 0 are obtained instead of the right-hand members of equations (25a) to (25f). These correspond to a section having an infinite γ.

Numerical Values of the Section-Length Parameter

$$\gamma = (AL^2/I)$$

Before beginning the actual computation of the numerical values of the influence coefficients, the limiting values of the section-length parameter γ must be established.

For a rectangular section

$$\gamma = 12\ (L/h)^2$$

By assuming

$$1/16 \text{ inch} < h < 1 \text{ inch} \tag{26a}$$

$$2 \text{ inches} < L < 100 \text{ inches} \tag{26b}$$

the approximate limits become

$$100 < \gamma < 3 \times 10^7 \tag{26c}$$

For a built-up section approximately

$$\gamma = 4\ (L/h)^2$$

By using

$$3 \text{ inches} < h < 20 \text{ inches},\ 3 \text{ inches} < L < 100 \text{ inches} \tag{26d}$$

the bounds on γ become

$$4 < \gamma < 4000 \tag{26e}$$

All other sections will have γ values lying between those for the two sections discussed. Hence, the over-all limits of γ may be given as 1 and 10^8.

Numerical Calculation of the Reactions at the Movable End

Numerical values of the influence coefficients were computed for the following values of the three parameters:

$$\beta = 0,\ 5,\ 15,\ 30,\ 45,\ 60,\ 75,\ 90,\ 120,\ 150,\ 180$$

$$\gamma = 10,\ 100,\ 500,\ 1000,\ 10{,}000, \text{ and } 10^6$$

$$\xi = 0.01,\ 0.1,\ 0.25$$

The numerical values are collected in table III. They are also presented in a graphical form in figures 14 to 31.

In these the abscissa is the angle β, the ordinate is proportional to the influence coefficient, and the parameter of the family of curves is γ. Three groups of figures are given corresponding to $\xi = 0.01$, 0.1, and 0.25, respectively.

In another group of graphs, not presented here, the influence coefficient was plotted against γ using β as a parameter. These curves together with figures 14 to 31 were used for cross-plotting the results in the form of graphs in which the abscissa is β, the ordinate is γ, and each curve of the family corresponds to a constant value of the influence coefficient. The latter curves are presented in figures 32 to 49. It is believed that figures 32 to 49 are the most convenient to use since in actual problems β and γ are given, or can be rapidly calculated from given data. If, however, in the actual problem the value of γ happens to coincide with one of the values of the parameter in figures 14 to 31, greater accuracy may be had by using these curves. Often it is found convenient to assume γ to be equal to one of the six values used.

The curves corresponding to different values of the parameter ξ differ but slightly. For this reason the three values used in the graphs suffice for all practical calculations and great accuracy is not necessary in determining the actual value of ξ in a problem.

It was found in part III (reference 1) that changes in the values of γ and ξ had little effect upon the final results of the calculations of the bending moments in fuselage rings as long as all the influence coefficients for a curved bar were calculated consistently for the same values of the parameters.

Fixed End Reactions

The reactions at the movable end being known, those at the fixed end can be determined from the conditions of equilibrium. (See fig. 1.) The requirement of equilibrium of moments about F gives:

$$N_F = N_M + R_M\, r \sin \beta + T_M\, r(1 - \cos \beta) \qquad (27a)$$

The condition for the equilibrium of forces in the direction of T_F is

$$T_F = T_M \cos \beta - R_M \sin \beta \tag{27b}$$

That in the direction of R_F is

$$-R_F = R_M \cos \beta + T_M \sin \beta \tag{27c}$$

It is convenient to transform the expression for N_F by dividing it by L:

$$(N_F/L) = (N_M/L) + R_M [(\sin \beta)/\beta] + T_M [(1 - \cos \beta)/\beta] \tag{27d}$$

For the straight bar these equations become:

$$T_F = T_M \tag{28a}$$

$$-R_F = T_M \tag{28b}$$

$$(N_F/L) = (N_M/L) + R_M \tag{28c}$$

In the three-unit problems the reactions at the movable end were denoted the influence coefficients for the movable end. Their values were calculated in the preceding articles. If these values are substituted for the reactions in equations (27) or (28), the influence coefficients for the fixed end are obtained. For instance

$$(\widehat{tr})_F = (\widehat{tr})_M \cos \beta - (\widehat{rr})_M \sin \beta \tag{29a}$$

$$-(\widehat{rt})_F = (\widehat{rt})_M \cos \beta + (\widehat{tt})_M \sin \beta \tag{29b}$$

$$[(\widehat{nn})_F/L] = [(\widehat{nn})_M/L] + (\widehat{rn})_M [(\sin \beta)/\beta] + (\widehat{tn})_M [(1 - \cos \beta)/\beta] \tag{29c}$$

Because of the validity of Maxwell's reciprocal theorem for the fixed end,

$$(\widehat{tr})_F = (\widehat{rt})_F$$

This relation, as well as two additional analogous ones, serve as a check on the accuracy of the numerical values obtained for the influence coefficients. Again, six distinct

influence coefficients exist. They are presented in table IV for the same values of the parameters β, γ, and ξ as were the influence coefficients for the movable end. The influence coefficients are graphed in figures 50 to 85. Figures 68 to 85 were obtained in the same manner as were those for the movable end.

THE EFFECT OF A UNIFORM SHEAR FLOW

When a uniform shear is applied along a curved bar which is part of a rigid frame, it is convenient to consider the two ends of the bar rigidly fixed, and to determine the three reaction components N, R, and T at each end. These reactions and the applied shear flow constitute a system of equilibrium. Forces and moments equal and opposite to the reactions can then be assumed to be acting upon the end points of the bar, and the displacements and stresses caused by them in the rigid frame can be determined by the procedure of systematic relaxations. These stresses superimposed upon the stresses caused by the first-mentioned system of equilibrium give the complete solution of the stress problem.

The problem of the determination of the end reactions caused by the shear flow is considered important enough to warrant the development of formulas and the graphic representation of the numerical values of the reactions. The case of a constant shear flow alone is considered, since it is always possible to subdivide a curved bar with a variable shear flow into segments along which the variation of the shear flow is small enough to be neglected in engineering calculations.

The reactions at the ends of the rigidly fixed curved bar can be calculated from equations (17) if the displacements δ_N, δ_R, and δ_T are set equal to zero. (In this case q is assumed to be different from zero.) The moment, radial force, and tangential force caused by a constant shear flow are denoted by the symbols n_q, r_q, and t_q, respectively. The sign convention is that shown in figure 1, and the values given are valid for the end of the bar toward which the shear flows. At the opposite end the reactions are equal in magnitude and opposite in sign because of the antisymmetry of the system. It should be noted that all the reactions considered here are acting from the support upon the curved bar.

The following symbols are introduced:

$$F_{q1} = (\beta^2/2) + \cos\beta - 1 \tag{30a}$$

$$F_{q2} = -\beta\cos\beta + [1 + (\beta^2/\gamma\xi)]\sin\beta - (\kappa\beta/2) + (\lambda/4)\sin 2\beta \tag{30b}$$

$$F_{q3} = (\beta^2/2) - \beta\sin\beta + (\lambda/2)\sin^2\beta + (\beta^2/\gamma\xi)(1 - \cos\beta) \tag{30c}$$

The solution of the three simultaneous linear equations is:

$$-n_q/L^2q = [(\widehat{nn})_M\ L/EI](F_{q1}/\beta^3) + [(\widehat{rn})_M(L^2/EI)](F_{q2}/\beta^4) + [(\widehat{tn})_M[(L^2/EI)]\ (F_{q3}/\beta^4) \tag{31a}$$

$$-r_q/Lq = [(\widehat{nr})_M\ (L^2/EI)](F_{q1}/\beta^3) + [(\widehat{rr})_M\ (L^3/EI)](F_{q2}/\beta^4) + [(\widehat{tr})_M\ (L^3/EI)]\ (F_{q3}/\beta^4) \tag{31b}$$

$$-t_q/Lq = [(\widehat{nt})_M\ (L^2/EI)](F_{q1}/\beta^3) + [(\widehat{rt})_M\ (L^3/EI)](F_{q2}/\beta^4) + [(\widehat{tt})_M\ (L^3/EI)]\ (F_{q3}/\beta^4) \tag{31c}$$

As in the previous case for small angles β few significant figures are obtained for F_{q1}/β^3, F_{q2}/β^4, and F_{q3}/β^4 even if a calculating machine is used. Consequently, series expansion must be resorted to. The expressions obtained follow:

$$(F_{q1}/\beta^3) = (0.04166\ldots\beta - 0.001388\ldots\beta^3 + 0.00002480158\beta^5 - \ldots) \tag{32a}$$

$$(F_{q2}/\beta^4) = (0.0333\ldots\beta - 0.005158730\beta^3 + 0.0003306878\beta^5$$
$$-0.0000125761\beta^7 + \ldots) + (1/\gamma)(-0.3333\ldots\beta$$
$$+0.06666\ldots\beta^3 - 0.006349206\beta^5 + 0.0003527336\beta^7 - \ldots)$$
$$+(1/\gamma\xi)(0.1666\ldots\beta - 0.05833\ldots\beta^3 + 0.006150793\beta^5$$
$$-0.0003499779\beta^7 + \ldots) \tag{32b}$$

$$(F_{q3}/\beta^4) = (0.013888\ldots\beta^2 - 0.0013888\ldots\beta^4 + 0.000067791\beta^6$$
$$-0.000002112734\beta^8 + \ldots) + (1/\gamma)(0.50 - 0.1666\ldots\beta^2$$
$$+0.0222\ldots\beta^4 - 0.0015873015\beta^6 + 0.00007054673\beta^8 - \ldots)$$
$$+(1/\gamma\xi)(0.125\beta^2 - 0.0208333\ldots\beta^4 + 0.0015625\beta^6$$
$$-0.00007027116\beta^8 + \ldots) \tag{32c}$$

If the shear and extensional strain energies are neglected, γ must be assumed to be infinitely large. The denominator determinant and the numerator determinants defining the fixed end reactions can then be given in the following simplified form, calculated from expressions given earlier:

$$\Delta_q = 2r \cos(\beta/2)(\beta - \sin\beta) \tag{33a}$$

$$-\Delta_{n_q} = qr^3 \cos(\beta/2)\,[\beta(\beta + \sin\beta) - 8\sin^2(\beta/2)] \tag{33b}$$

$$\Delta_{r_q} = 2qr^2 \cos(\beta/2)\,[(2 + \cos\beta) - 3\sin\beta] \tag{33c}$$

$$-\Delta_{t_q} = 2qr^2 \sin(\beta/2)\,[-(\cos\beta + 1) + 2\sin\beta] \tag{33d}$$

A simple relation is found to exist between t_q and r_q

$$-t_q/(Lq) = \left\{1 - [r_q/(Lq)]\,\beta\right\} \tan(\beta/2)/\beta \qquad (33s)$$

For small angles β it is again necessary to expand the functions in power series. The final expressions for the fixed end reactions then become, after the resulting numeratprs are divided by the resulting denominator determinants:

$$-n_q/(L^2q) = \beta(0.008333\ldots + 0.00011904762\beta^2 + 0.00000099205\beta^4$$
$$-0.0000000004439\beta^6 - 0.000000000100\beta^8 - \ldots) \qquad (34a)$$

$$r_q/(Lq) = \beta(0.1 + 0.000238096\beta^2 - 0.0000079367\beta^4$$
$$-0.000000022934\beta^6 - 0.00000000120\beta^8 - \ldots) \qquad (34b)$$

$$-t_q/(Lq) = \left\{1 - [r_q/(Lq)]\beta\right\}[0.5 + 0.041666\ldots\beta^2 + 0.0041666\ldots\beta^4$$
$$+0.0004216269\beta^6 + 0.0000427138\beta^8 + \ldots] \qquad (34c)$$

Numerical values of the reactions were calculated and are presented in a graphical form. It was found that they are almost independent of the cross-sectional properties, especially in the case of r_q and t_q. Hence these two reactions were calculated only for the case when $\gamma = 1000$. Figures 92 and 93 contain the values of r_q and t_q plotted against β for the three conditions $\xi = 0.01$, 0.10, and 0.25. The variation of n_q was more apparent. For this reason values of n_q are plotted in figures 86 to 88 corresponding to five different values of the parameter γ. A cross plot of the results is presented in figures 89 to 91 in which the abscissa is β, the ordinate γ, and each curve corresponds to a constant value of n_q.

It would appear that the shear problem just discussed is statically determinate since there are three equations

and three unknowns. However, two of the equilibrium equations are interdependent. A useful relation can be obtained in the following way:

Equilibrium in the direction of T of all the forces acting upon the curved bar requires that

$$T(1 + \cos \beta) - R \sin \beta + [\sin \beta/\beta \ (Lq)] = 0 \qquad (35a)$$

In the direction of R

$$R(\cos \beta - 1) + T \sin \beta + (Lq) \ [(1 - \cos \beta)/\beta] = 0 \qquad (35b)$$

The condition of moment equilibrium about F is

$$2N + Lt \ [(1 - \cos \beta)/\beta] + LR \ (\sin \beta/\beta) + (L^2 q)[(\beta - \sin \beta)/\beta^2] = 0 \qquad (35c)$$

If R is eliminated from equation (35c) and either of equations (35a) or (35b), there results the simple expression

$$-\beta(N/L) = 0.5(Lq) + T \qquad (35d)$$

Hence, only one reaction due to shear need be computed from equations (31), (33), or (34) after which the other two may be readily found with the aid of equations (35). Equations not used can serve then for checking the numerical values. This procedure was adopted in the preparation of this report.

CONCLUSIONS

1. Influence coefficients are presented for curved bars the median line of which is an arc of a circle to be used in calculations by Southwell's procedure of systematic relaxations. The influence coefficient $(\widehat{tr})_F$ is defined as the reaction from the support upon the fixed end F of the curved bar in the tangential $(t-)$ direction caused by a unit displacement of the movable end M of the curved bar in the radial $(r-)$ direction. (See fig. 1.) The influence

coefficients $(\widehat{nn})_F$ $(\widehat{nr})_F$ $(\widehat{nt})_F$, $(\widehat{rn})_F$ $(\widehat{rr})_F$ $(\widehat{rt})_F$, $(\widehat{tn})_F$, and $(\widehat{tt})_F$ are defined in an analogous manner. Because of Maxwell's reciprocal theorem $(\widehat{nr})_F = (\widehat{rn})_F$, $(\widehat{nt})_F = (\widehat{tn})_F$, and $(\widehat{rt})_F = (\widehat{tr})_F$. The influence coefficient $(\widehat{tr})_M$ differs from $(\widehat{tr})_F$ insofar as it represents the reaction at the movable rather than the fixed end.

The influence coefficients depend upon the parameters

$$\beta, \quad \gamma = AL^2/I, \quad \text{and} \quad \xi = A^*/A$$

2. It is suggested that the numerical values of the influence coefficients for the movable end be obtained in one of the following four ways:

(a) Read the value of the influence coefficient from the appropriate diagram of figures 32 to 49 corresponding to the actual value of β and γ, and the closest approximate value of ξ. Expressions for ξ are given in equations (10), (11), (12), and (13).

(b) If γ happens to be close to one of the values 10, 100, 500, 1000, 10^4, or 10^6, greater accuracy may be had by using the appropriate diagram of figures 14 to 31. Deviations from the actual value of γ and ξ do not seem to have great effect upon the results of the displacement and stress calculations by the relaxation method, as long as the same value of γ (and ξ) is used for determining all the influence coefficients.

(c) If γ has one of the values listed under item (b), and at the same time the angle β subtended by the curved bar is equal to one of the values listed in table III, the influence coefficient can be obtained directly (and accurately) from the table.

(d) If it is desirable to obtain accurately the value of an influence coefficient in a case when β, γ, or ξ have values not listed in table III, equations (23) in conjunction with figures 3 to 13 may be used if the angle is not greater than 90°. If it is greater, equations (21) should be used. Instead of figures 3 to 13, table II can be used for greater accuracy if β happens to have a value listed in it.

3. The influence coefficients at the fixed end can be obtained in a like manner from figures 49 to 85 or table IV. For their calculation equations (27) can be used.

4. The reaction n_q is the moment acting from the support upon one end of the curved beam of circular median line caused by a uniform shear flow q flowing toward the same end. The radial reaction r_q and the tangential reaction t_q are defined in an analogous manner. (See fig. 1.) The reactions at the other end of the curved bar are equal in magnitude and opposed in sign. The reactions depend upon the same parameters as the influence coefficients.

5. It is suggested that the reactions be determined in one of the following four ways:

(a) Read the value of the reaction from the appropriate diagram of figures 86 to 93.

(b) Use equations (31) for the calculation of the reactions. If β is small, the values of the functions represented by the symbols F can best be calculated from the series expansions given in equations (32).

(c) For most purposes it is permissible to disregard the effect of the cross-sectional properties when the fixed end reactions of the uniform shear flow are determined. In such cases equations (33) can be used. If the angle β is small, these equations should be replaced by equations (34) obtained through series expansion.

In both methods suggested under items (b) and (c), equations (35) may also be employed.

(d) The following simplified formulas apply in good approximation:

$$-n_q/(L^2q) = 0.008333\ldots\beta\ (1 + 0.014286\beta^2)$$

$$r_q/(Lq) = 0.1\beta$$

$$-t_q/(Lq) = 0.5(1 - 0.01666\ldots\beta^2)$$

Polytechnic Institute of Brooklyn,
Brooklyn, N. Y., October 1944.

REFERENCE

1. Hoff, N. J., Libby, Paul A., and Klein, Bertram: Numerical Procedures for the Calculation of the Stresses in Monocoques. III - Calculation of the Bending Moments in Fuselage Frames. NACA TN No. 998, 1945.

Table I

Influence Functions F
(For Original Functions)

β (in degrees)	$F_{\Delta 1}$	$F_{\Delta 2}$	$F_{\Delta 3}$	$F_{\Delta 4}$	
60	0.28709519	-0.5235988	-0.19634955	0.4330127	
75	0.5607328	-0.9702037	-0.3053277	0.7159258	
90	0.9689462	-1.5707963	-0.3926991	1.0001	
120	2.2967615	-3.1415927	-0.3926991	1.2990381	
150	4.4858555	-4.8852433	-0.163624625	0.9330127	
180	7.75156928	-6.2831853	-0.0	0.0	
β	F_{nn1}	F_{nn2}	F_{nn3}	F_{nn4}	F_{nn5}
60	0.28709519	-0.3755128	-0.19634955	0.54797243	-0.2617994
75	0.5607328	-0.5336223	-0.3053277	1.0071267	-0.7190966
90	0.9689462	-0.5296087	-0.3926991	1.5707963	-1.5707963
120	2.2967615	0.7947103	-0.3926991	2.5204995	-4.7123892
150	4.4858555	5.5447647	-0.1636246	2.1384097	-9.1159881
180	7.75156928	15.503139	0.0	0.0	-12.566371

β	F_{rn1}	F_{rn2}	F_{rn3}	F_{tn1}	F_{tn2}	F_{tn3}
60	0.2870952	0.1932170	-0.4748516	-0.10402739	-0.1626019	0.2741557
75	0.8312089	0.4329818	-1.226720	-0.3847425	-0.5189796	0.9412962
90	1.937892	0.7041918	-2.467401	-1.1061418	-1.233701	2.467401
120	6.890285	0.5960325	-5.698219	-5.6425451	-3.633988	9.8696054
150	16.741468	-0.9544533	-6.394769	-19.002061	-4.7416075	23.865605
180	31.0062767	0 0	0.0	-48.704546	0.0	39.4784176

Table I
(contd)

Influence Functions F
(For Original Functions)

β (in degrees)	F_{rr1}	F_{rr2}	F_{rr3}	F_{tr1}	F_{tr2}
60	0.62967039	0.2 6036657	-0.86128575	-0.4509682	0.4972636
75	1.9216011	0.3669987	-2.0926831	-1.369658	1.605772
90	4.7815577	0	-3.8757845	-3.044034	3.8757845
120	20.149448	-4.16586509	-6.8902845	-7.215489	11.934323
150	61.491148	-10.1705687	-4.48586275	-5.8719818	16.74147
180	153.00984	0	0	0	0

β	F_{tt1}	F_{tt2}	F_{tt3}
60	0.62967039	-0.26036657	-0.2870953
75	1.9216011	-0.3669987	-1.2321526
90	4.7815577	0	-3.8757845
120	20.149448	4.16586509	-20.670854
150	61.491148	10.1705687	-62.480008
180	153.00984	0	-124.025108

Table II

Influence Functions f
(For Series Expansions)

β (in degrees)	$f_{\Delta 1}$ x 10^6	$f_{\Delta 2}$ x 10^2	$f_{\Delta 3}$ x 10^4	$f_{\Delta 4}$ x 10^3	f_{nn1} x 10^5	f_{nn2}	f_{nn3} x 10^4	f_{nn4} x 10^3	f_{rn1} x 10^4	f_{rn2}	f_{rn3} x 10^3	
0	0	8.333...	0	0	0	0.333...	0	0	0	0.5	0	
5	0.8808	8.3249	0.6341	0.6340	0.79264	0.33293	2.7474	0.6340	0.31699	0.49905	0.63373	
15	7.8836	8.2576	5.6713	5.6596	7.0886	0.32974	24.537	5.6596	2.8303	0.49151	5.6402	
30	30.984	8.0354	22.209	22.027	27.751	0.31923	95.628	22.027	11.020	0.46696	21.723	
45	67.697	7.6815	48.236	47.358	60.230	0.30260	206.04	47.358	23.700	0.42895	45.870	
60	115.23	7.2185	81.613	79.021	101.79	0.27611	344.66	79.021	39.540	0.38140	74.546	
75	171.18	6.6750	119.67	113.87	149.01	0.24839	497.95	113.87	56.900	0.32894	103.62	
90	230.98	6.083	159.75	148.68	198.08	0.21830	651.46	148.68	74.023	0.27627	129.01	
β	f_{tn1} x 10^4	f_{tn2} x 10^2	f_{tn3} x 10^3		f_{rr1} x 10^3	f_{rr2} x 10^2		f_{tr1} x 10^2	f_{tr2} x 10^2		f_{tt1} x 10^2	f_{tt2} x 10^2
0	0	0	0		0	0		0	0		8.333...	0
5	6.0486	2.1784	7.2465		0.16905	0.25346		0.36296	4.3523		8.3143	0.25346
15	17.871	6.4584	21.100		1.5082	2.2535		1.0735	12.7936		8.1635	2.2535
30	33.925	12.4110	58.032		5.8580	8.6503		2.0455	23.873		7.6718	8.6503
45	46.546	17.420	47.279		12.552	18.169		2.8270	31.831		6.9090	18.169
60	54.500	21.172	46.512		20.819	29.325		3.3523	35.810		5.9639	29.325
75	57.056	23.515	34.928		29.751	40.450		3.5825	35.638		4.9630	40.452
90	54.056	24.467	13.337		38.389	50.081		3.5249	31.829		4.1242	50.007

Table IIIa

Influence Coefficient $\widehat{mm}_{\alpha\alpha}(L/EI)$
(For Movable End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	1.517	1.518	1.521	1.532	1.546	1.560	1.572	1.580	1.578	1.550	1.495
100	3.027	3.032	3.069	3.181	3.327	3.473	3.597	3.684	3.759	3.699	3.562
500	3.737	3.763	3.956	4.461	5.007	5.45	5.757	5.945	6.058	5.930	5.634
1000	3.863	3.914	4.282	5.137	5.894	6.407	6.707	6.860	6.884	6.673	6.301
10^4	3.986	4.462	6.393	7.842	8.284	8.407	8.406	8.335	8.066	7.668	7.161
10^6	4.000	8.649	8.938	8.943	8.888	8.812	8.701	8.572	8.239	7.808	7.278
	$\xi = (A^*/A) = 0.10$										
10	1.231	1.231	1.235	1.246	1.258	1.268	1.275	1.279	1.279	1.267	1.242
100	2.364	2.369	2.407	2.508	2.618	2.707	2.768	2.801	2.803	2.743	2.595
500	3.419	3.445	3.632	4.075	4.489	4.778	4.952	5.042	5.049	4.952	4.637
1000	3.679	3.730	4.087	4.845	5.444	5.799	5.986	6.092	6.060	5.906	5.524
10^4	3.964	4.444	6.343	7.715	8.130	8.260	8.236	8.170	7.907	7.532	7.026
10^6	4.000	8.562	8.937	8.939	8.886	8.803	8.701	8.625	8.237	7.806	7.276
	$\xi = (A^*/A) = 0.01$										
10	1.025	1.025	1.028	1.040	1.032	1.032	1.032	1.035	1.032	1.030	1.028
100	1.231	1.236	1.261	1.288	1.299	1.300	1.305	1.305	1.300	1.287	1.262
500	1.882	1.907	2.029	2.154	2.203	2.221	2.224	2.22	2.210	2.157	2.039
1000	2.364	2.413	2.648	2.878	2.963	2.992	2.996	2.986	2.932	2.848	2.685
10^4	3.679	4.131	5.561	6.340	6.529	6.567	6.542	6.481	6.282	6.056	5.622
10^6	4.000	8.543	8.951	8.910	8.857	8.781	8.672	8.542	8.211	7.784	7.255

Table IIIb

Influence Coefficient $-\widehat{rn}_M(L^2/EI)$
(For Movable End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	1.034	1.037	1.055	1.109	1.179	1.246	1.293	1.312	1.246	1.055	.7806
100	4.054	4.082	4.2997	4.944	5.690	6.574	7.212	7.621	7.742	7.076	5.918
500	5.474	5.627	6.769	9.767	12.95	15.44	17.07	17.92	17.91	16.38	13.88
10^3	5.725	6.033	8.236	13.31	17.73	20.60	22.12	22.70	22.04	19.86	16.75
10^4	5.971	8.830	20.38	28.97	31.43	31.90	31.56	30.73	28.15	24.69	20.58
10^6	6.000	33.39	35.58	35.49	34.96	34.22	33.21	32.03	29.06	25.38	21.10
	$\xi = (A^*/A) = 0.10$										
10	.4615	.4644	.4856	.5426	.6038	.6497	.6737	.6759	.6238	.5069	.3621
10^2	2.727	2.757	2.976	3.554	4.163	4.621	4.887	4.973	4.722	4.0881	3.137
500	4.839	4.991	6.095	8.721	11.11	12.70	13.56	13.87	13.37	12.08	9.839
10^3	5.357	5.664	7.996	12.29	15.77	17.79	18.72	18.95	18.07	16.26	13.42
10^4	5.928	8.772	20.09	28.30	30.60	31.03	30.66	29.85	27.34	24.02	19.97
10^6	6.000	33.38	35.57	35.47	34.95	34.18	33.21	32.25	29.05	25.37	21.103
	$\xi = (A^*/A) = 0.01$										
10	.0496	.0525	.0675	.0839	.08845	.0891	.0875	.0842	.0738	.0576	.0401
10^2	.4615	.4910	.6395	.7936	.8462	.8452	.8377	.8067	.7088	.5478	.3938
500	1.765	1.912	2.636	3.359	3.602	3.641	3.580	3.453	3.062	2.558	1.832
10^3	2.727	3.0207	4.423	5.762	6.201	6.277	6.180	5.976	5.337	4.481	3.372
10^4	5.357	8.068	16.63	21.21	22.21	22.23	21.83	21.16	19.30	16.99	13.84
10^6	6.000	33.24	35.41	35.30	34.79	34.06	33.06	31.87	28.92	25.26	21.01

Table IIIc

Influence Coefficient $\widehat{tn}_M(L^2/EI)$
(For Movable End)

β(in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	0	.02736	.07691	.1216	.1139	.04994	-.06202	-.2077	-.5410	-.8484	-1.059
100	0	.5474	1.574	2.734	3.180	3.176	2.611	1.737	-.3996	-2.500	-4.251
500	0	3.367	9.427	15.24	16.87	15.69	13.11	10.02	3.809	-1.526	-5.718
1000	0	6.925	18.67	27.49	27.76	24.11	19.29	14.45	5.901	-.8587	-5.988
10^4	0	65.32	109.0	84.99	60.20	43.21	31.18	22.12	9.144	.1739	-6.252
10^6	0	627.3	224.3	109.19	68.65	47.16	33.30	23.39	9.637	.3279	-6.282
	$\xi = (A^*/A) = 0.10$										
10	0	.05208	.1449	.2264	.2287	.1715	.08111	-.02358	-.2345	-.4114	- .5299
100	0	.6025	1.628	2.664	2.828	2.424	1.730	.9255	-.6728	-2.040	-3.013
500	0	3.381	9.194	13.79	14.07	12.19	9.555	6.822	1.864	-2.162	-5.162
1000	0	6.913	18.14	25.11	23.98	19.96	15.44	11.19	4.002	-1.545	-5.668
10^4	0	65.06	107.2	82.64	58.30	41.78	30.22	21.30	8.721	.01998	-6.216
10^6	0	627.1	224.2	109.13	68.63	47.10	33.30	23.54	9.632	.3262	-6.283
	$\xi = (A^*/A) = 0.01$										
10	0	.06618	.1315	.1175	.08074	.05087	.02697	.007791	-.02404	-.04638	-.06241
100	0	.6647	1.312	1.160	.8086	.5118	.2848	.09447	-.2056	-.4278	-.5728
500	0	3.323	6.471	5.660	3.998	2.658	1.616	.7762	-.5191	-1.492	-2.099
1000	0	6.648	12.66	10.89	7.732	5.265	3.381	1.886	-.3828	-2.033	-3.147
10^4	0	61.94	85.23	58.40	39.52	27.65	19.47	13.38	4.668	-1.395	-5.713
10^6	0	624.0	223.0	108.5	68.26	46.88	33.11	23.23	9.561	.3007	-6.277

Table IIId

Influence Coefficient $\widetilde{rr}_M(L^3/EI)$
(For Movable End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	2.069	2.085	2.210	2.587	3.097	3.618	4.058	4.355	4.542	4.276	3.852
100	8.108	8.286	9.659	13.74	18.54	24.22	28.51	31.55	33.80	32.32	29.20
500	10.95	11.88	18.80	37.02	56.34	71.45	81.28	86.49	87.09	79.67	68.51
10^3	11.45	13.31	26.62	57.20	83.79	100.9	109.9	113.3	109.4	97.77	82.67
10^4	11.94	29.10	98.43	149.8	164.3	166.6	164.7	158.9	142.8	122.9	106.7
10^6	12.00	176.3	189.4	188.6	185.2	180.2	173.7	166.1	147.8	126.5	104.2
	$\xi = A^*/A = 0.10$										
10	.9231	.9408	1.074	1.433	1.829	2.143	2.340	2.424	2.343	2.053	1.787
100	5.455	5.636	6.991	10.57	14.40	17.37	19.24	20.13	19.81	17.99	15.49
500	9.677	10.60	17.30	33.23	47.76	57.49	62.87	65.01	63.15	57.48	48.55
10^3	10.71	12.57	25.44	52.54	73.51	85.62	93.32	92.72	88.17	79.09	66.24
10^4	11.85	28.94	96.86	145.9	159.5	161.7	159.8	153.8	138.4	119.4	98.55
10^6	12.00	176.3	189.3	188.5	185.1	179.96	173.8	167.2	147.7	126.5	104.1
	$\xi = A^*/A = 0.01$										
10	0.09917	.1169	.2069	.3042	.3355	.3423	.3365	.3238	.2842	.2381	.1963
100	0.9230	1.100	1.995	2.926	3.261	3.298	3.275	3.15	2.777	2.334	1.943
500	3.529	4.415	8.774	13.14	14.68	15.01	14.79	14.27	12.69	11.01	9.042
10^3	5.455	7.220	15.66	23.73	26.48	27.07	26.71	25.8	23.10	19.95	16.64
10^4	10.71	26.98	78.37	105.8	111.8	111.8	109.3	105.4	94.76	82.85	68.27
10^6	12.00	175.5	188.4	187.5	184.2	179.2	172.8	165.2	147.0	125.9	103.66

Table IIIe

Influence Coefficient $-\widehat{tr}_M(L^3/EI)$
(For Movable End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	0	.3447	1.002	1.808	2.294	2.442	2.307	1.979	1.099	.3523	0
100	0	3.991	11.55	20.56	25.04	27.07	25.58	22.28	13.62	5.576	0
500	0	21.16	59.36	96.74	108.7	103.7	90.36	74.09	42.38	17.20	0
1000	0	42.54	114.9	170.2	173.7	153.7	127.0	100.3	54.93	21.86	0
10^4	0	392.9	656.6	513.8	366.7	266.8	196.9	145.1	73.85	28.43	0
10^6	0	3764	1347	658.4	417.0	290.1	209.4	152.5	76.69	29.37	0
	$\xi = (A^*/A) = 0.10$										
10	0	.3931	1.110	1.837	2.085	1.982	1.690	1.326	.6338	.1705	0
100	0	4.091	11.49	18.77	21.07	19.96	17.16	13.78	7.327	2.621	0
500	0	21.13	57.64	87.39	91.12	81.79	68.08	53.94	29.38	11.62	0
1000	0	42.41	111.5	155.5	150.7	128.6	103.66	80.52	43.13	17.05	0
10^4	0	391.3	645.3	499.8	355.4	258.3	191.3	140.3	71.29	27.50	0
10^6	0	3763	1347	658.0	416.8	289.7	209.3	153.6	76.66	29.36	0
	$\xi = (A^*/A) = 0.01$										
10	0	.4057	.8143	.7546	.5581	.4051	.2911	.2162	.0835	.01911	0
100	0	4.056	8.107	7.413	5.532	3.984	2.892	2.031	.8597	.2109	0
500	0	20.25	39.71	35.65	26.52	19.37	14.07	10.06	4.560	1.423	0
1000	0	40.36	77.32	67.90	50.19	36.72	26.84	19.39	9.147	3.077	0
10^4	0	372.6	513.6	354.4	243.0	173.9	127.2	93.18	46.90	18.03	0
10^6	0	3745	1340	654.5	414.6	288.5	208.2	151.5	76.25	29.20	0

Table IIIf

Influence Coefficient $\widehat{tt}_M(L^3/EI)$
(For Movable End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\xi = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	10	9.965	9.691	8.851	7.686	6.438	5.294	4.355	3.274	3.056	3.326
100	100	99.53	95.89	84.96	68.61	55.49	42.22	31.55	18.06	13.00	13.35
500	500	495.5	461.9	372.1	273.8	191.2	129.7	86.49	38.15	20.09	17.97
1000	1000	985.8	884.2	646.7	431.2	278.5	178.0	113.3	46.01	22.04	18.81
10^4	10^4	9011	4999	1930	897.7	474.7	269.6	158.9	57.61	24.48	19.64
10^6	10^6	86225	10245	2469	1019	515.2	285.9	166.1	59.24	24.81	19.67
	$\xi = (A^*/A) = 0.10$										
10	10	9.925	9.358	7.796	6.000	4.432	3.245	2.424	1.611	1.469	1.495
100	100	99.15	92.76	75.58	56.53	40.42	28.44	20.13	11.35	8.911	9.464
500	500	493.6	447.5	336.0	230.0	151.9	99.35	65.01	29.22	17.24	16.22
1000	1000	982.1	857.8	591.3	374.9	234.2	148.6	92.72	38.37	20.03	17.81
10^4	10^4	8975	4914	1877	870.3	459.9	262.3	153.8	55.99	24.16	19.53
10^6	10^6	86196	10242	2468	1019	514.5	285.8	167.2	59.22	24.81	19.74
	$\xi = (A^*/A) = 0.01$										
10	10	9.390	6.285	2.919	1.452	0.8068	0.4919	0.3468	0.1878	0.1669	0.1947
100	100	93.82	62.51	28.61	14.33	7.899	4.825	3.150	1.784	1.521	1.800
500	500	467.2	305.1	136.7	67.71	37.38	22.33	14.27	7.422	6.081	6.594
1000	1000	929.9	592.8	259.0	126.9	69.48	41.09	25.81	12.54	9.292	9.885
10^4	10^4	8543	3912	1333	597.8	312.6	177.5	105.4	40.60	20.40	17.95
10^6	10^6	85782	10187	2455	1013	512.3	284.3	165.2	58.97	24.75	19.72

Table IVa

Influence Coefficient $\widehat{m}_F(L/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	.4828	.4834	.4882	.5036	.5262	.5532	.5823	.6122	.6745	.7433	.8212
100	-1.027	-1.021	-.9766	-.8406	-.66	-.4474	-.2467	-.0622	.2719	.5657	.8565
500	-1.737	-1.710	-1.509	-.9658	-.3587	.172	.585	.8841	1.380	1.7152	1.993
1000	-1.863	-1.810	-1.430	-.5377	.2854	.881	1.302	1.602	1.997	2.267	2.488
10^4	-1.986	-1.508	.434	1.923	2.433	2.656	2.771	2.856	2.975	3.077	3.180
10^6	-2.000	2.653	2.950	2.995	3.010	3.032	3.052	3.068	3.124	3.195	3.279
	$\xi = (A^*/A) = 0.10$										
10	.7692	.7698	.7739	.7855	.7994	.8125	.8239	.8341	.8530	.8740	.905
100	-.3636	-.3582	-.3236	-.2047	-.0755	.0424	.1409	.2246	.3686	.5086	.6768
500	-1.419	-1.393	-1.197	-.7255	-.2647	.0896	.357	.5565	.8565	1.108	1.351
1000	-1.679	-1.626	-1.259	-.468	.1884	.617	.9171	1.149	1.453	1.698	1.916
10^4	-1.964	-1.480	.4268	1.839	2.322	2.535	2.630	2.727	2.848	2.931	3.069
10^6	-2.000	2.569	2.948	2.993	3.009	3.026	3.056	3.082	3.123	3.194	3.277
	$\xi = (A^*/A) = 0.01$										
10	.9752	.9757	.9782	.9904	.9823	.9828	.9831	.9825	.9791	.9860	.9882
100	.7692	.7742	.7995	.8270	.8389	.8455	.8485	.8515	.8597	.877	.8975
500	.1177	.08252	.2658	.3949	.4507	.4787	.4973	.5146	.5546	.6056	.7030
1000	-.3636	-.3142	-.07614	.1628	.2634	.3149	.3498	.3821	.4511	.5433	.6818
10^4	-1.679	-1.226	.2143	1.026	1.272	1.382	1.453	1.528	1.657	1.817	1.985
10^6	-2.000	2.552	2.929	2.971	2.985	2.998	3.024	3.039	3.101	3.174	3.259

Table IVb

Influence Coefficient $\widehat{rm}_F(L^2/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = \Lambda L^2/I$	$\xi = (\Lambda^*/\Lambda) = 0.25$										
10	1.034	1.031	.9961	.8996	.7533	.5798	.3946	.2077	-.1543	-.4897	-.7806
100	4.054	4.019	3.746	2.912	1.775	.5365	-.6552	-1.737	-3.525	-4.878	-5.918
500	5.474	5.312	4.098	.8360	-2.776	-5.868	-8.246	-10.02	-12.25	-13.42	-13.88
1000	5.725	5.407	3.124	-2.221	-7.091	-10.58	-12.91	-14.45	-16.13	-16.77	-16.75
10^4	5.971	3.097	-8.531	-17.41	-20.29	-21.47	-21.95	-22.12	-21.99	-21.47	-20.58
10^6	6.000	-21.58	-23.49	-23.86	-23.82	-23.73	-23.57	-23.39	-22.88	-22.14	-21.10
	$\xi = (\Lambda^*/\Lambda) = 0.10$										
10	0.4615	.4622	.4319	.3567	.2653	.1763	.0960	.0236	-.1088	-.2333	-.3621
100	2.727	2.694	2.441	1.746	.9437	.2108	-.4063	-.9255	-1.778	-2.520	-3.137
500	4.839	4.678	3.508	.6600	-2.097	-4.205	-5.720	-6.822	-8.297	-9.362	-9.839
1000	5.357	5.040	2.824	-1.909	-5.804	-8.391	-10.07	-11.19	-12.50	-13.31	-13.42
10^4	5.928	3.088	-8.288	-16.82	-19.59	-20.67	-21.25	-21.30	-21.22	-20.81	-19.97
10^6	6.000	-21.49	-23.61	-23.85	-23.81	-23.70	-23.57	-23.54	-22.87	-22.13	-21.10
	$\xi = (\Lambda^*/\Lambda) = 0.01$										
10	0.0496	.04662	.03084	.01386	.00546	.00050	-.00341	-.007791	-.01608	-.02666	-.0401
100	.4615	.4323	.2772	.1073	.02662	-.02063	-.05829	-.09447	-.1764	-.2605	-.3938
500	1.765	1.623	.8645	.0791	-.2794	-.4813	-.6343	-.7762	-1.081	-1.469	-1.832
1000	2.727	2.418	.9956	-.4555	-1.082	-1.421	-1.666	-1.886	-2.337	-2.864	-3.372
10^4	5.357	2.639	-5.997	-10.83	-12.24	-12.83	-13.16	-13.38	-13.68	-14.01	-13.84
10^6	6.000	-21.27	-23.52	-23.70	-23.66	-23.57	-23.42	-23.23	-22.74	-22.02	-21.01

Table IVc

Influence Coefficient $\widehat{tn}_F(L^2/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	0	.1176	.3473	.6598	.9145	1.104	1.233	1.312	1.349	1.262	1.059
100	0	.9011	2.633	4.840	6.226	7.281	7.641	7.621	6.904	5.703	4.251
500	0	3.845	10.86	18.08	21.09	21.22	19.88	17.92	13.61	9.509	5.718
1000	0	7.424	20.17	30.46	32.17	29.90	26.36	22.70	16.14	10.67	5.988
10^4	0	65.84	110.6	88.09	64.80	49.23	38.55	30.73	19.81	12.19	6.252
10^6	0	627.8	225.8	112.3	73.27	53.22	40.70	32.03	20.35	12.40	6.282
	$\xi = (A^*/A) = 0.10$										
10	0	.09236	.2656	.4674	.5887	.6484	.6717	.6759	.6575	.6097	.5299
100	0	.8405	2.342	4.084	4.943	5.214	5.168	4.973	4.426	3.811	3.013
500	0	3.804	10.46	16.30	17.81	17.09	15.57	13.87	10.64	7.902	5.162
1000	0	7.381	19.54	27.89	28.10	25.39	22.08	18.95	13.65	9.470	5.668
10^4	0	65.57	108.7	85.72	62.86	47.76	36.86	29.85	19.32	12.03	6.216
10^6	0	627.57	225.8	112.2	73.24	53.15	40.70	32.25	20.34	12.40	6.283
	$\xi = (A^*/A) = 0.01$										
10	0	.07050	.1445	.1437	.1196	.1026	.09147	.0842	.07593	.06895	.06241
100	0	.7049	1.433	1.401	1.170	.9879	.8829	.8067	.7166	.6444	.5728
500	0	3.483	6.933	6.581	5.374	4.482	3.876	3.455	2.911	2.571	2.099
1000	0	6.886	13.38	12.31	9.852	8.069	6.845	5.976	4.814	4.002	3.147
10^4	0	62.40	86.63	61.12	43.65	33.08	26.12	21.16	14.35	9.700	5.713
10^6	0	624.6	224.6	111.7	72.87	52.93	40.51	31.87	20.26	12.37	6.277

Table IVd

Influence Coefficient $\widehat{rr}_F(L^3/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	-2.068	-2.047	-1.875	-1.336	-0.5676	0.3061	1.178	1.979	3.223	3.879	3.852
100	-8.110	-7.906	-6.340	-1.615	4.595	11.34	17.33	22.28	28.69	30.78	29.20
500	-10.95	-9.988	-2.798	16.31	37.03	54.09	66.24	74.09	80.25	77.60	68.51
10^3	-11.45	-9.553	4.026	35.54	63.58	82.67	94.21	100.3	102.3	95.59	82.61
10^4	-11.94	5.253	74.86	127.2	143.1	147.8	147.5	145.1	135.4	120.7	101.6
10^6	-12.00	152.4	165.7	165.9	163.9	161.2	157.3	152.5	140.3	124.3	104.2
	$\xi = (A^*/A) = 0.10$										
10	-0.924	-0.9029	-0.7499	-0.3225	0.1812	0.6449	1.027	1.326	1.720	1.863	1.787
100	-5.46	-5.258	-3.7789	0.2279	4.714	8.603	11.59	13.78	16.25	16.89	15.49
500	-9.68	-8.722	-1.795	14.92	30.66	42.09	49.49	53.94	57.2	55.59	48.55
10^3	-10.99	-8.825	4.294	32.27	54.59	68.56	75.97	80.52	81.44	77.02	66.24
10^4	-11.85	5.275	73.47	123.5	138.5	142.9	143.4	140.3	130.9	117.2	98.55
10^6	-12.00	152.3	165.7	165.8	163.9	160.9	157.2	153.6	140.3	124.2	104.1
	$\xi = (A^*/A) = 0.01$										
10	-0.099	-0.0811	0.0109	0.1116	0.1574	0.1797	0.1940	0.2162	0.2144	0.2114	0.1963
100	-0.924	-0.7430	0.1708	1.172	1.606	1.801	1.946	2.0311	2.133	2.055	1.943
500	-3.528	-2.634	1.8011	6.444	8.374	9.276	9.765	10.06	10.29	10.25	9.042
10^3	-5.46	-3.674	4.886	13.39	16.77	18.27	19.01	19.39	19.47	18.82	16.64
10^4	-10.72	5.586	57.23	85.57	92.80	94.75	94.55	93.18	87.99	80.77	68.27
10^6	-12.00	151.6	164.7	164.9	162.9	160.3	156.3	151.5	139.5	123.7	103.7

Table IVe

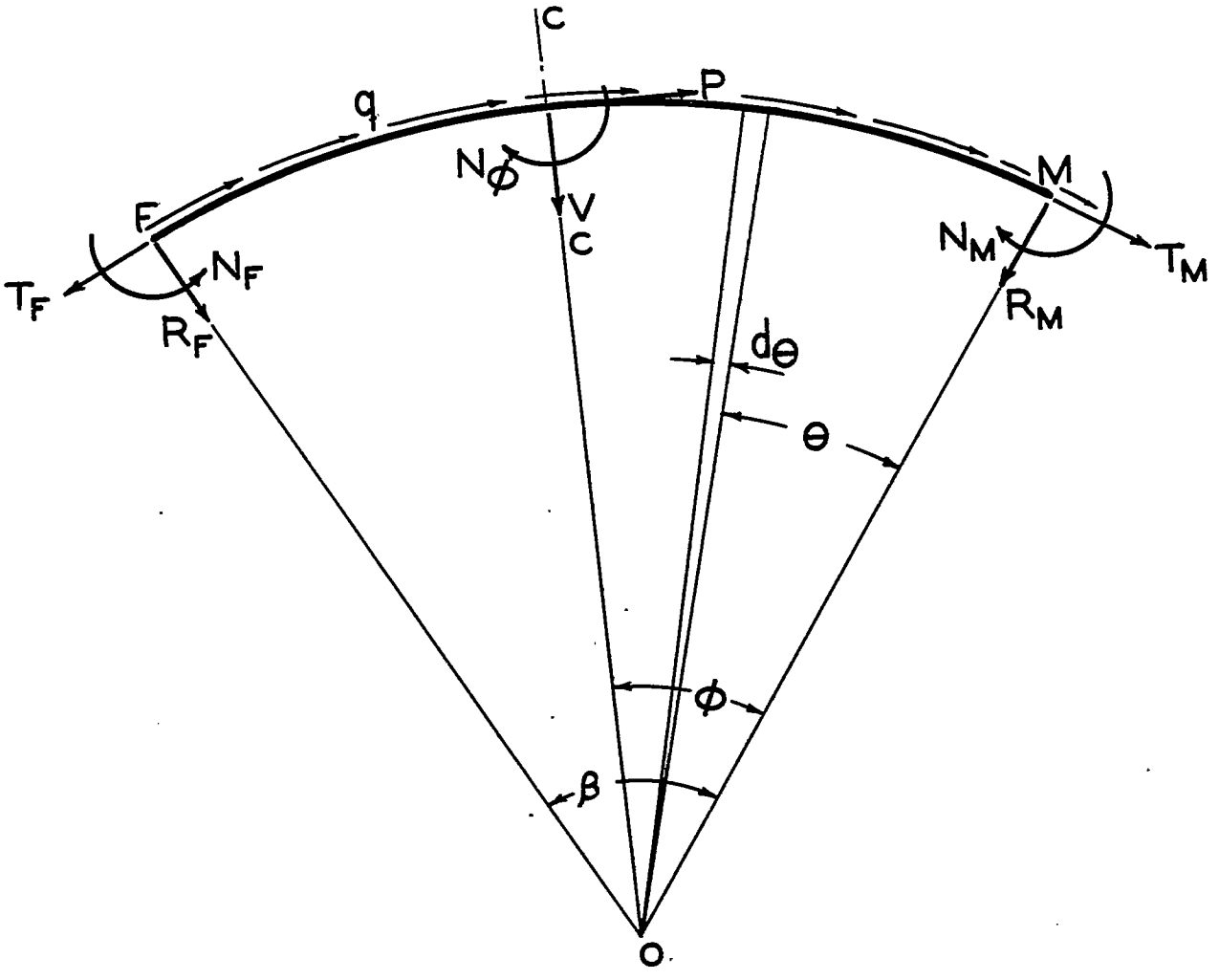
Influence Coefficient $-\widehat{tr}_F(L^3/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$	$\xi = (A^*/A) = 0.25$										
10	0	.5251	1.540	2.859	3.812	4.354	4.517	4.355	3.384	1.833	0
100	0	4.698	13.66	24.67	30.81	34.52	34.16	31.55	22.46	11.33	0
500	0	22.11	62.20	102.3	118.7	113.7	101.9	86.49	54.23	24.94	0
1000	0	43.54	117.9	176.0	182.1	164.2	139.0	113.3	67.31	29.95	0
10^4	0	393.95	659.7	519.9	375.5	277.7	209.4	158.9	86.82	36.85	0
10^6	0	3765	1350	664.3	425.8	301.1	222.0	166.1	89.65	37.84	0
	$\xi = (A^*/A) = 0.10$										
10	0	.4734	1.350	2.308	2.768	2.847	2.697	2.424	1.712	.8821	0
100	0	4.566	12.91	21.54	25.08	25.03	23.03	20.13	13.49	6.726	0
500	0	21.97	60.15	92.30	98.20	90.69	78.34	65.01	40.00	18.68	0
1000	0	43.34	114.3	161.0	158.6	138.5	116.7	92.72	54.80	24.78	0
10^4	0	392.3	648.4	505.8	364.1	269.1	203.8	153.8	84.14	35.90	0
10^6	0	3764	1350	664.1	425.6	300.7	221.9	167.2	89.62	37.83	0
	$\xi = (A^*/A) = 0.01$										
10	0	.4143	.8401	.8060	.6318	.4961	.4001	.3238	.2044	.1025	0
100	0	4.136	8.347	7.883	6.218	4.849	3.912	3.150	1.975	.9844	0
500	0	20.55	40.62	37.46	29.13	22.68	17.93	14.27	8.708	4.273	0
1000	0	40.84	78.74	70.67	54.22	41.81	32.75	25.81	15.43	7.311	0
10^4	0	373.4	516.4	359.8	250.9	183.8	138.5	105.4	58.61	25.81	0
10^6	0	3746	1343	660.6	423.4	299.4	220.8	165.2	89.19	37.67	0

Table IVf

Influence Coefficient $\widehat{tt}_p(L^3/EI)$
(For Fixed End)

β (in degrees)	0°	5°	15°	30°	45°	60°	75°	90°	120°	150°	180°
$\gamma = AL^2/I$											
	$\xi = (A^*/A) = 0.25$										
10	10	9.957	9.620	8.570	7.057	5.334	3.599	1.979	-0.6855	-2.470	-3.326
100	100	99.50	96.61	83.86	66.22	51.19	35.63	22.28	2.767	-8.474	-13.35
500	500	49.55	461.5	370.7	270.5	185.4	120.9	74.09	17.63	-8.798	-17.97
10^3	10^3	985.8	883.8	645.1	427.7	272.3	168.7	100.3	24.57	-8.155	-18.81
10^4	10^4	9011.	4999.	1958.	894.1	468.4	259.9	145.1	35.15	-6.987	-19.64
10^6	10^6	86225.	10245.	2468.	1015.	508.8	276.2	152.5	36.80	-6.804	-19.67
	$\xi = (A^*/A) = 0.10$										
10	10	9.922	9.326	7.670	5.717	3.933	2.472	1.326	-0.2568	-1.188	-1.495
100	100	99.13	92.58	74.84	54.87	37.50	23.94	13.78	0.6702	-6.407	-9.464
500	500	493.6	447.2	334.7	227.1	146.8	91.47	53.94	10.83	-9.117	-16.22
10^3	10^3	982.0	857.4	589.9	371.7	228.5	138.6	80.52	18.16	-8.817	-17.81
10^4	10^4	8974.	4913.	1876.	866.7	453.7	252.6	140.3	33.74	-7.176	-19.53
10^6	10^6	86196.	10241.	2466.	1015.	508.2	276.1	153.6	36.78	-6.806	-19.74
	$\xi = (A^*/A) = 0.01$										
10	10	9.390	6.281	2.905	1.421	0.7542	0.4085	0.2162	-0.0216	-0.1350	-0.1947
100	100	93.82	62.47	28.48	14.04	7.399	4.042	2.031	-1.476	-1.212	-1.800
500	500	467.2	305.0	136.2	66.63	35.47	19.37	10.06	0.2380	-4.555	-6.594
10^3	10^3	929.9	592.6	258.2	125.2	66.54	36.56	19.39	1.652	-6.508	-9.885
10^4	10^4	8543.	3912.	1332.	594.5	306.9	168.8	93.18	20.32	-8.650	-17.95
10^6	10^6	85782.	10187.	2453.	1010.	506.1	274.7	151.5	36.21	-6.837	-19.72



FORCES AND MOMENTS ACTING ON CURVED BAR.

