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RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

VI - COMPUTED TEMPERATURE DISTRIBUTION THROUGH CROSS SECTION OF WATER-COOLED TURBINE BLADE

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RESEARCH MEMORANDUM

COOLING OF GAS TURBINES

VI - COMPUTED TEMPERATURE DISTRIBUTION THROUGH CROSS SECTION OF WATER-COOLED TURBINE BLADE

By John N. B. Livingood and Eldon W. Sams

SUMMARY

A theoretical analysis of the cross-sectional temperature distribution of a water-cooled turbine blade was made using the relaxation method to solve the differential equation derived from the analysis. The analysis was applied to a specific turbine blade and the studies included investigations of the accuracy of simple methods to determine the temperature distribution along the mean line of the rear part of the blade, of the possible effect of varying the perimetric distribution of the hot-gas-to-metal heat-transfer coefficient, and of the effect of changing the thermal conductivity of the blade metal for a constant cross-sectional-area blade with two 1/4-inch diameter coolant passages. In order to determine the effect of having a coolant passage nearer the trailing edge, the temperature at the trailing edge of the same size blade with five coolant passages was found for several thermal conductivities and lengths of the rear part of the blade. The distance from the trailing edge to the nearest coolant passage was limited by minimum wall clearance and minimum passage diameter. A similar investigation was also made to determine the effect of doubling the blade size and using seven coolant passages.

The results indicated that a good approximation to the temperature distribution along the mean line of the rear part of a turbine blade of exact shape may be obtained by a calculation of the temperature distribution through a wedge of comparable dimensions. The temperature distribution through the cross section of a constant-area water-cooled turbine blade was only slightly changed by use of a variable hot-gas-to-metal heat-transfer coefficient but was affected by changes in the thermal conductivity of the metal. A reduction in length of the rear part of the blade from 0.365 inch to 0.240 inch resulted in a 350° F drop in trailing-edge temperature for the blade with a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft) and five coolant passages. Doubling the blade size had no appreciable effect on the

[redacted]

blade temperature distribution. These results indicated that a coolant passage about one-fourth inch from the trailing edge would probably have the effect of keeping the trailing-edge temperature under the limit set by current metals.

INTRODUCTION

The NACA is conducting an investigation of methods of cooling gas turbines, including both indirect and direct blade cooling, in order to show that the application of a proper method of cooling will permit the use of gas temperatures higher than the approximate 1500° F limit that current uncooled metals require. Indirect blade cooling was investigated in references 1 to 3. In reference 1, it was shown that indirect cooling and application of a ceramic coating near the blade root section would have negligible effects in the upper half of a turbine blade unless metals with very large thermal conductivities are used. An increase in the cooling surface by the addition of air-cooling fins to the turbine rotor (reference 2) showed that the critical blade temperature could be reduced about 350° F. Rim cooling, demonstrated in reference 3 in terms of turbine operating conditions, showed that an increase of only 200° F in the allowable gas temperature could be obtained for most turbine blades. Several cooling methods were compared in reference 4, and the suggestion of the good possibilities of direct cooling led to an extensive investigation of this type of cooling.

Liquid cooling was investigated in reference 5. A radial temperature distribution near the coolant passages from turbine axis to turbine blade tips was obtained. Effects of variations in blade length, coolant-passage length, type of coolant, and coolant flow were studied. For water as coolant, with a flow of 6.42 pounds per second and an average temperature of 200° F, the blade temperatures near the coolant passages through most of the liquid-cooled part of the blades were found to be only about one-fifth the effective gas temperature. Another investigation was then made to determine blade temperatures away from the coolant passages (reference 6). The rear part of the blade was replaced by approximate shapes (wedge, rectangle, and right prism), and temperature distributions through the center of these revealed that temperatures near the blade trailing edge approached the effective gas temperature. Because air cooling is less complex than liquid cooling, direct cooling by air was investigated (reference 7). Although air cooling was found superior to indirect cooling, it did not offer as extensive possibilities for large gas-temperature increases as liquid cooling.

In order to obtain more information concerning the temperature distribution in a liquid-cooled turbine blade, a theoretical analysis of the temperature distribution through the cross section of a water-cooled turbine blade of exact shape was made. The solution of the differential equation that resulted was obtained by applying the relaxation method. The purpose of the present report is to present this analysis and its application to determine blade cross-sectional temperature distributions. A comparison of the temperature distributions through the cross section of the rear part of a water-cooled turbine blade of exact shape with that of an approximate shape is made for a set of chosen turbine operating conditions. Temperature distributions are compared for a constant and variable heat-transfer coefficient around the blade perimeter for a water-cooled turbine blade with two coolant passages and a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft). Temperature distributions are also compared for thermal conductivities of 15 and 100 Btu/(hr)(sq ft)(°F/ft) for the two-passage turbine blade. In order to determine the gain that would be obtained by moving a coolant passage nearer the trailing edge, the effect of the variation of the length of the rear part of the blade was investigated for several thermal conductivities for the same size blade as that used above (two passages) but with five coolant passages, and for a double-sized blade with seven coolant passages. In each of the latter cases, the coolant passage nearest the trailing edge has a 0.079-inch diameter, the smallest practical size.

SYMBOLS

The following symbols are used in the analysis:

k thermal conductivity of metal, Btu/(hr)(sq ft)(°F/ft)

q_i local heat-transfer coefficient between hot gases and metal, Btu/(hr)(sq ft)(°F)

q_o heat-transfer coefficient between metal and coolant, Btu/(hr)(sq ft)(°F)

T metal temperature, (°F)

T_e effective gas temperature, (°F)

T_l liquid-coolant temperature, (°F)

θ temperature difference, $T_e - T$, (°F)

ANALYSIS

This investigation was restricted to a turbine-blade cross section and hence was two dimensional. Justification for this restriction is presented in references 5 and 6. In reference 5 it is shown that there is little or no radial temperature change through most of a liquid-cooled turbine blade. This fact is further verified by a three-dimensional investigation in reference 6, which shows very little blade-temperature change in the radial direction except near the blade root; the heat flows directly to the liquid coolant throughout most of the blade.

On the basis of this two-dimensional restriction, the temperature at any point on the blade cross section may be found by solving the Laplace differential equation given in terms of the temperature difference θ

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$$

where x and y are Cartesian coordinates. The outer-edge boundary condition is

$$\frac{\partial\theta}{\partial n} = \frac{q_i}{k}\,\theta$$

and the coolant-passage boundary condition is

$$\frac{\partial\theta}{\partial n} = \frac{q_o}{k}\left(T_e - T_l - \theta\right)$$

where $\partial\theta/\partial n$ denotes a partial derivative normal to the boundary.

A solution to this problem was found by applying the relaxation method (reference 8). Temperature estimates for the blade leading edge were obtained by a calculation of the temperature distribution between concentric circles and for the trailing edge by a calculation of the temperature distribution through a wedge of comparable dimensions by the method used in reference 6. From these estimates, and approximations obtained from them by use of the boundary conditions, values of θ were found and inserted at each point of a square network of points. Residuals, which may be considered as interior heat sinks, were then calculated for each net point and reduced in value by a series of suitable changes in θ at the net points. Each change in θ at any point effected a redistribution of residuals according

to a fixed relaxation pattern. Changes in θ were calculated until all residuals were reduced to minimum values. From these revised values of θ, corresponding temperature values were inserted at each net point and isotherms were drawn to show the temperature distribution through the turbine-blade cross section.

The local heat-transfer coefficients around the blades were determined by using Reynolds analogy between transfer of heat and transfer of momentum. Boundary-layer theory was used to determine the friction coefficients for turbulent and laminar boundary layers. In the process, it was necessary to determine the point of transition from the laminar to the turbulent boundary layer in the blade. The methods of Squire (reference 9) for applying recognized boundary-layer theory in combination with Reynolds analogy to obtain heat-transfer coefficients over airfoil shapes were in general used. No attempt to explain the method of obtaining the local heat-transfer coefficients other than that given is made because the problem is complete in itself and the detail was not needed for the purpose of the present report.

The above analysis was applied to a specific turbine using a gas flow of 55 pounds per second, an effective gas temperature of 2000° F, a coolant (water) flow of 6.42 pounds per second, and an average coolant temperature of 200° F. For the given gas flow, the average hot-gas-to-metal heat-transfer coefficient was calculated by the methods of references 7 and 10 and found to be 217 Btu/(hr)(sq ft)(°F).

A turbine blade of constant cross-sectional area containing two 1/4-inch diameter coolant passages and having a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft) was considered first. The rear part of the blade was replaced by a wedge of comparable dimensions, and a temperature distribution obtained for this wedge by the method of reference 6. This distribution was then used as a first approximation to the temperature distribution along the mean line of the rear part of the blade of exact shape; the relaxation method was applied, and a temperature distribution obtained for the exact blade shape. The metal-to-coolant heat-transfer coefficient for the given coolant flow and passage diameter was calculated by the methods of reference 10 and was found to be 2370 Btu/(hr)(sq ft)(°F).

For this same blade, temperature distributions were obtained for a constant average and for a perimetric variable hot-gas-to-metal heat-transfer coefficient in order to determine the effect of this variable heat-transfer coefficient on the temperature distribution.