FIG. 1.

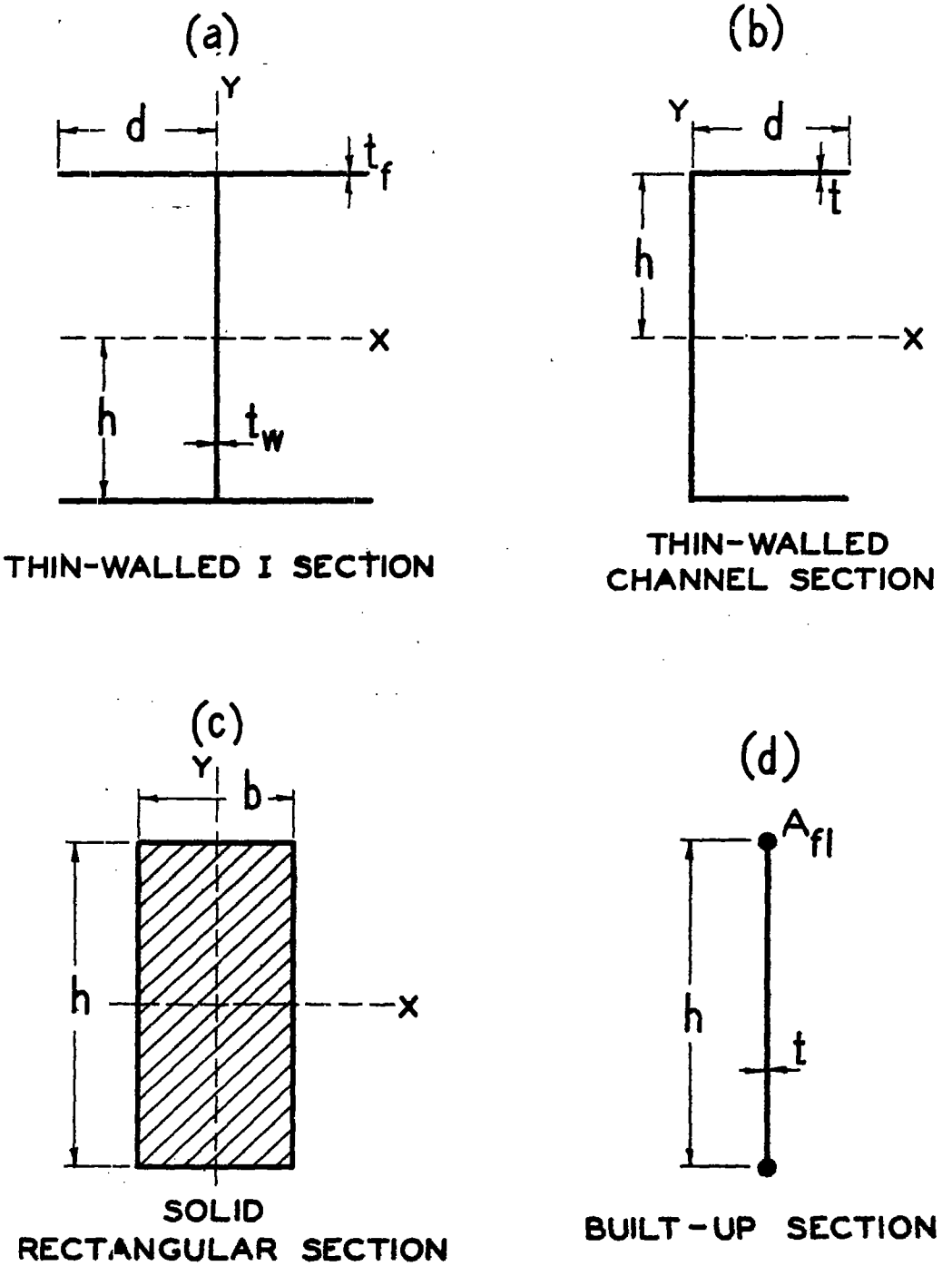


FIG. 2. COMMON SECTIONS

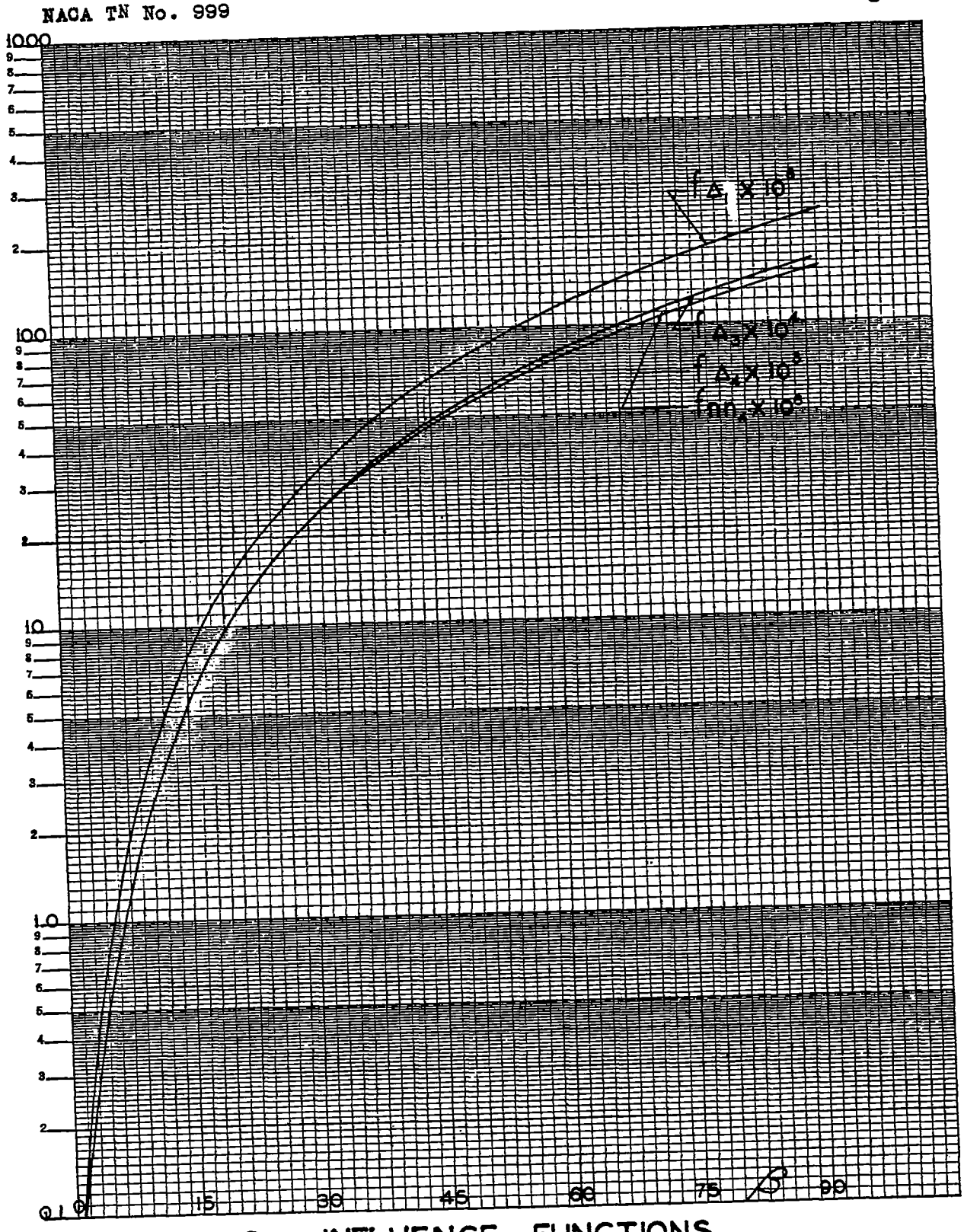


FIG. 3. INFLUENCE FUNCTIONS

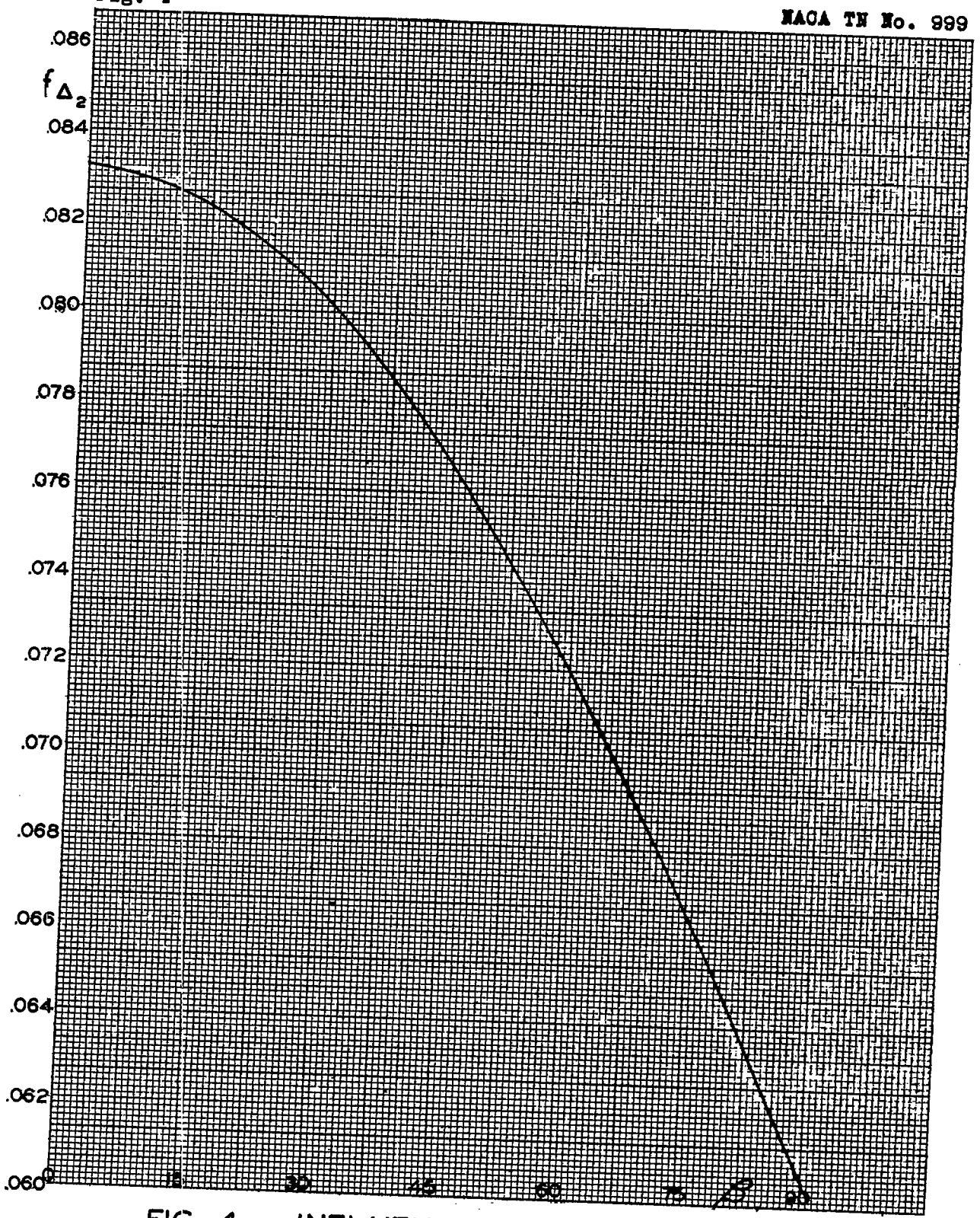


FIG. 4. INFLUENCE FUNCTION

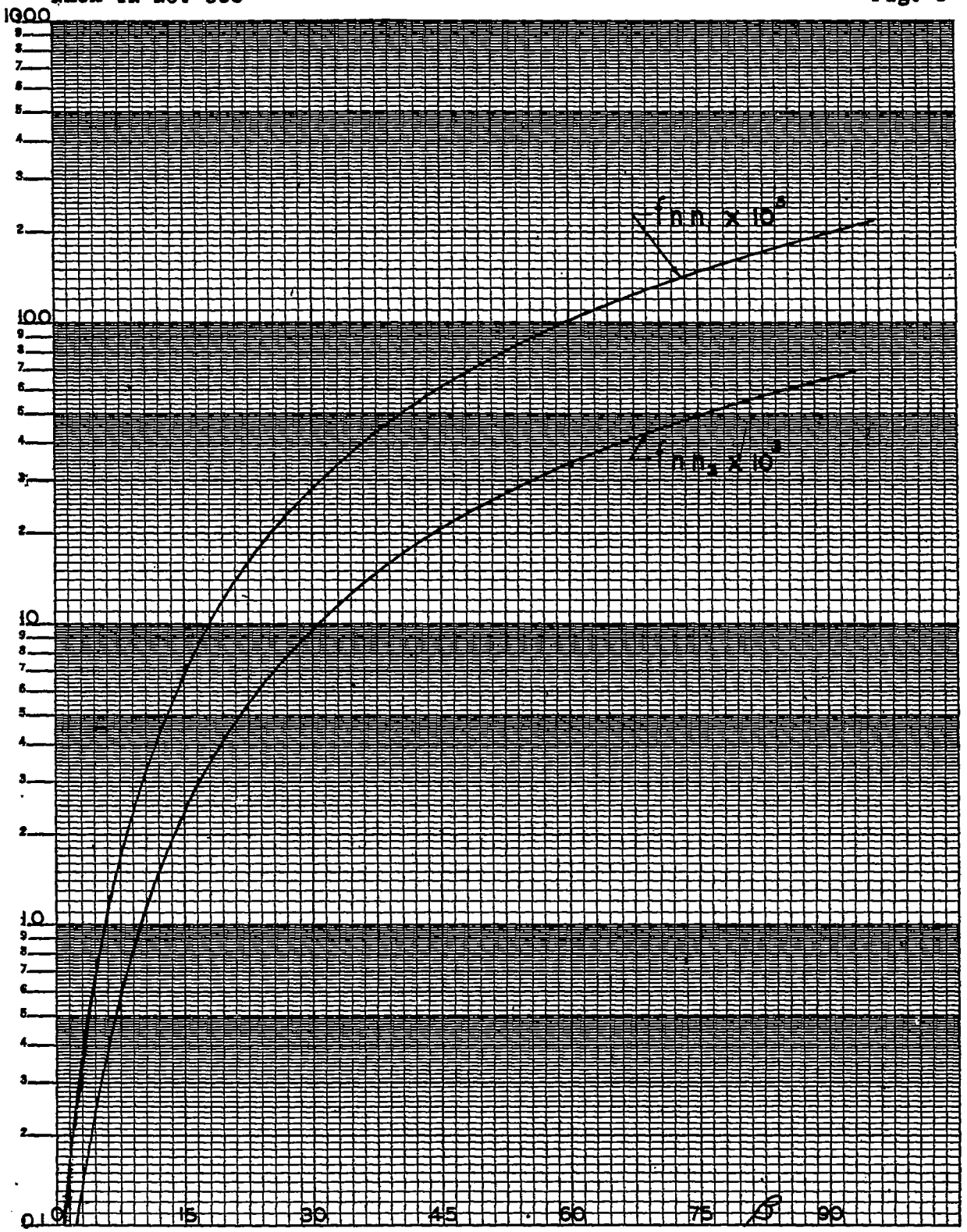


FIG. 5. INFLUENCE FUNCTIONS

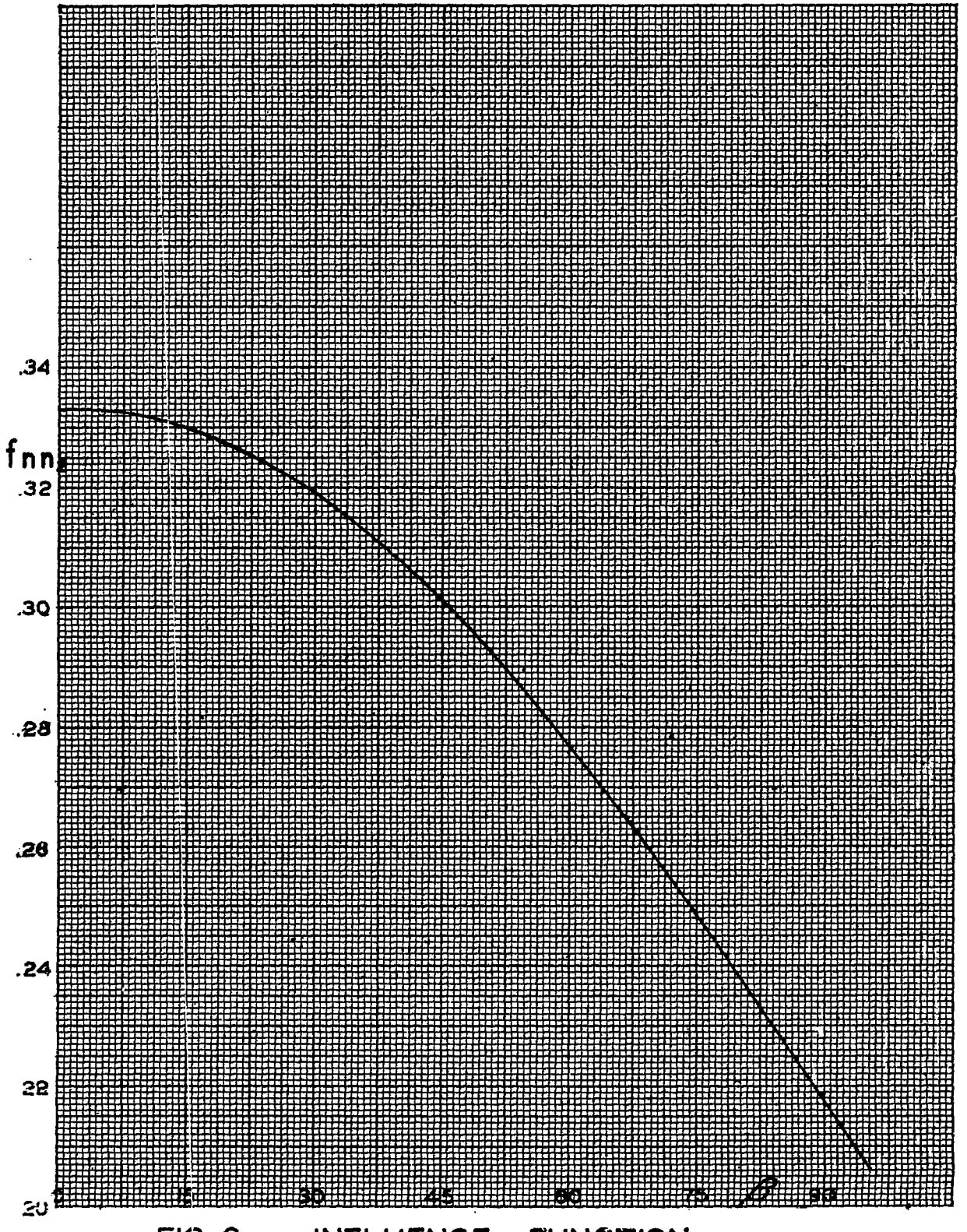


FIG. 6. INFLUENCE FUNCTION

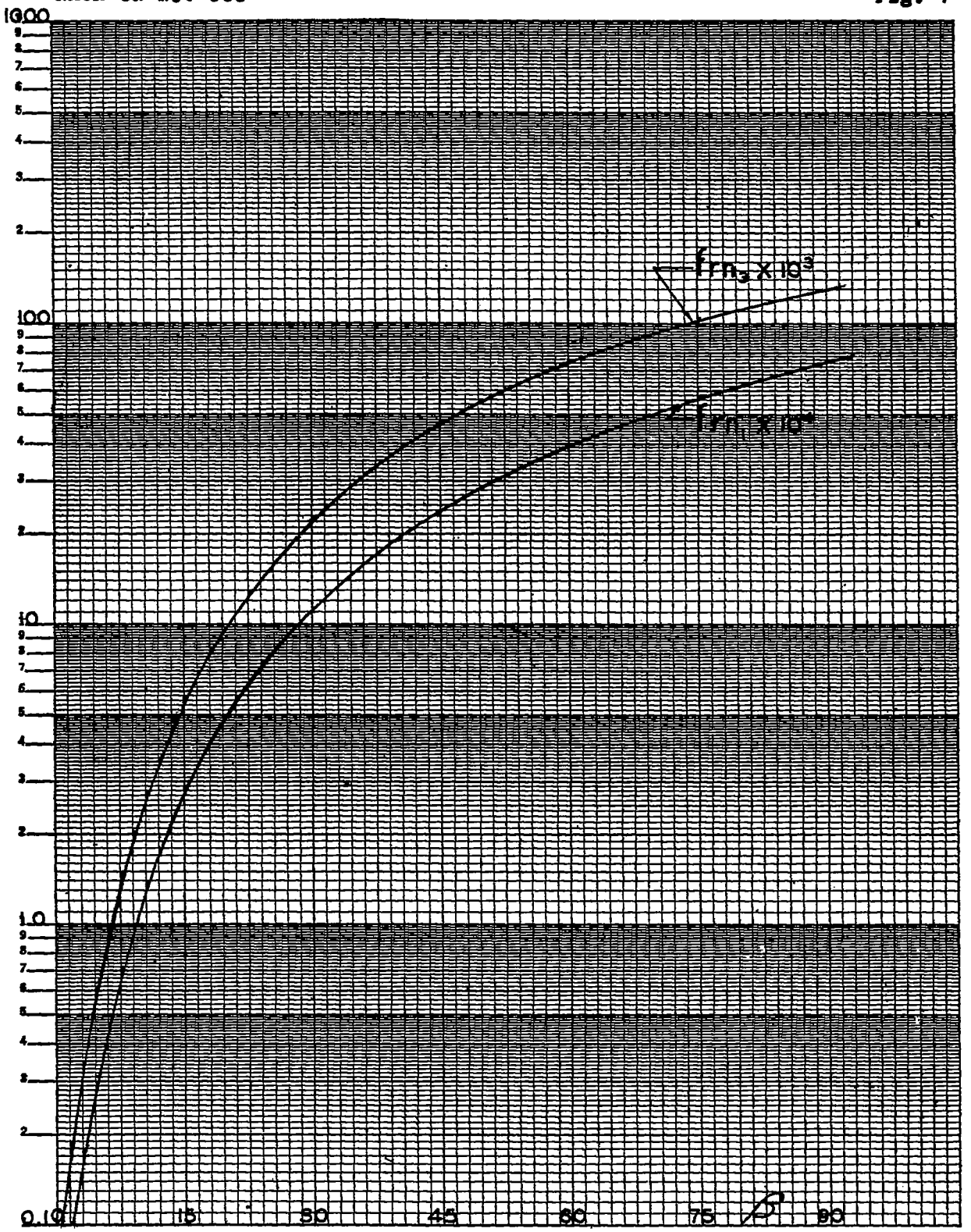


FIG. 7. INFLUENCE FUNCTIONS

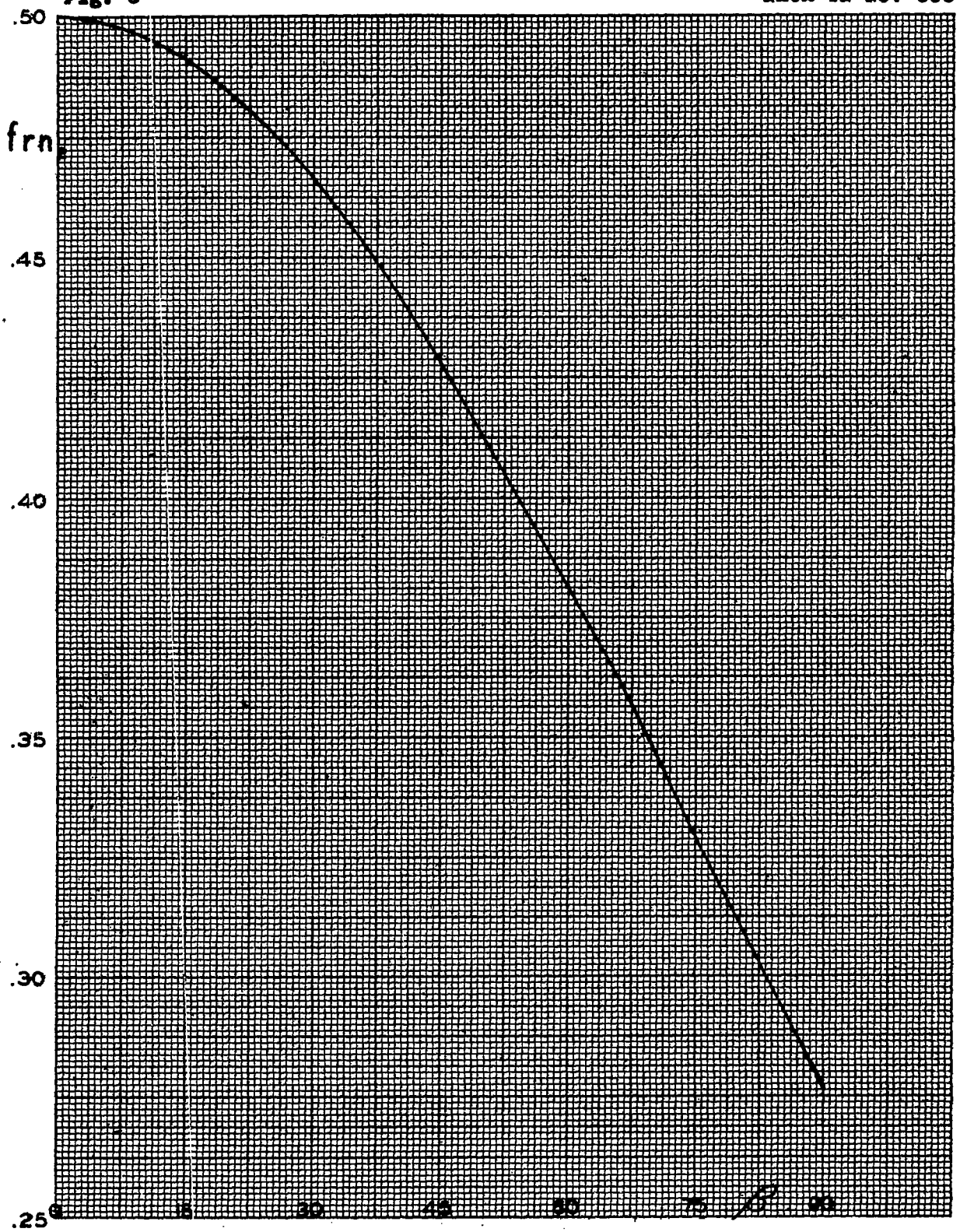


FIG. 8. INFLUENCE FUNCTION

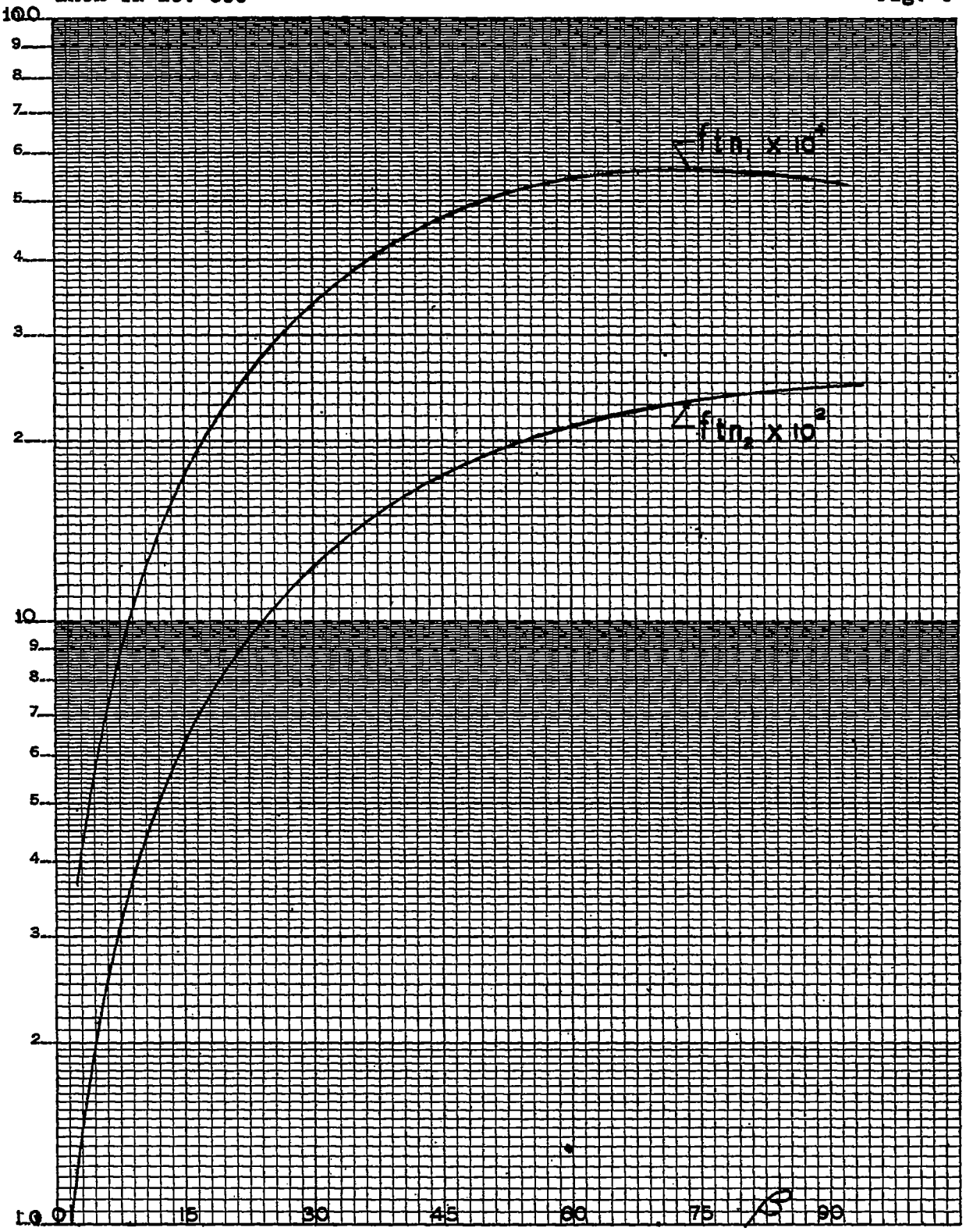


FIG. 9. INFLUENCE FUNCTIONS

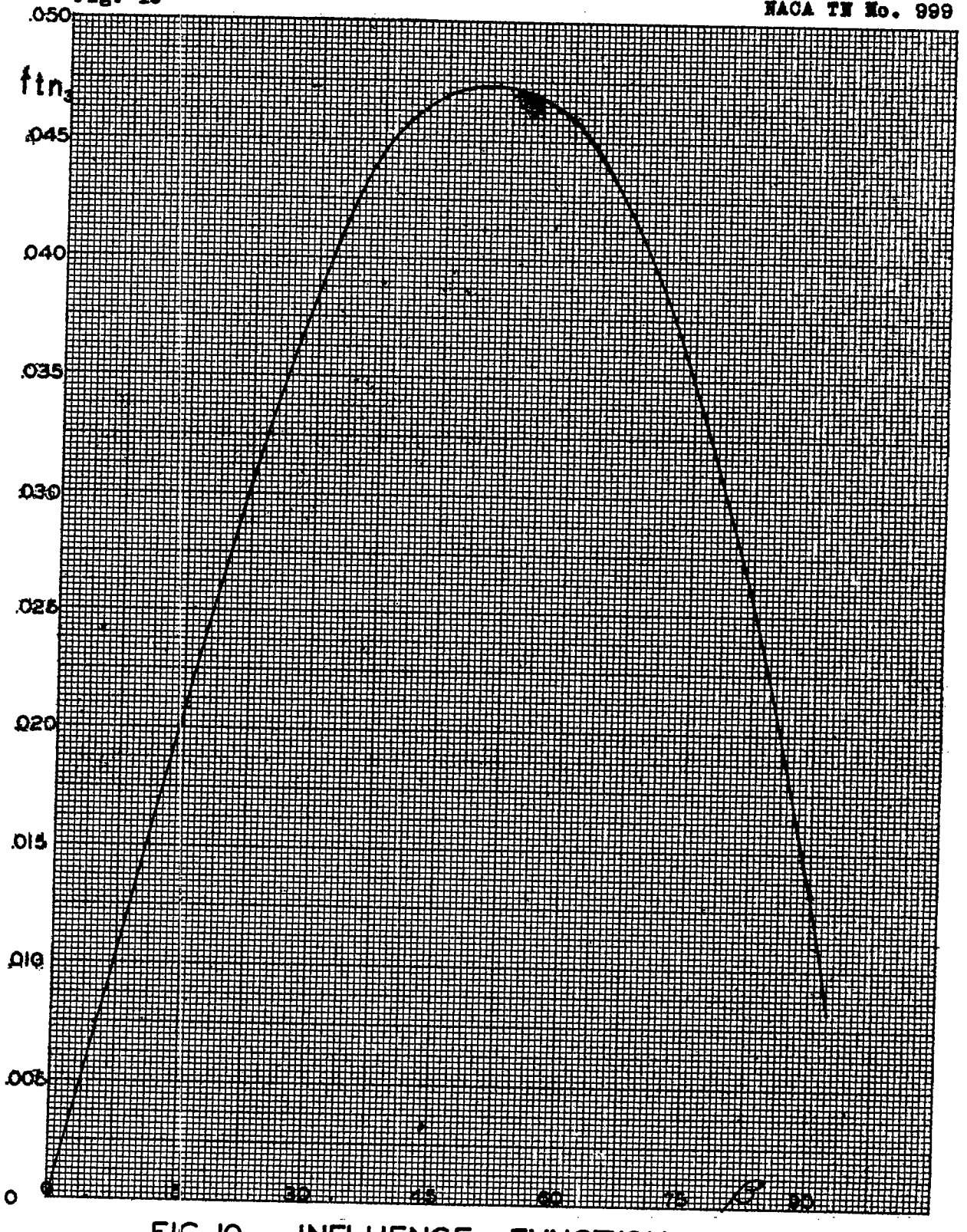


FIG. 10. INFLUENCE FUNCTION

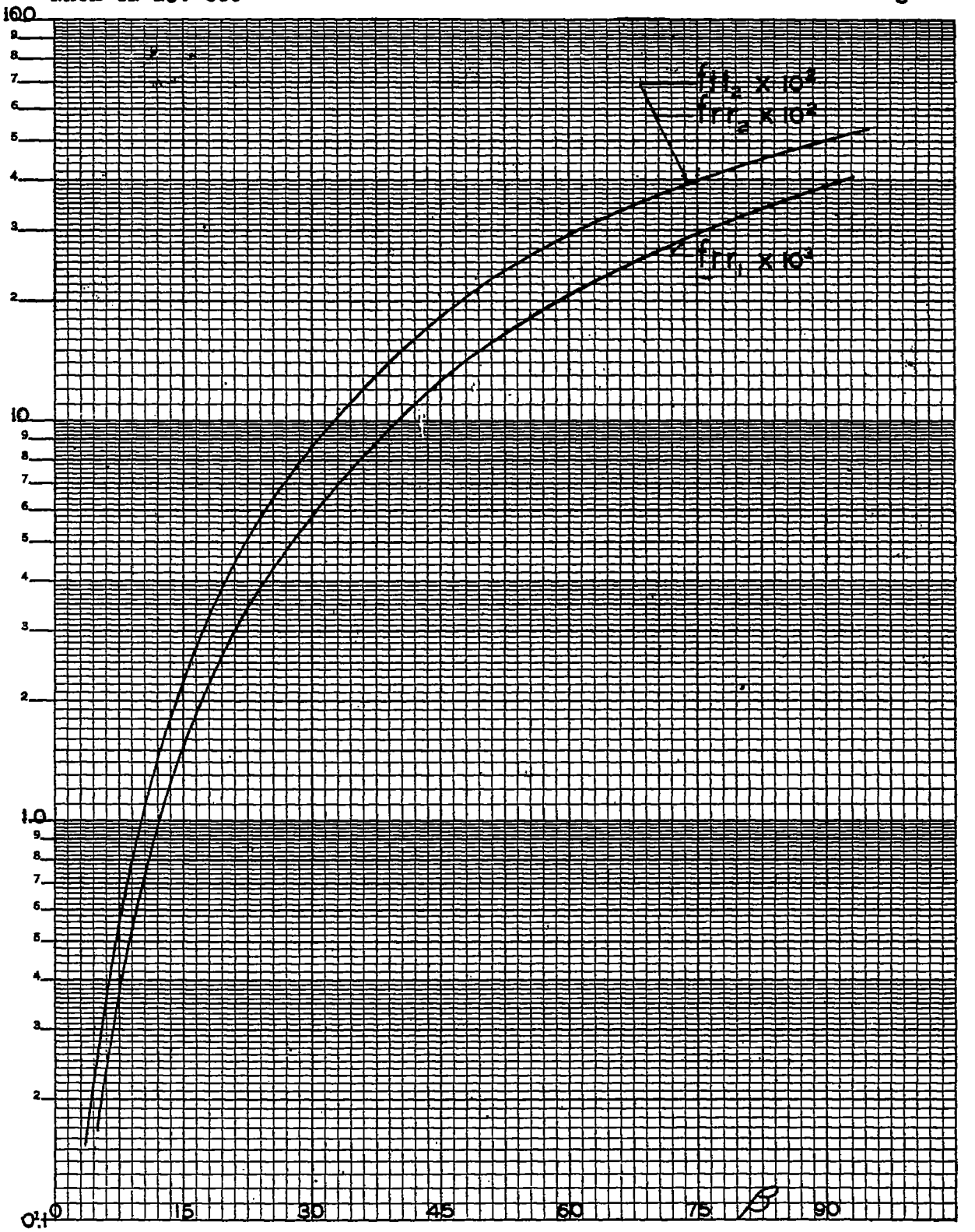


FIG. 11. INFLUENCE FUNCTIONS

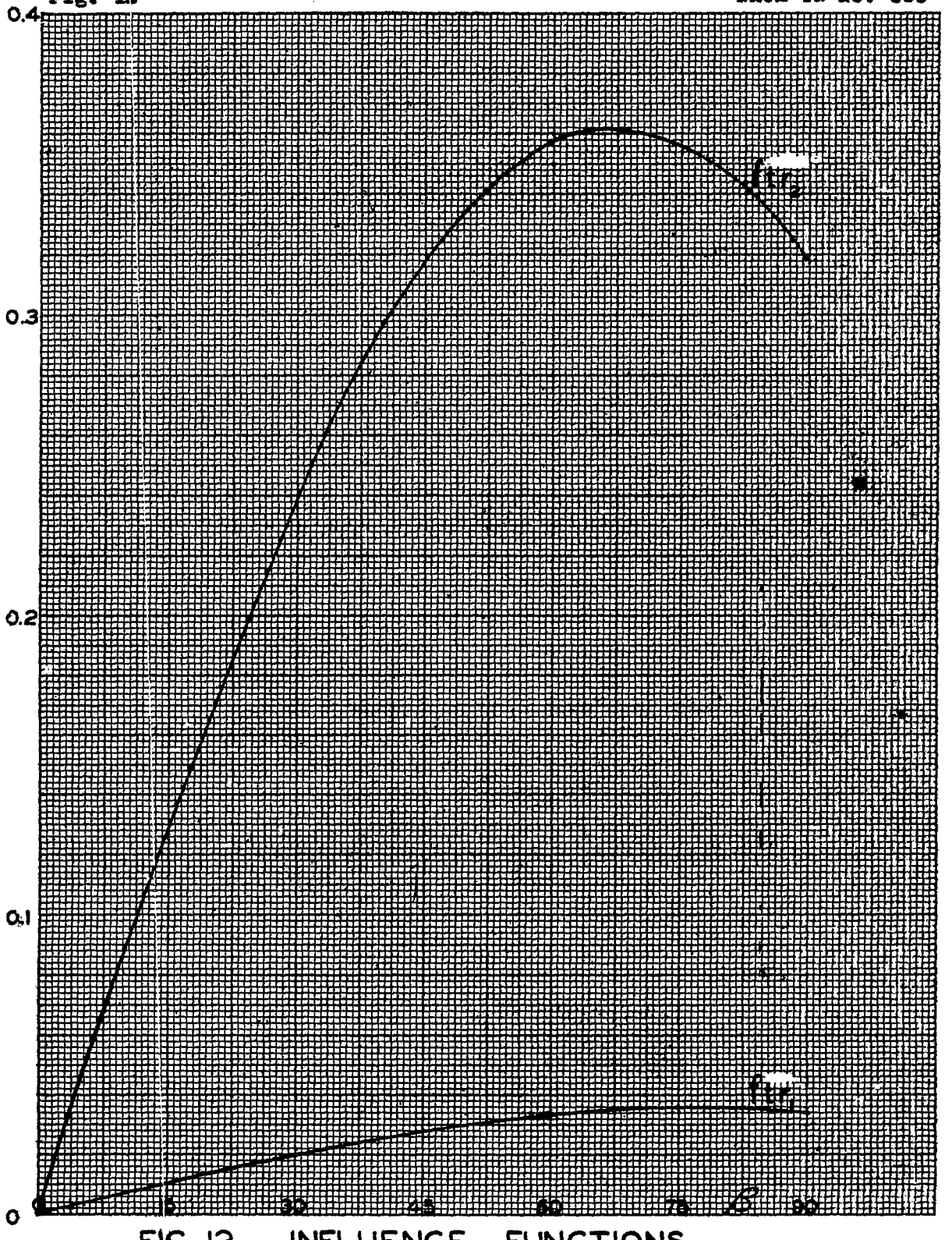


FIG. 12. INFLUENCE FUNCTIONS

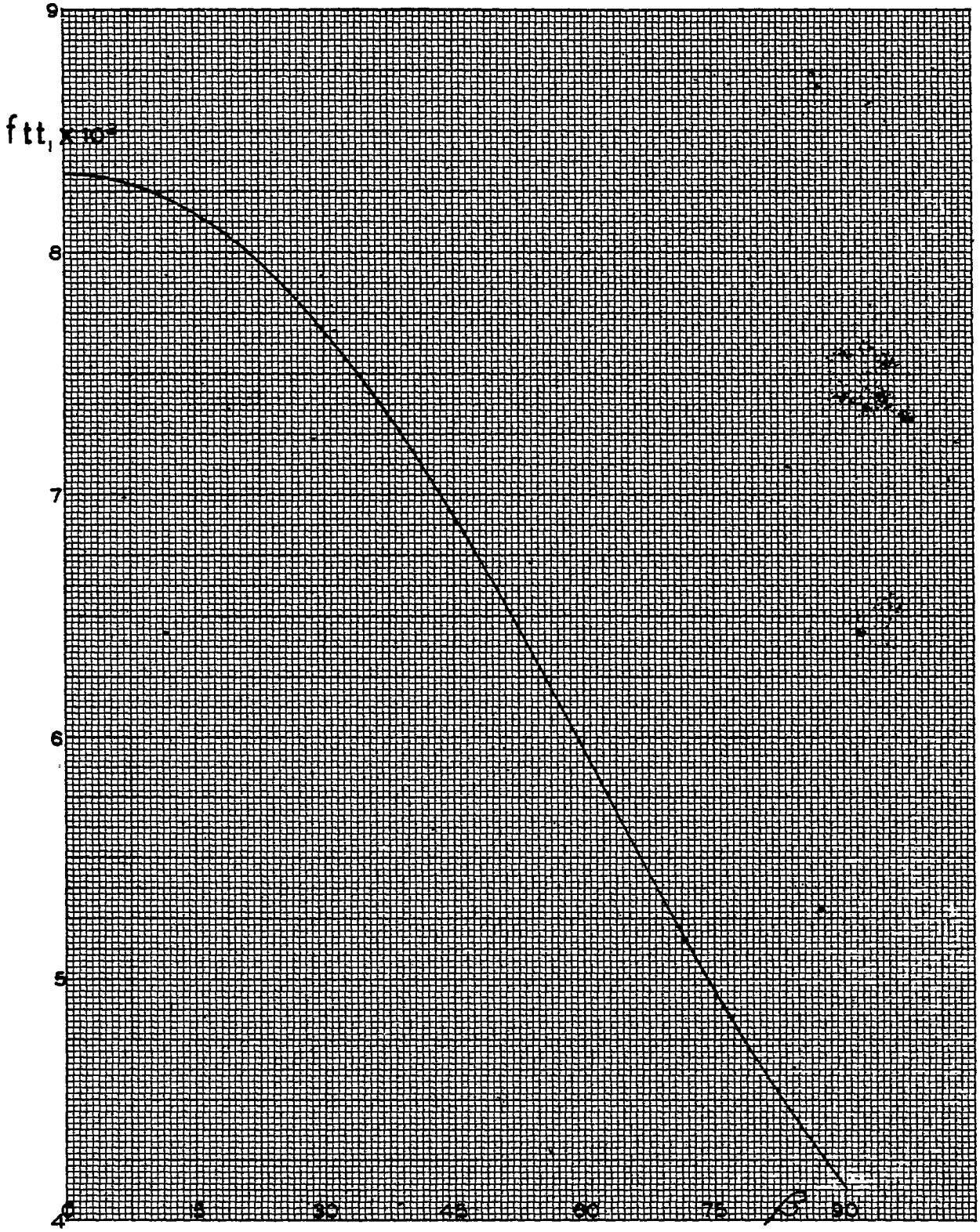


FIG. 13. INFLUENCE FUNCTION

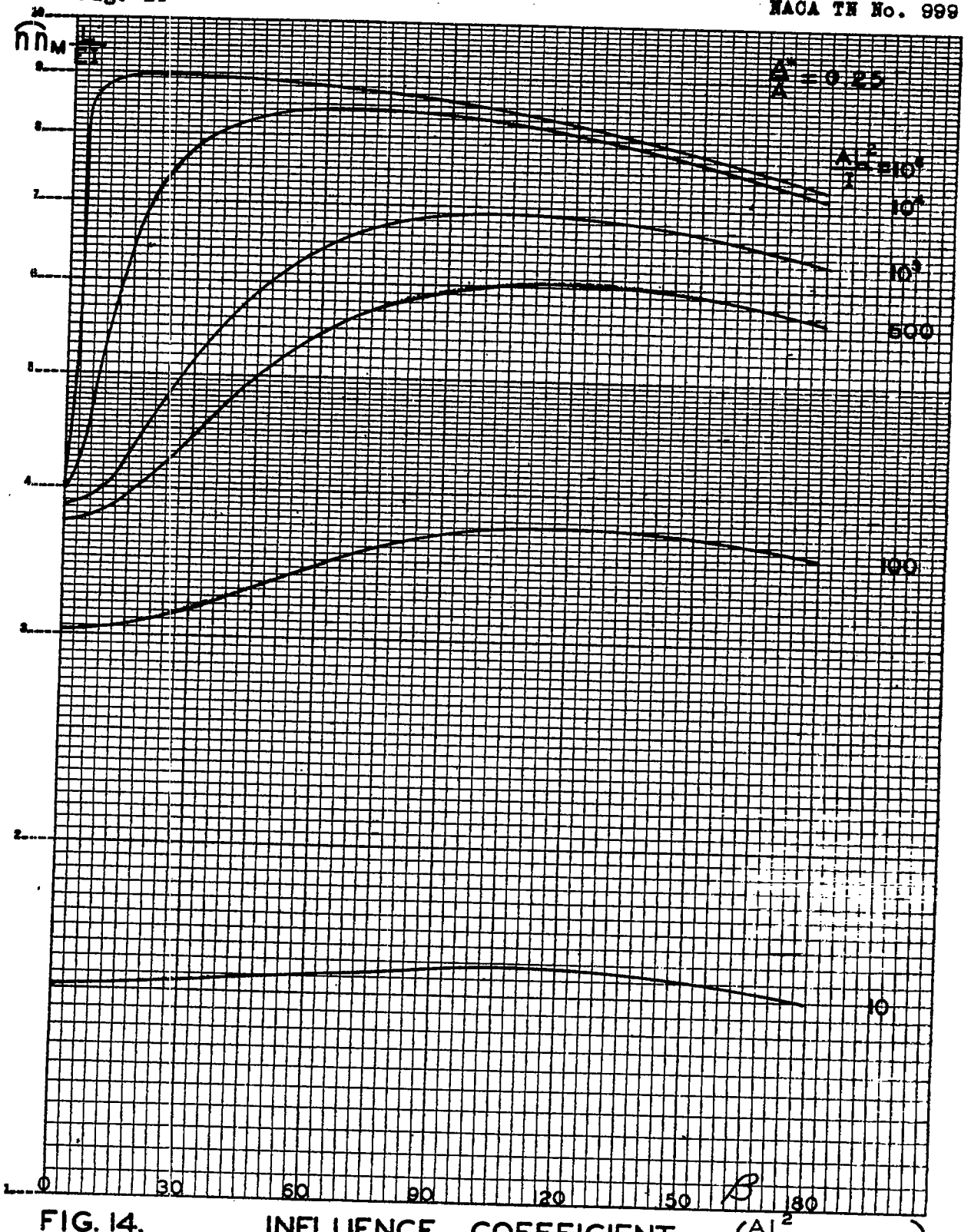


FIG. 14. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

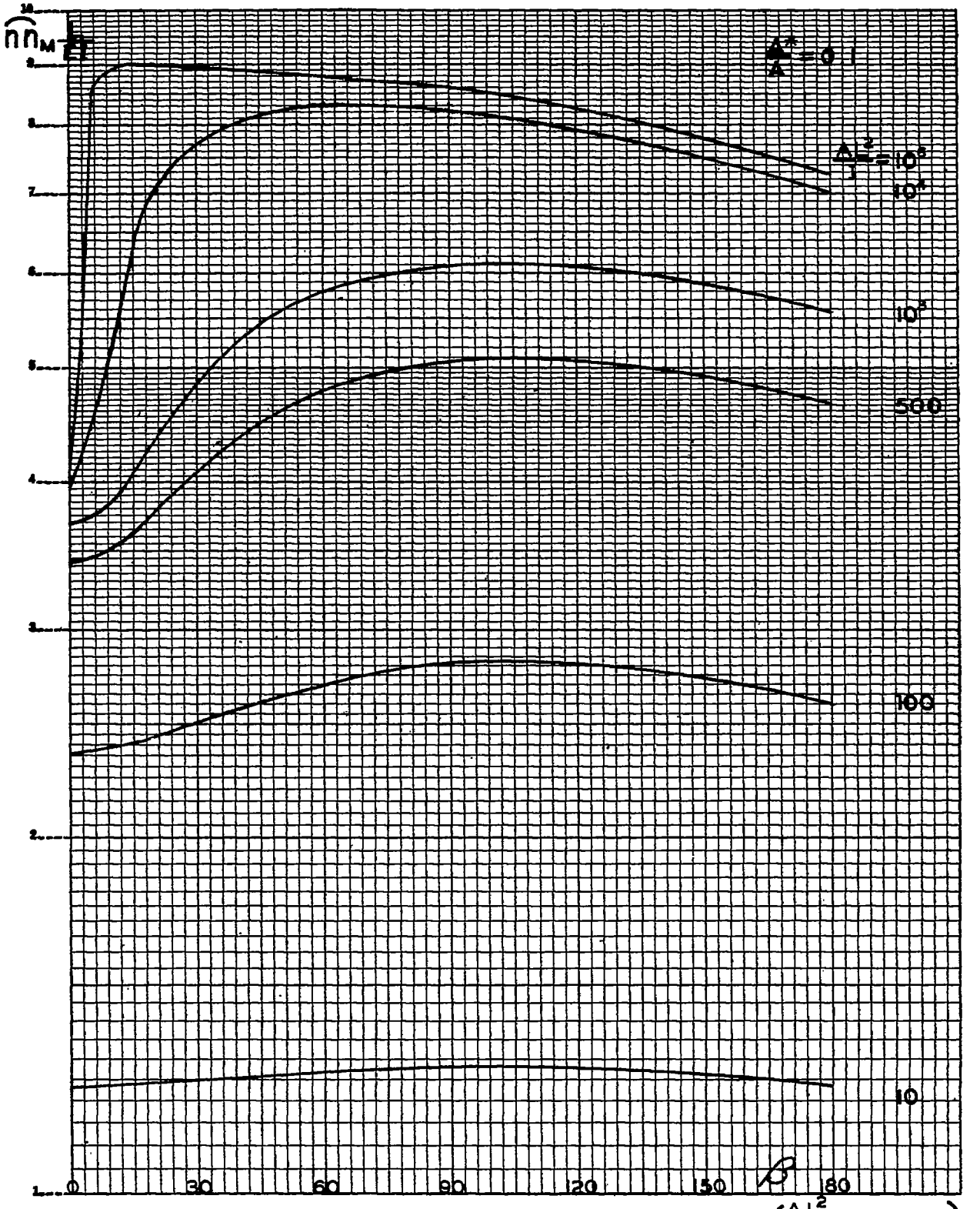


FIG. 15. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

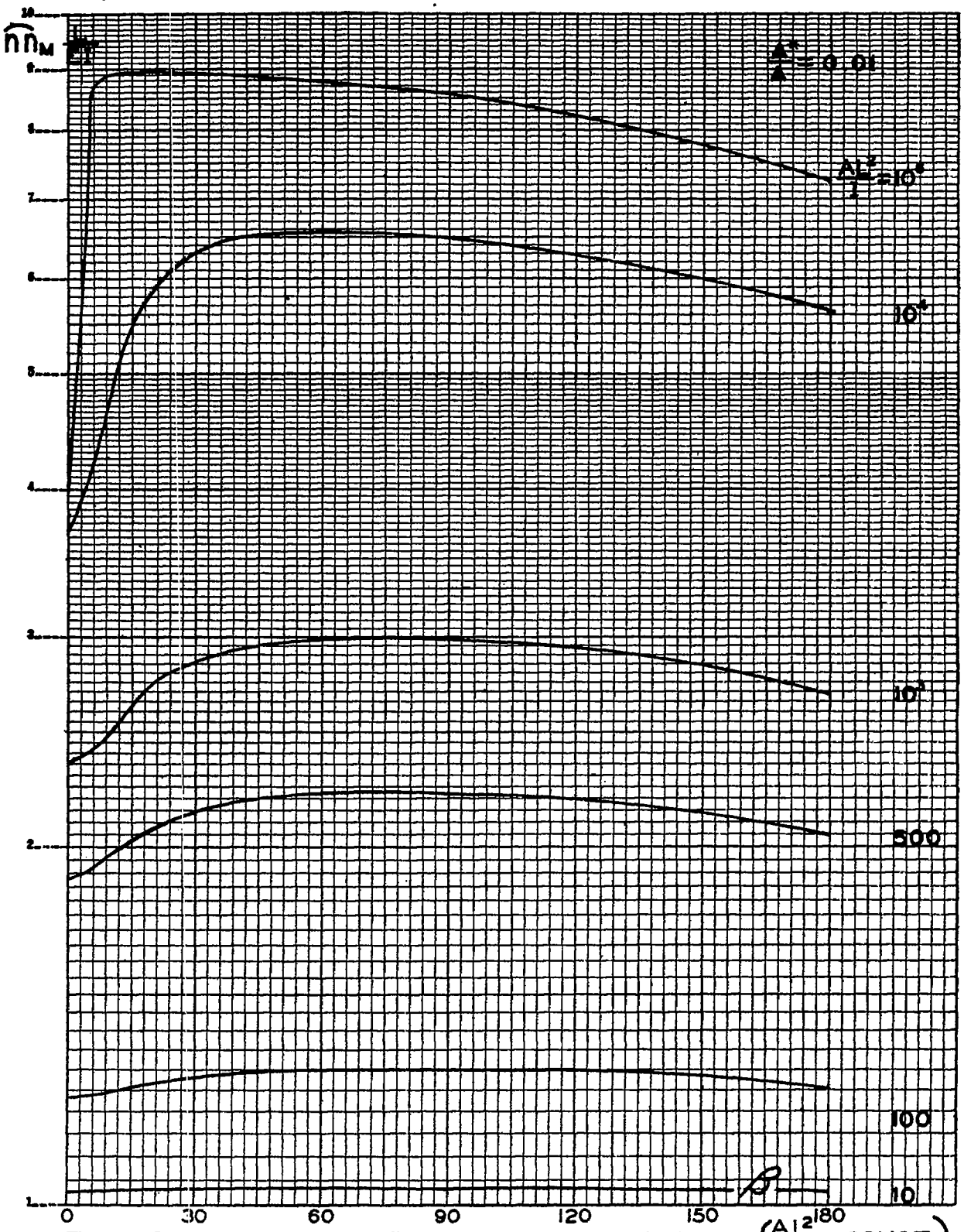


FIG. 16. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

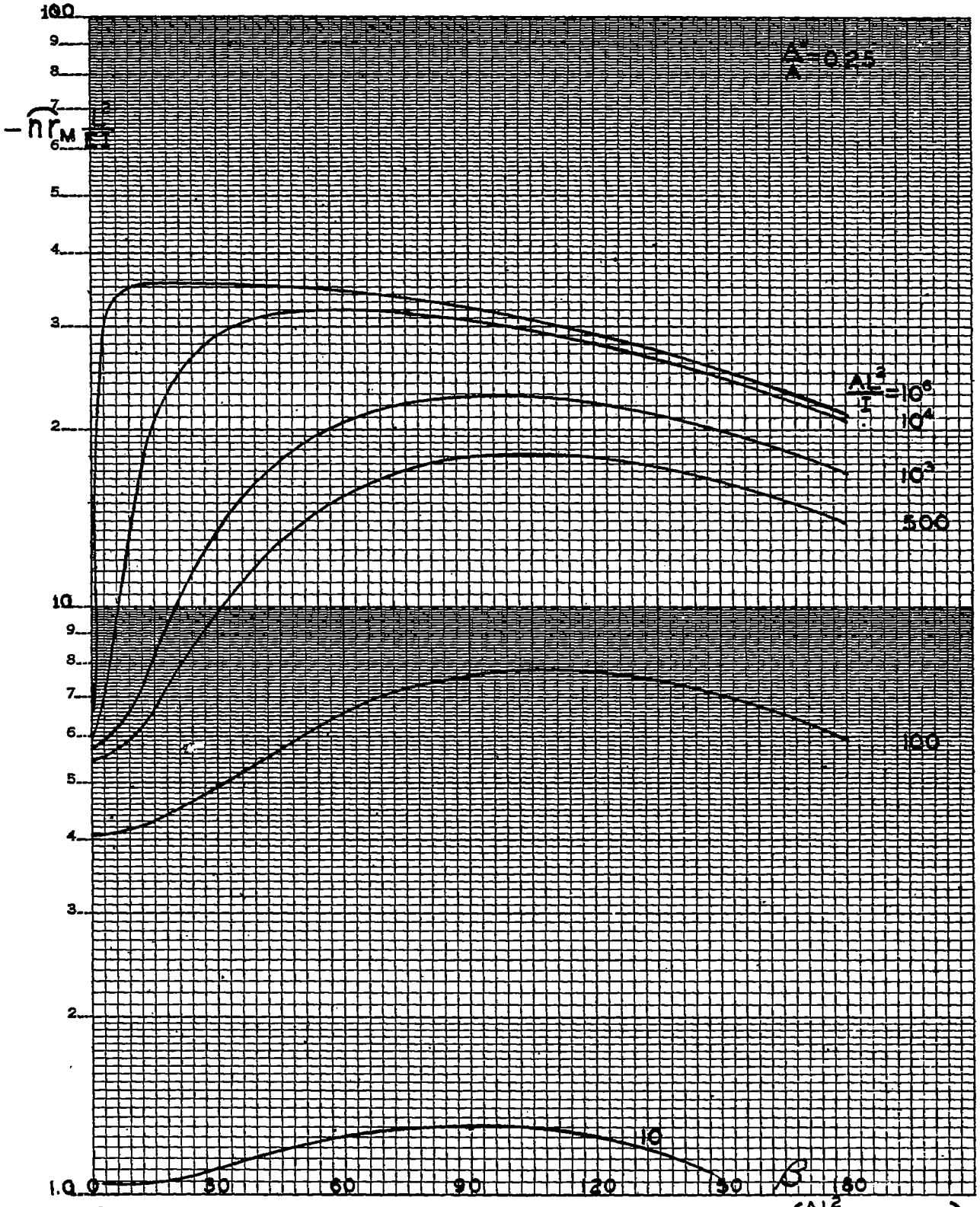


FIG. 17. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

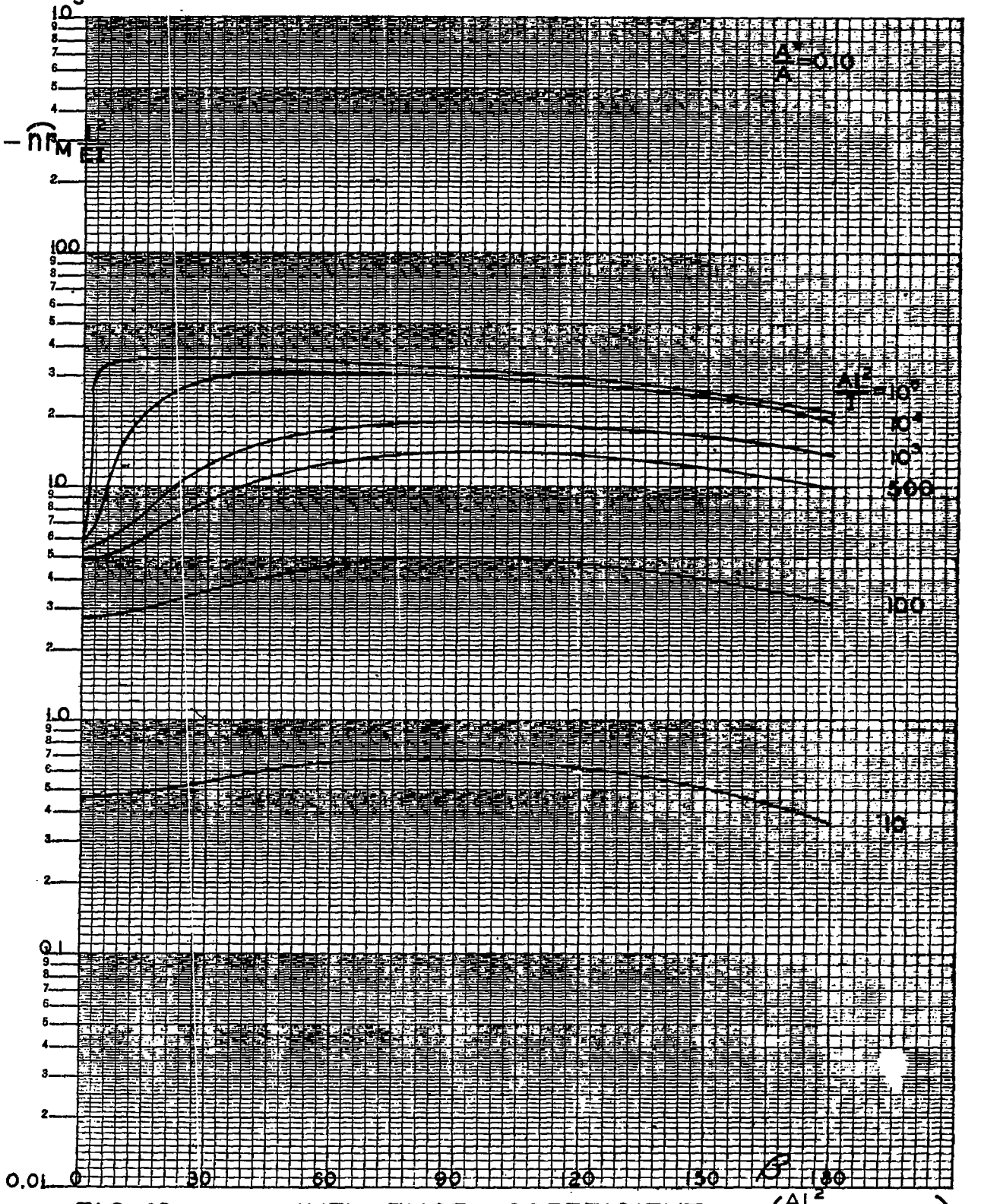


FIG. 18. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

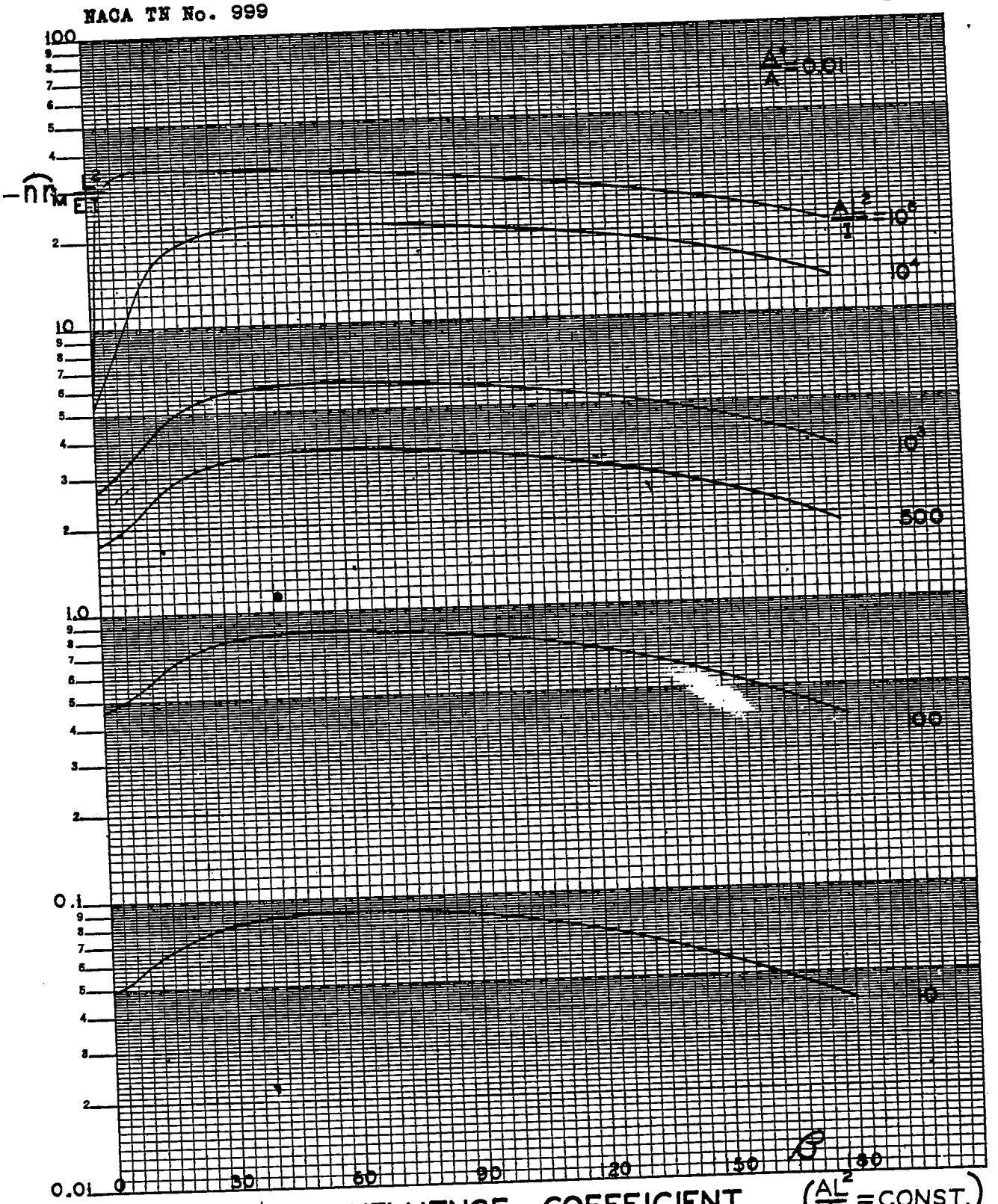


FIG. 19. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

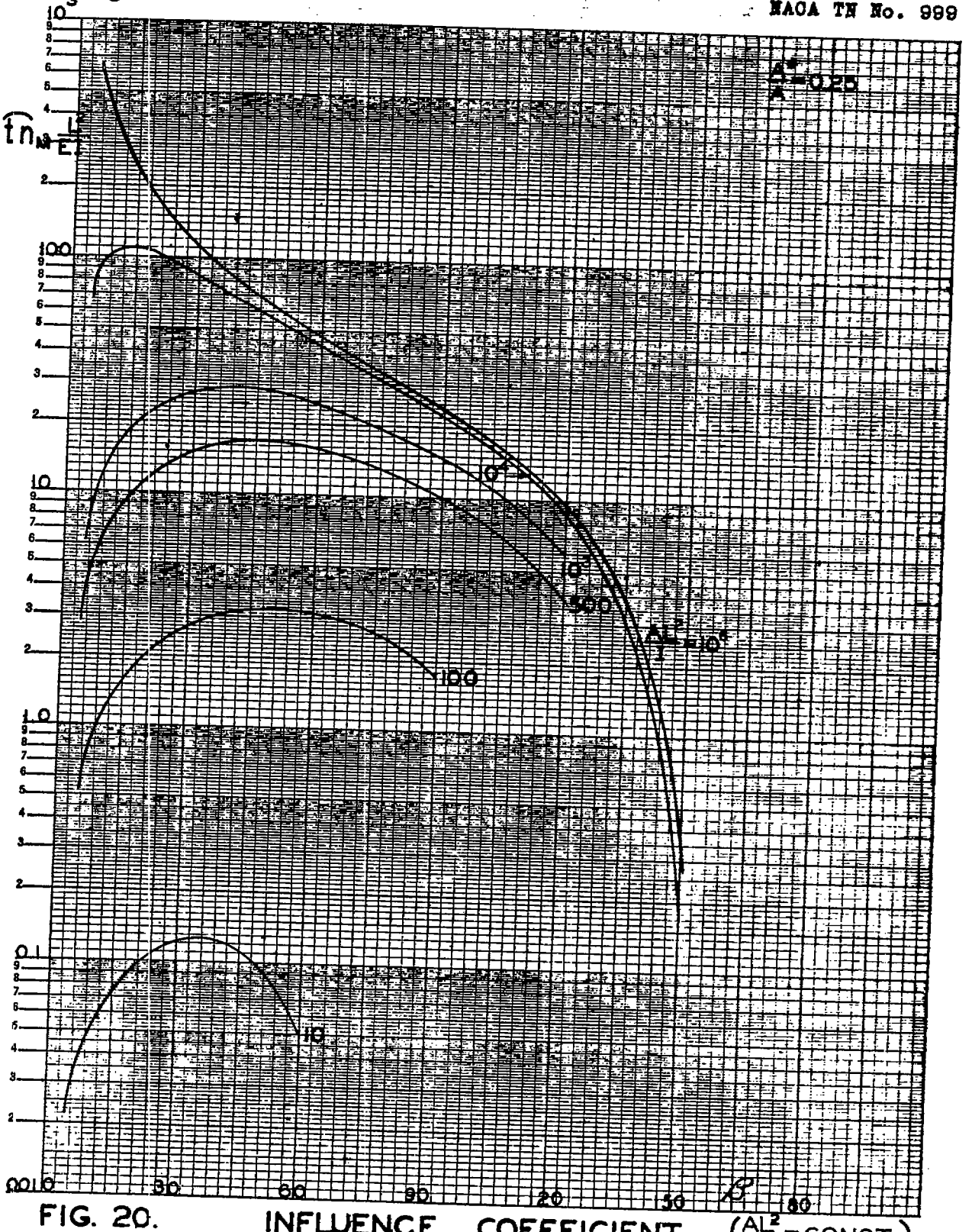


FIG. 20. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

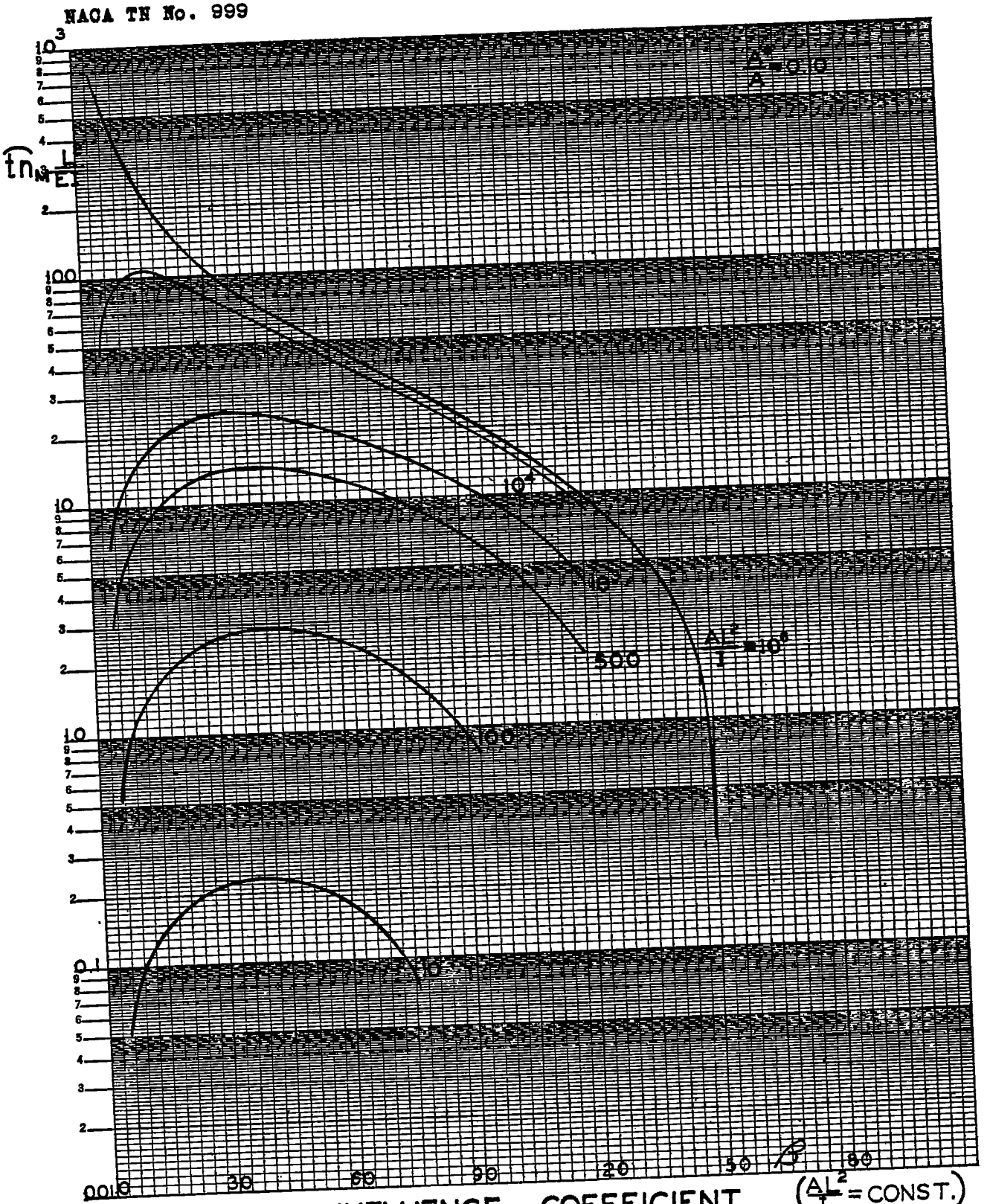


FIG. 21. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

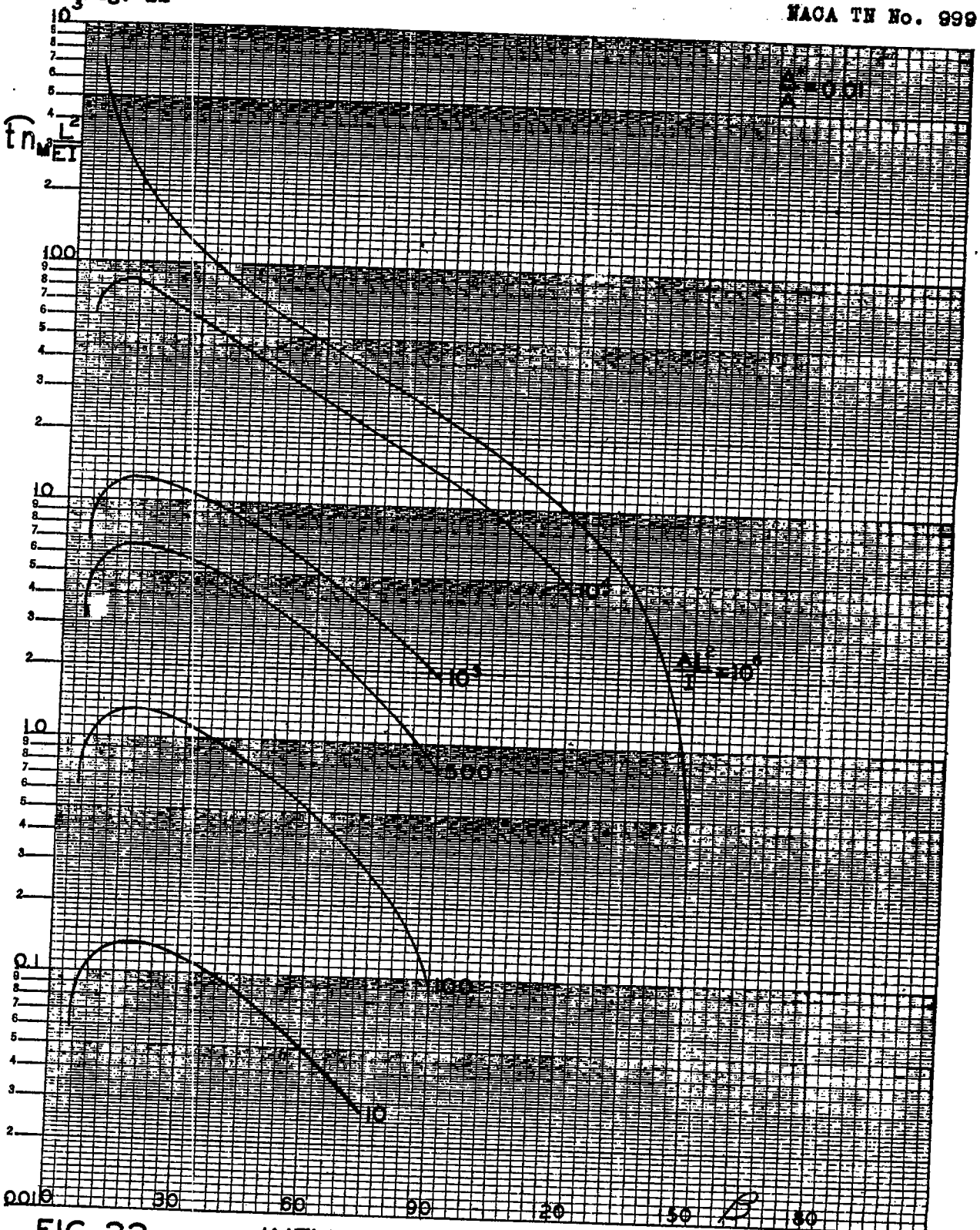


FIG. 22. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

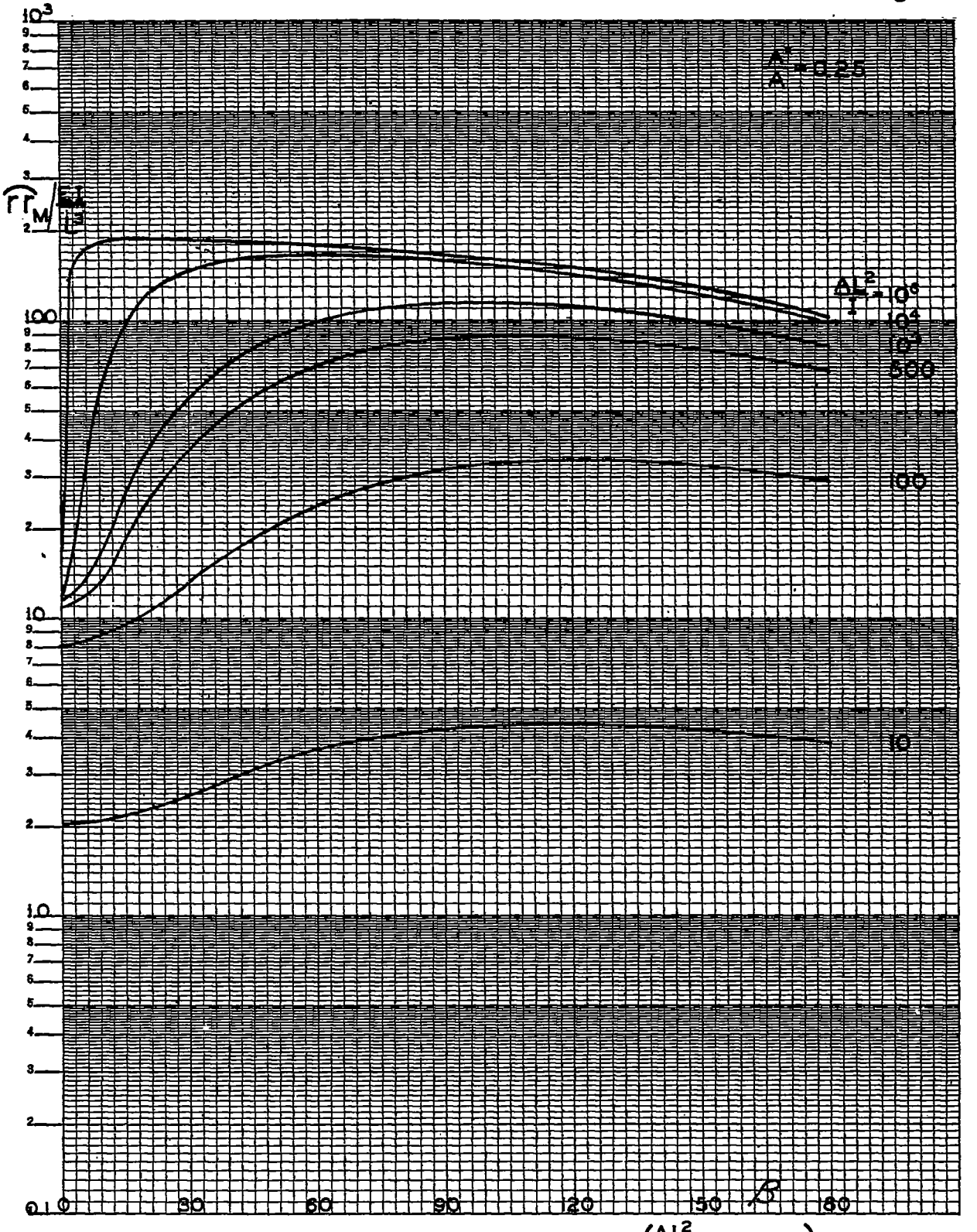


FIG. 23. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

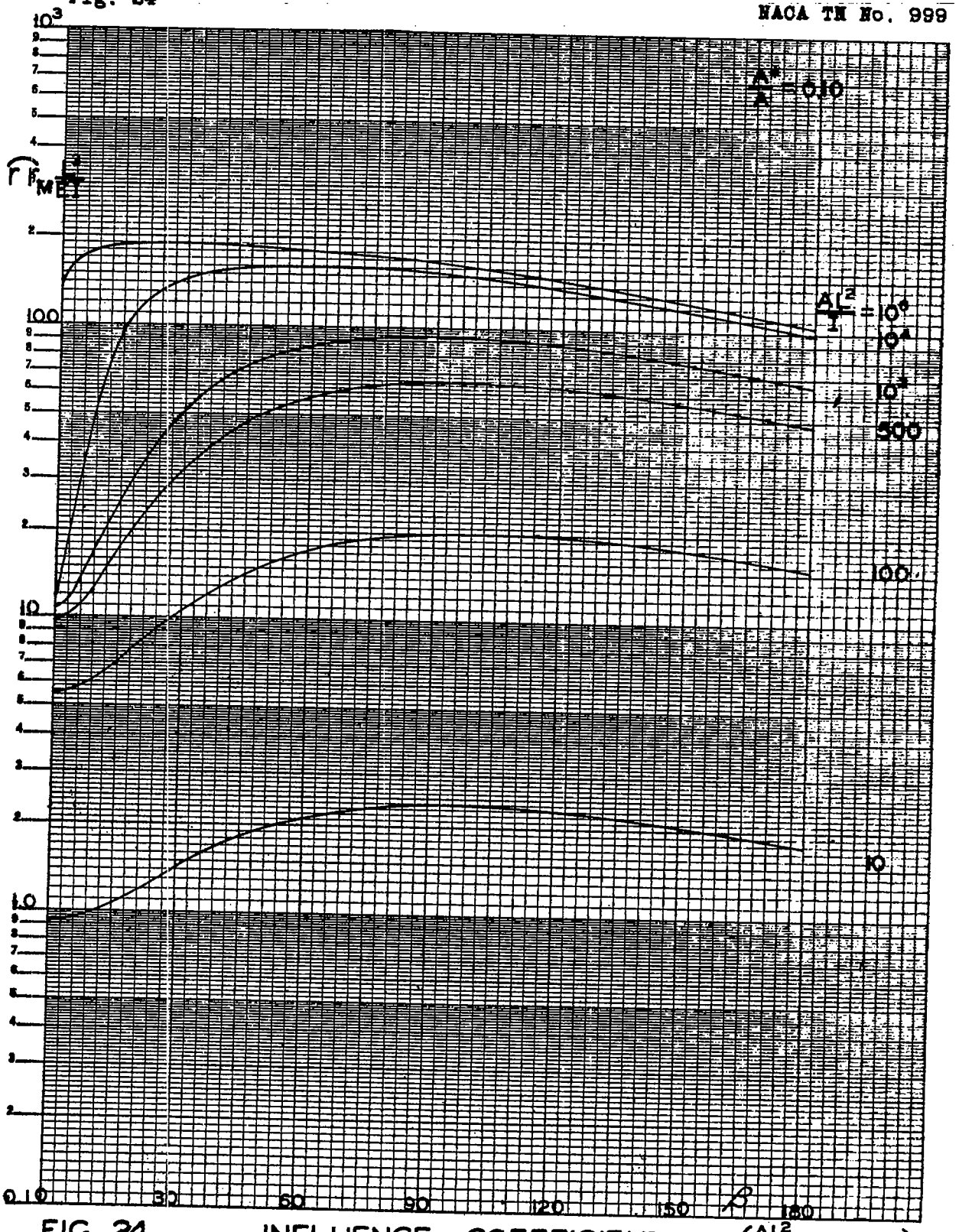


FIG. 24. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

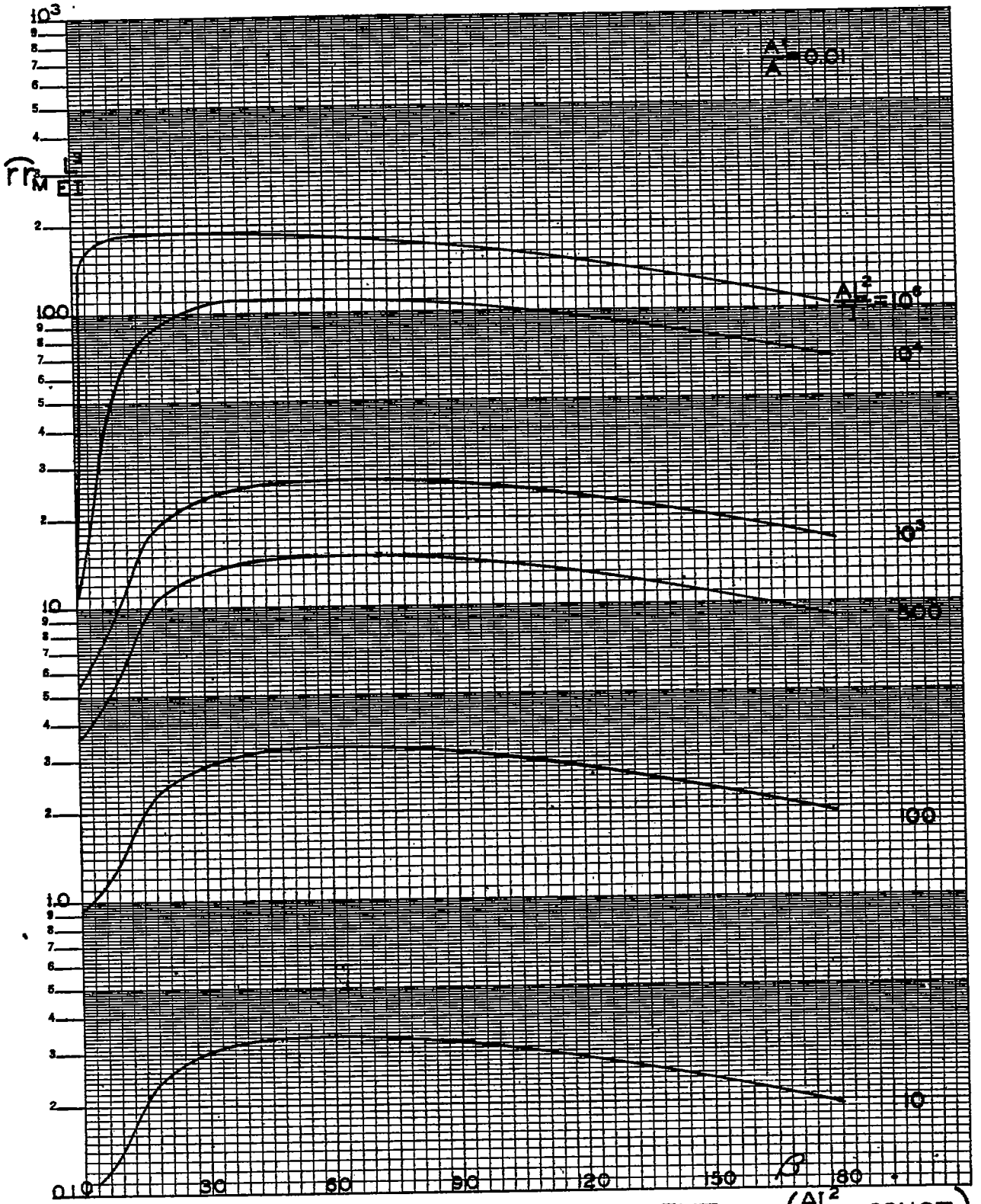


FIG. 25 INFLUENCE COEFFICIENT $\left(\frac{Al^2}{I} = \text{CONST.}\right)$

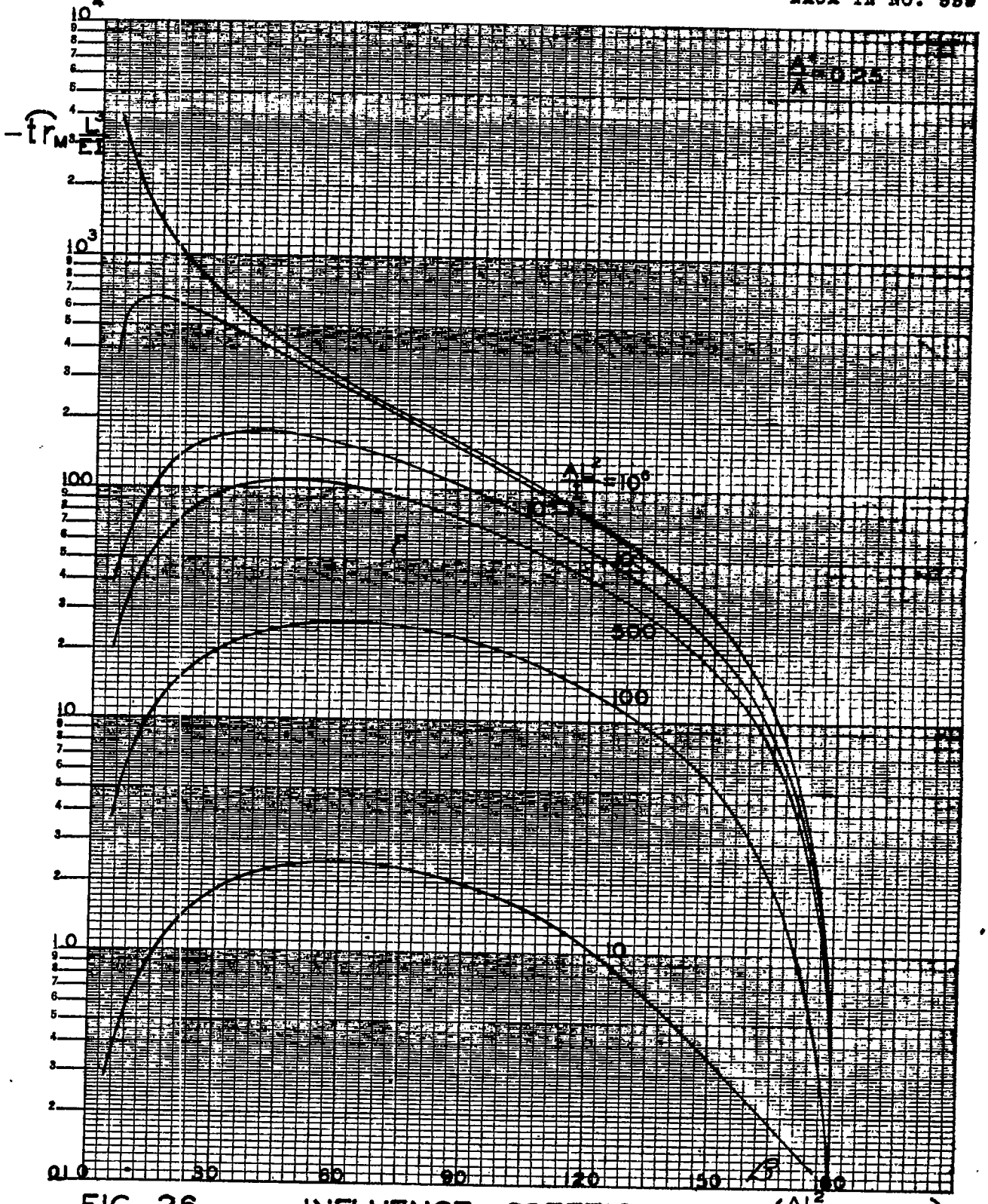


FIG. 26. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

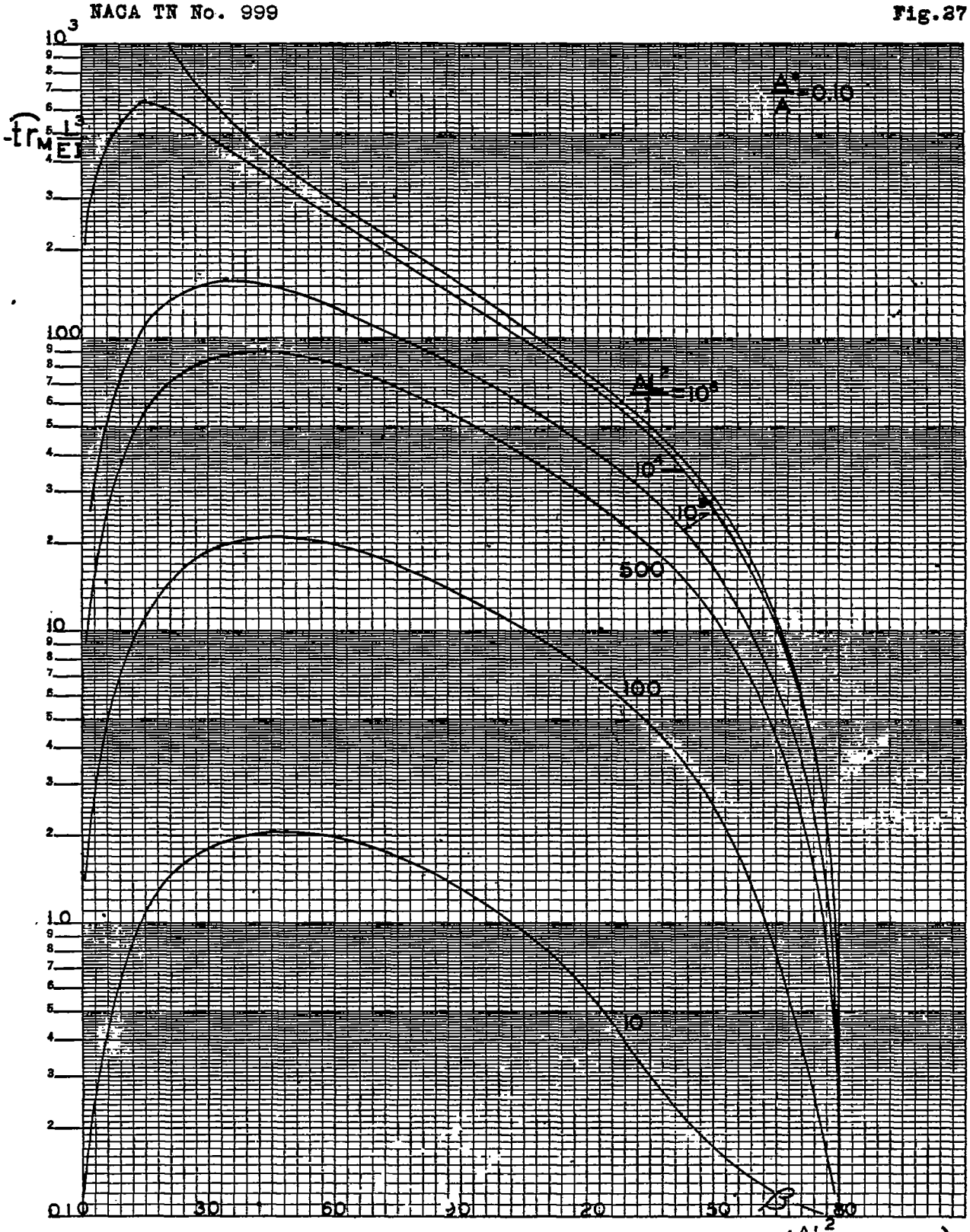


FIG. 27. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

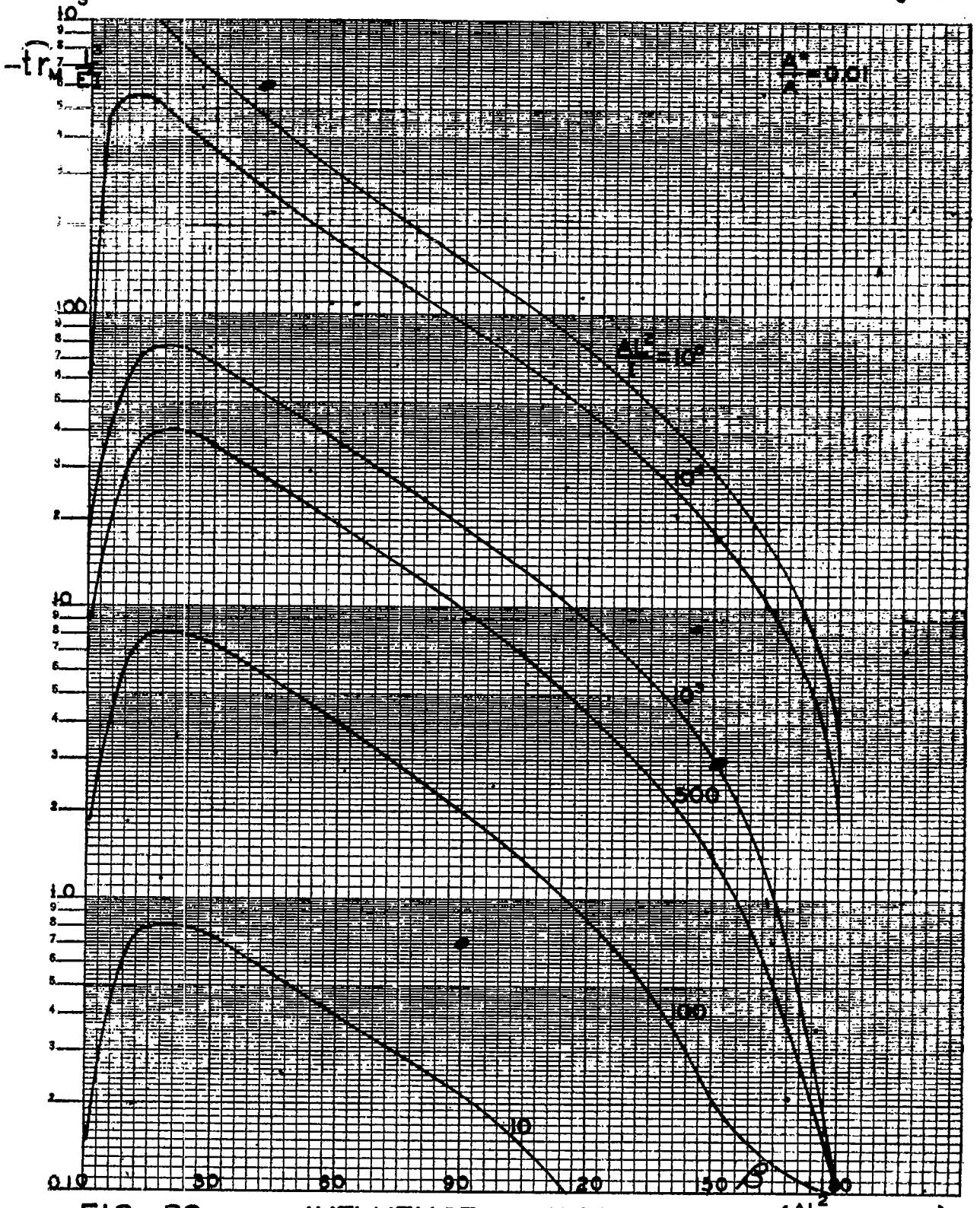