A temperature distribution was obtained for a two-passage blade (as above), but with a thermal conductivity of 100 Btu/(hr)(sq ft)(°F/ft) for the constant average heat-transfer coefficient of 217 Btu/(hr)(sq ft)(°F) so that a comparison between this and a previously found temperature distribution for a blade with a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft) could be made to determine the effect of varying thermal conductivity on the blade temperature distribution.

The effect of the variation of length of the rear part of the blade was then investigated. The same size blade, with five coolant passages, was considered. The distance from the trailing edge to the nearest coolant passages was limited by minimum wall clearance and minimum passage diameter. The metal-to-coolant heat-transfer coefficient for this passage was calculated by the method of reference 10 and was found to be 2134 Btu/(hr)(sq ft)(°F). Thermal conductivities of 15, 30, and 60 Btu/(hr)(sq ft)(°F/ft) were considered, and trailing-edge temperatures were obtained for various lengths of the rear part of the blade.

A similar investigation of trailing-edge temperatures for a double-sized blade with seven coolant passages was made for the same conditions as before, except that the metal-to-coolant heat-transfer coefficient in this case was found to be 2412 Btu/(hr)(sq ft)(°F).

RESULTS AND DISCUSSION

The cross section of the turbine blade containing two 1/4-inch diameter coolant passages is shown in figure 1, together with the perimetric variation of the hot-gas-to-metal heat-transfer coefficient. Values of this heat-transfer coefficient varied from 390 Btu/(hr)(sq ft)(°F) at the blade leading edge to 182 Btu/(hr)(sq ft)(°F) near the blade trailing edge; i.e., the ratio of the greatest to the least heat-transfer coefficients was approximately 2 to 1.

Figure 2 shows the temperature distribution through the two-passage blade cross section, calculated on the basis of the variable hot-gas-to-metal heat-transfer coefficient. In spite of the large heat-transfer coefficients around the leading edge, the leading-edge temperature remained below 1200° F. The trailing-edge temperature, however, was found to be 1700° F, or 300° F below the effective gas temperature.

The temperature distribution obtained for the same blade by use of the constant average heat-transfer coefficient was superimposed on that based on the perimetric variable heat-transfer coefficient and is shown in figure 3. The effect of the use of the average

constant heat-transfer coefficient was a lowering of temperatures by about 200° F near the blade leading edge, and a raising of the trailing-edge temperature by about 75° F. Inasmuch as the leading-edge temperatures are already sufficiently low for the use of current metals, and because the increase in trailing-edge temperature was not excessive, it was concluded that the use of the average constant heat-transfer coefficient for further investigation would result in good approximations for the blade temperature distributions yet to be determined.

The effect of varying thermal conductivity is shown in figure 4 where the temperature distribution for the blade with a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft) is superimposed on that for the blade with a thermal conductivity of 100 Btu/(hr)(sq ft)(°F/ft). Use of the higher thermal conductivity resulted in a general lowering of the blade temperatures ranging from a 300° F drop at the leading edge to a 600° F drop at the trailing edge. The leading-edge temperature for this higher-conductivity metal was found to be 660° F; the trailing-edge temperature was 1140° F.

Temperature distributions were obtained for the rear part of a turbine blade of exact shape and for one approximated by a wedge. A comparison of the results showed that the temperatures along the axis of the wedge were in good agreement with those along a mean line in the rear part of the blade of exact shape. A maximum difference of only 2° in 1140° F was found for the blade with a thermal conductivity of 100 Btu/(hr)(sq ft)(°F/ft).

The calculations discussed in the preceding paragraphs were all obtained for an effective gas temperature of 2000° F. According to some German results, however, strong cooling of the blade leading edge would cool the boundary layer in the trailing part of the blade, and a difference of several hundred degrees in effective gas temperature might result around the blade periphery. On this basis, the trailing-edge temperatures as found above may be those corresponding to an effective gas temperature somewhat higher than 2000° F near the blade trailing part.

The turbine blade containing five coolant passages, used in determining the effect of length of the rear part of the blade on blade temperature distributions, is shown in figure 5. Maximum length of the rear part of the blade is shown in this figure. Results of the investigations for turbine blades with five coolant passages are shown in figure 6. For the maximum length of the rear part of the blade and a conductivity of 15 Btu/(hr)(sq ft)(°F/ft), the trailing-edge temperature was about 1630° F; a reduction in

length of the rear part of the blade from 0.365 inch to 0.240 inch resulted in a trailing-edge temperature of about 1280° F. Doubling the conductivity resulted in decreases of 250° F and 200° F, respectively, in the trailing-edge temperatures for the maximum length of the rear part of the blade (0.365 in.) and for a length of the rear part of the blade of 0.240 inch. Further corresponding reductions resulted from increasing the thermal condictivity to 60 Btu/(hr)(sq ft)(°F/ft). The improvement in trailing-edge temperatures obtained by moving a cooling passage nearer the trailing edge was diminished because the ratio of the heating area to the cooling area is limited, and the metal-to-coolant heat-transfer coefficient decreased with the decrease in coolant-passage diameter.