FIG. 28. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

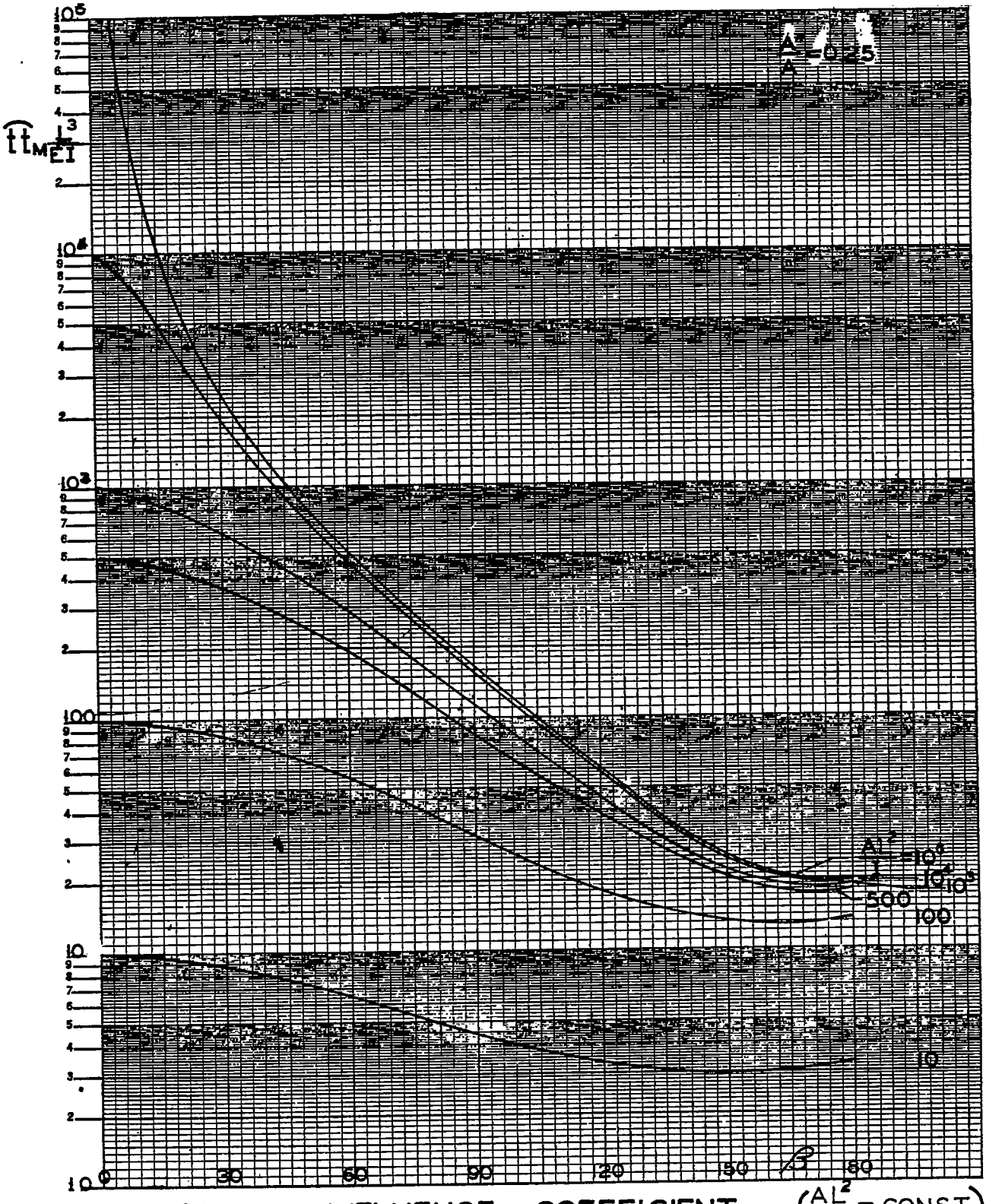


FIG. 29. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

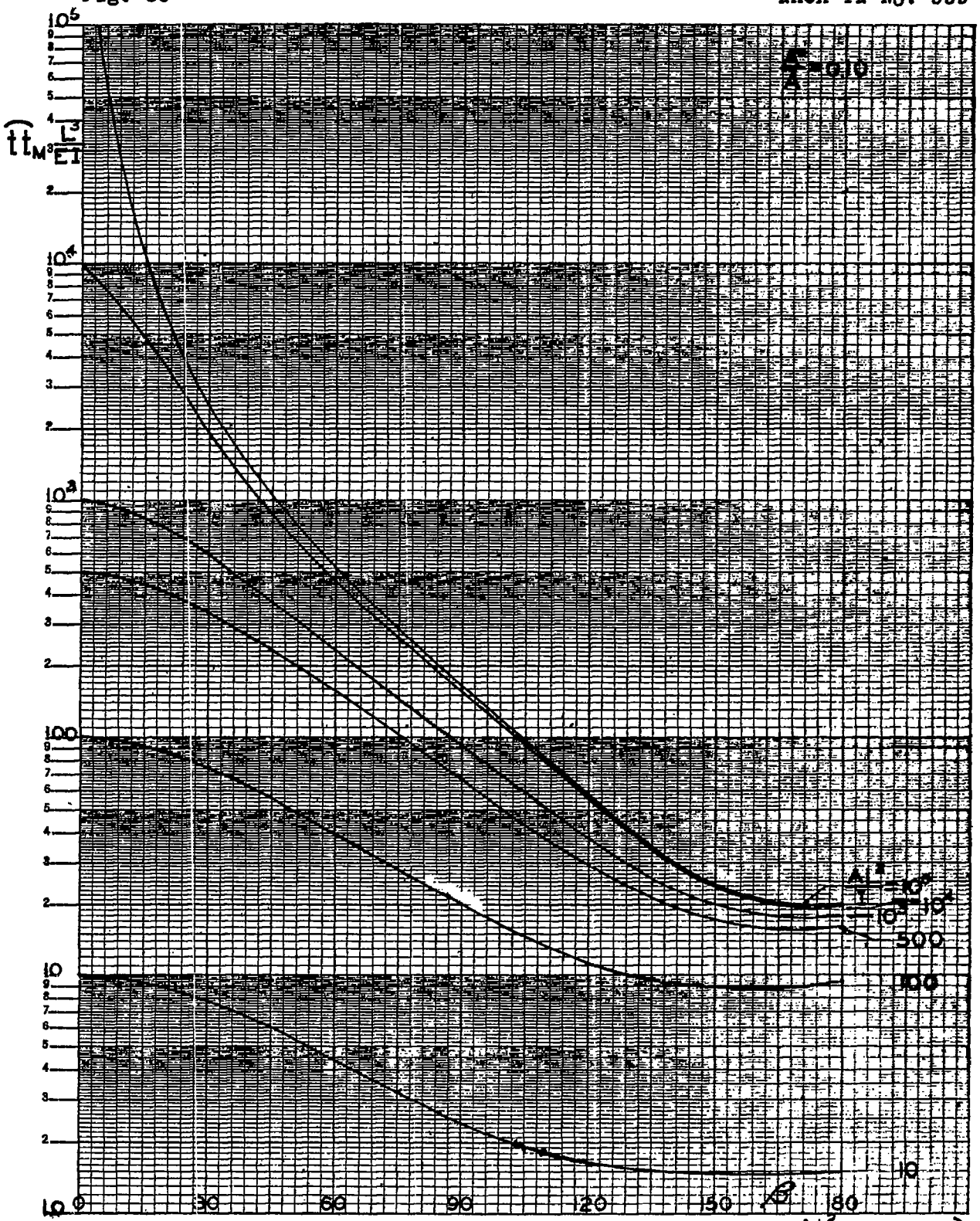


FIG. 30. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

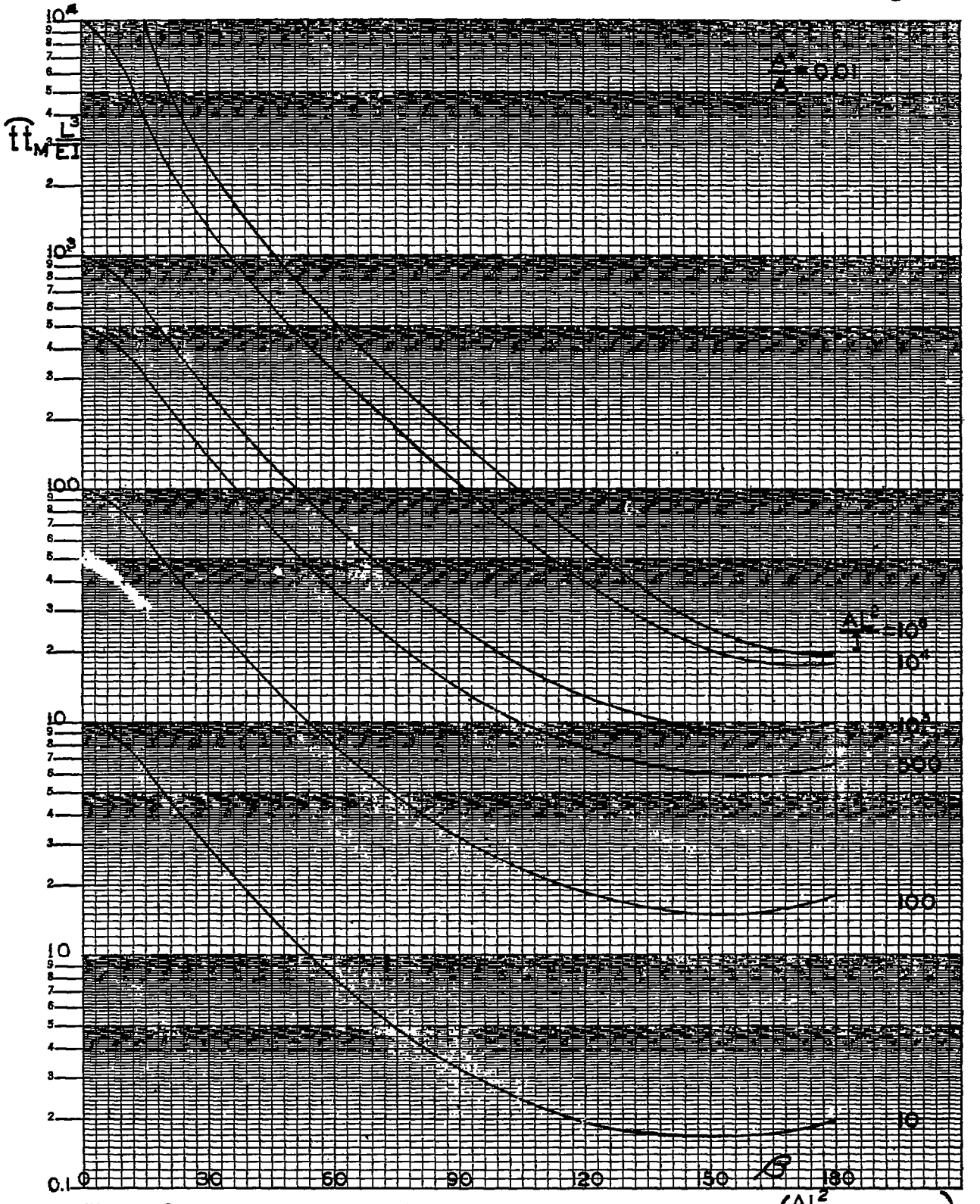


FIG. 31. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

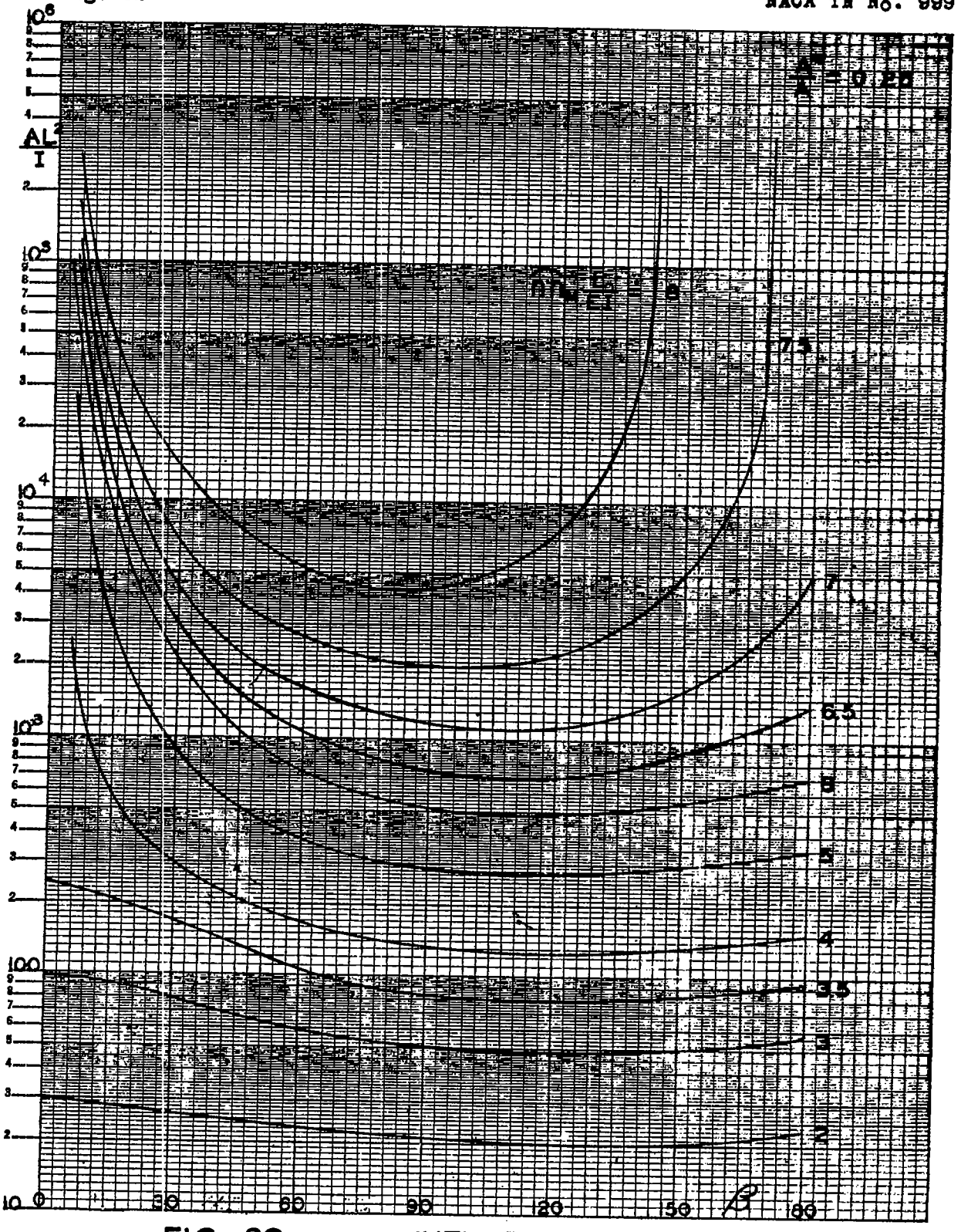


FIG. 32. INFLUENCE COEFFICIENT

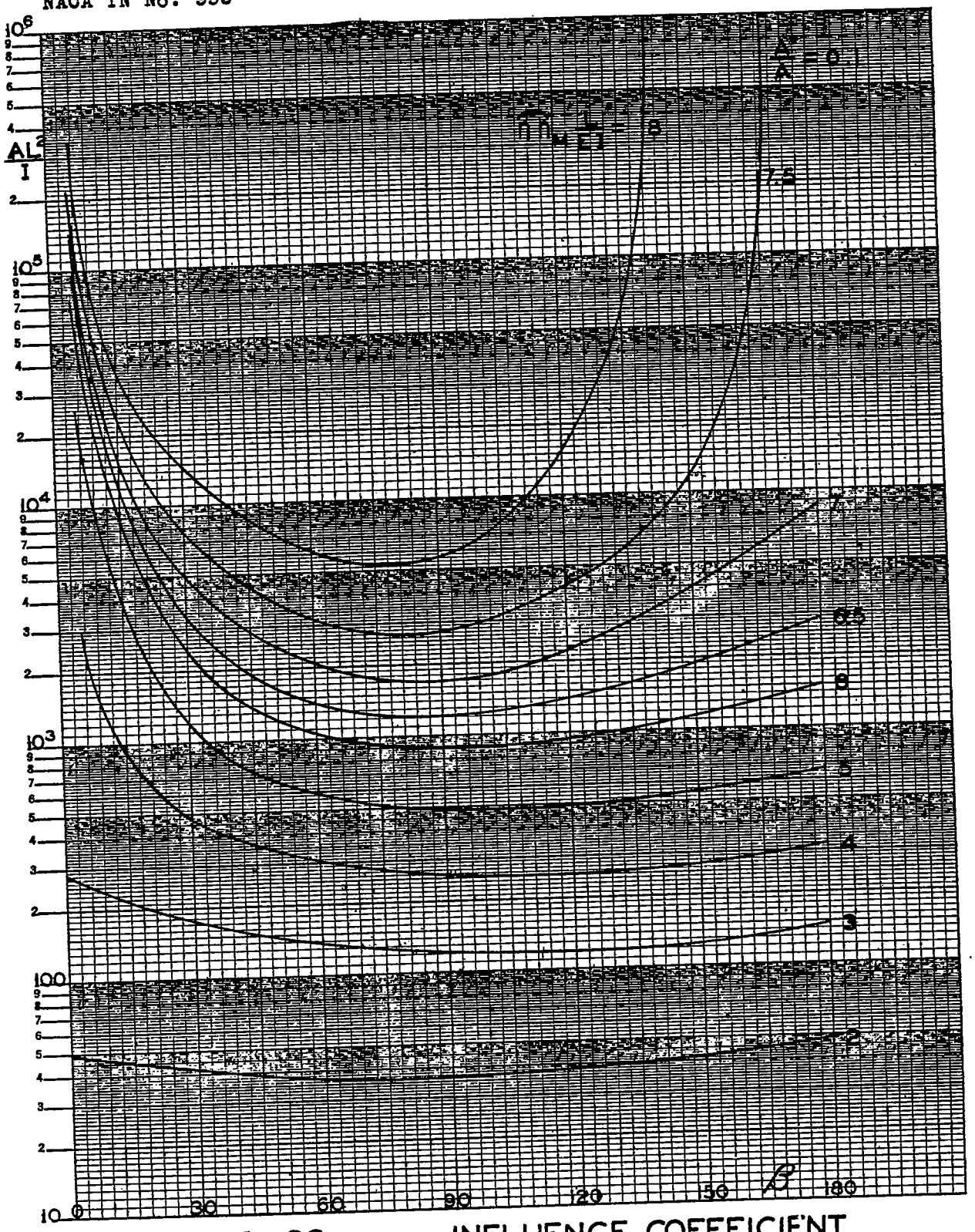


FIG. 33. INFLUENCE COEFFICIENT

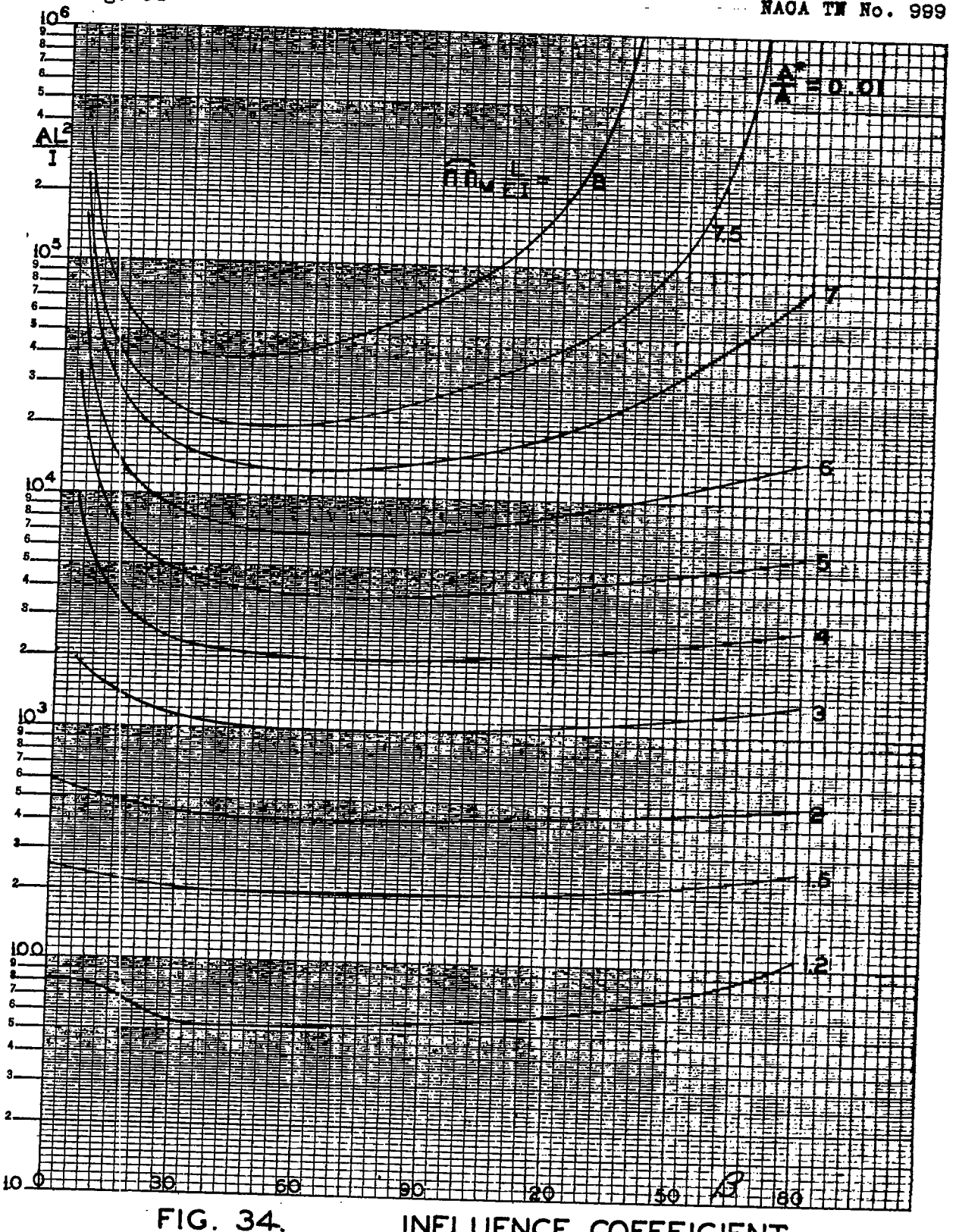


FIG. 34. INFLUENCE COEFFICIENT

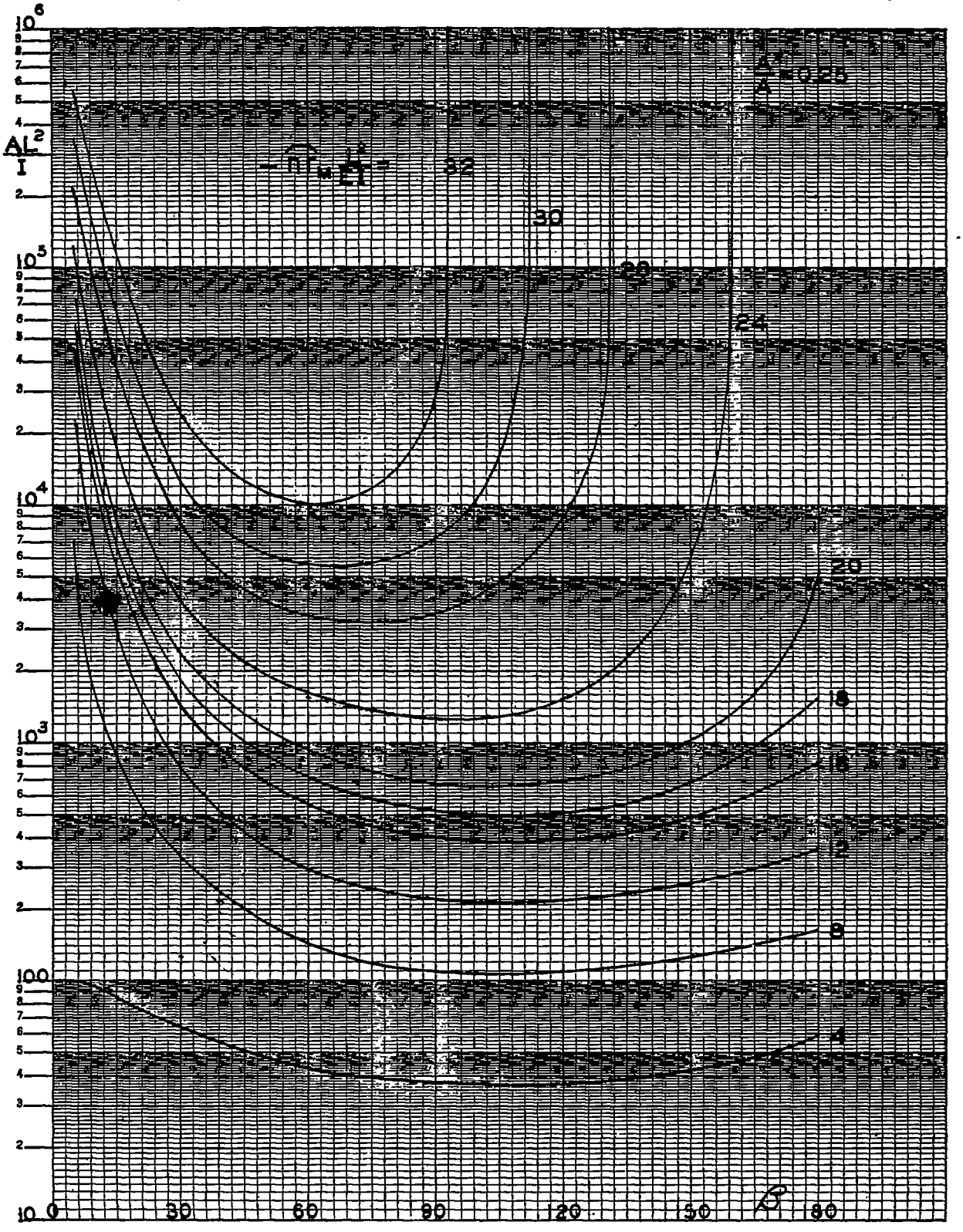


FIG. 35. INFLUENCE COEFFICIENT

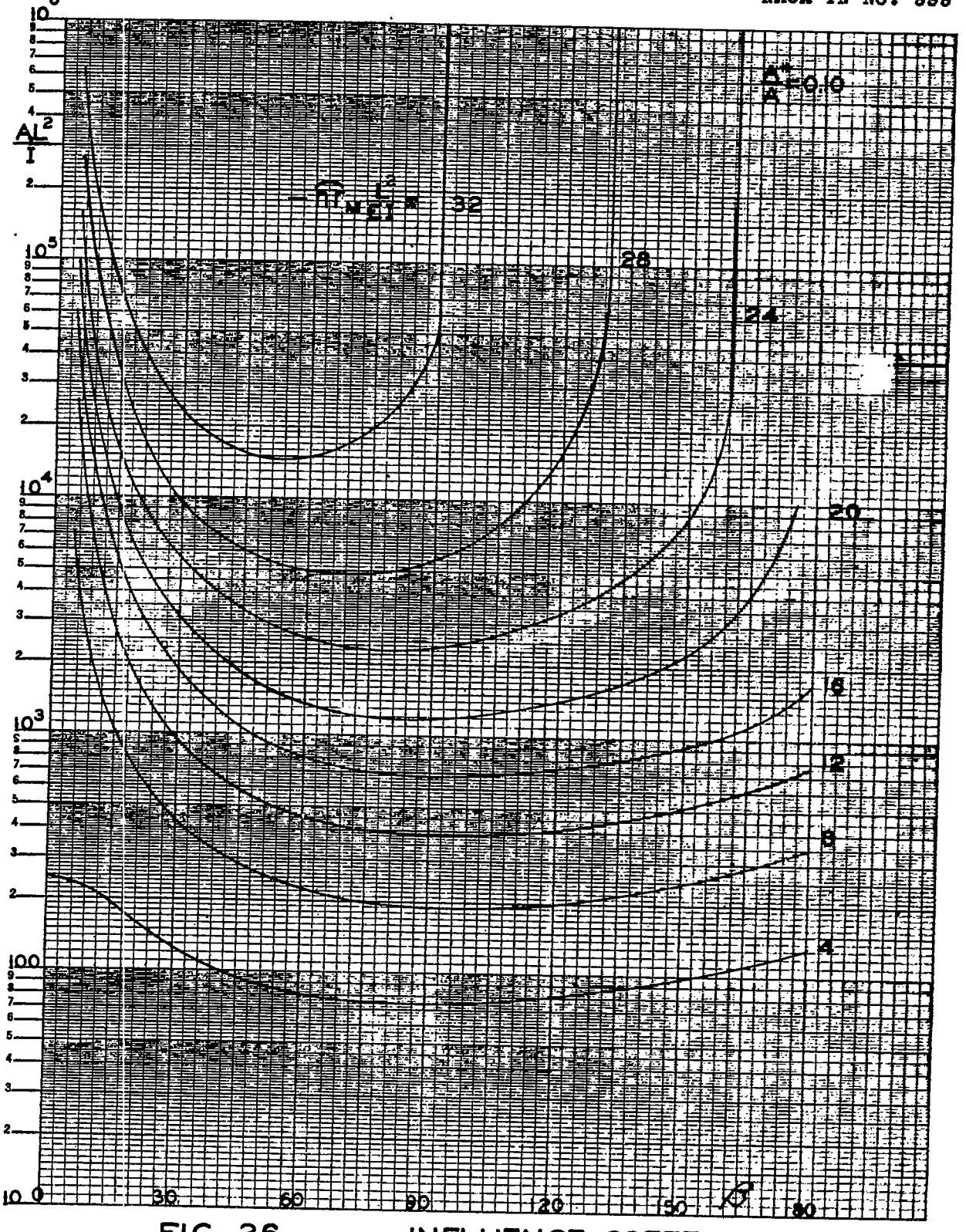


FIG. 36. INFLUENCE COEFFICIENT

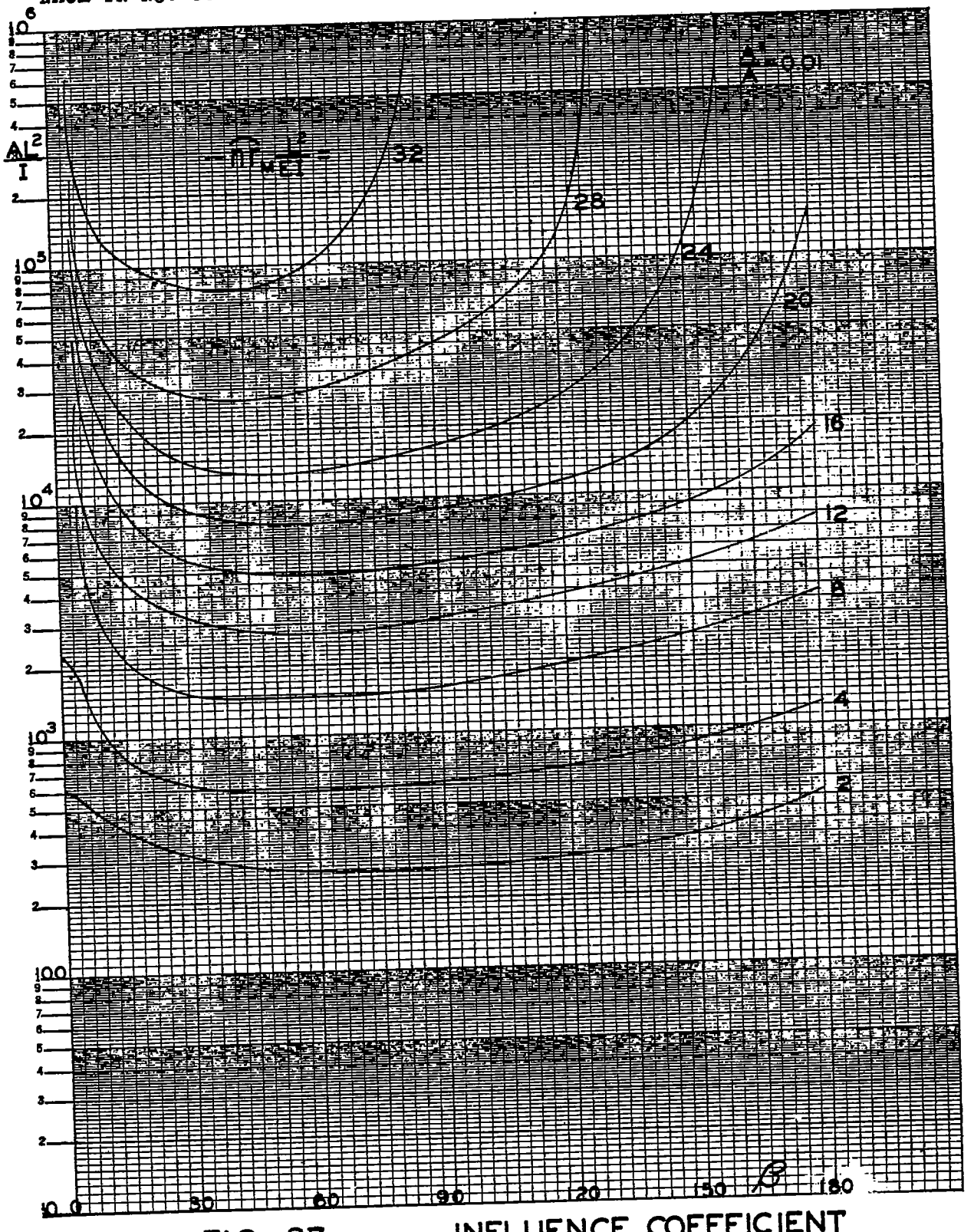


FIG. 37. INFLUENCE COEFFICIENT

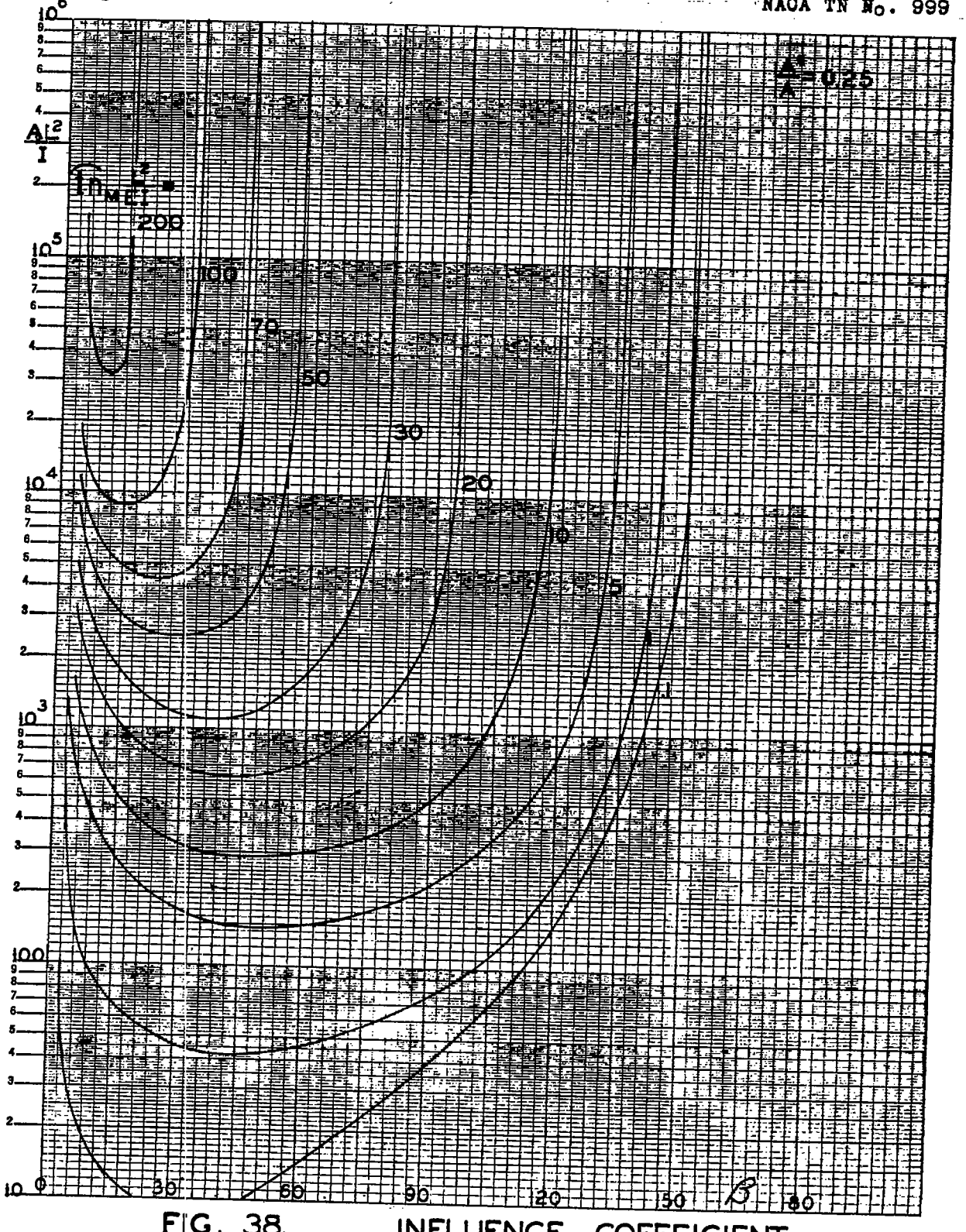


FIG. 38. INFLUENCE COEFFICIENT

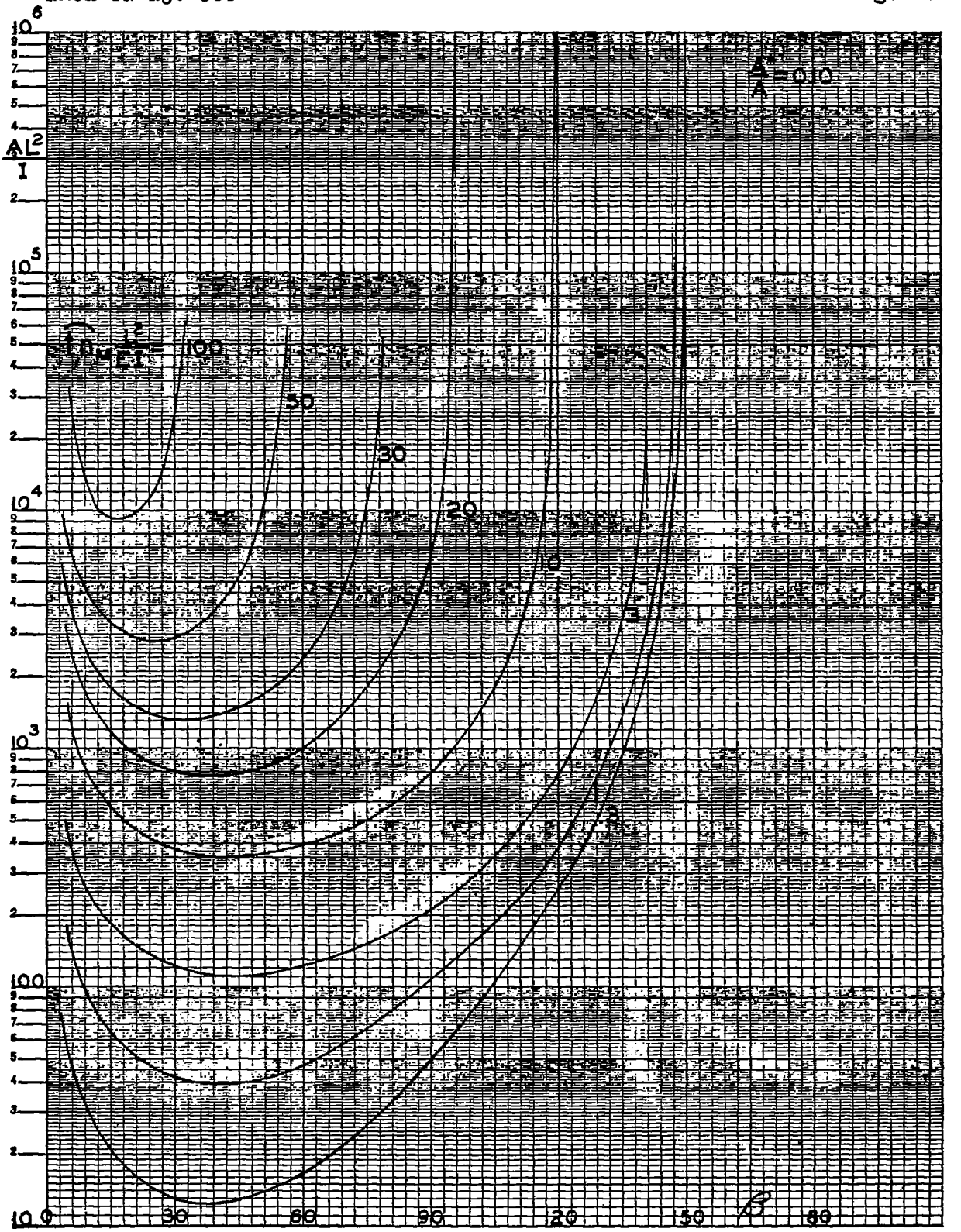


FIG. 39. INFLUENCE COEFFICIENT

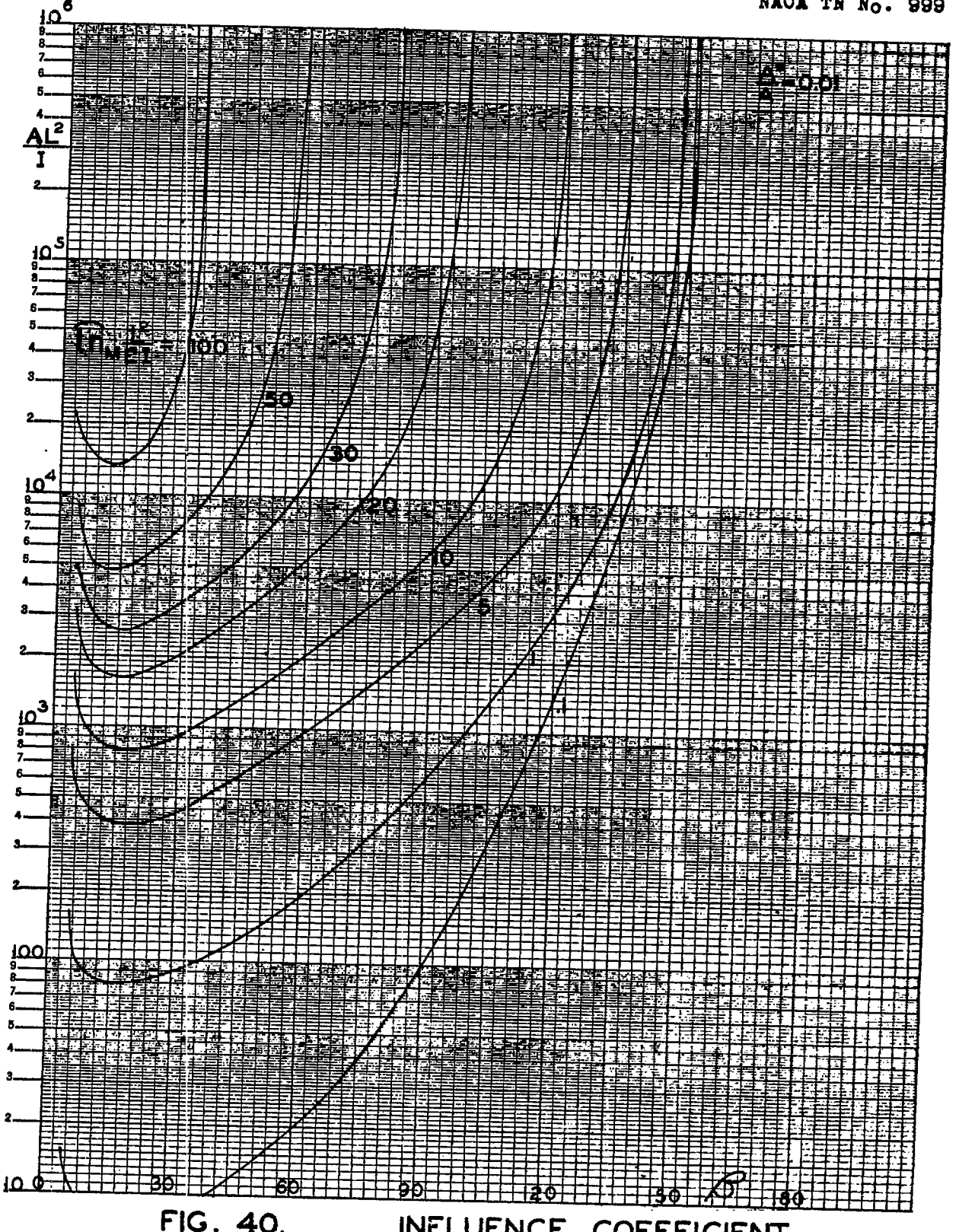


FIG. 40. INFLUENCE COEFFICIENT

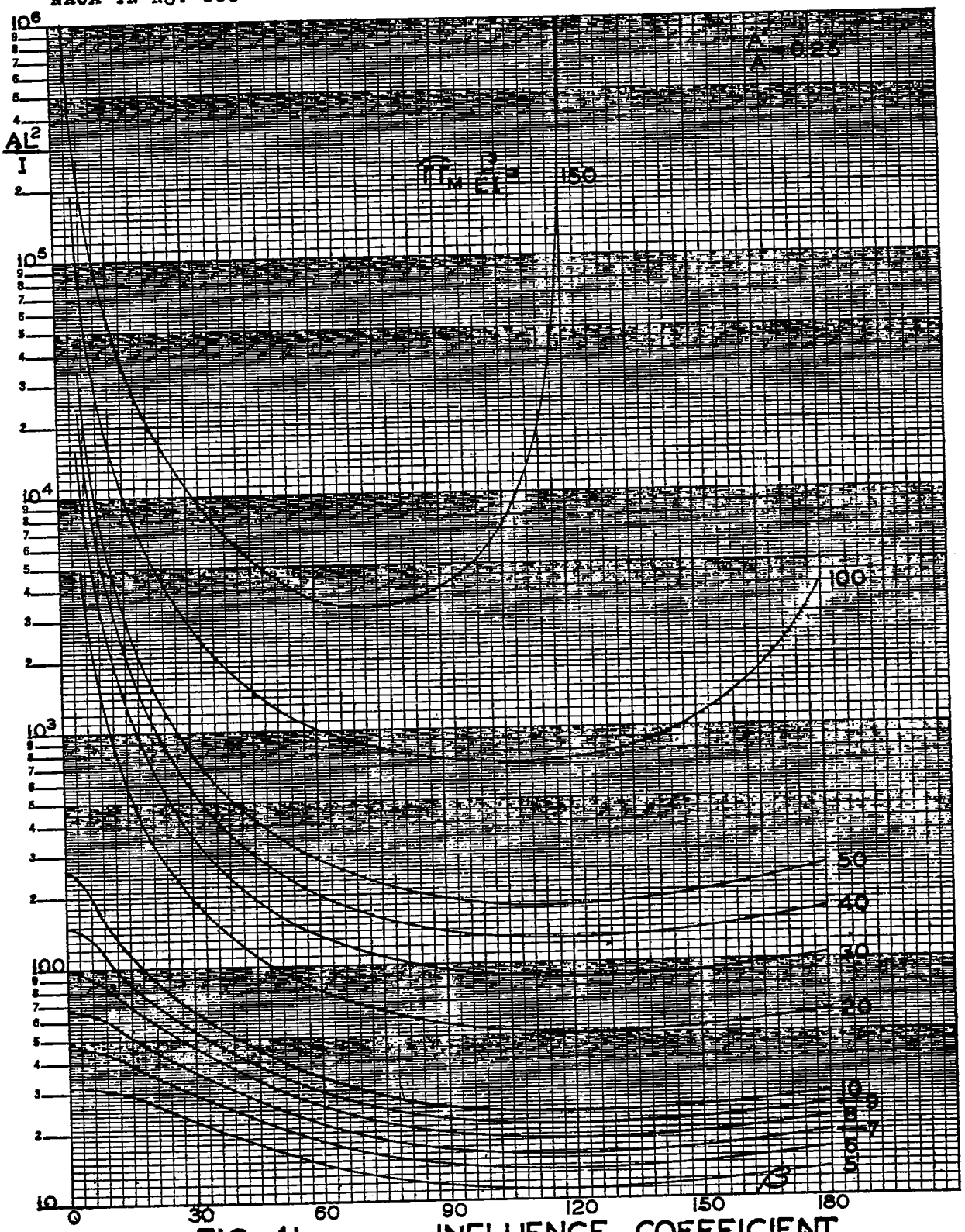


FIG. 41. INFLUENCE COEFFICIENT

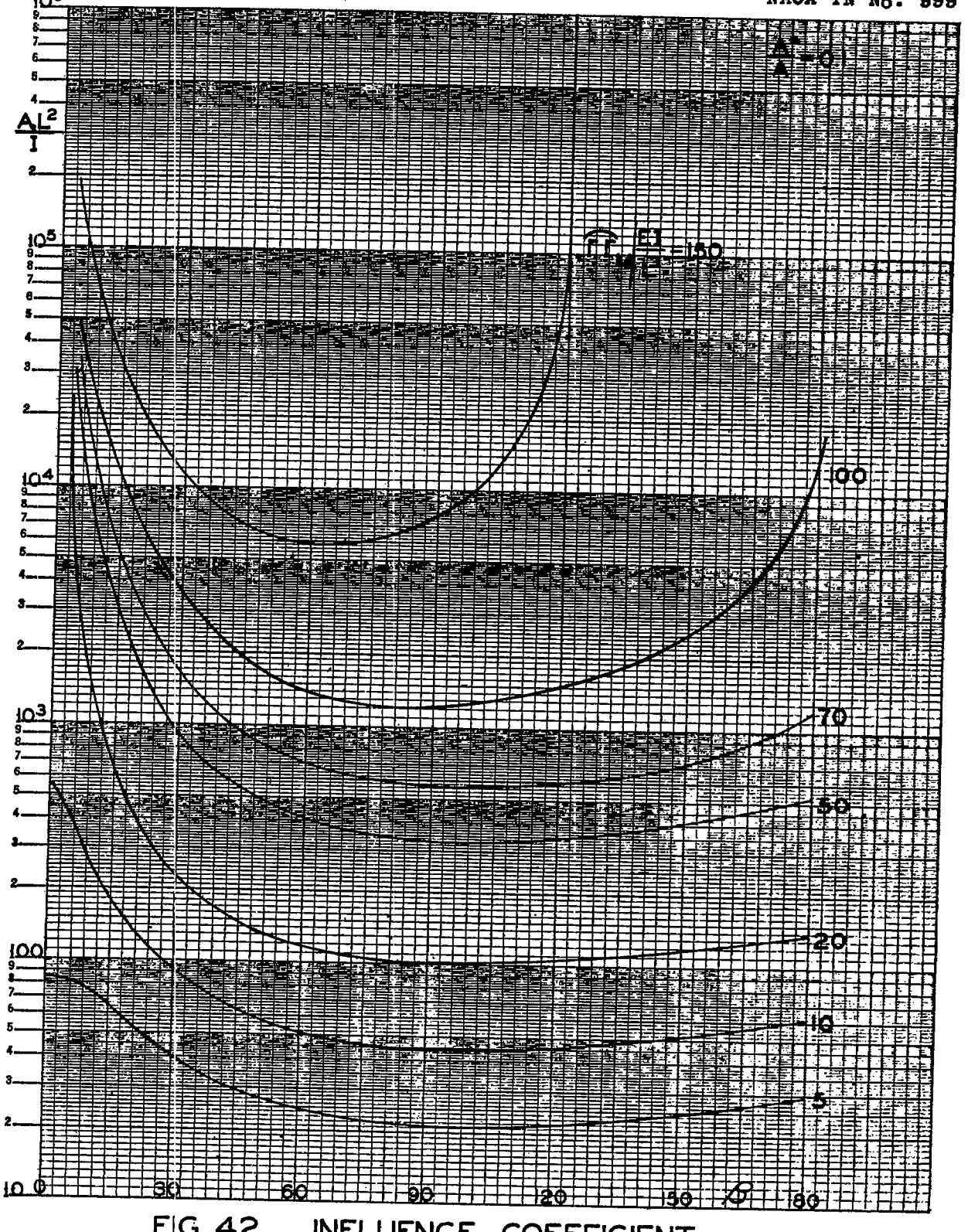


FIG. 42. INFLUENCE COEFFICIENT

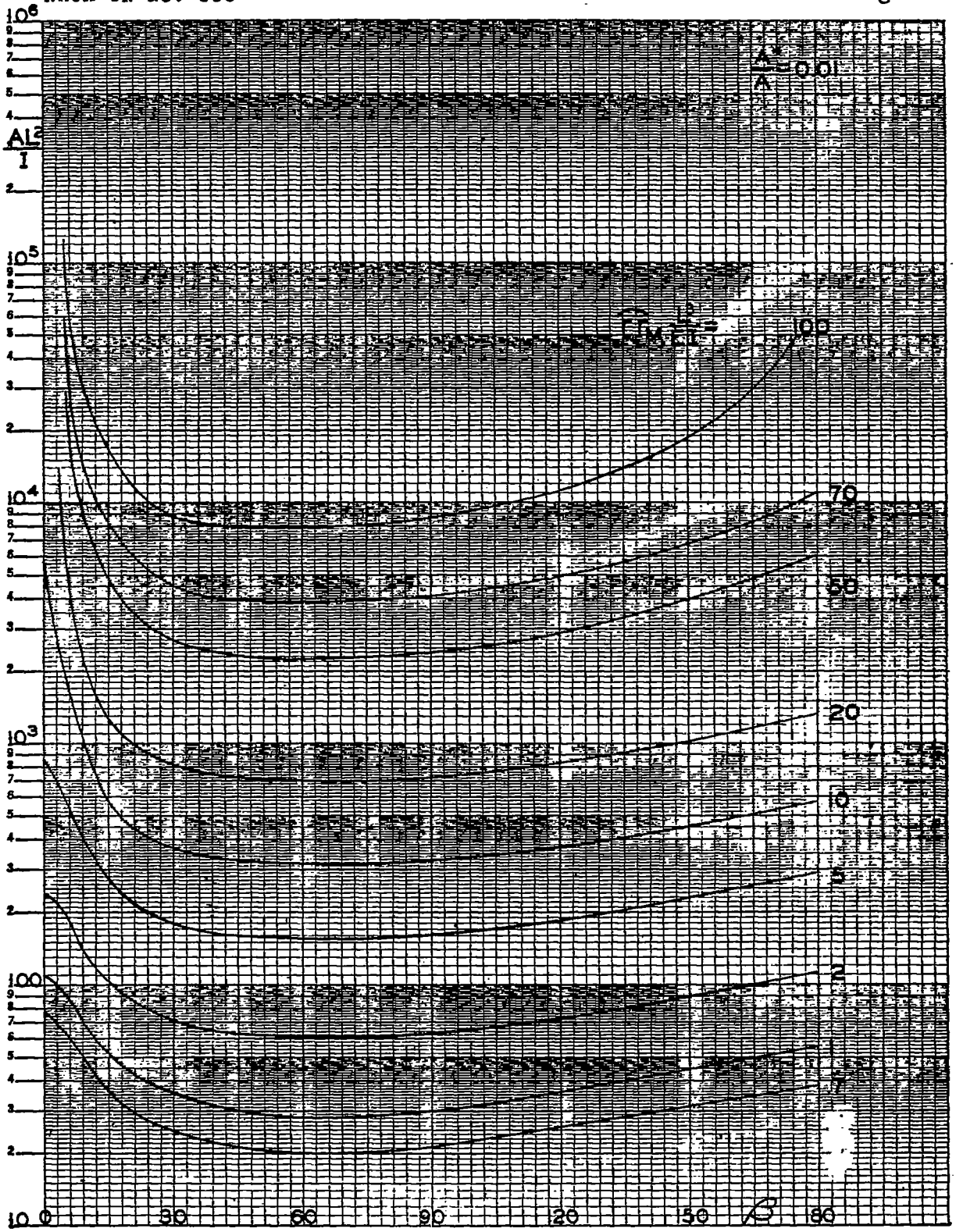


FIG. 43. INFLUENCE COEFFICIENT

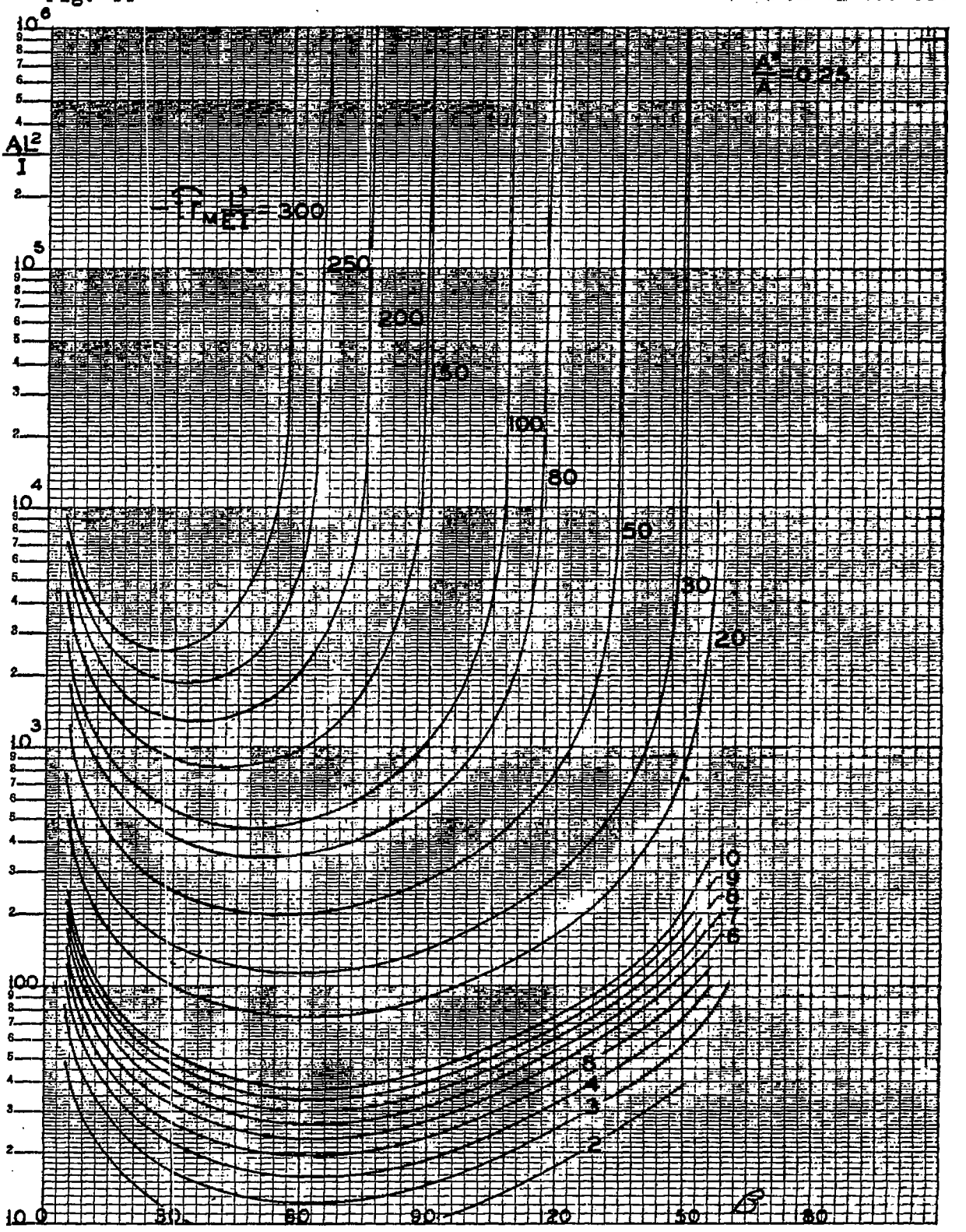


FIG. 44. INFLUENCE COEFFICIENT

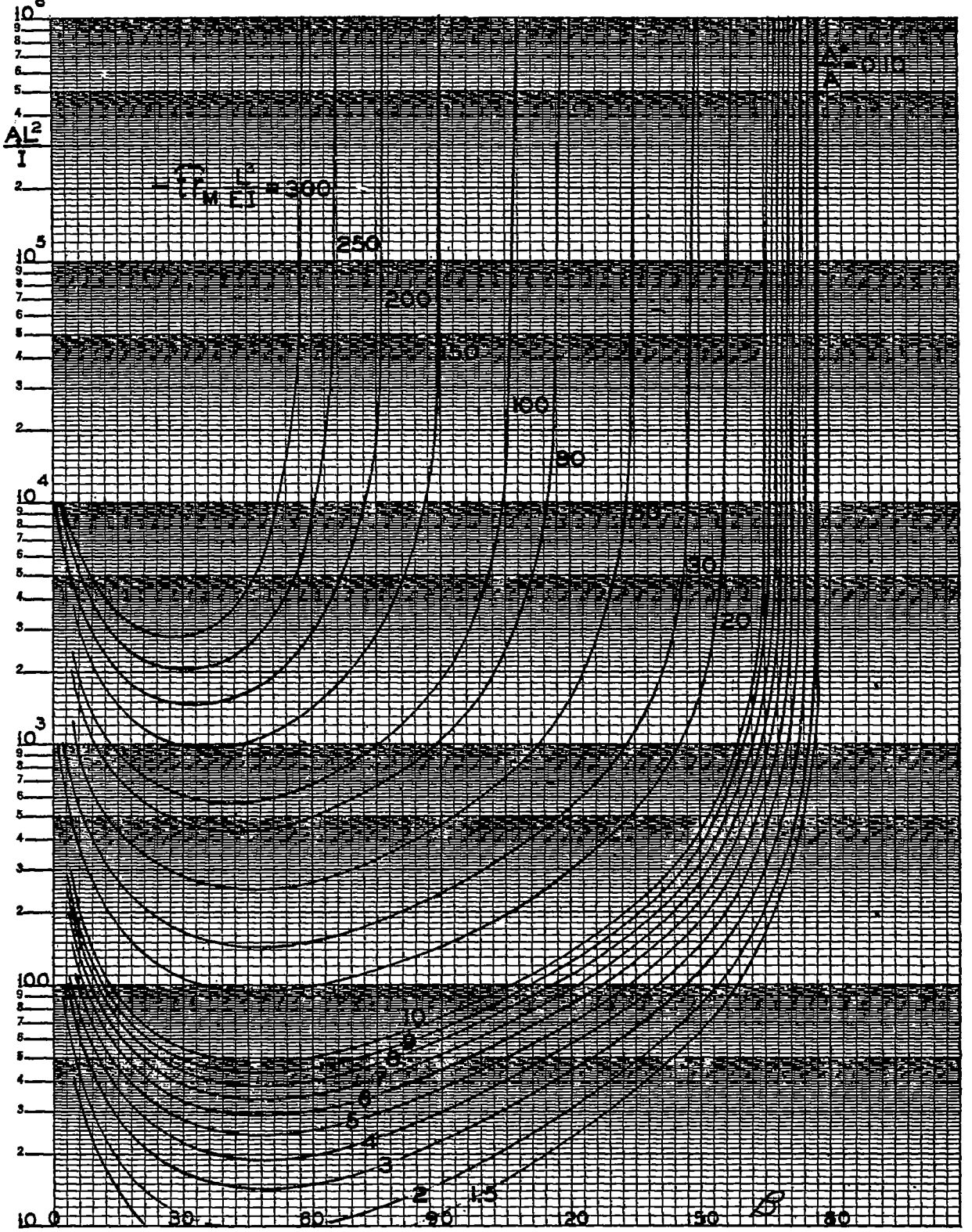


FIG. 45. INFLUENCE COEFFICIENT

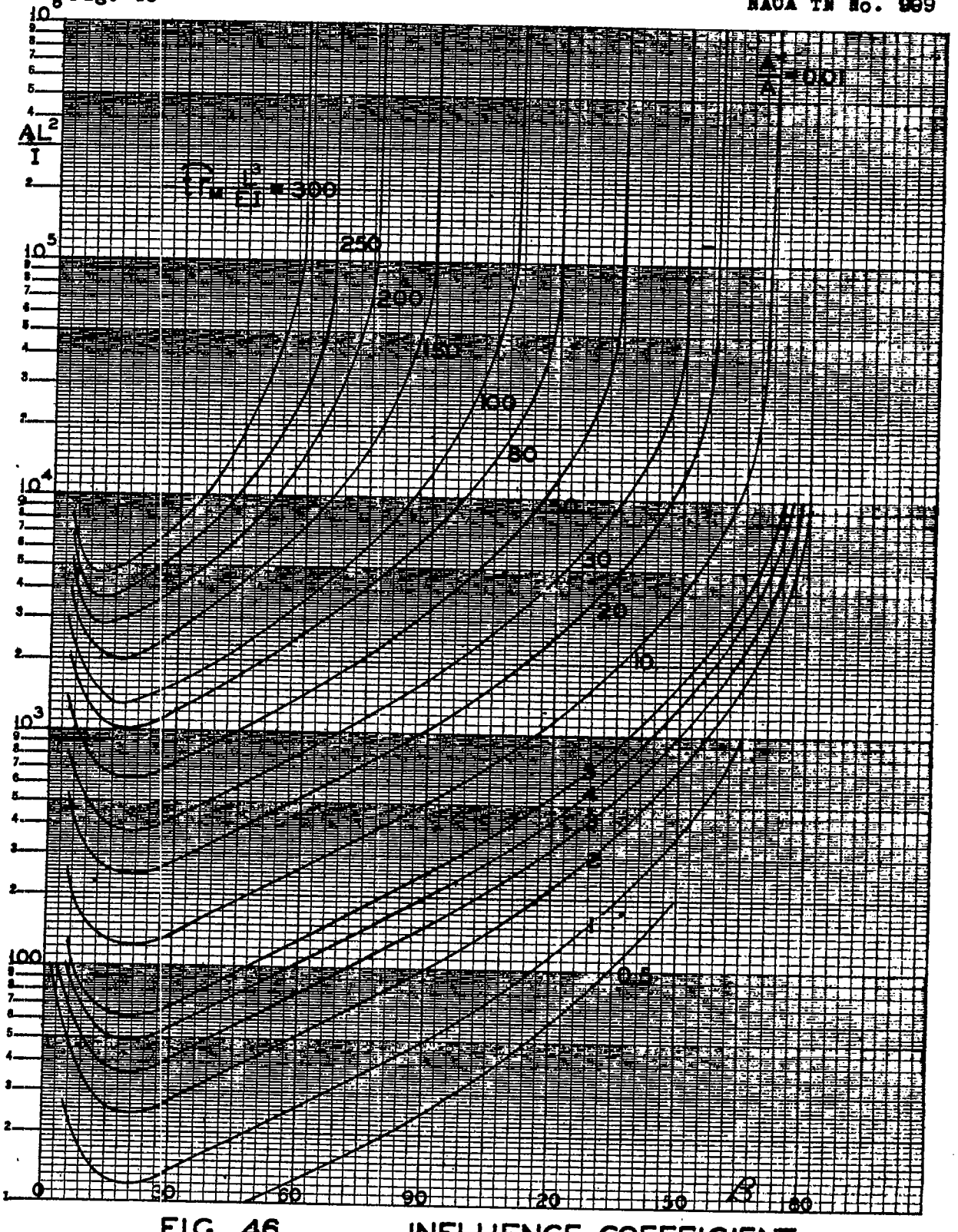


FIG. 46. INFLUENCE COEFFICIENT

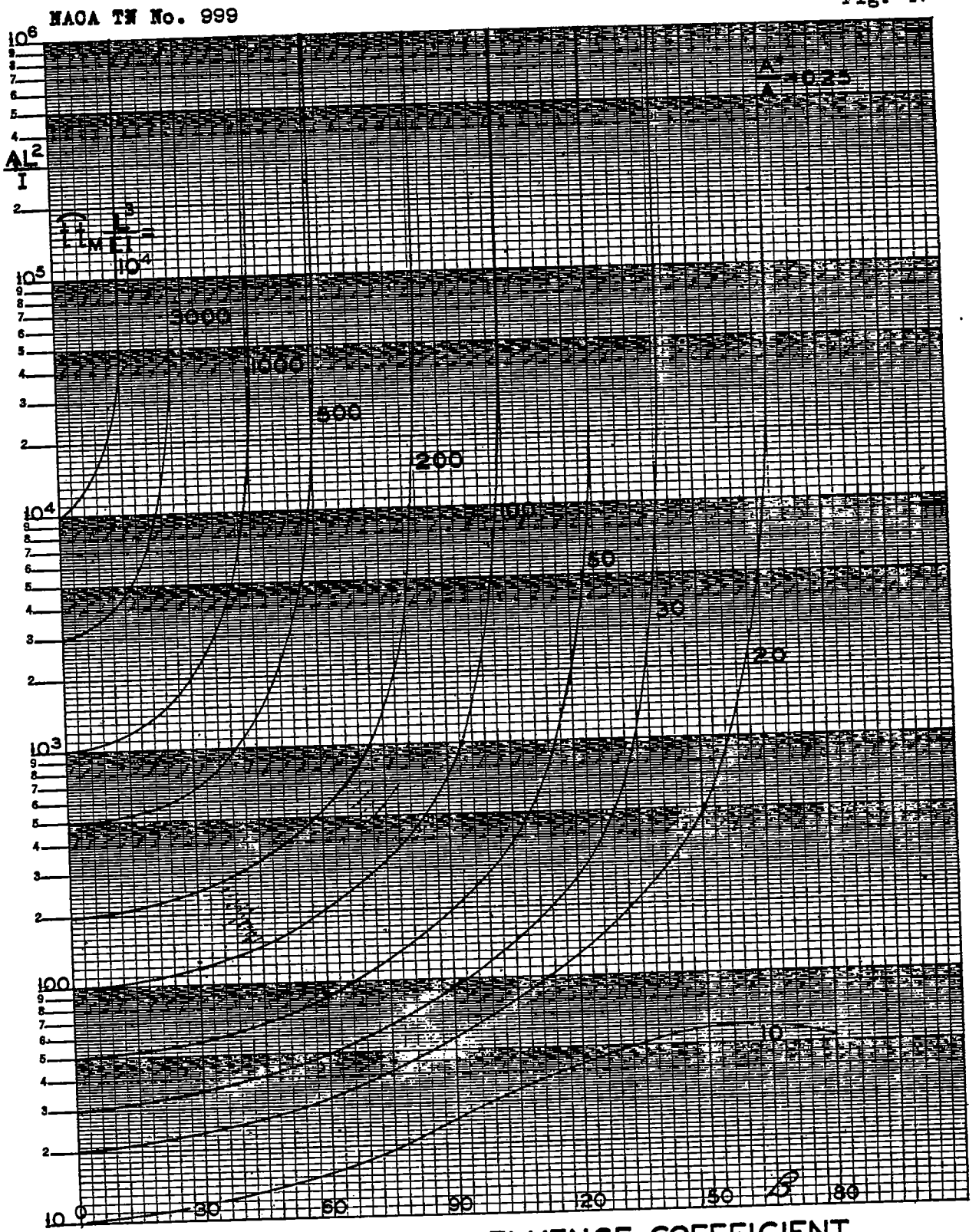


FIG. 47. INFLUENCE COEFFICIENT

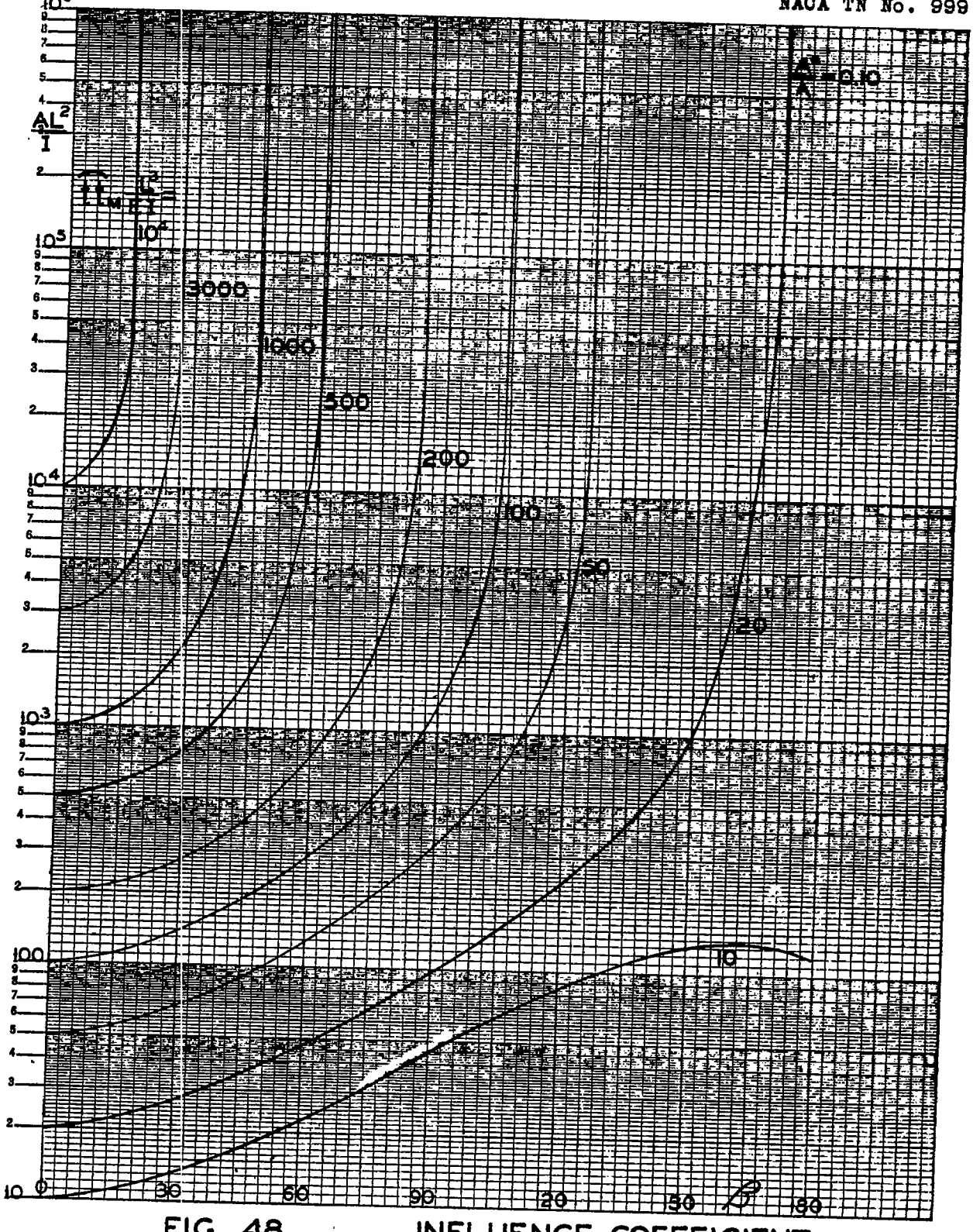


FIG. 48. INFLUENCE COEFFICIENT

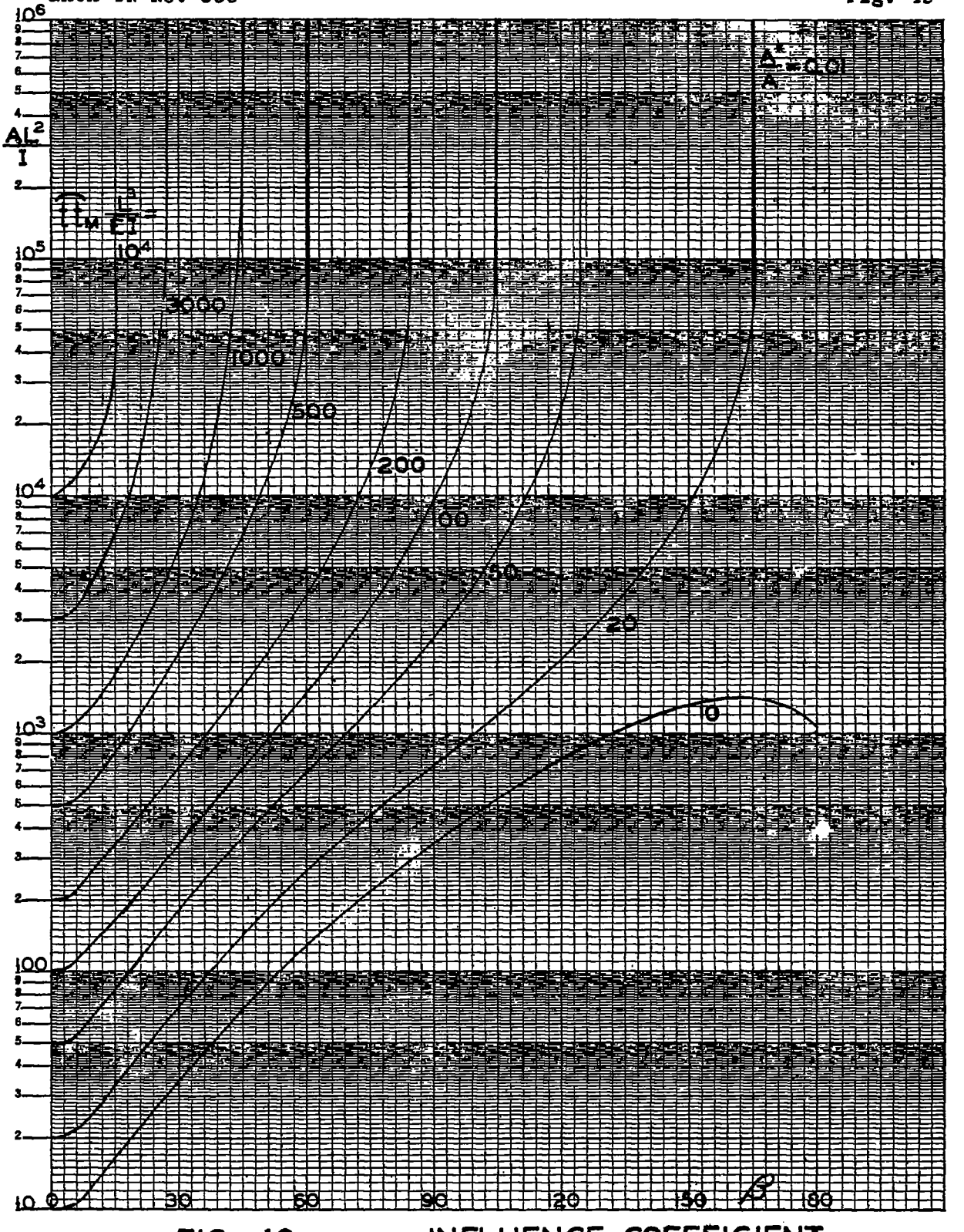


FIG. 49. INFLUENCE COEFFICIENT

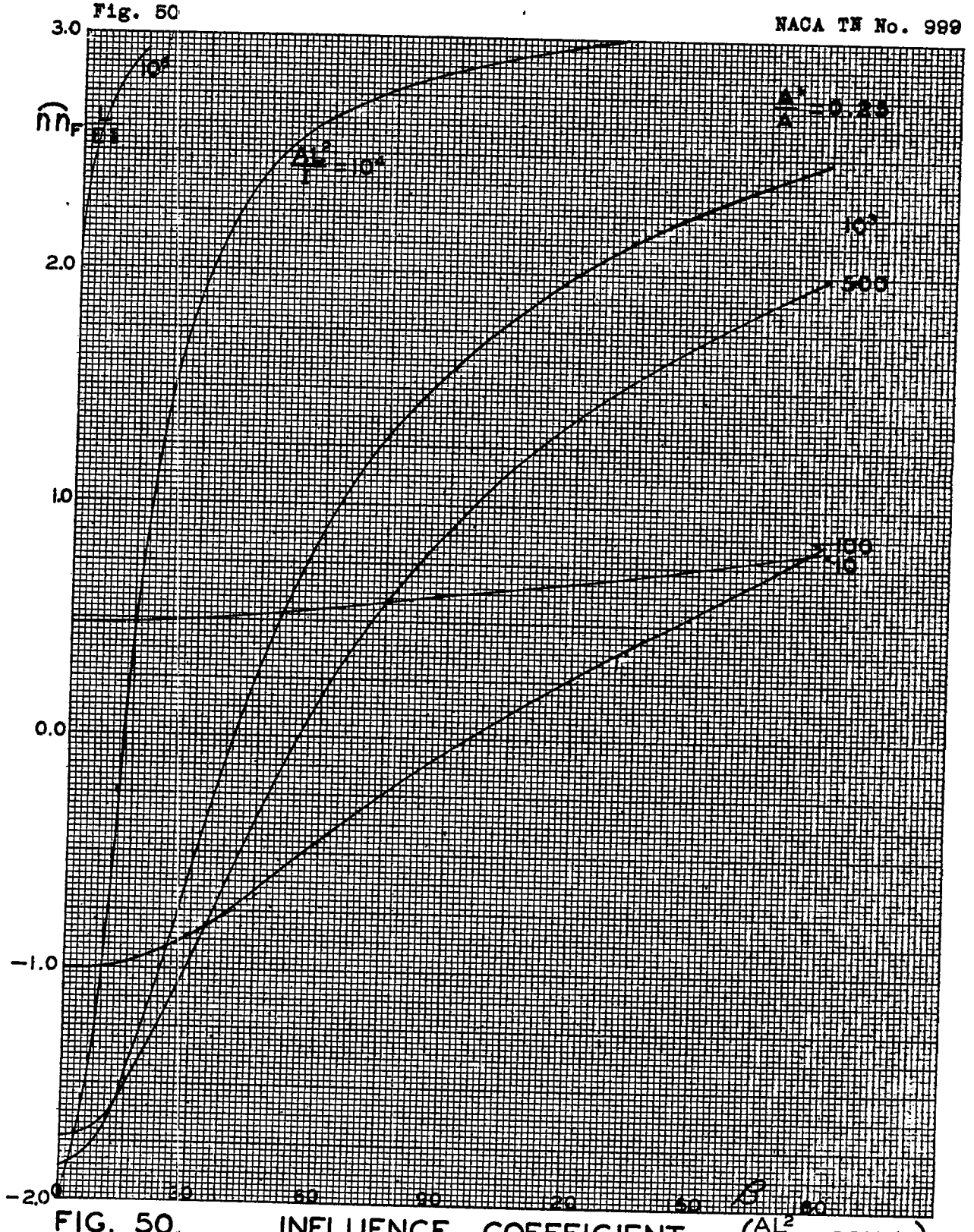


FIG. 50. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

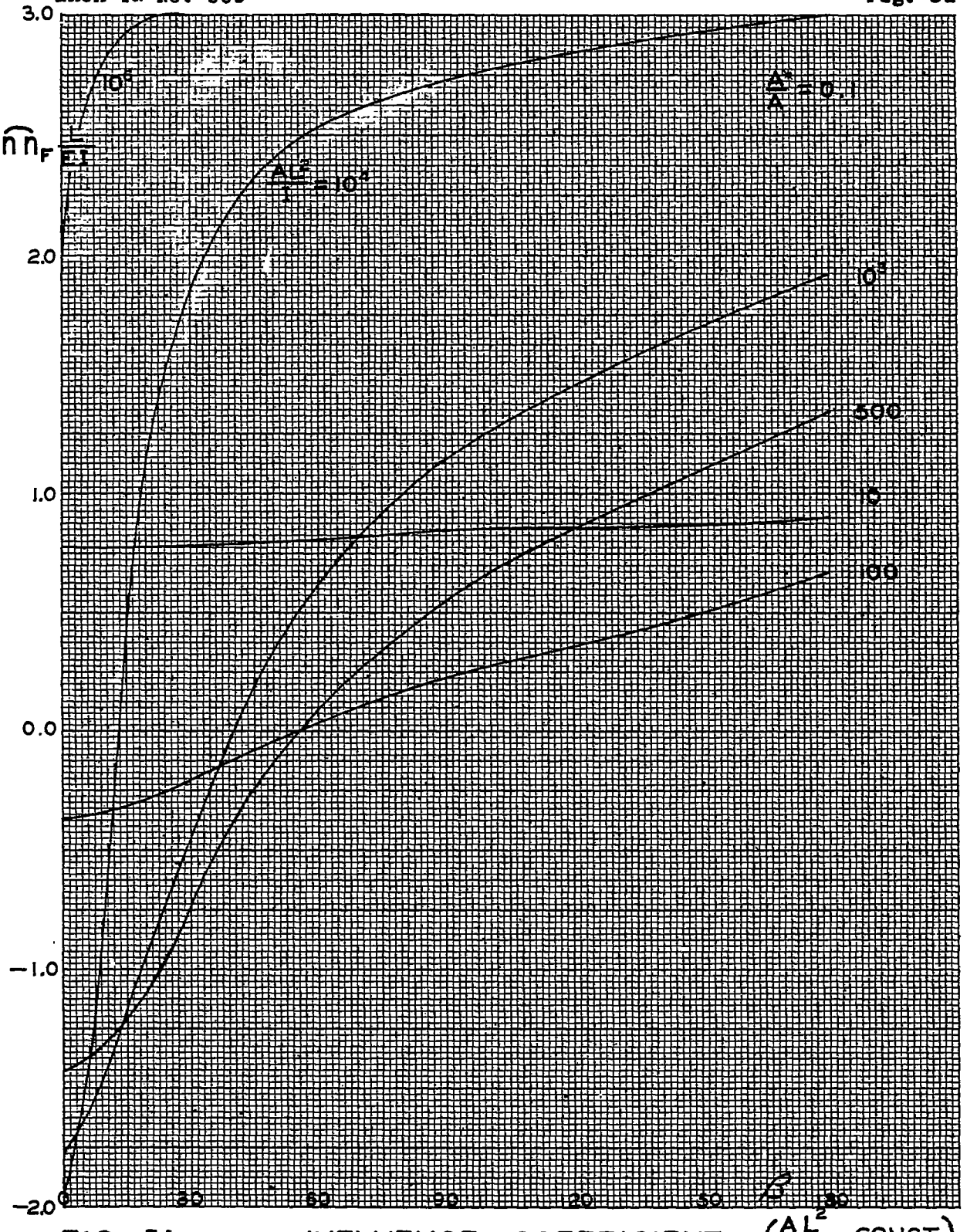


FIG. 51. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

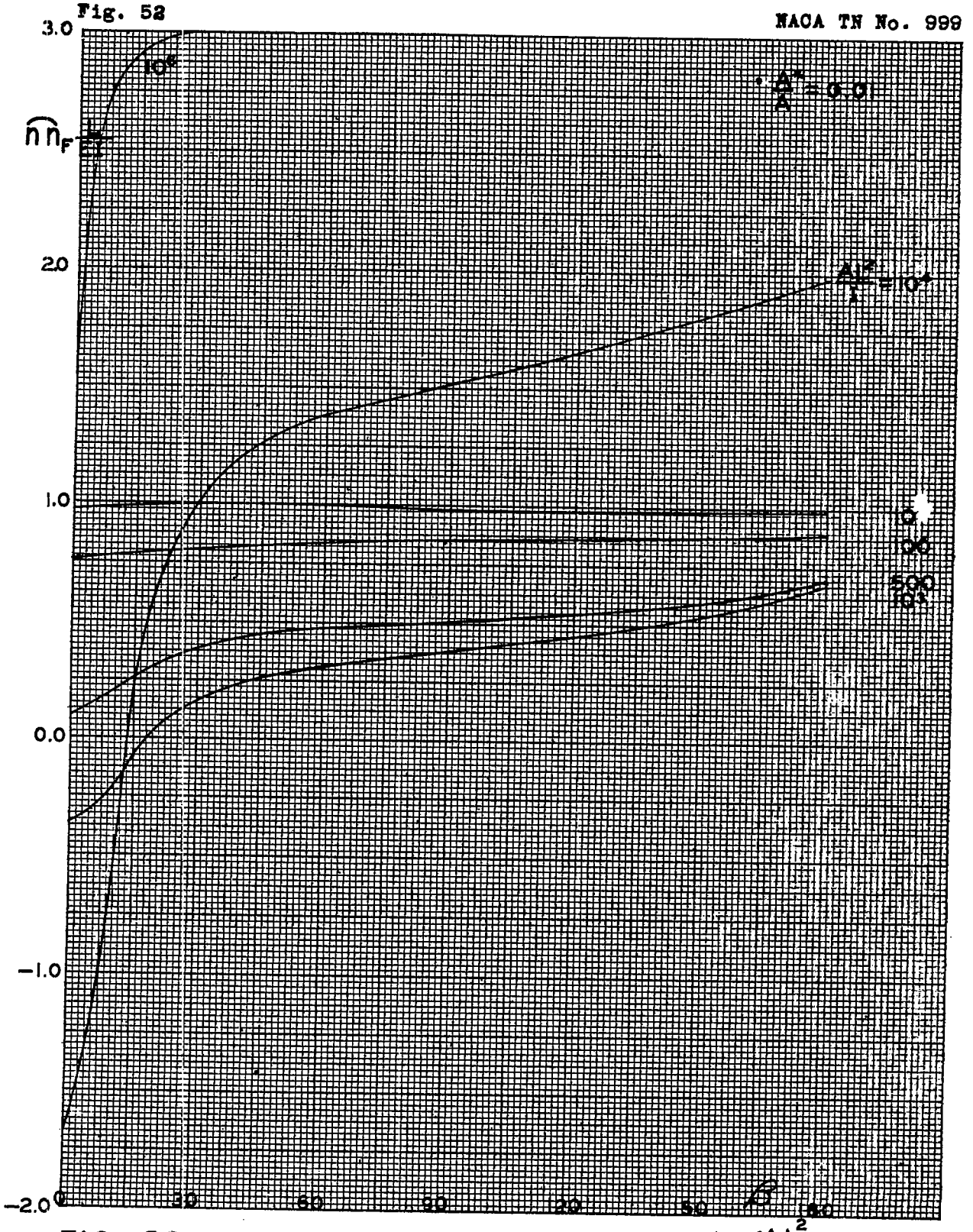


FIG. 52. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

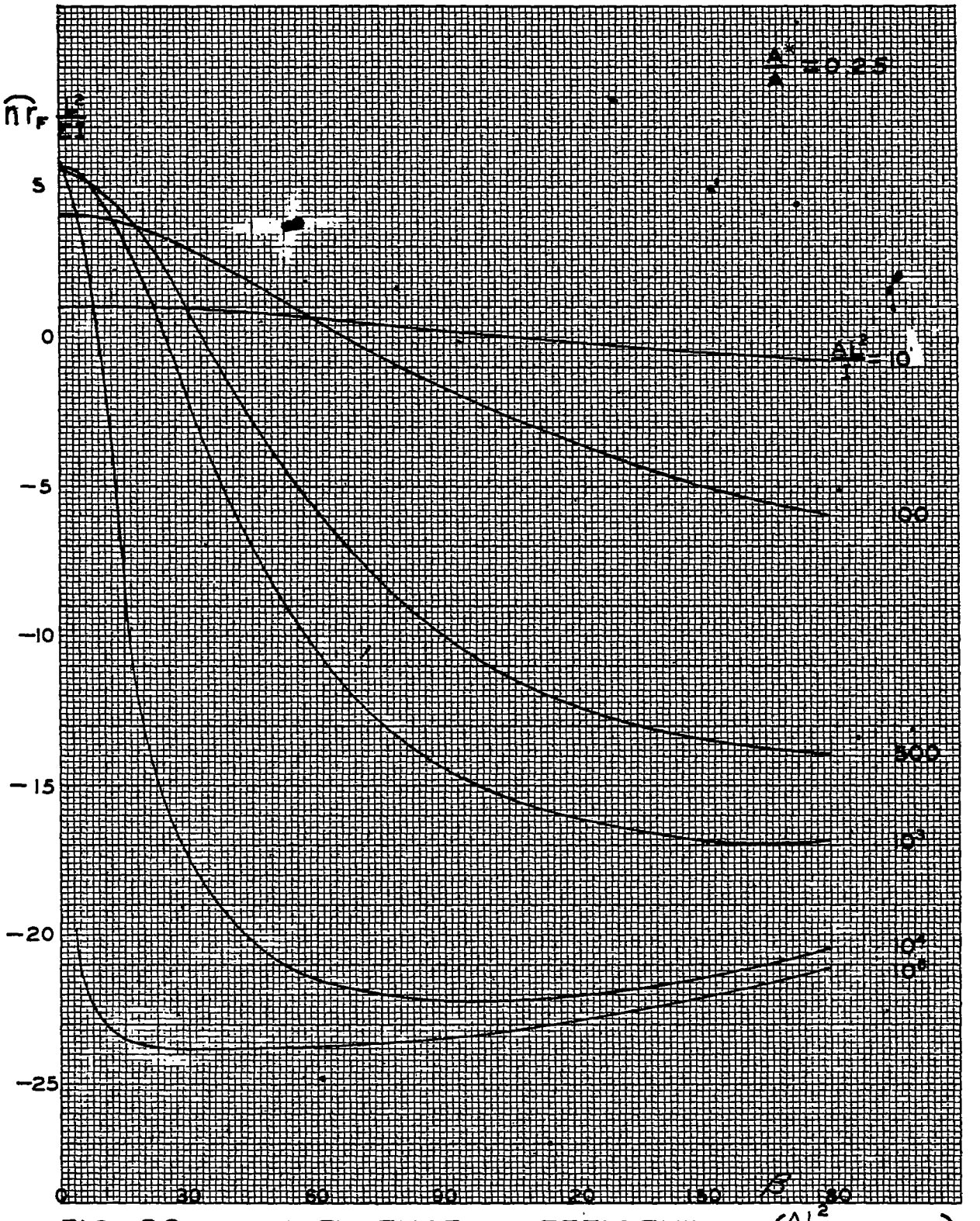


FIG. 53. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

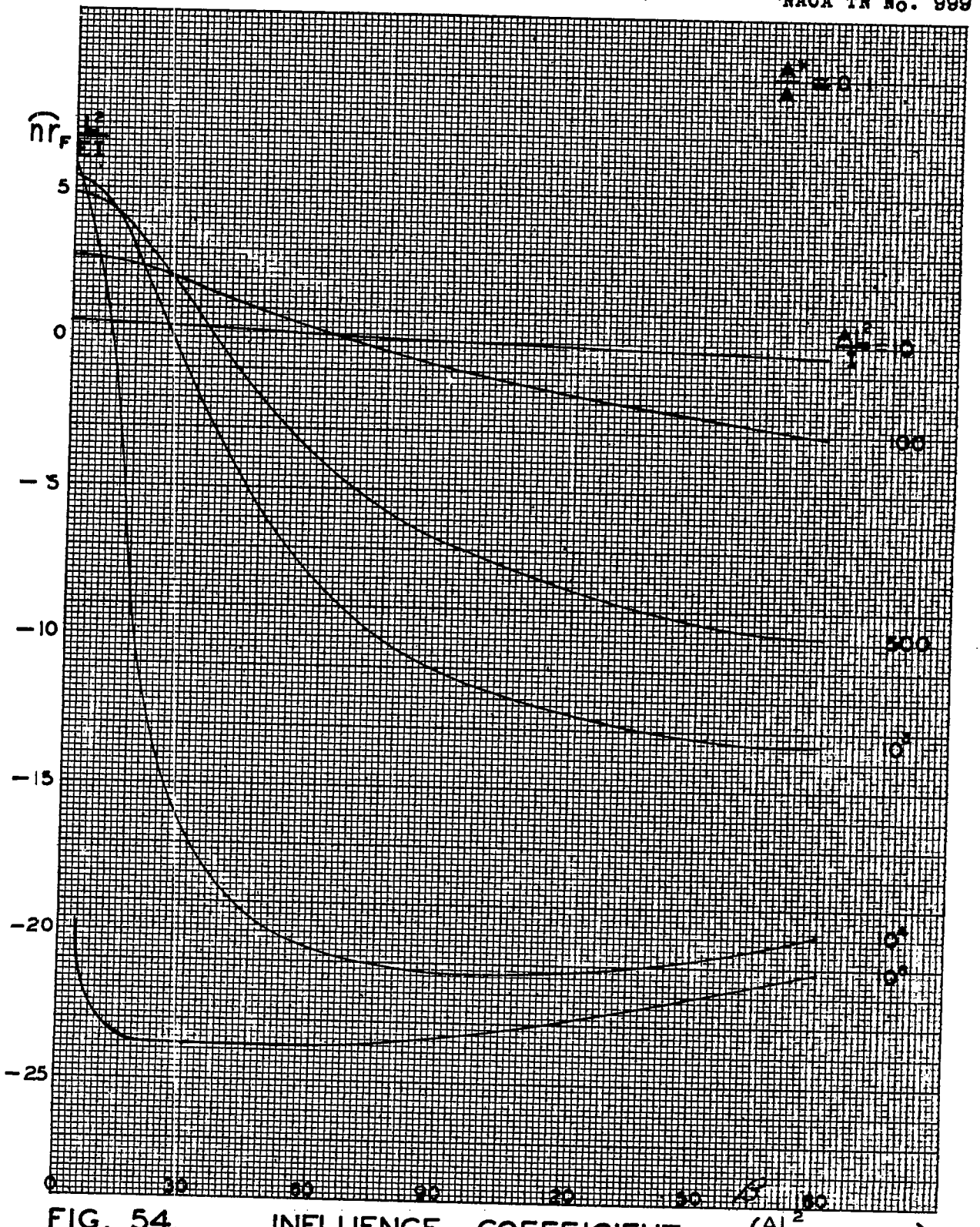


FIG. 54. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

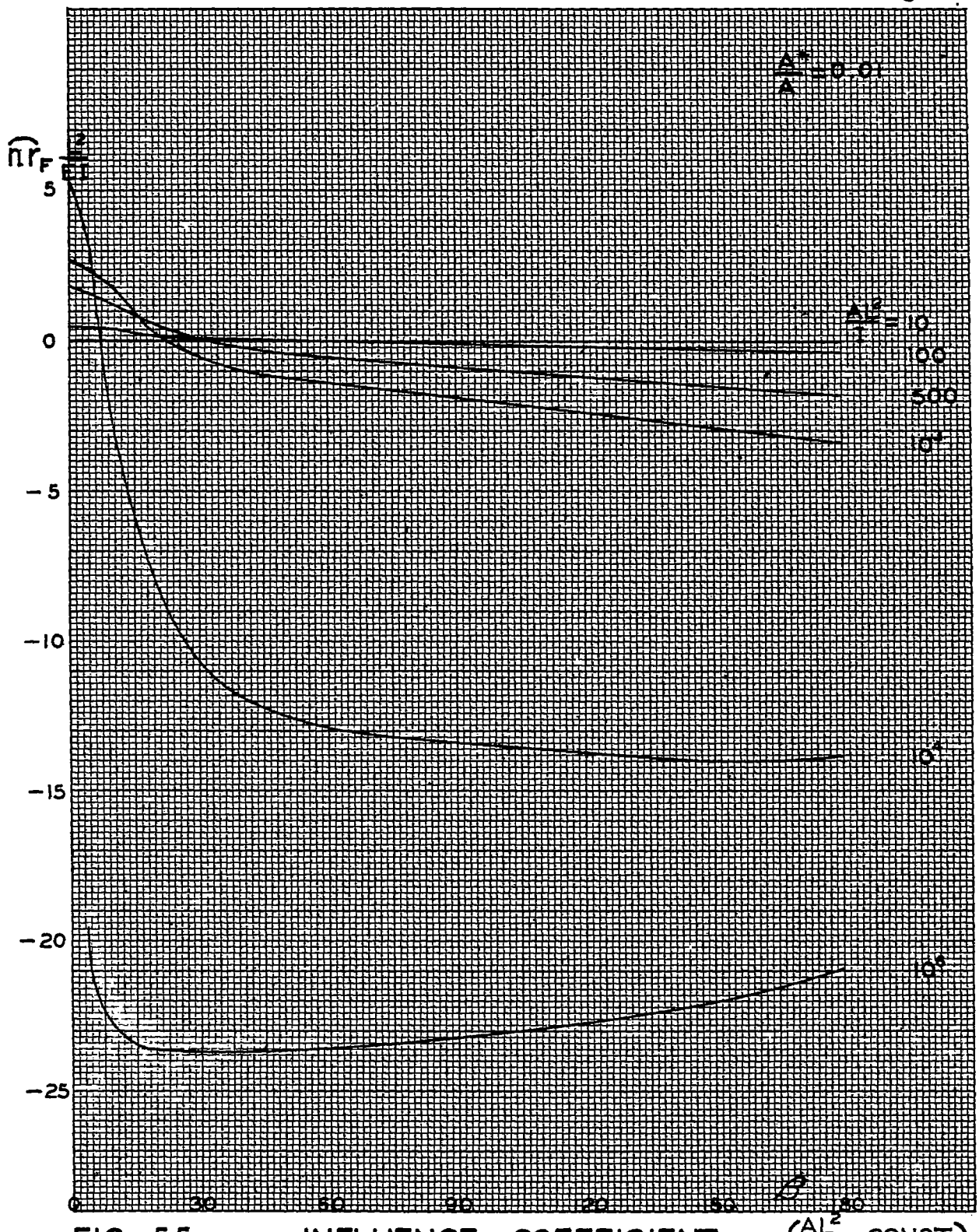


FIG. 55. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

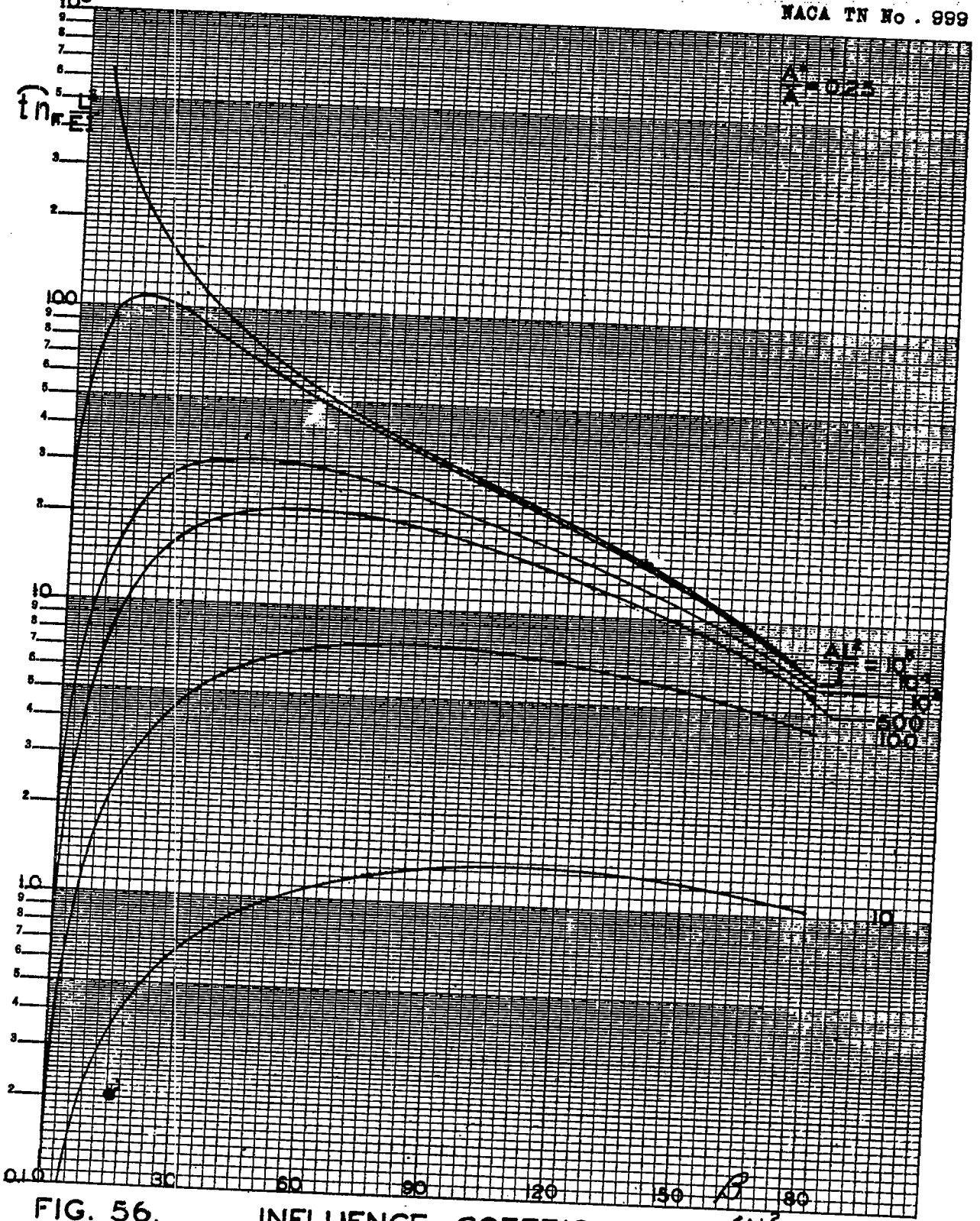


FIG. 56. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