Similar investigations were made for the double-sized blade with seven coolant passages, shown with maximum tail length in figure 7. Results similar to those for the five-passage blade were obtained (fig. 8).

SUMMARY OF RESULTS

Application of the theoretical analysis of the temperature distribution through a cross section of a water-cooled turbine blade to a specific turbine revealed the following results:

1. The temperature distributions along the mean line of the rear part of a turbine blade of exact shape may be accurately obtained by a calculation of the temperature distribution through a wedge of comparable dimensions.

2. The temperature distribution through a cross section of a constant-area water-cooled turbine blade was slightly changed by use of a calculated variation in the hot-gas-to-metal heat-transfer coefficient instead of a constant average hot-gas-to-metal heat-transfer coefficient.

3. The temperature distribution through a cross section of a water-cooled turbine blade was greatly responsive to changes in the thermal conductivity of the metal.

4. A reduction in length of the rear part of the blade from 0.365 inch to 0.240 inch reduced the trailing-edge temperatures about 350° F for the five-passage blade with a thermal conductivity of 15 Btu/(hr)(sq ft)(°F/ft).

5. Blade size had no appreciable effect on the blade temperature distribution if additional cooling passages accompanied any increase in blade size.

6. These results indicated that a coolant passage about one-fourth inch from the trailing edge would probably have the effect of keeping the trailing-edge temperature under the limit set by current metals.

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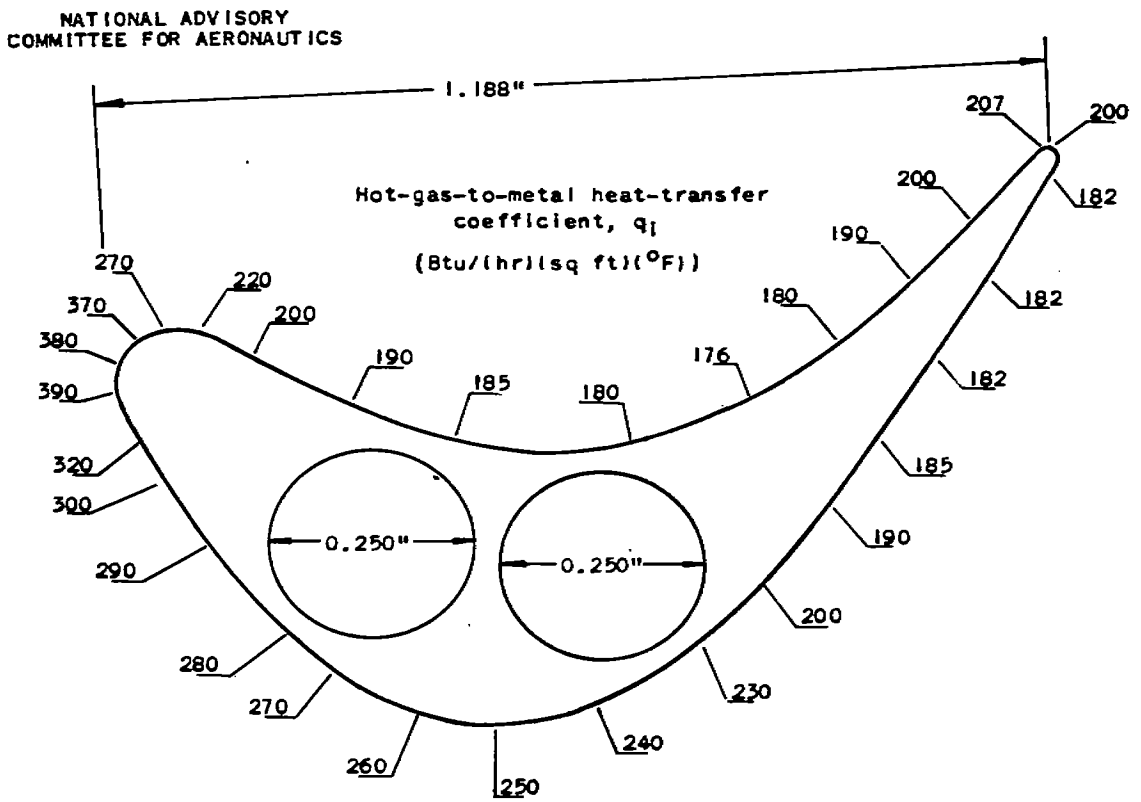

Figure 1. - Cross section of turbine blade showing variation of heat-transfer coefficient around blade perimeter.