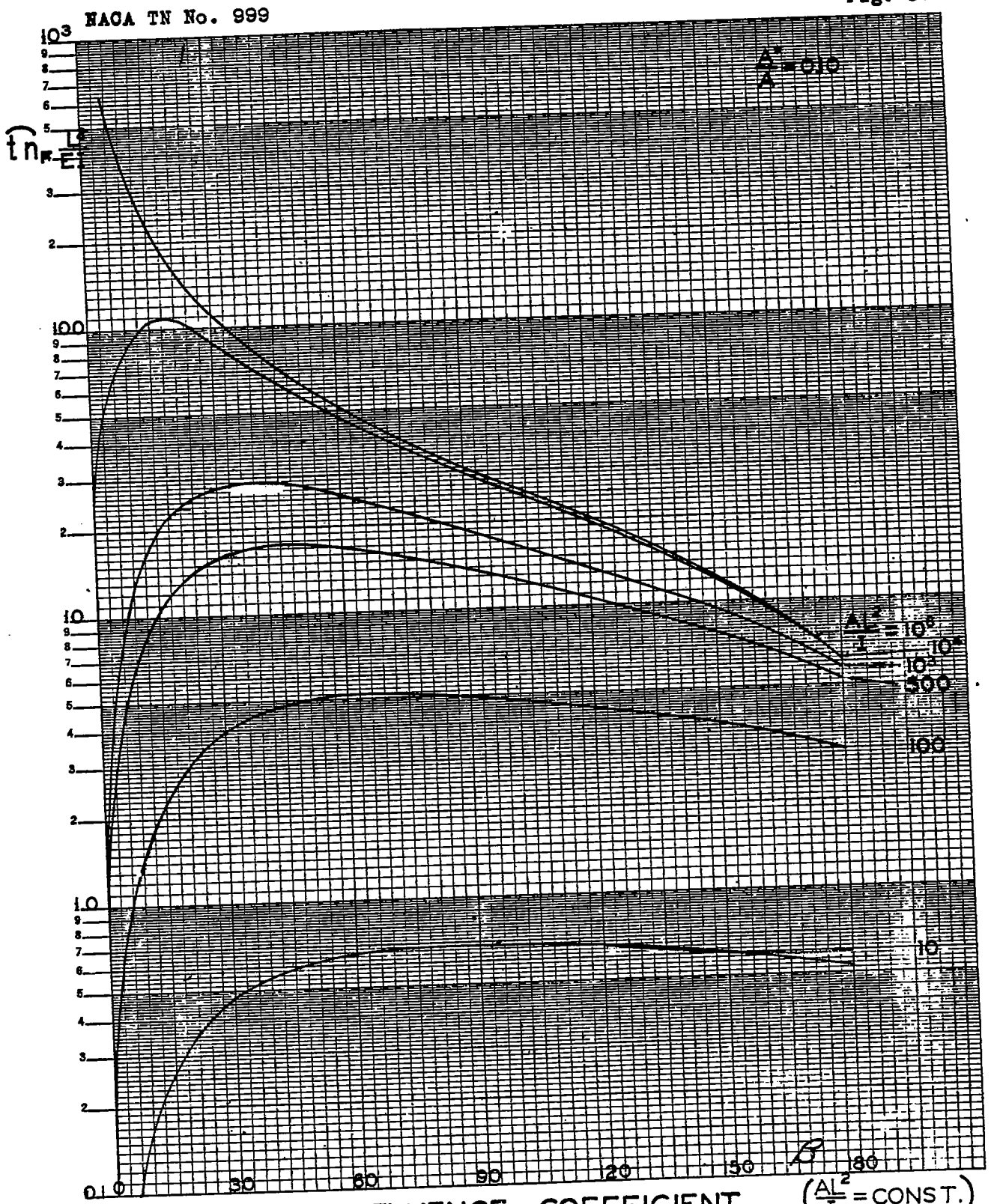


FIG. 57. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

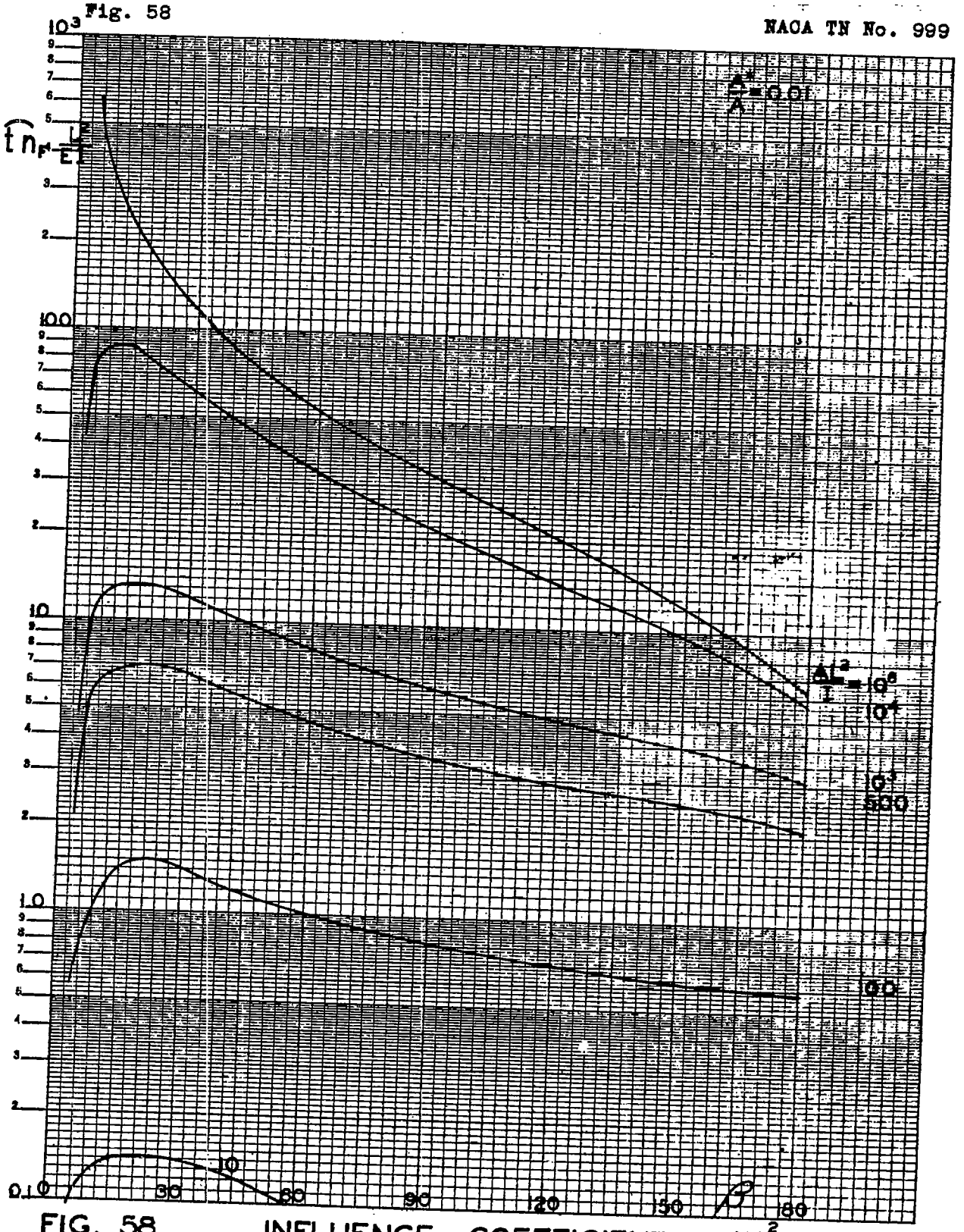


FIG. 58. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

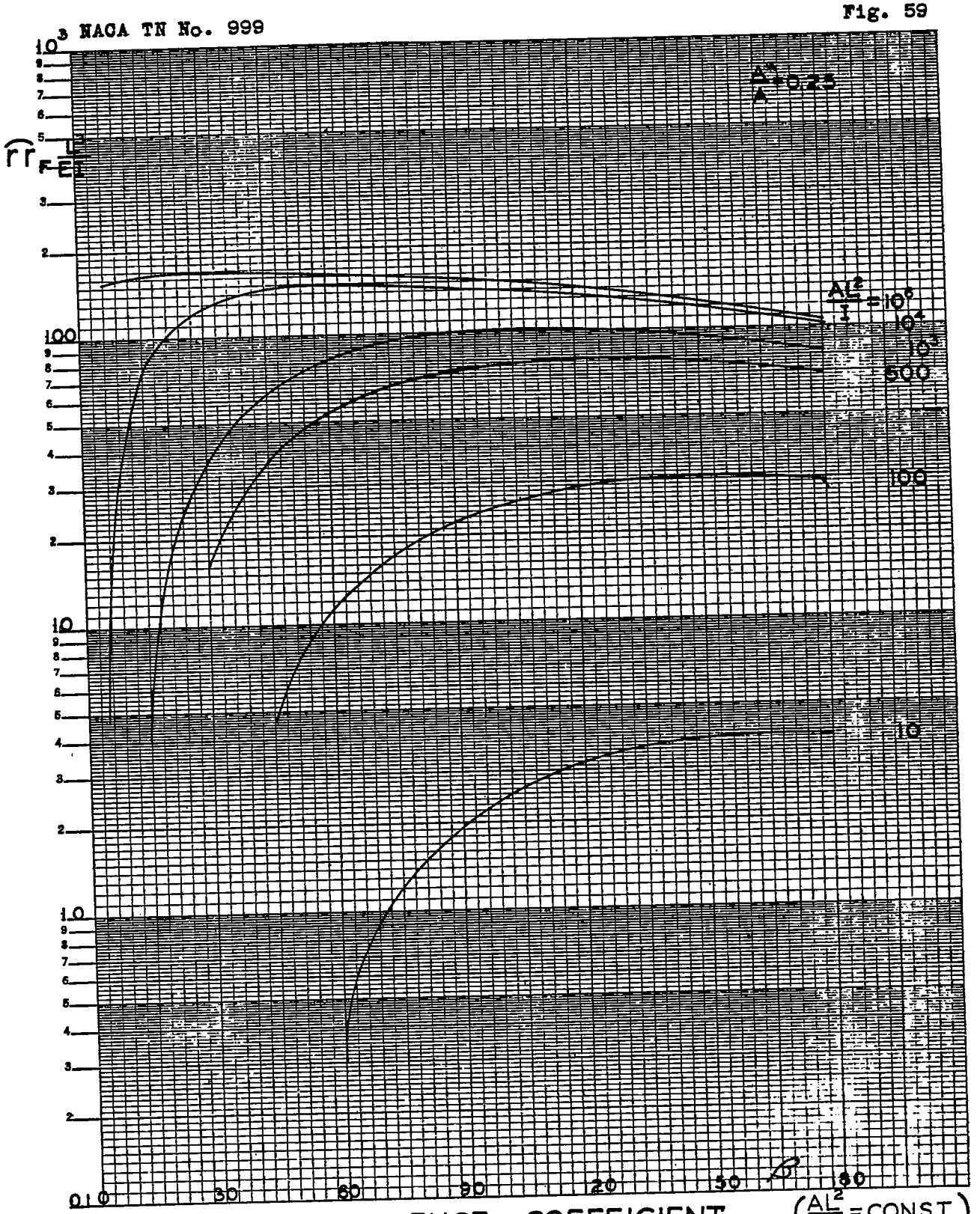


FIG. 59. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

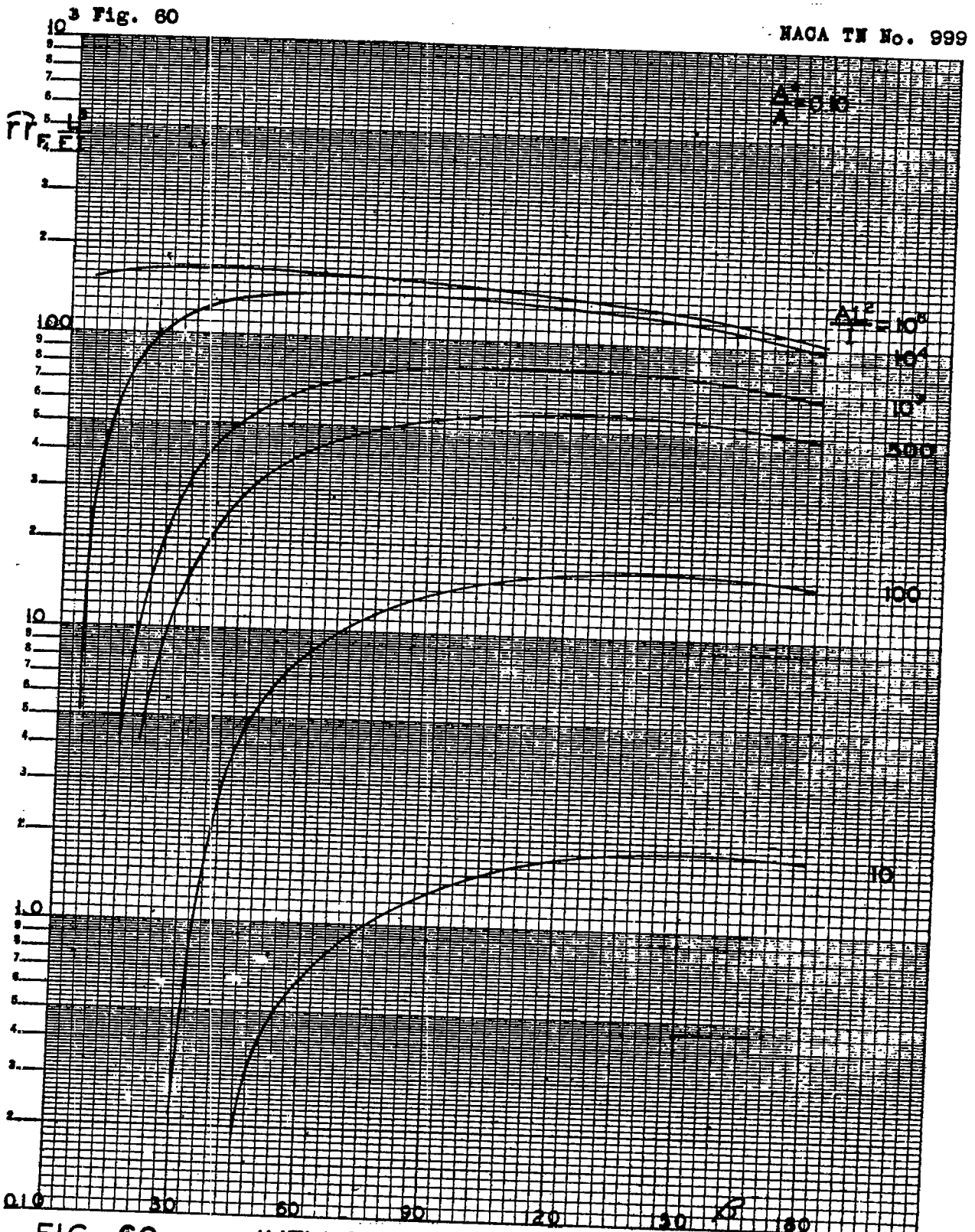


FIG. 60. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

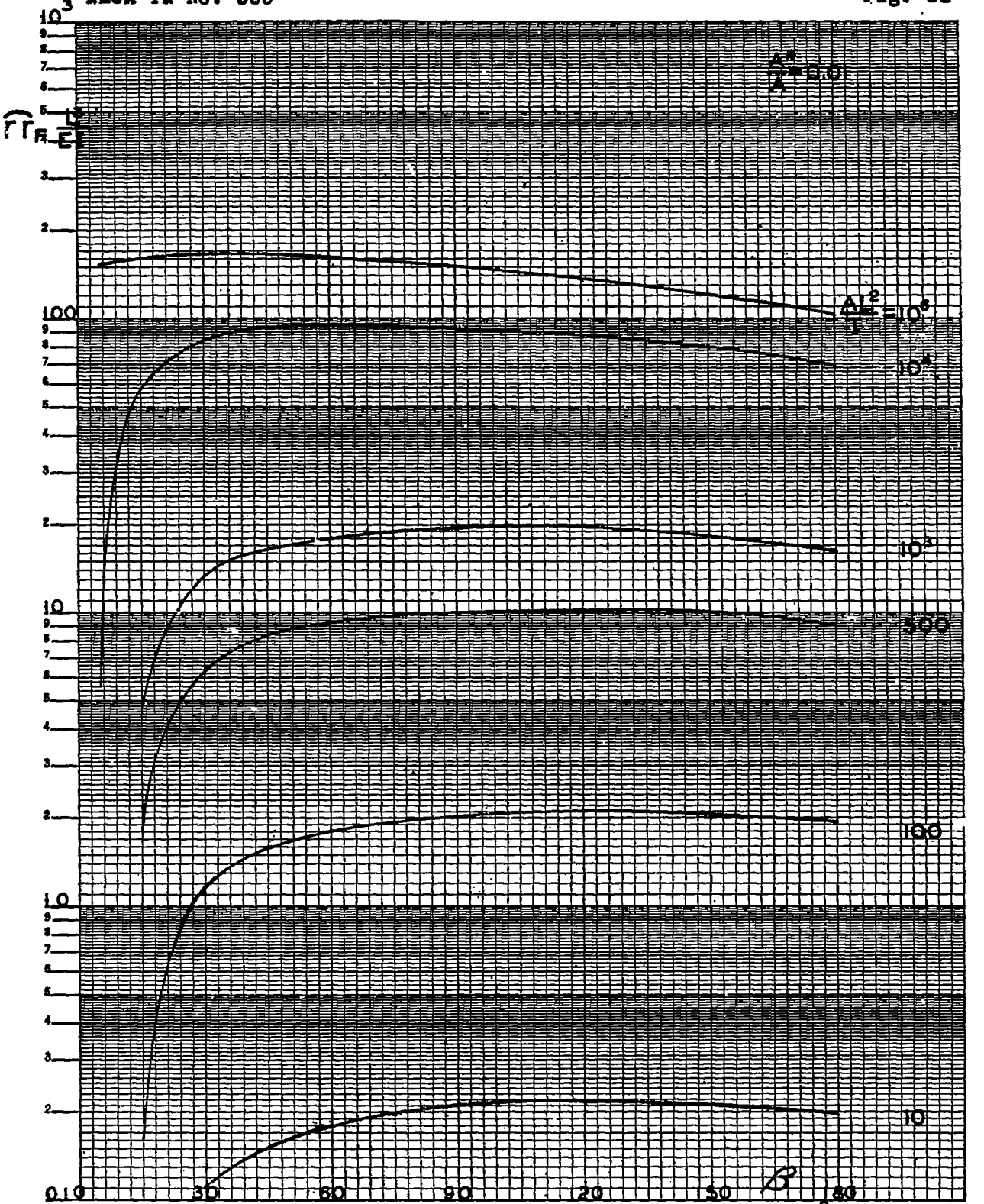


FIG. 61. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

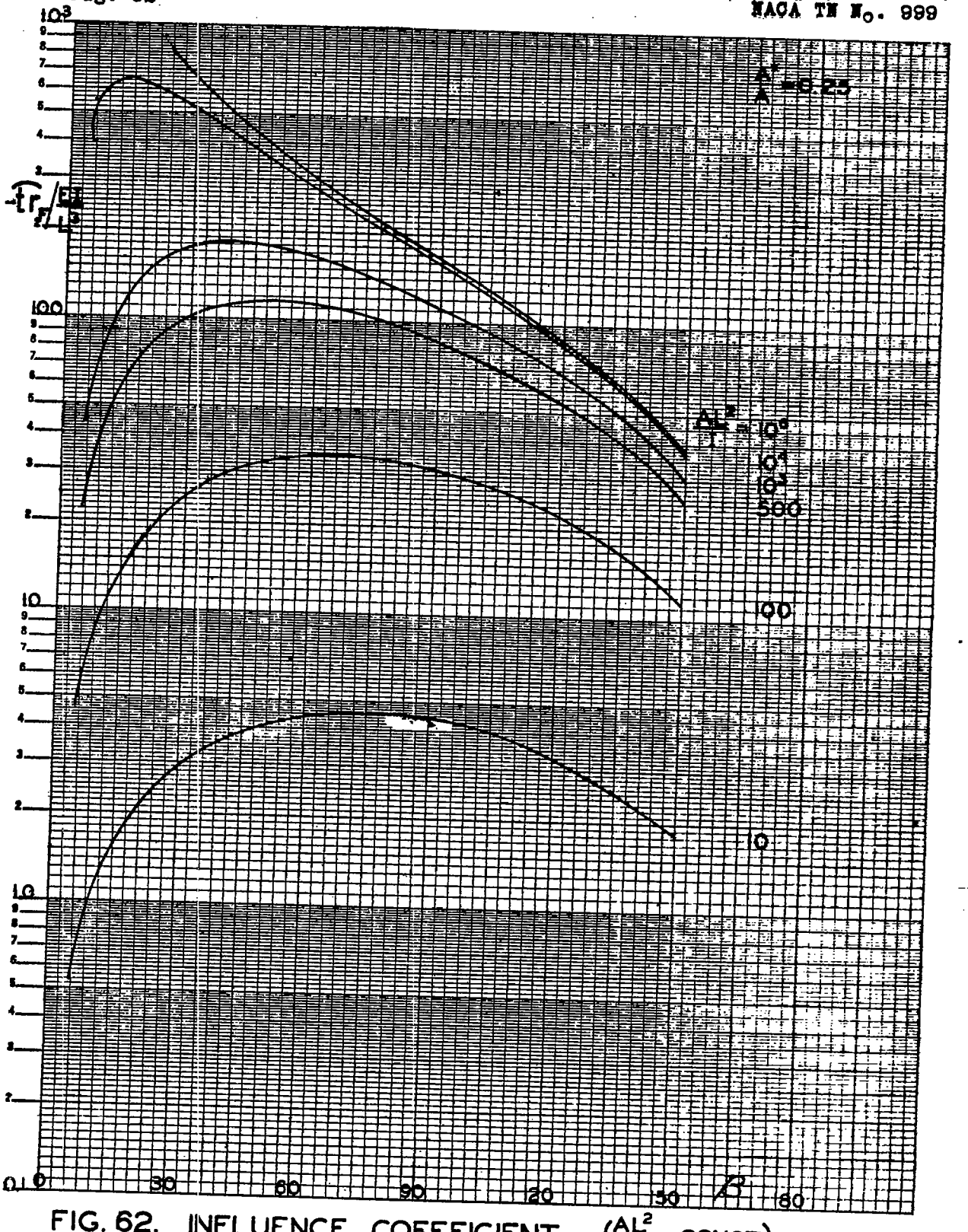


FIG. 62. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

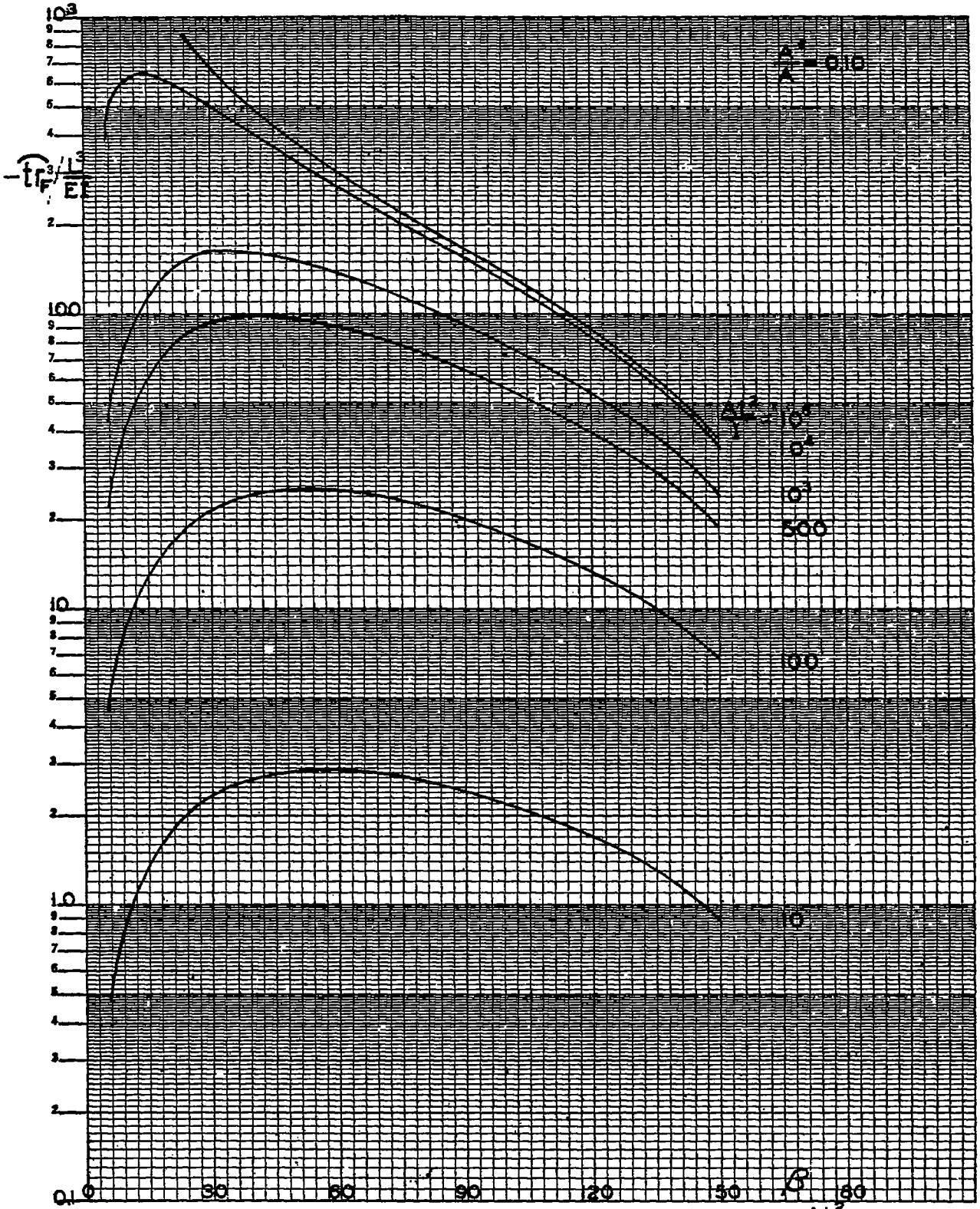


FIG. 63. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

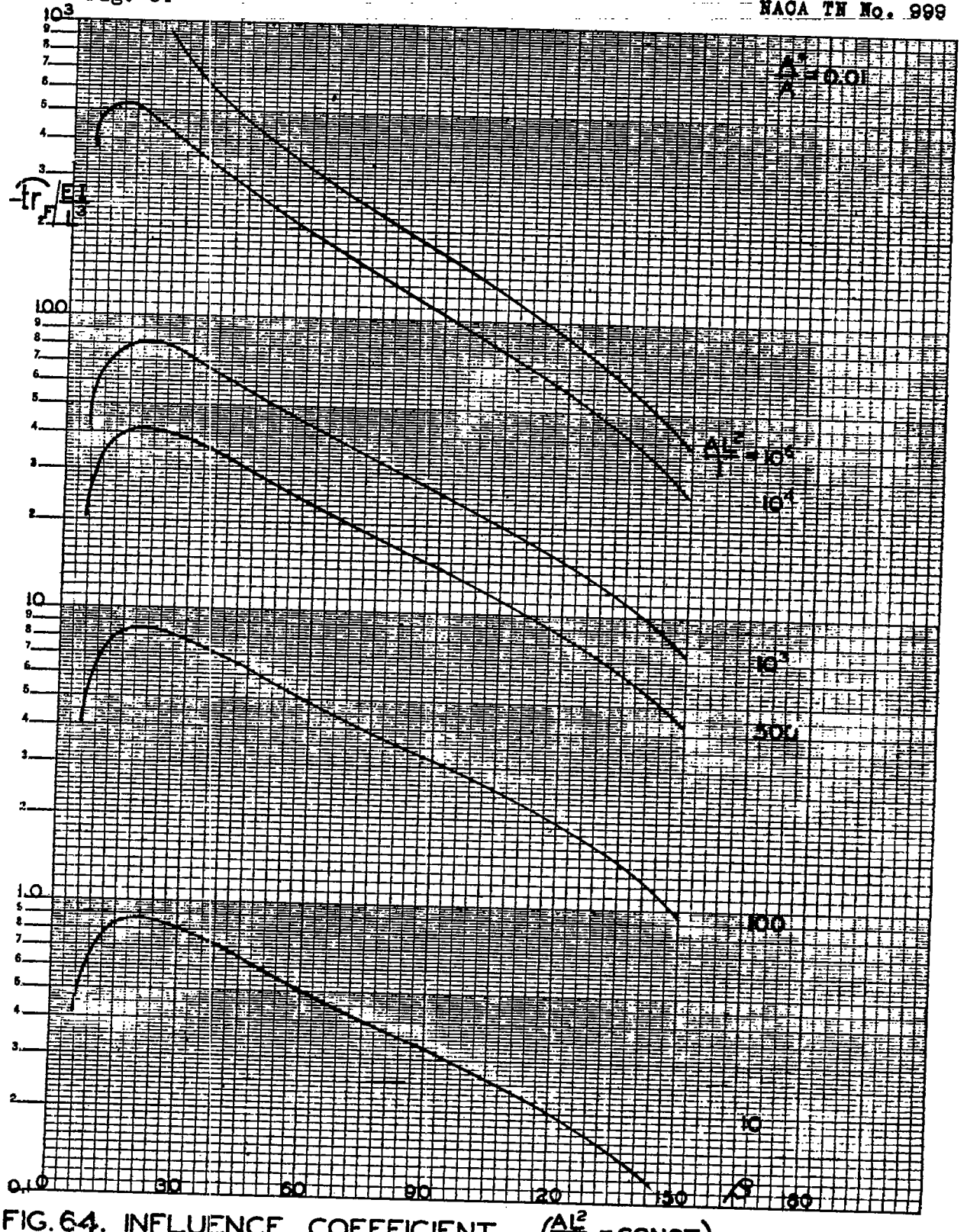


FIG. 64. INFLUENCE COEFFICIENT $\left(\frac{Al^2}{I} = \text{CONST.}\right)$

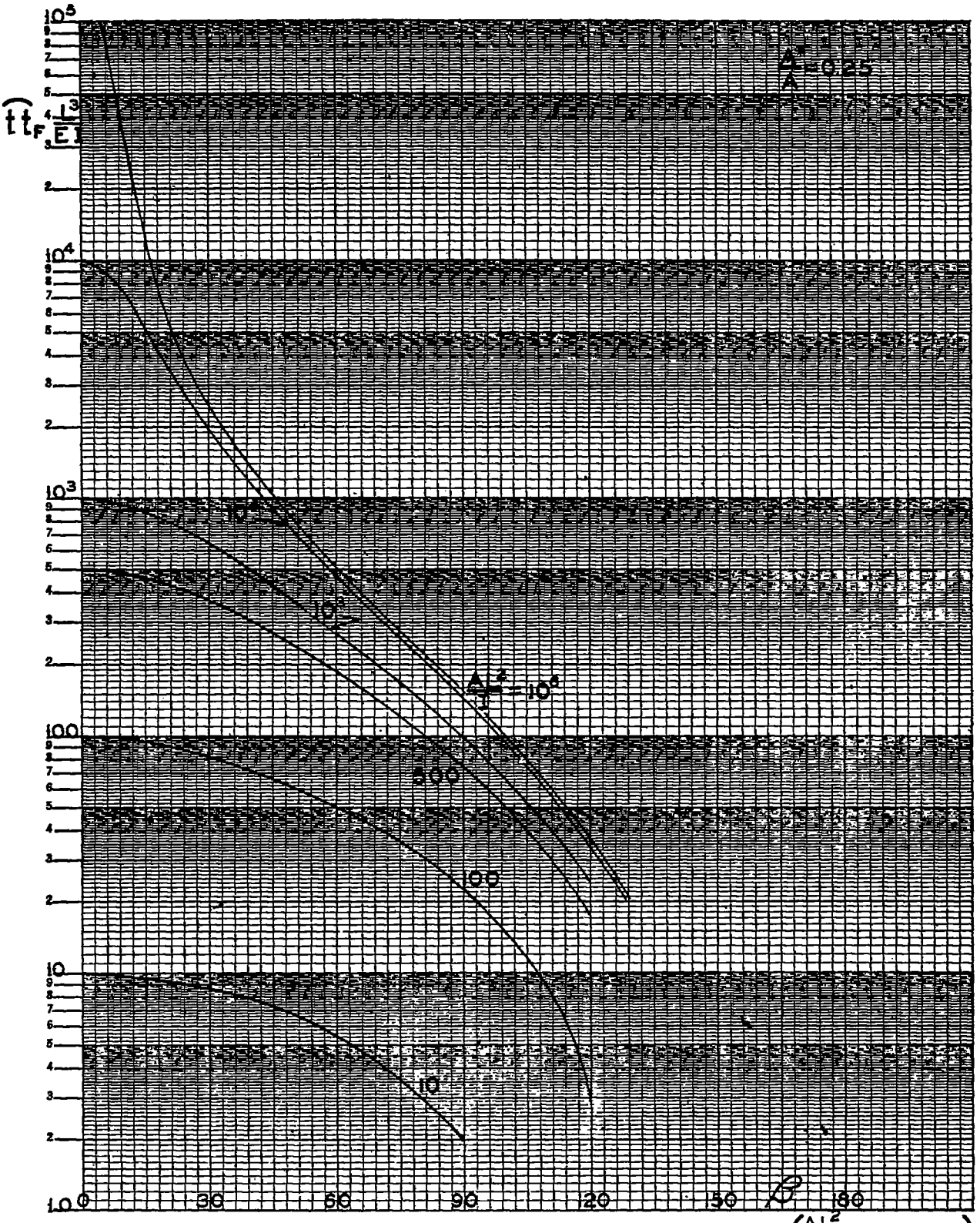


FIG. 65. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

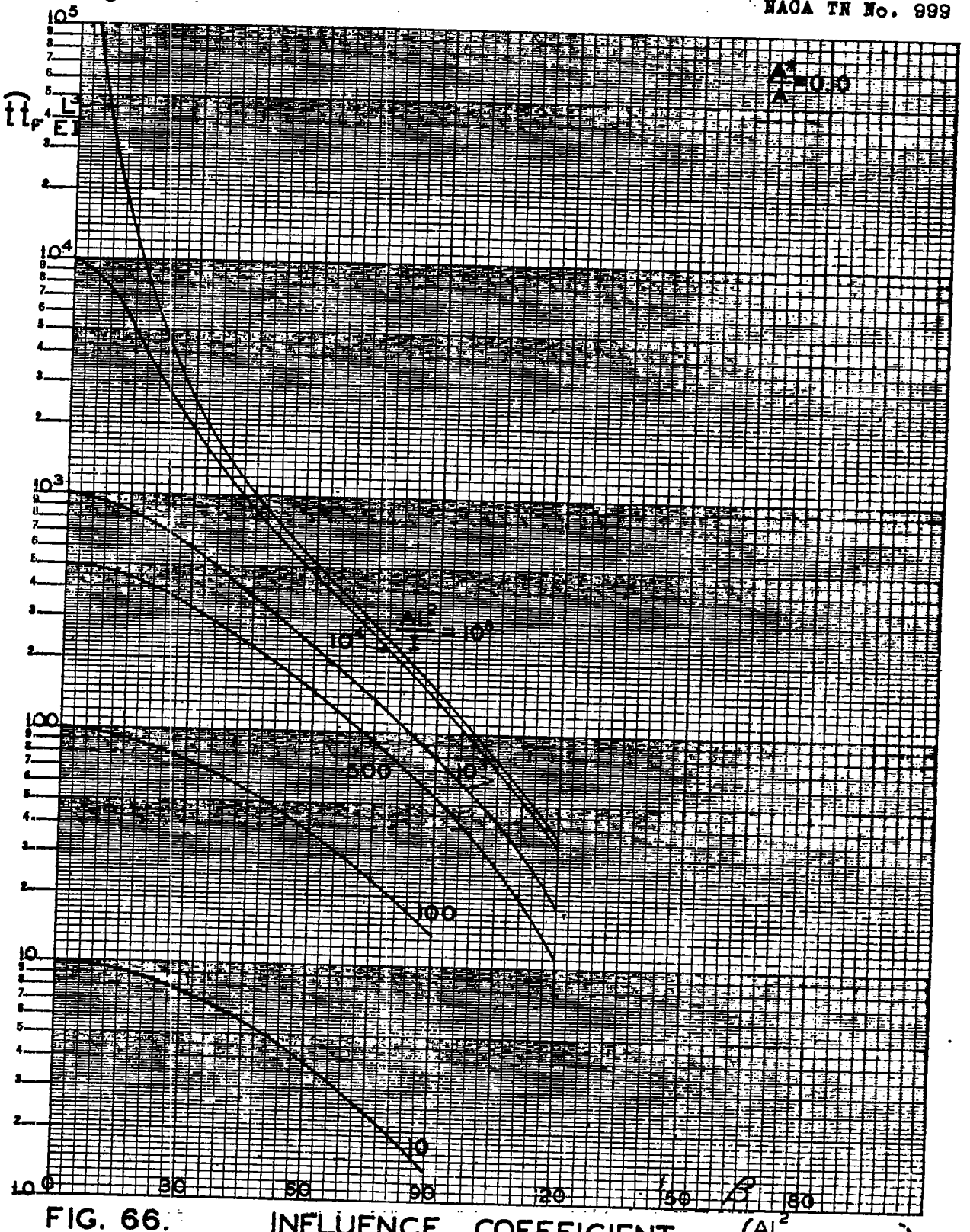


FIG. 66. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

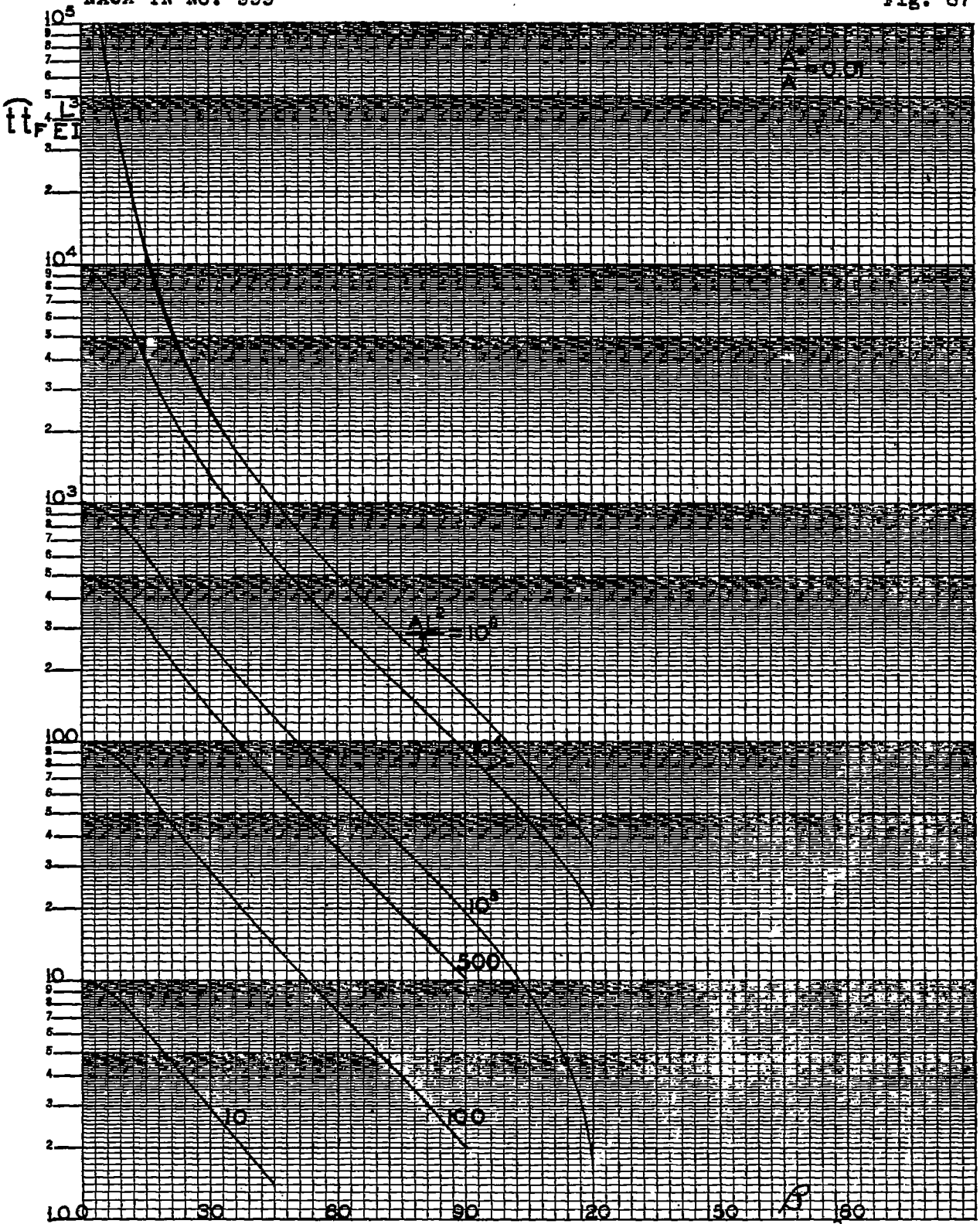


FIG. 67. INFLUENCE COEFFICIENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