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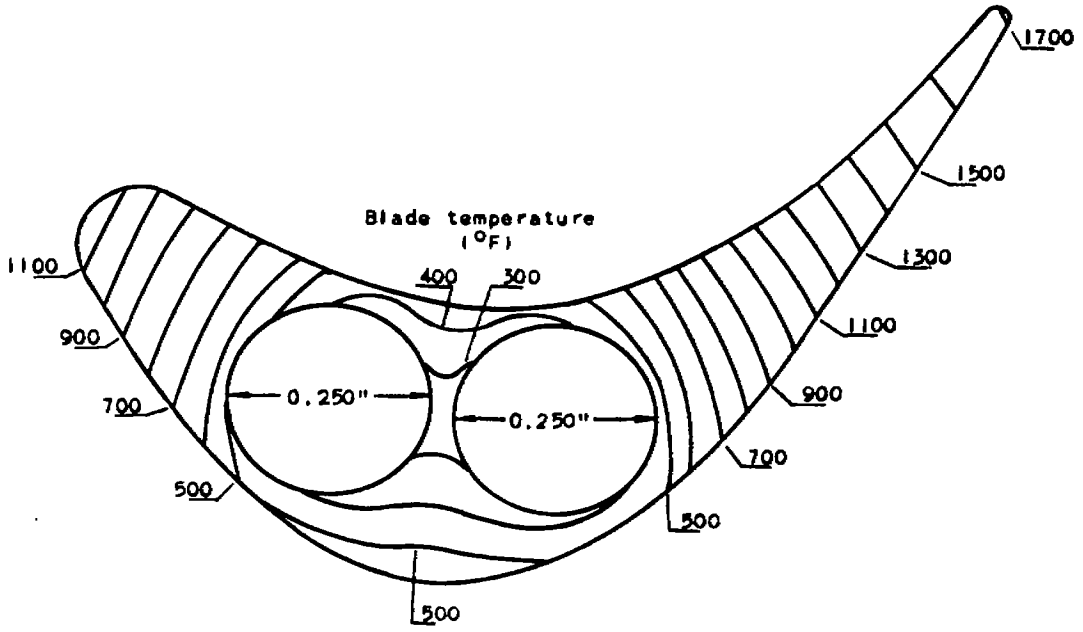

Figure 2. - Temperature distribution through cross section of water-cooled turbine blade for variable hot-gas-to-metal heat-transfer coefficient q_1. Thermal conductivity of metal k, 15 Btu/(hr)(sq ft)(°F/ft); effective gas temperature T_e, 2000° F; average water temperature T_1, 200° F.

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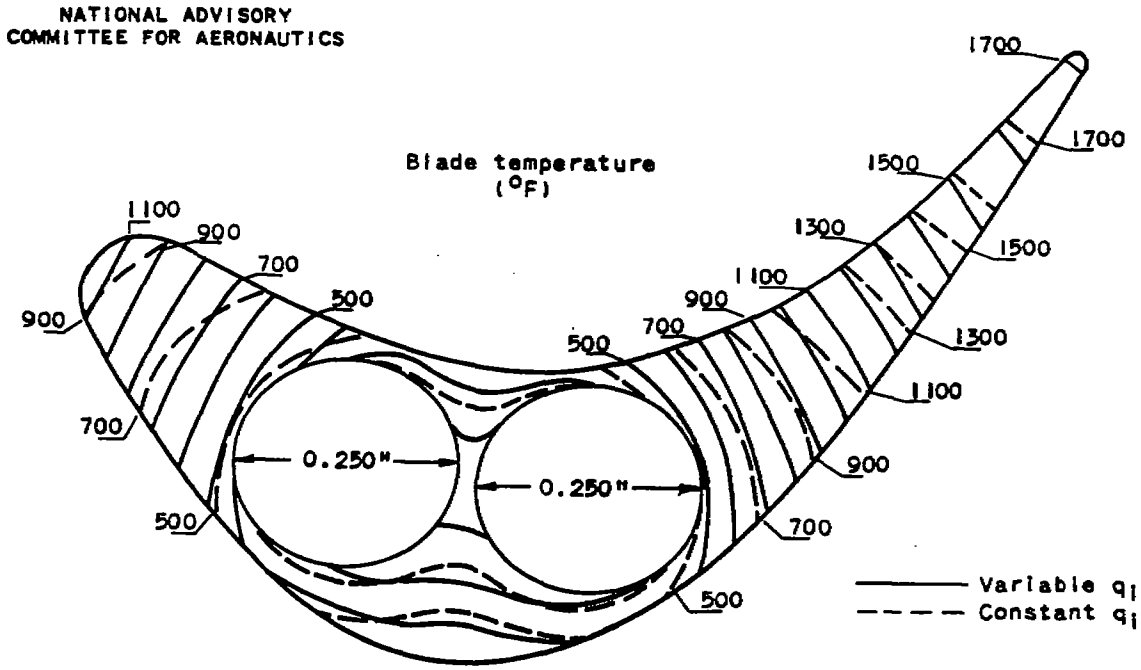

Figure 3. - Effect of variation of hot-gas-to-metal heat-transfer coefficient q_i, on temperature distribution through cross section of water-cooled turbine blade. Thermal conductivity of metal k, 15 Btu/(hr)(sq ft)(°F/ft); effective gas temperature T_e, 2000° F; average water temperature T_l, 200° F.

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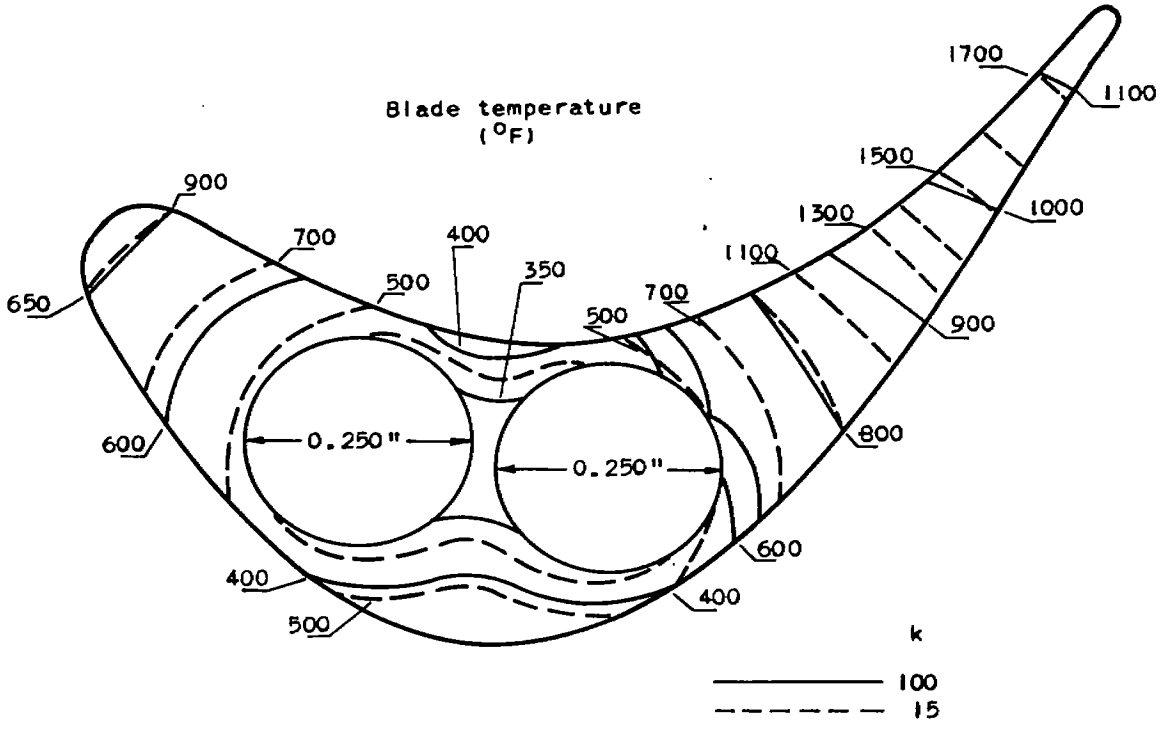

Figure 4. - Effect of variation of thermal conductivity k on temperature distribution through water-cooled turbine blade. Hot-gas-to-metal heat-transfer coefficient q_i, 217 Btu/(hr)(sq ft)(°F); effective gas temperature T_e, 2000° F; average water temperature T_l, 200° F.