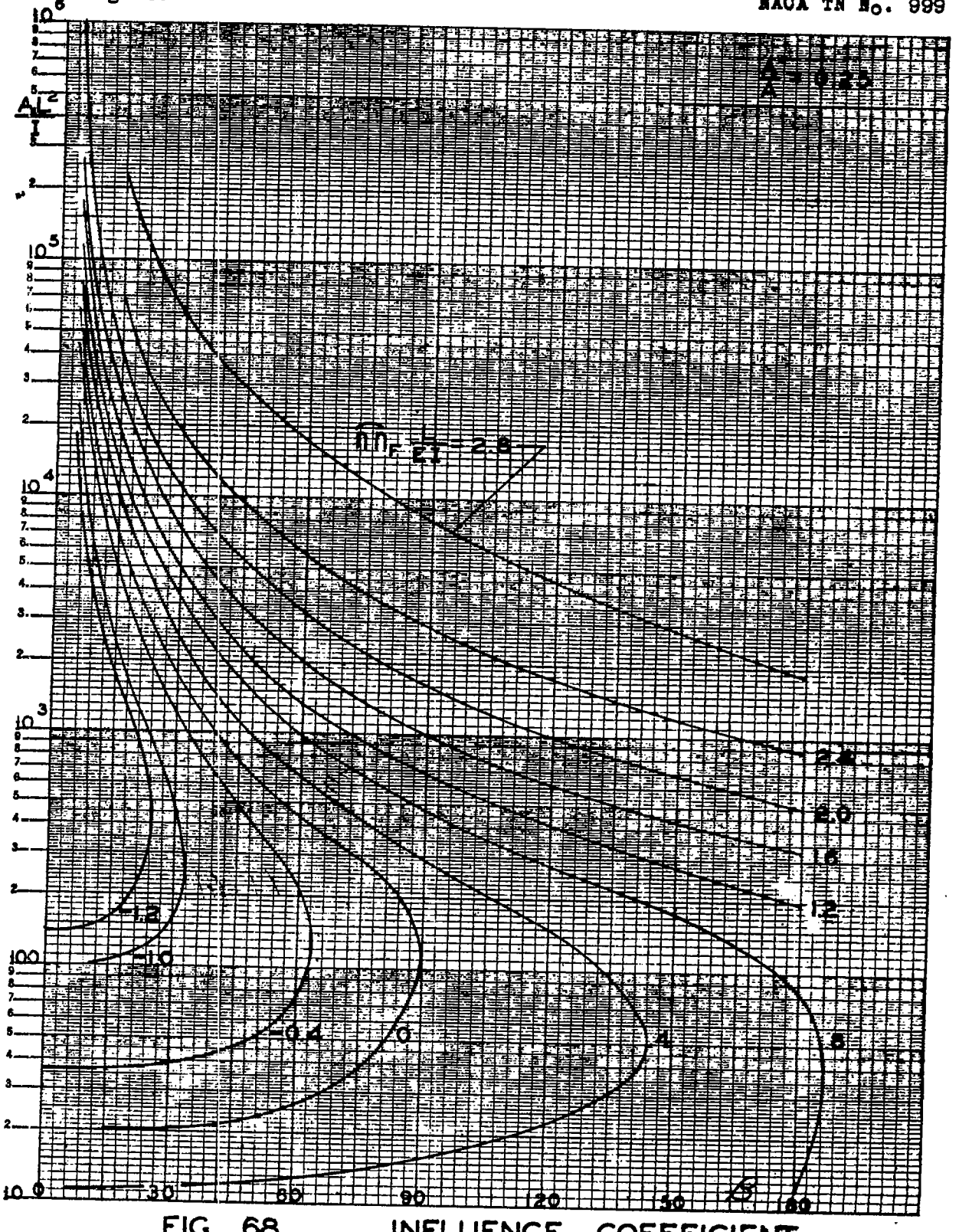


FIG. 68. INFLUENCE COEFFICIENT

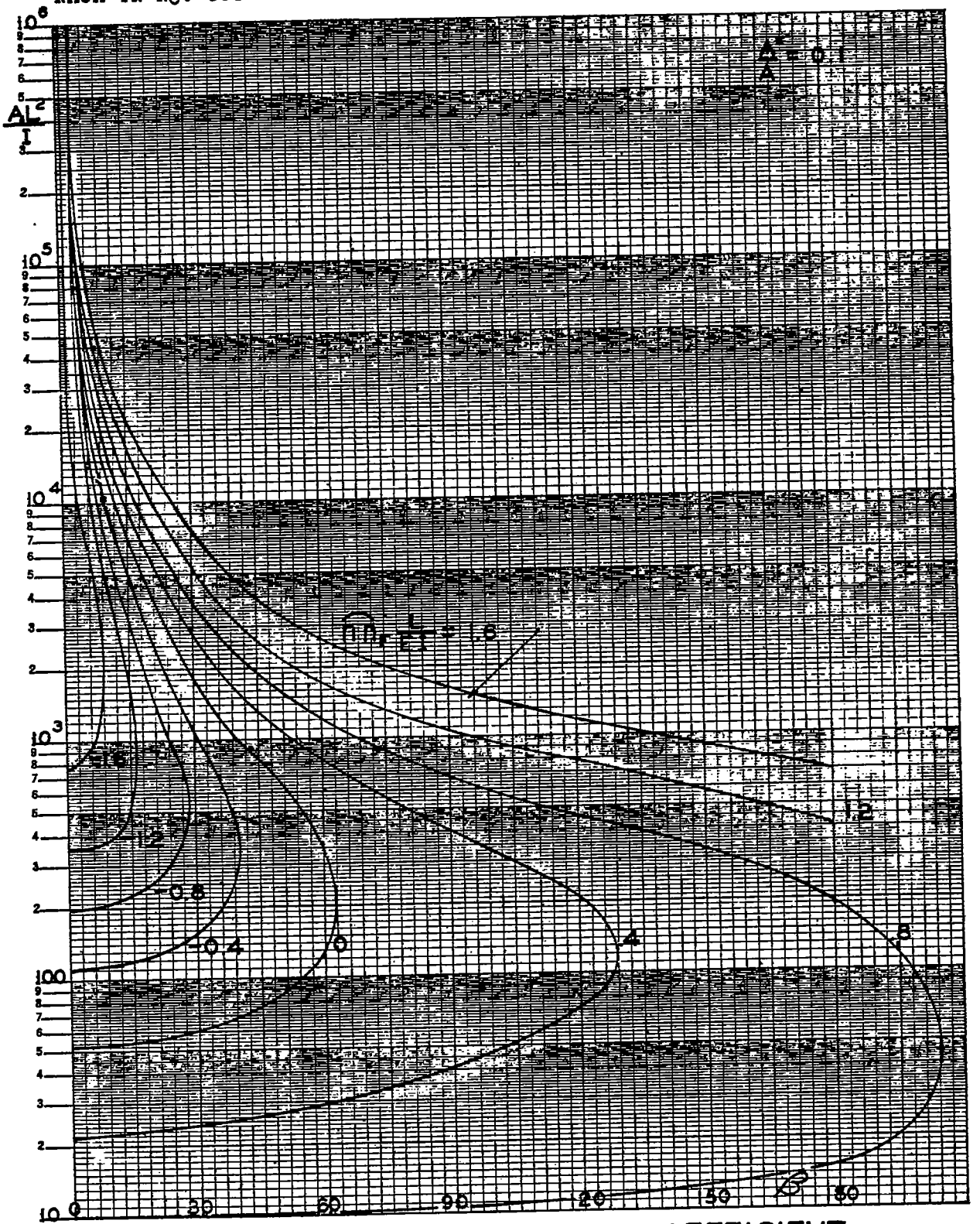


FIG. 69. INFLUENCE COEFFICIENT

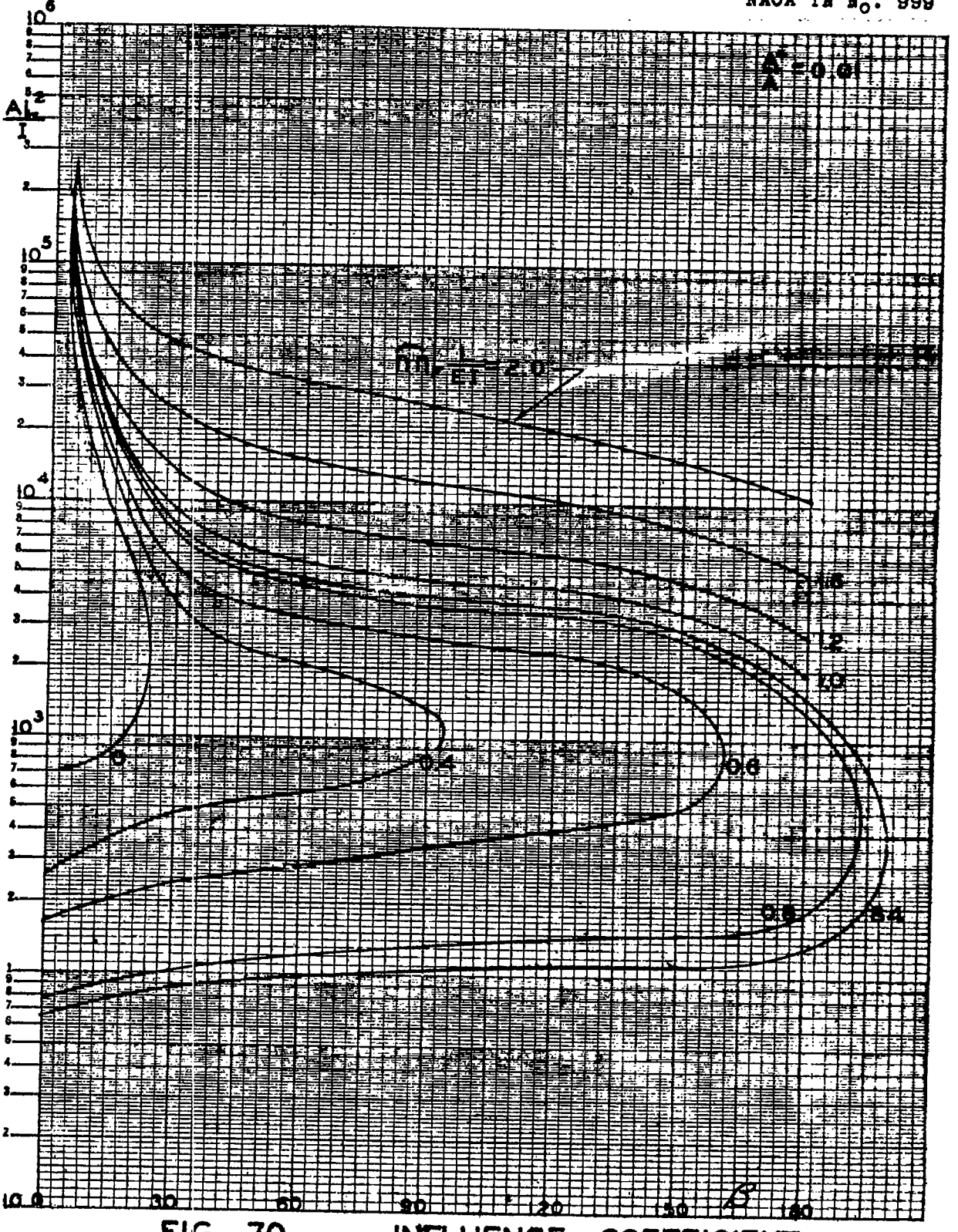


FIG. 70. INFLUENCE COEFFICIENT

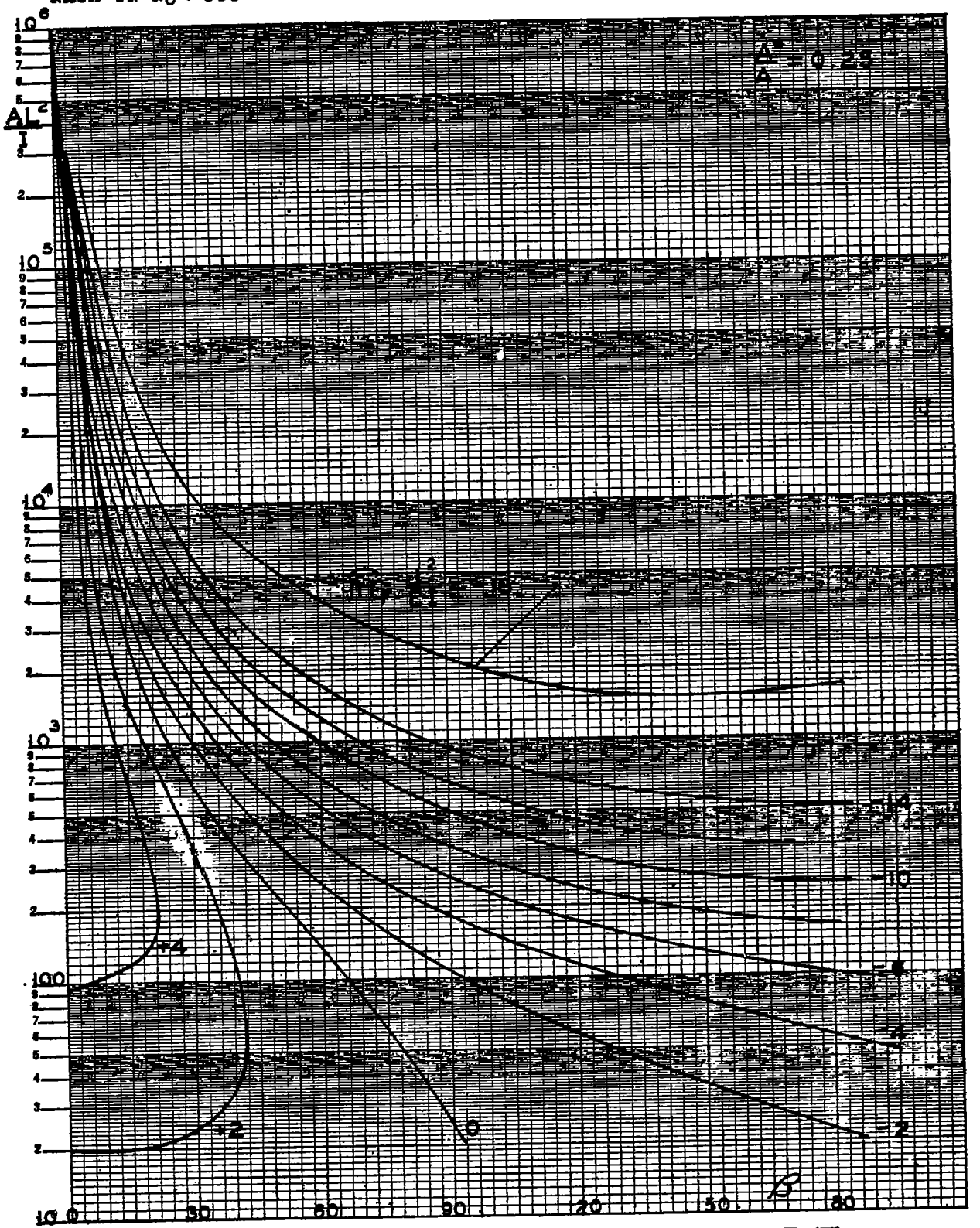


FIG. 71. INFLUENCE COEFFICIENT

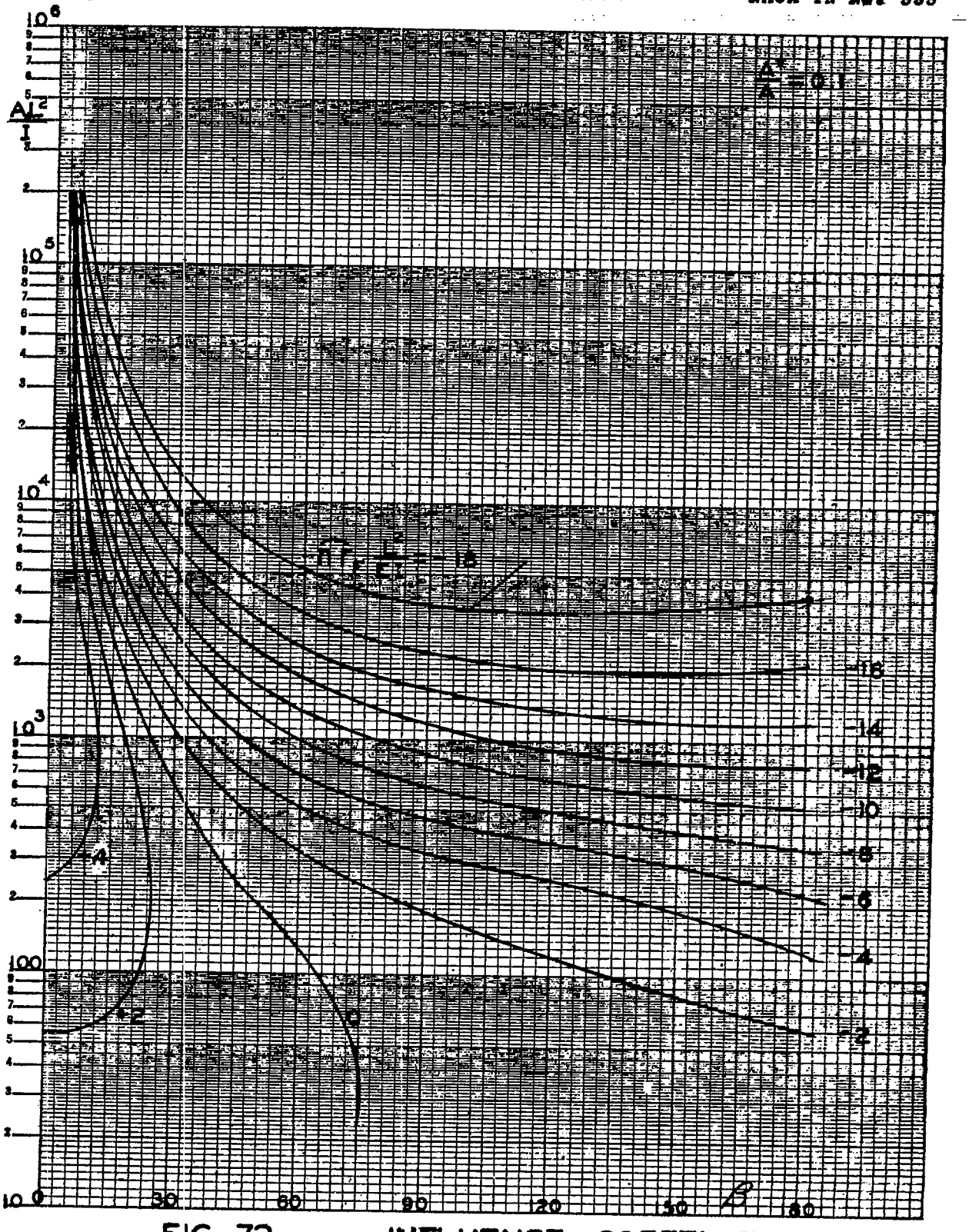


FIG. 72. INFLUENCE COEFFICIENT

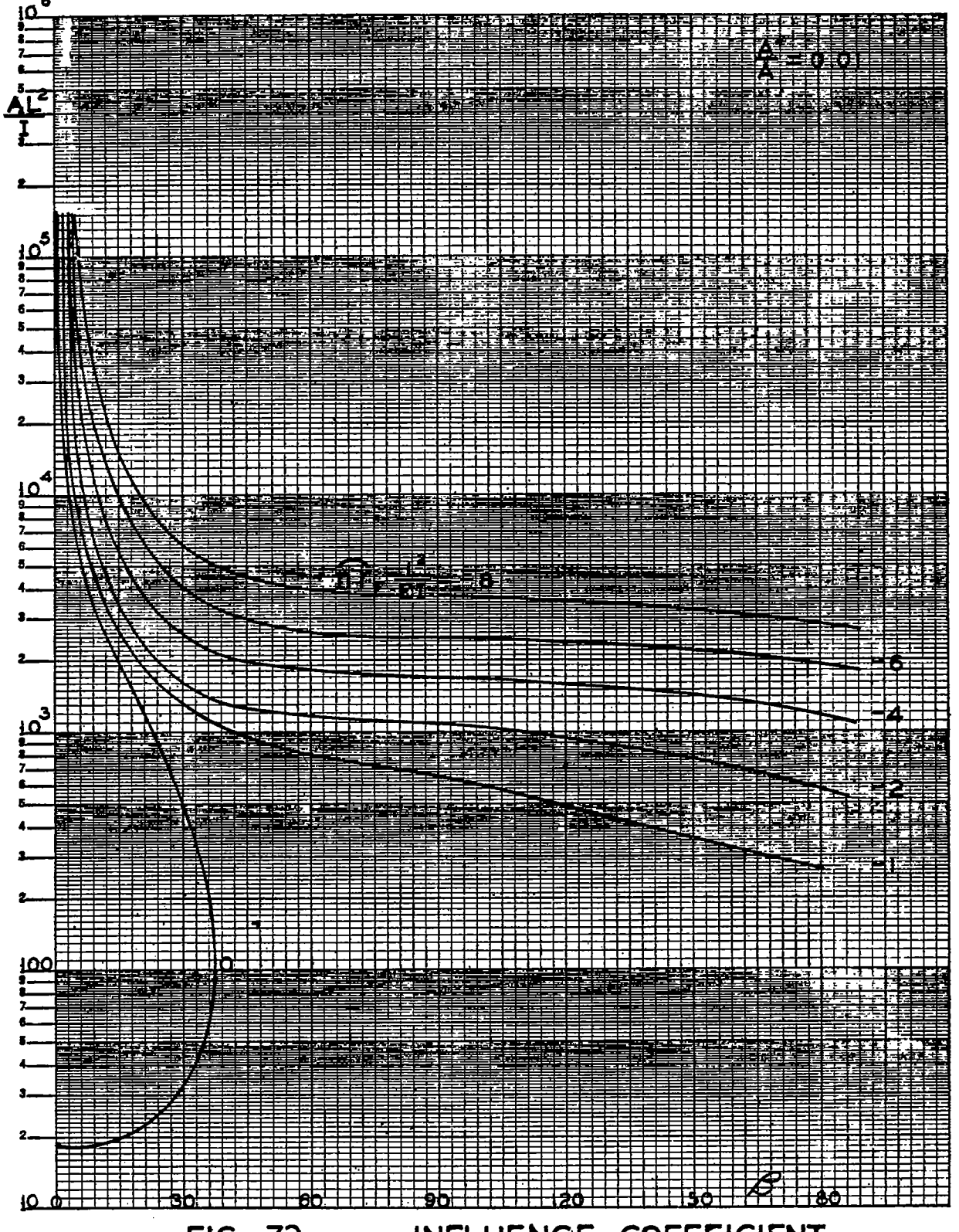


FIG. 73. INFLUENCE COEFFICIENT

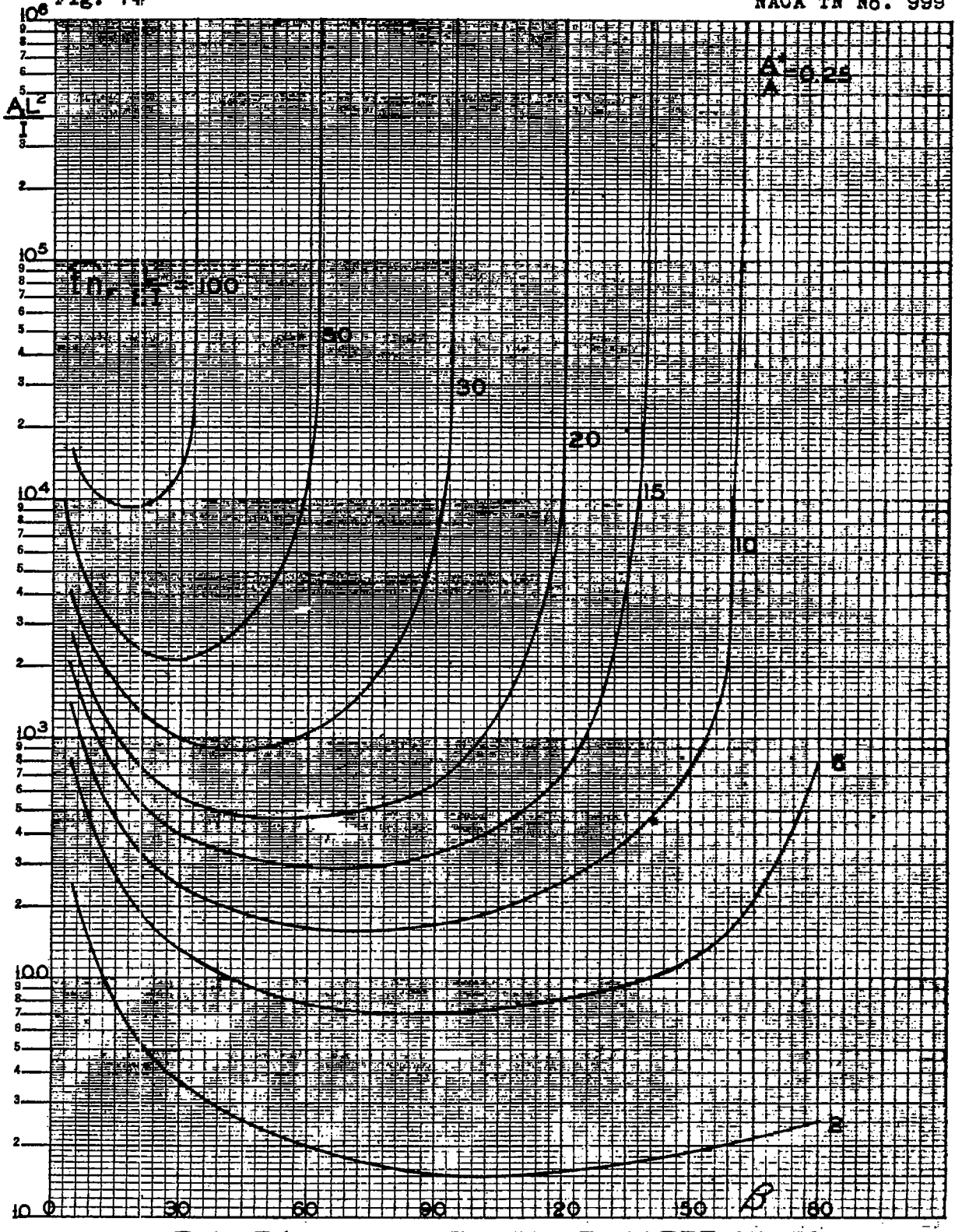


FIG. 74. INFLUENCE COEFFICIENT

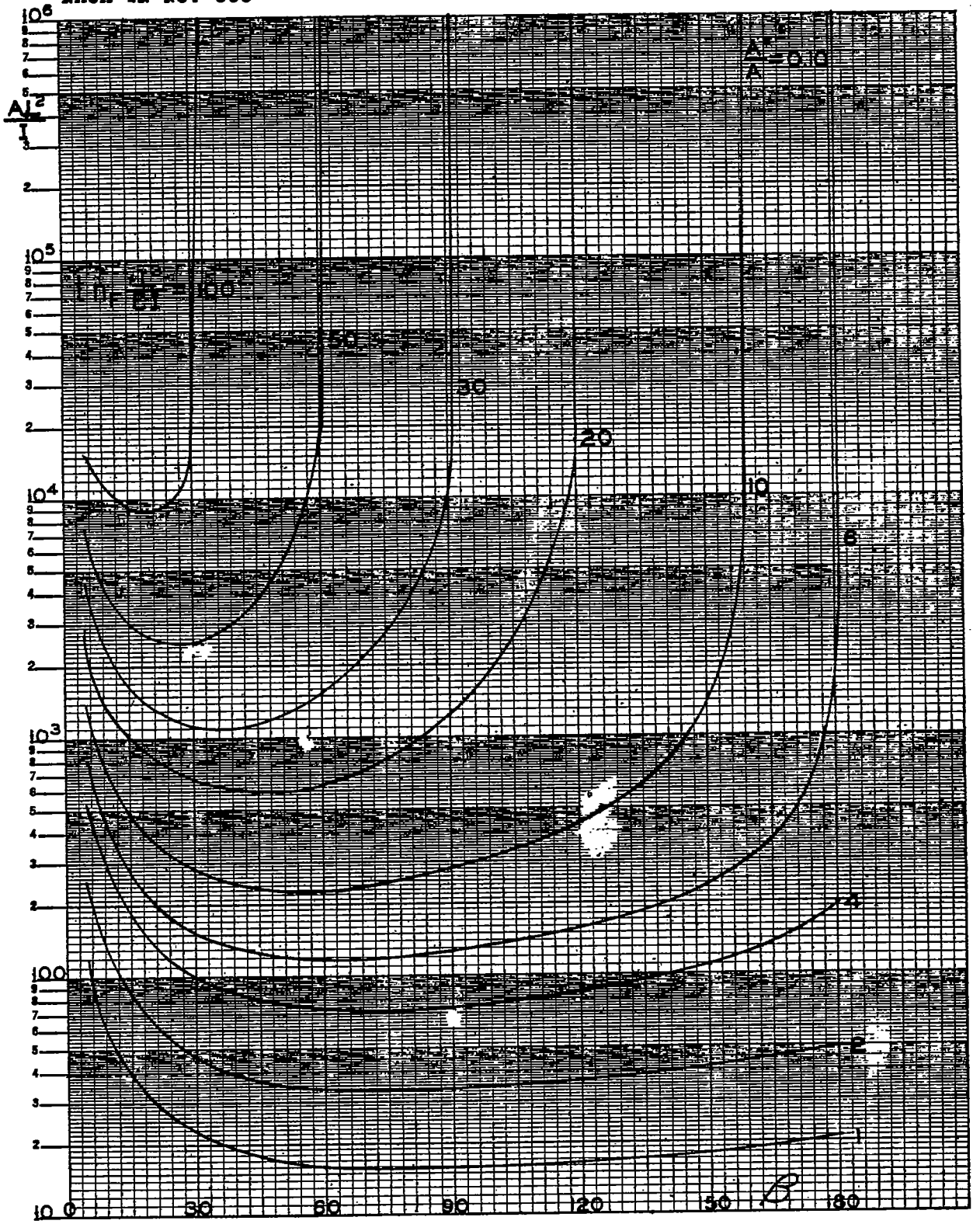


FIG. 75. INFLUENCE COEFFICIENT

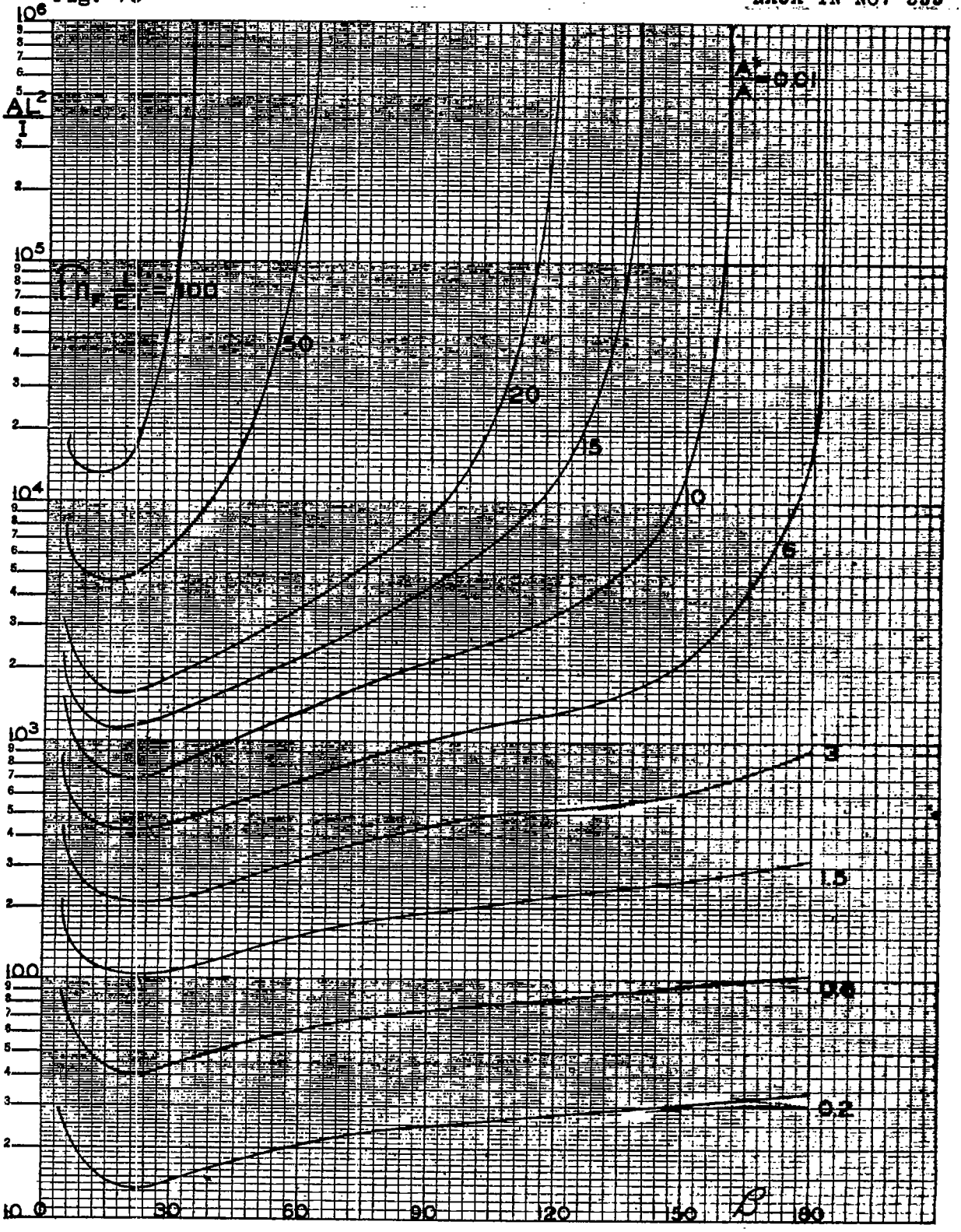


FIG. 76. INFLUENCE COEFFICIENT

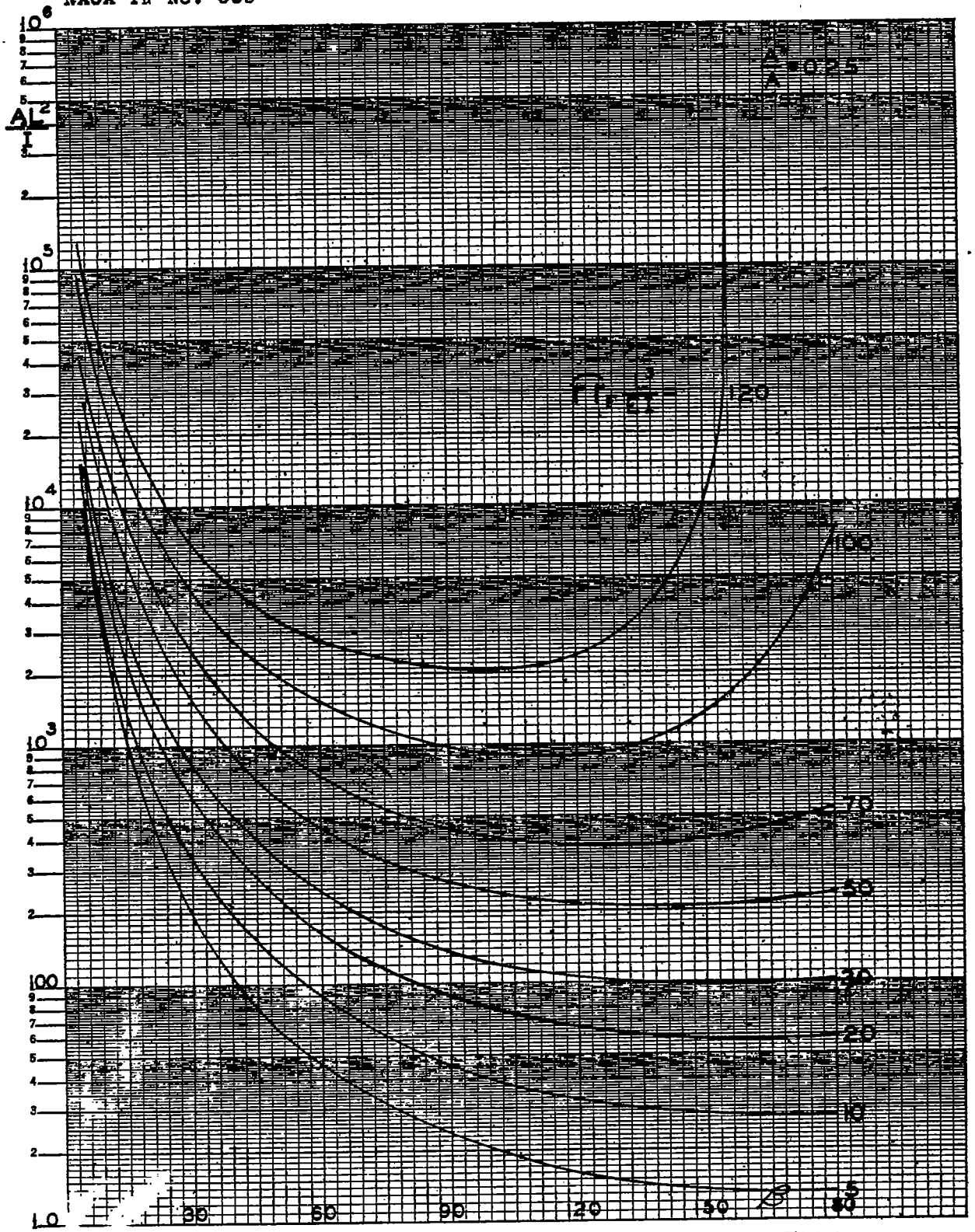


FIG. 77. INFLUENCE COEFFICIENT

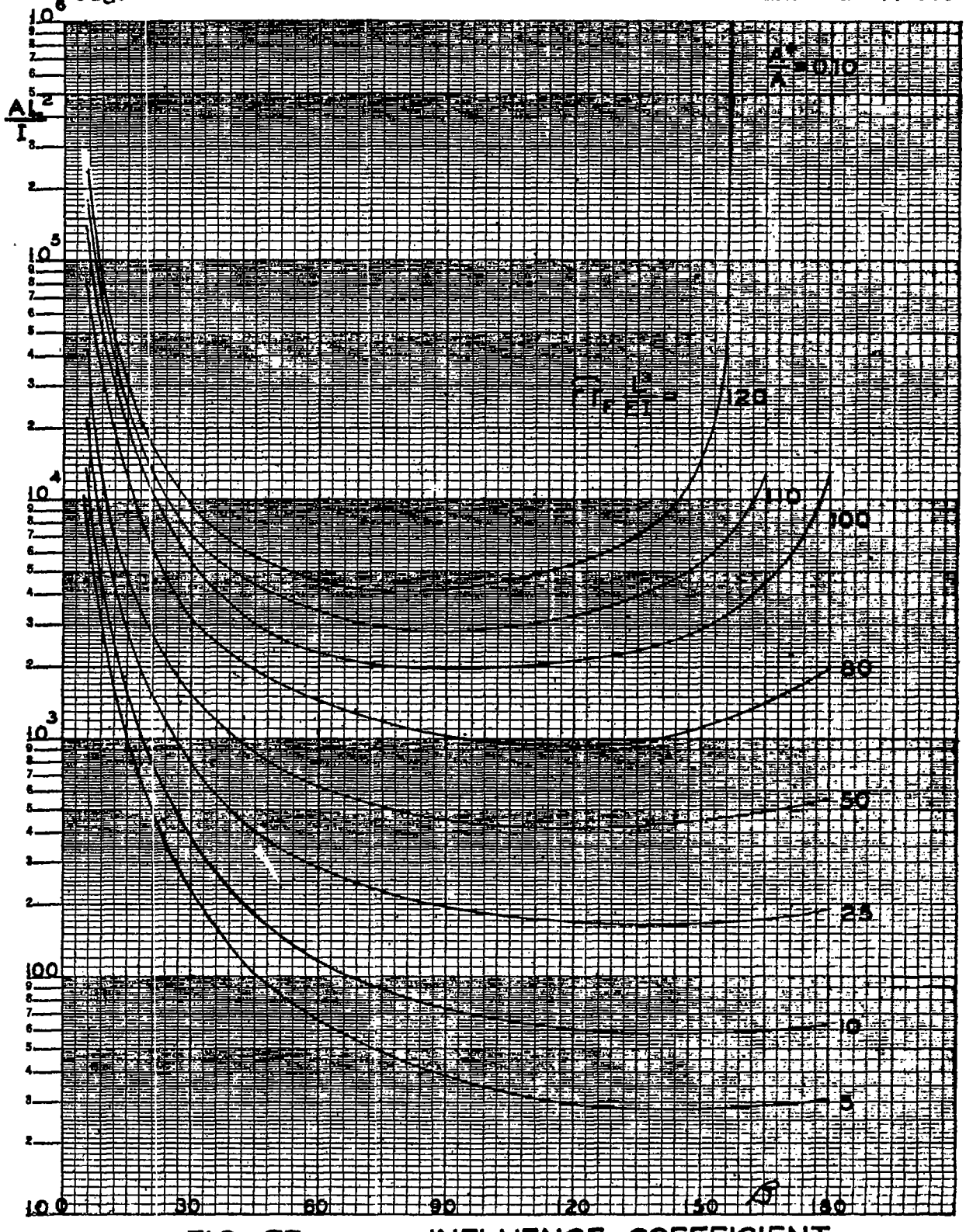


FIG. 78. INFLUENCE COEFFICIENT

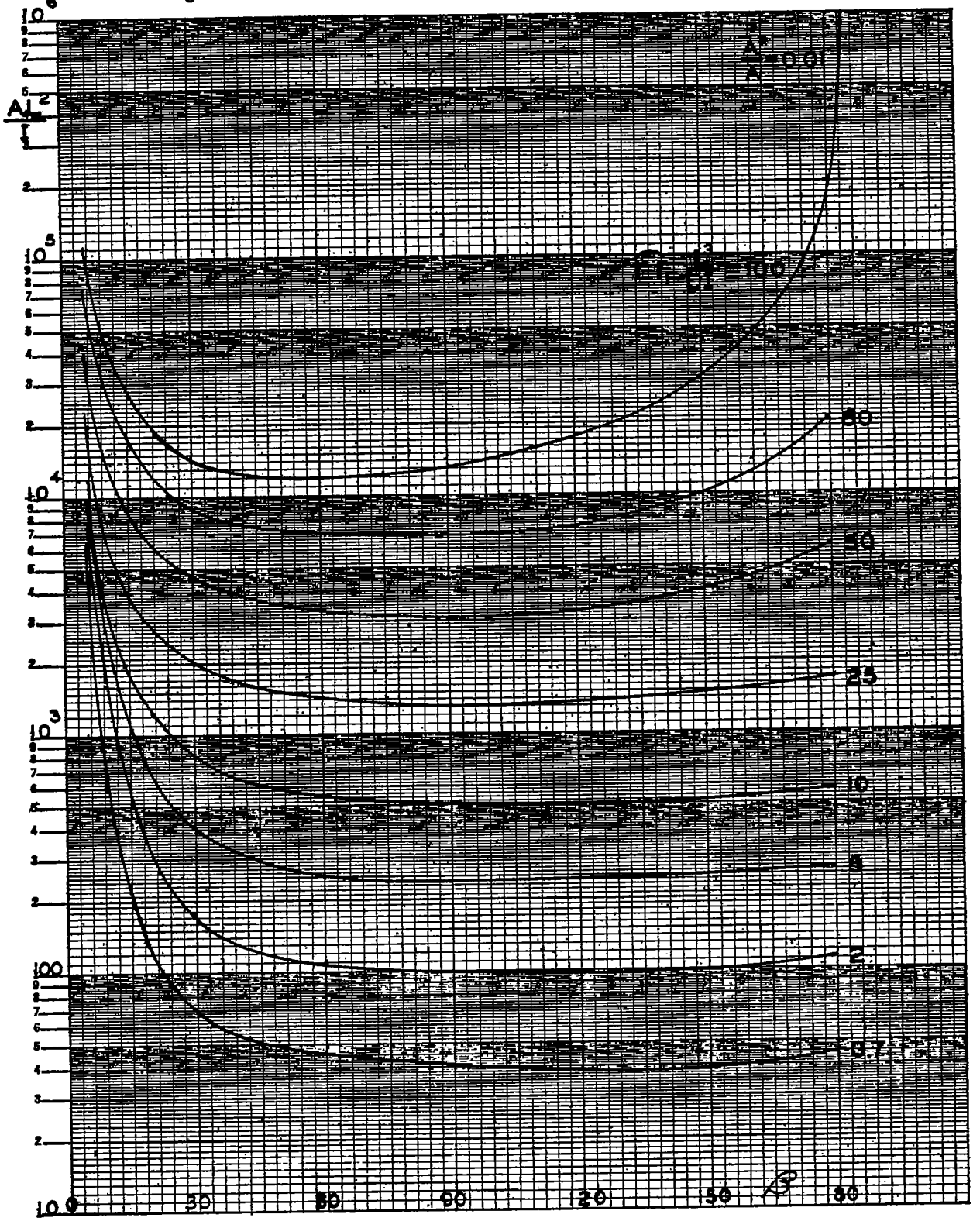


FIG. 79. INFLUENCE COEFFICIENT

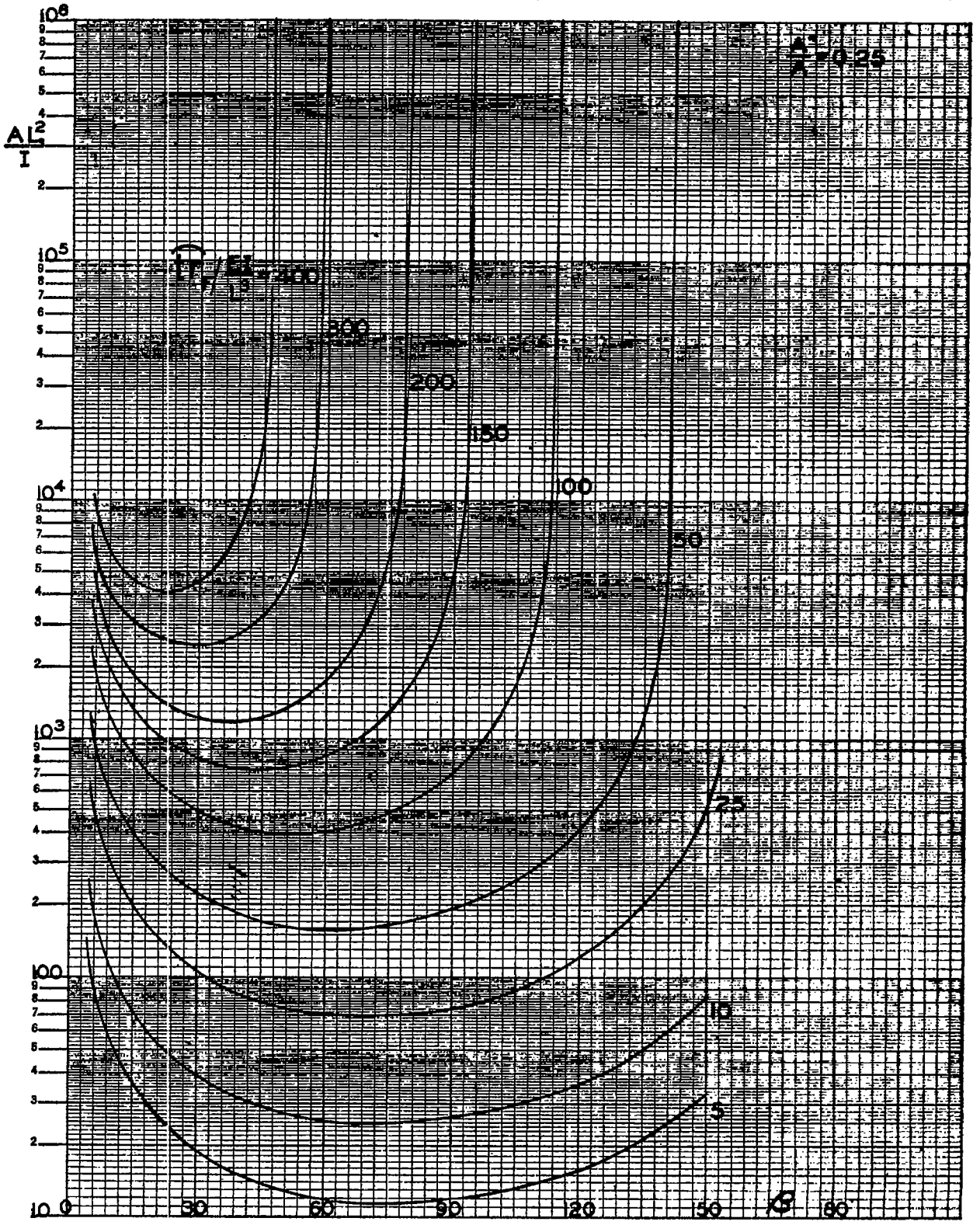


FIG. 80. INFLUENCE COEFFICIENT

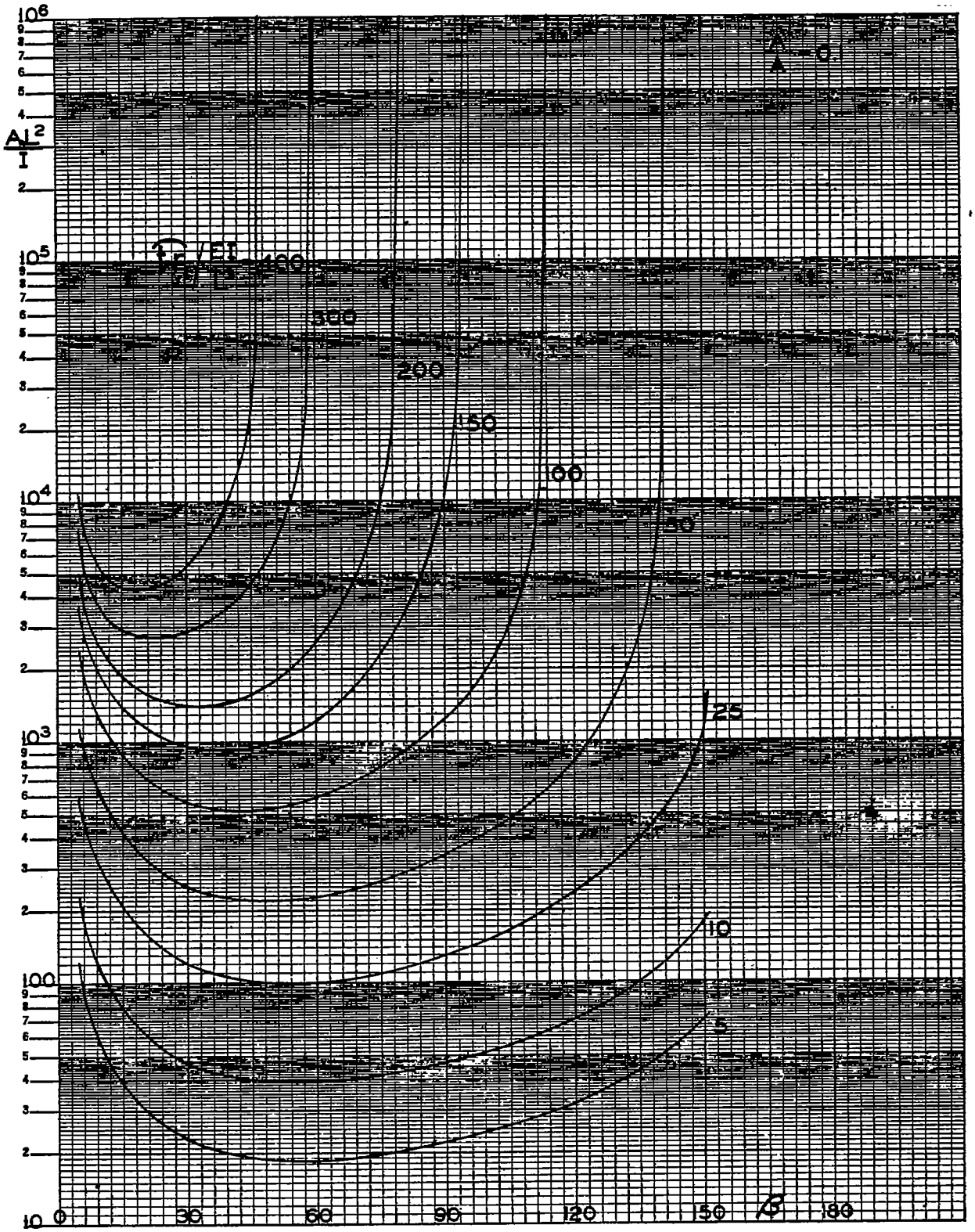


FIG. 81. INFLUENCE COEFFICIENT

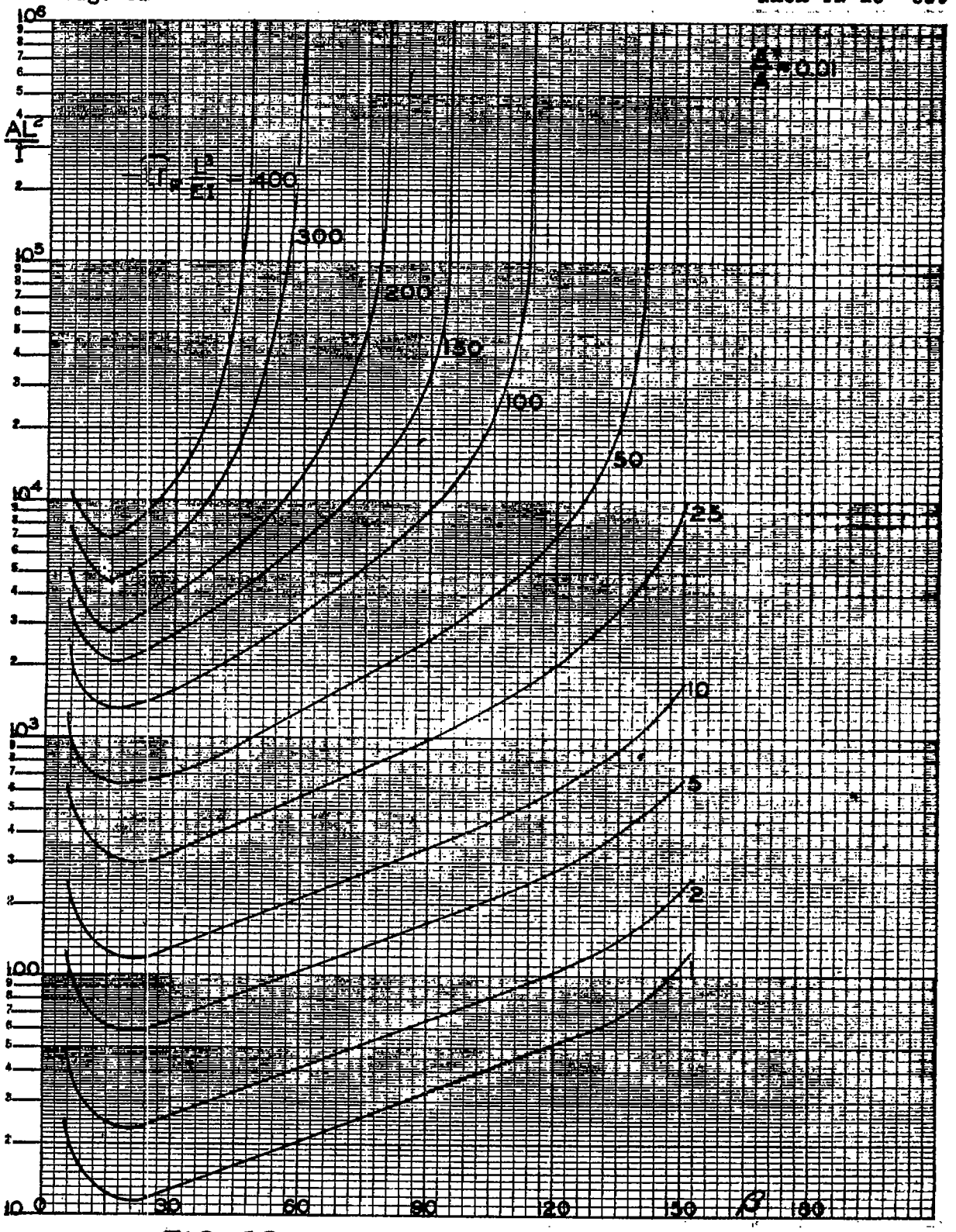


FIG. 82. INFLUENCE COEFFICIENT

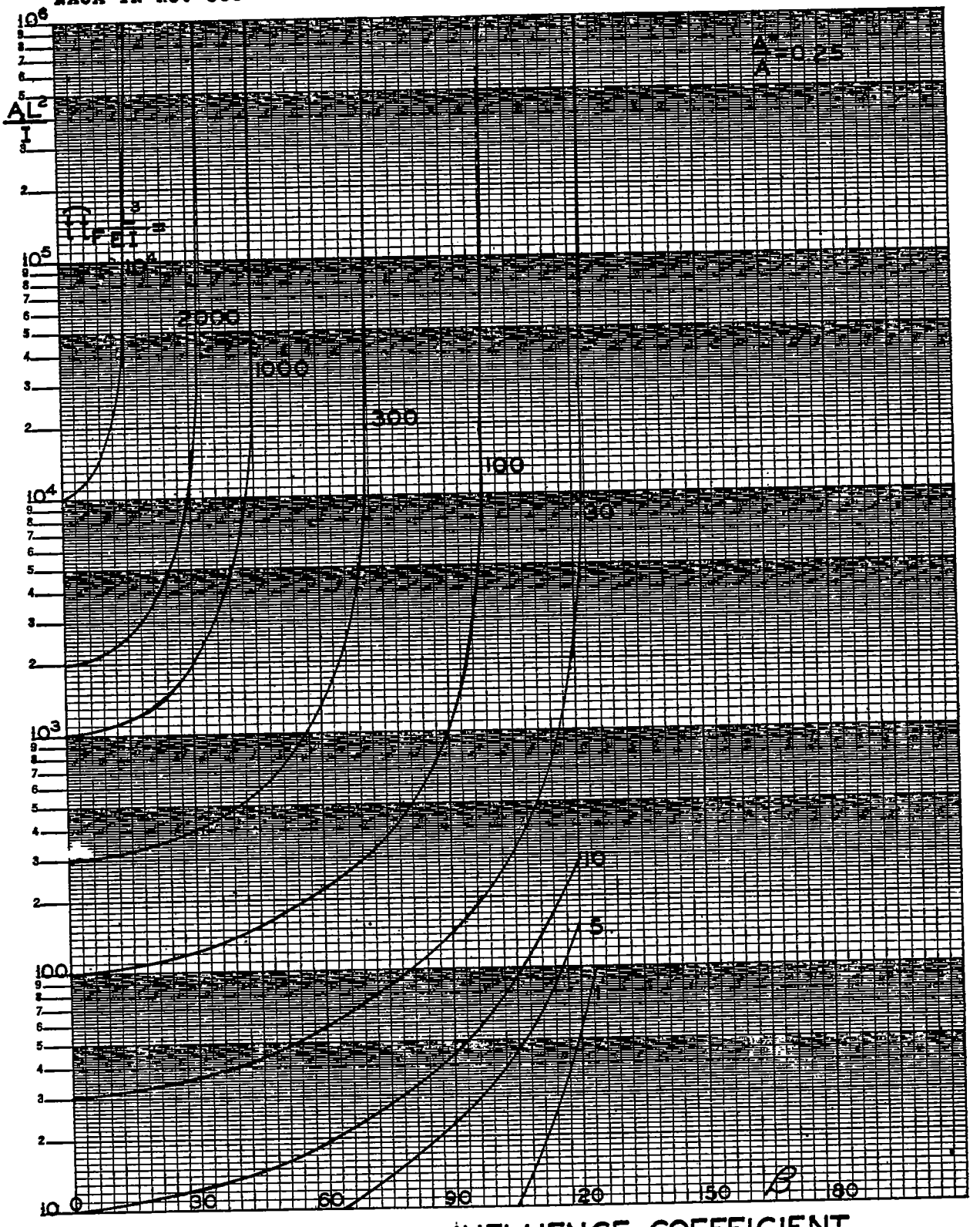


FIG. 83. INFLUENCE COEFFICIENT

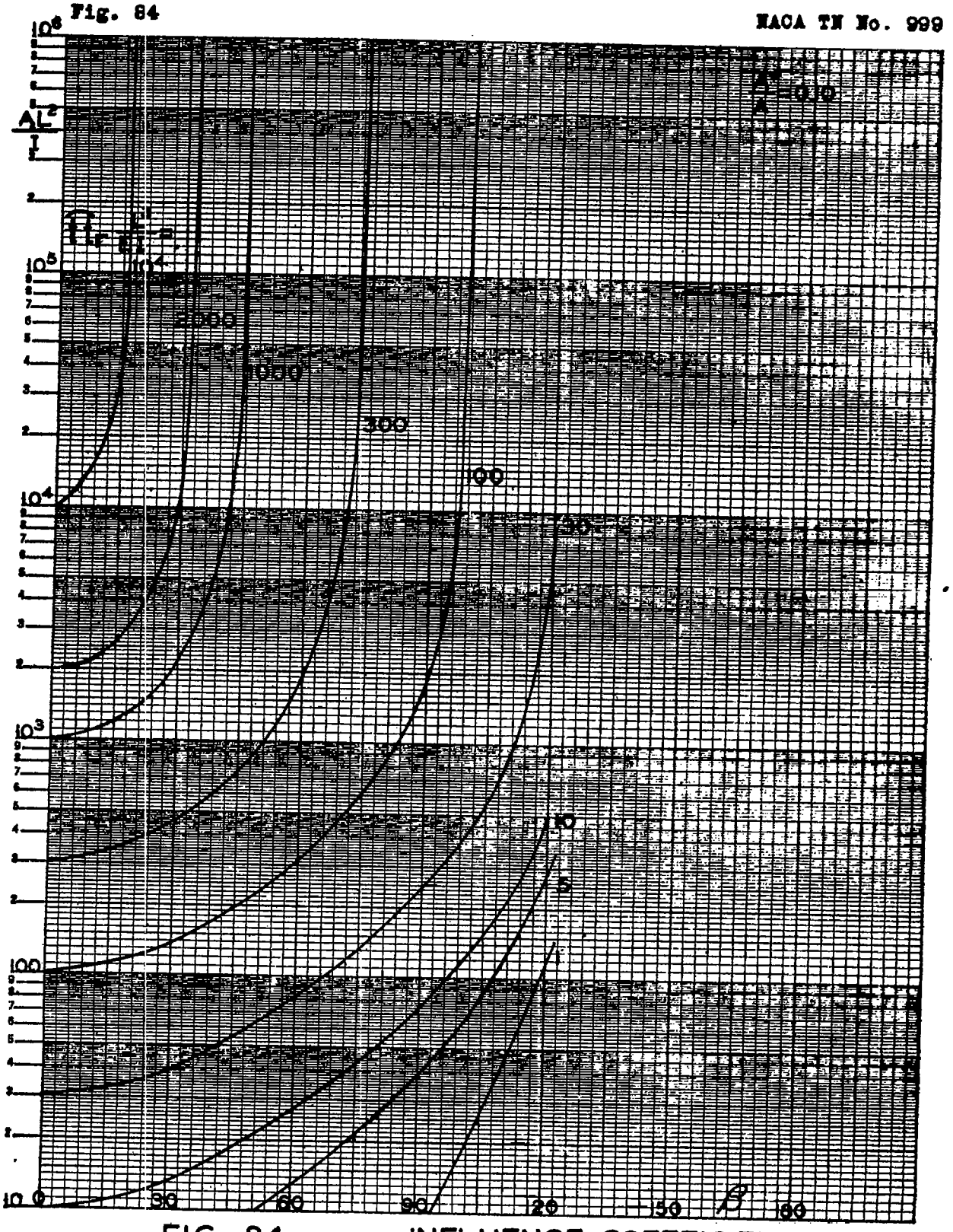


FIG. 84. INFLUENCE COEFFICIENT

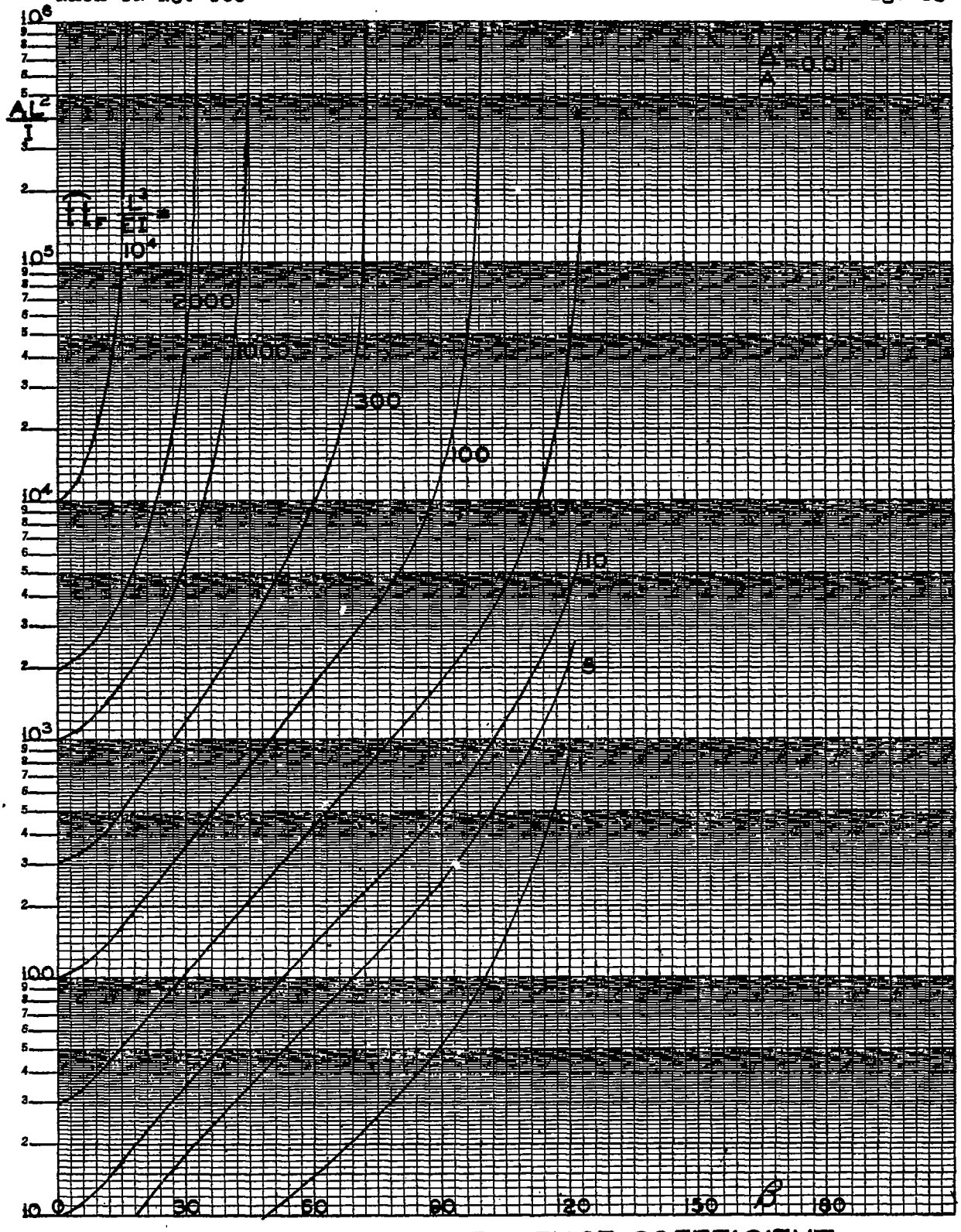


FIG. 85. INFLUENCE COEFFICIENT

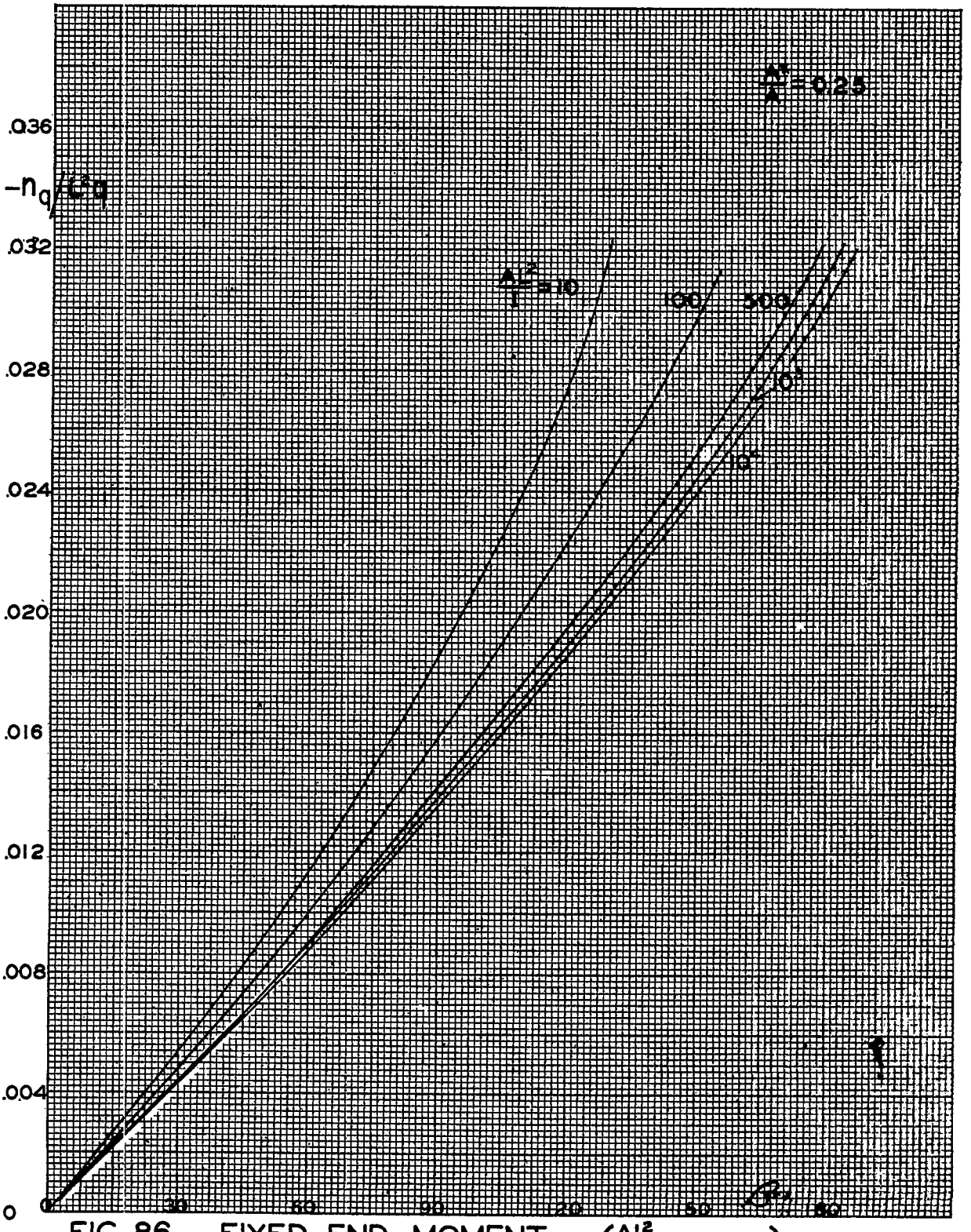


FIG. 86. FIXED END MOMENT $\left(\frac{AL^2}{I} = \text{CONST.}\right)$

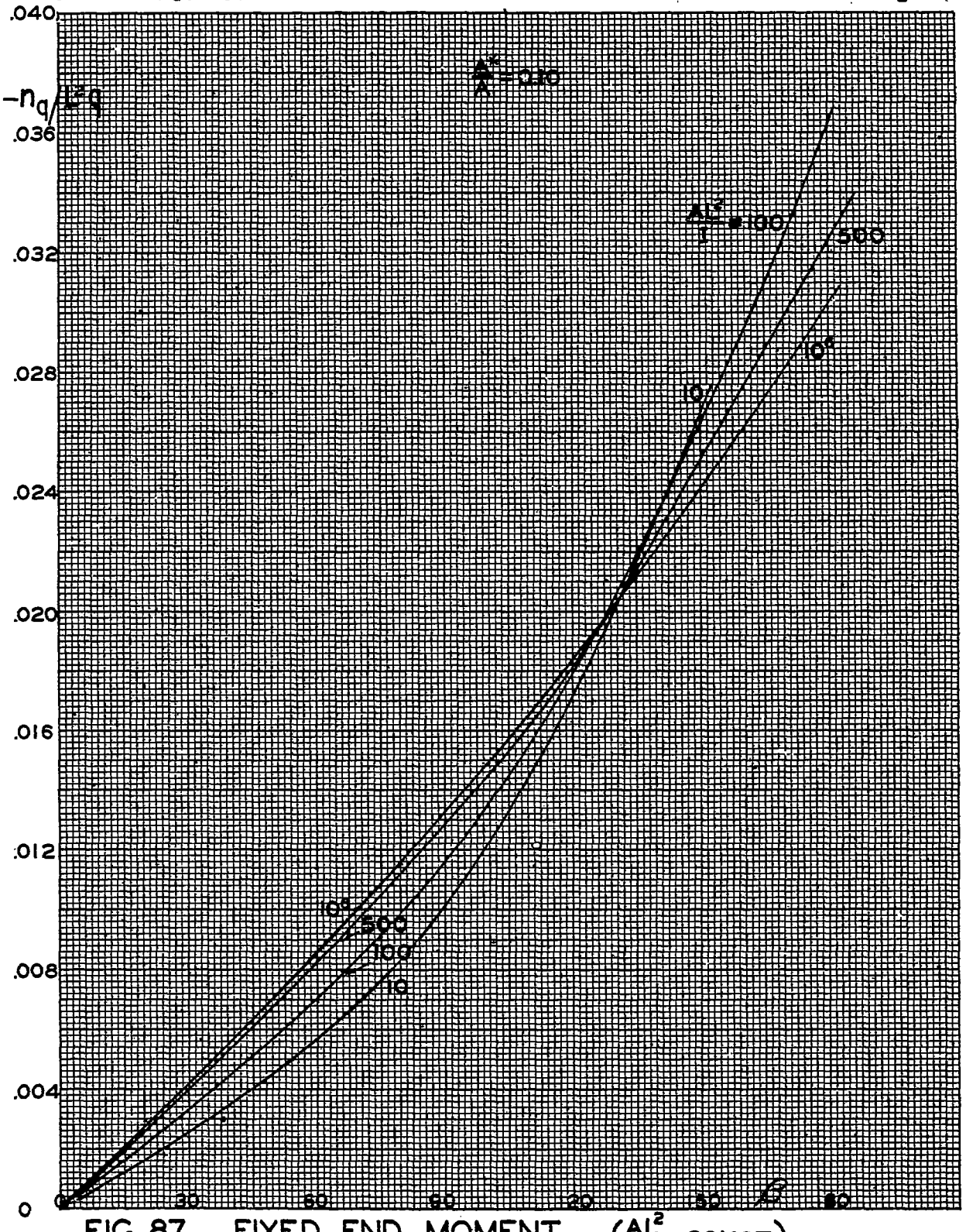


FIG. 87. FIXED END MOMENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

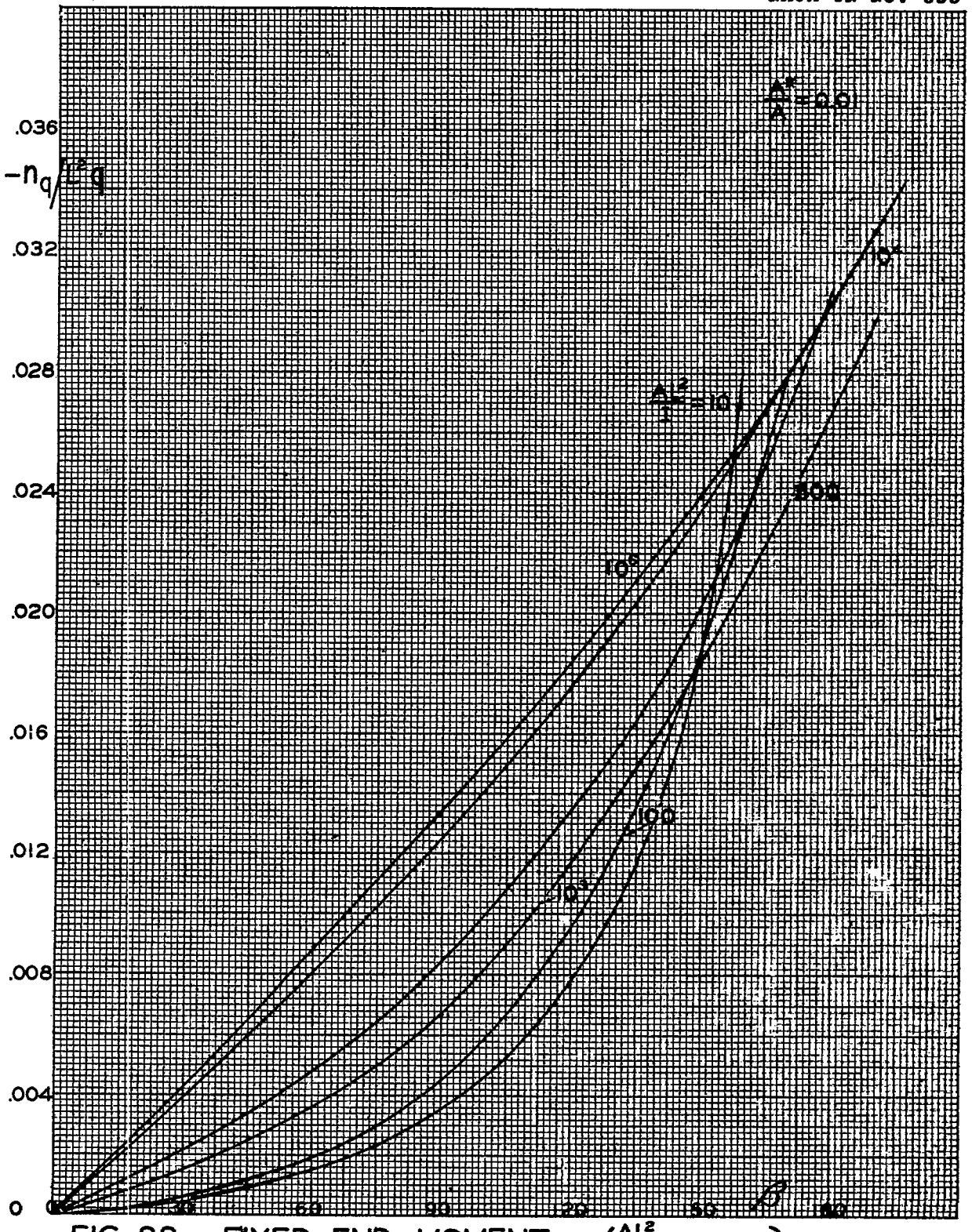


FIG. 88. FIXED END MOMENT $\left(\frac{AL^2}{I}=\text{CONST.}\right)$

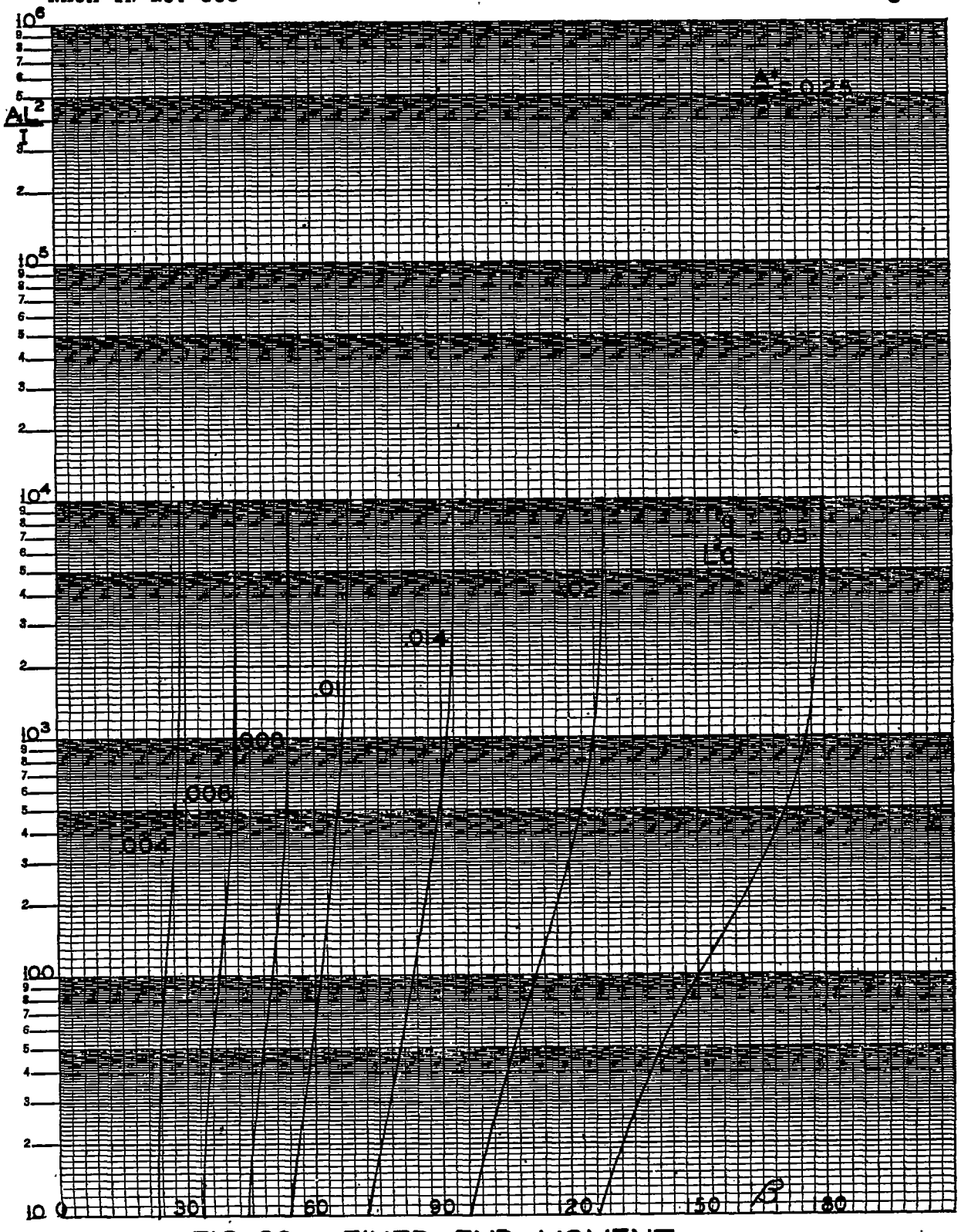


FIG. 89. FIXED END MOMENT

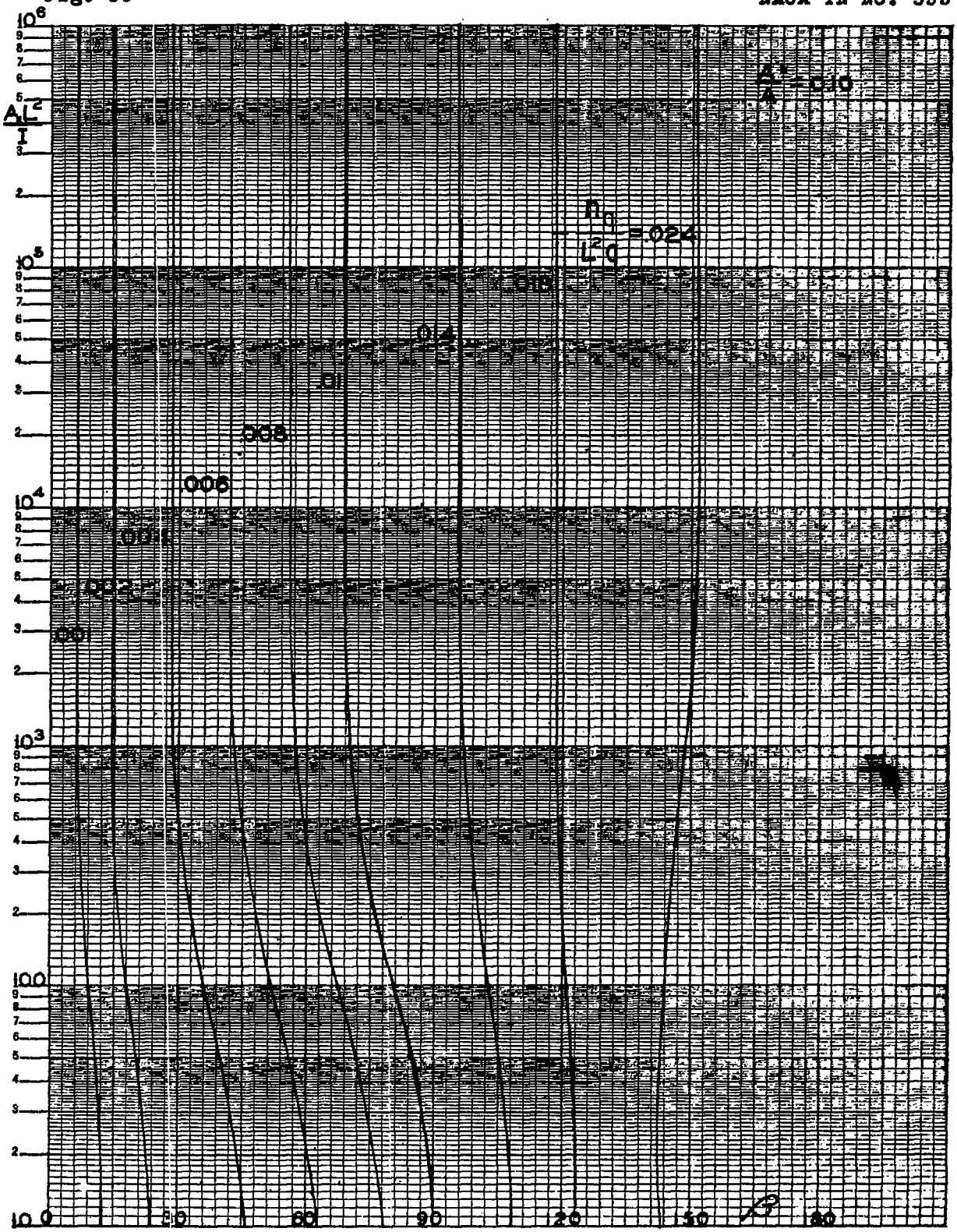


FIG. 90. FIXED END MOMENT

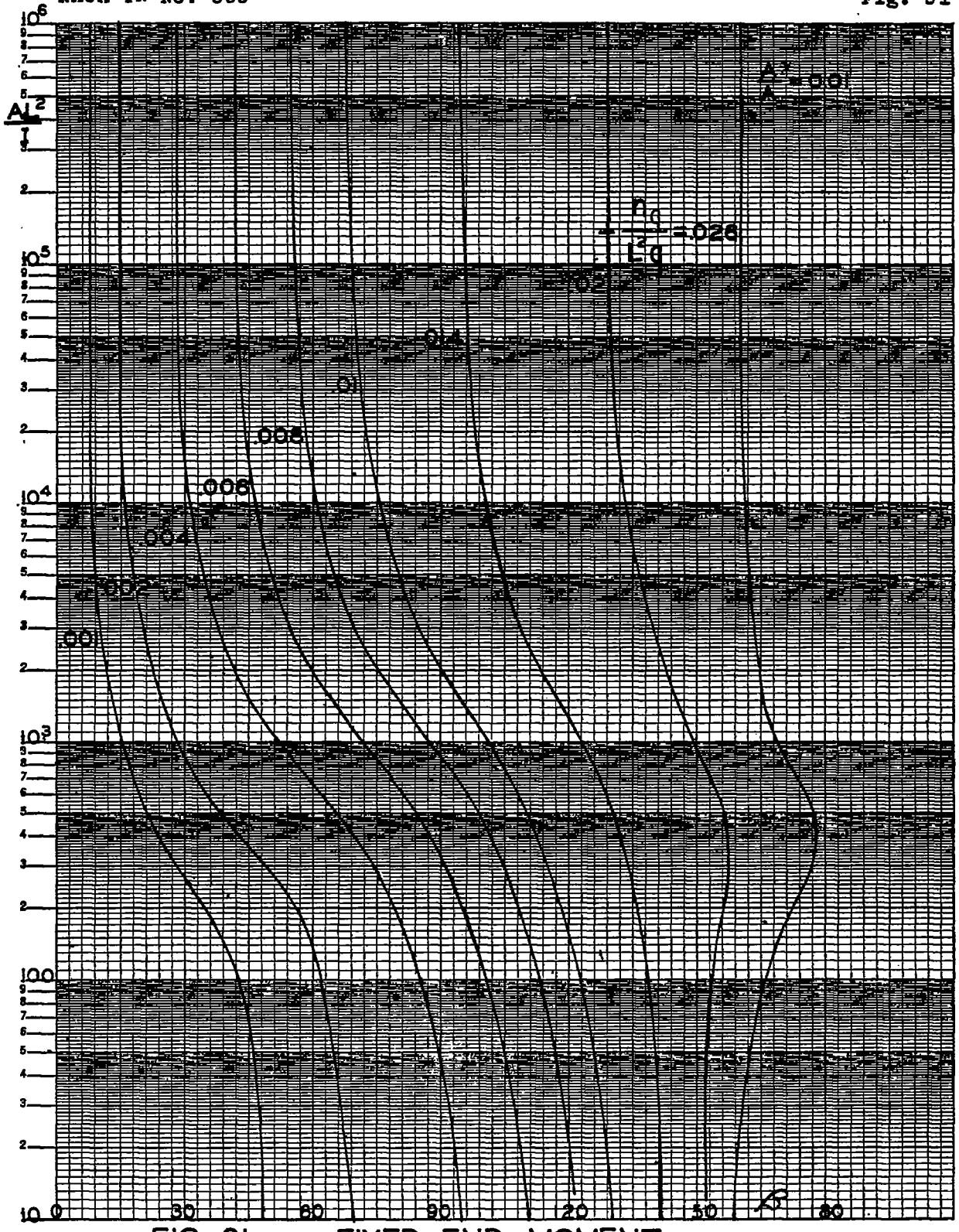


FIG. 91. FIXED END MOMENT

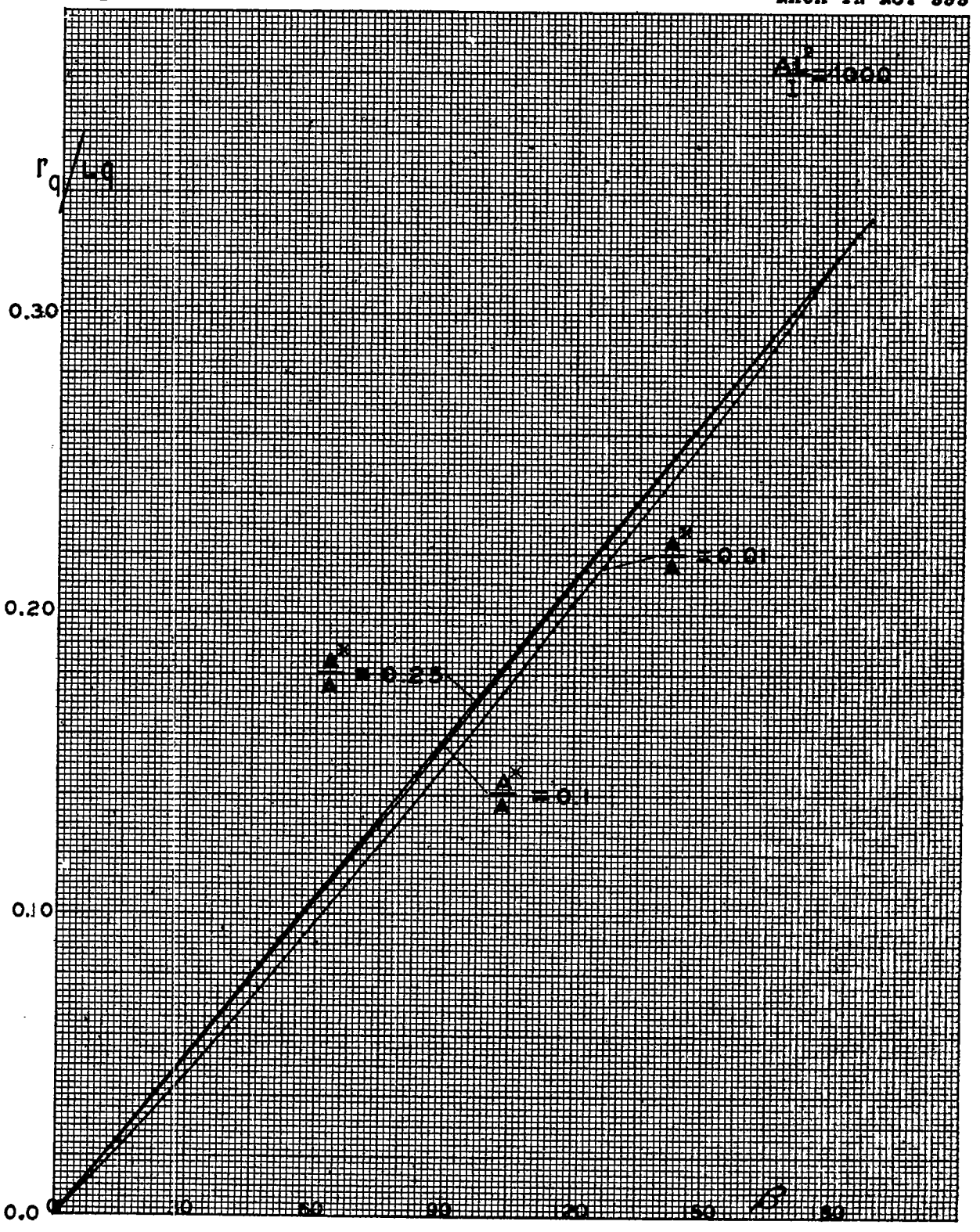


FIG. 92. FIXED END FORCE

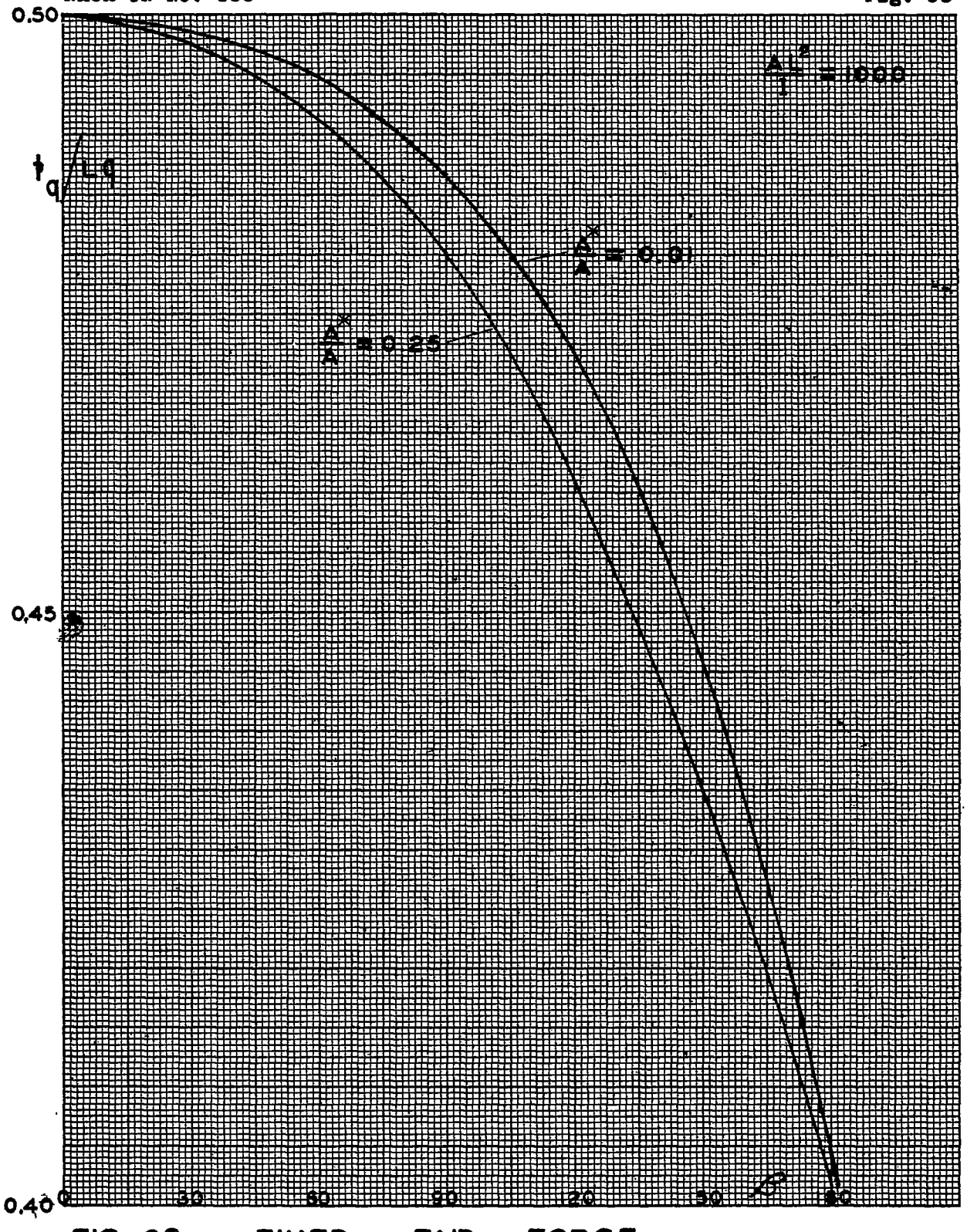


FIG. 93. FIXED END FORCE