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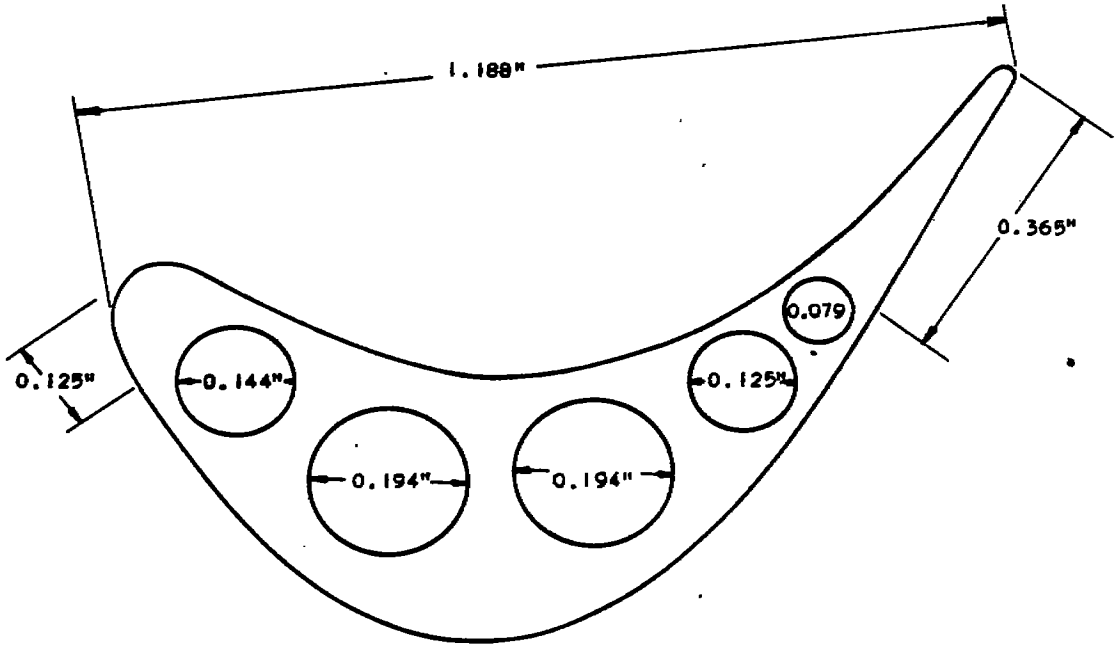

Figure 5. - Cross section of water-cooled turbine blade showing location and size of five coolant passages and blade dimensions with maximum length of rear part of blade.

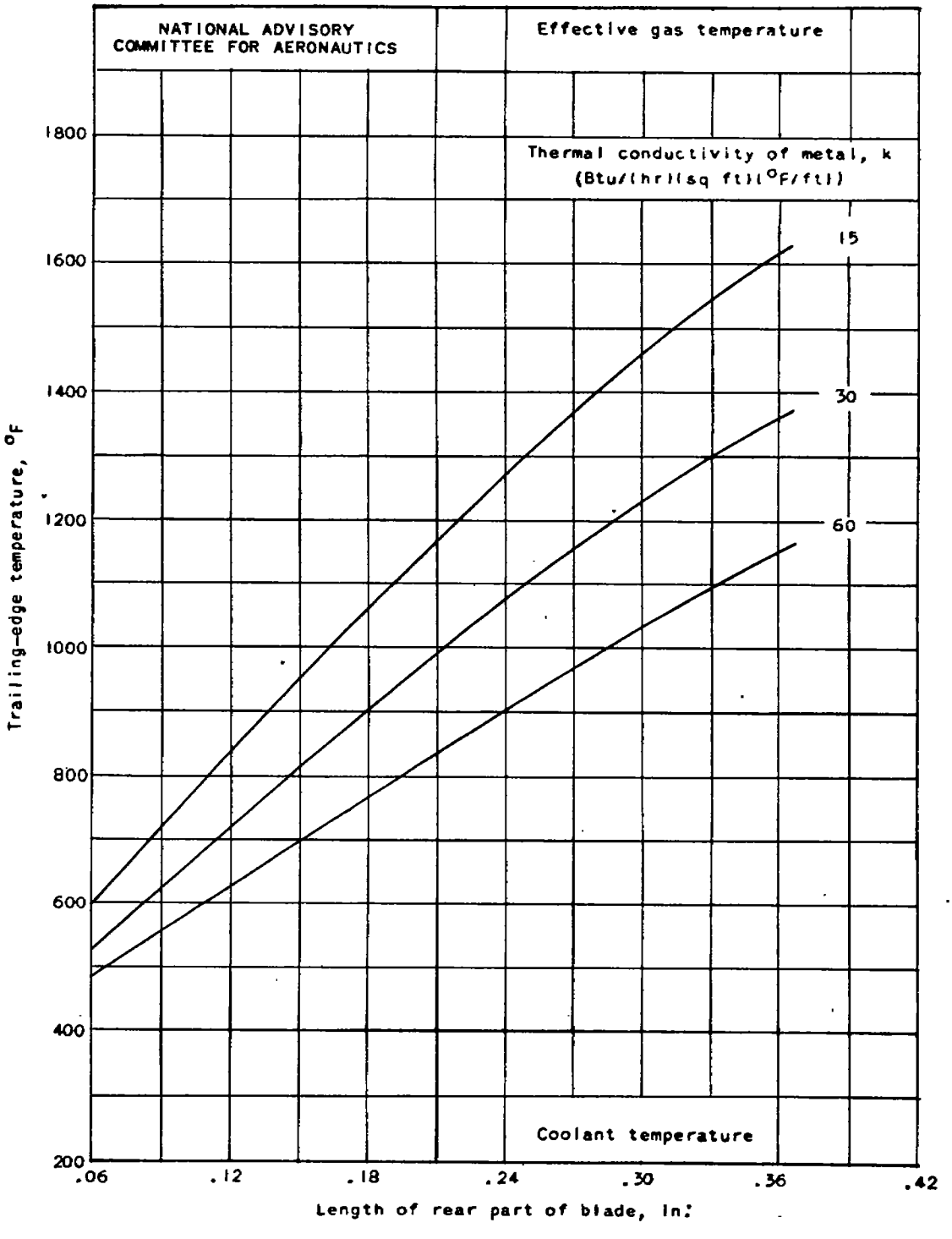

Figure 6. - Trailing-edge temperature as function of length of rear part of blade for water-cooled turbine blades with five coolant passages and various thermal conductivities.

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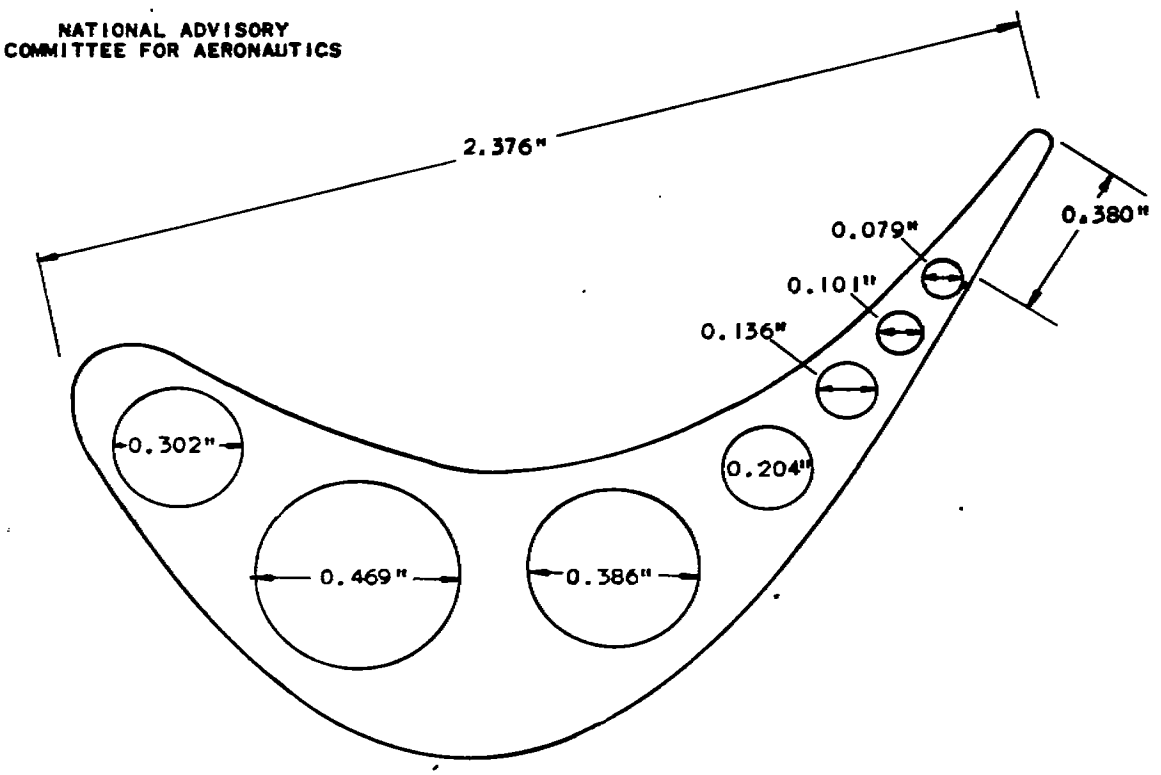

Figure 7. - Cross section of water-cooled turbine blade showing location and size of seven coolant passages and blade dimensions with maximum length of rear part of blade.

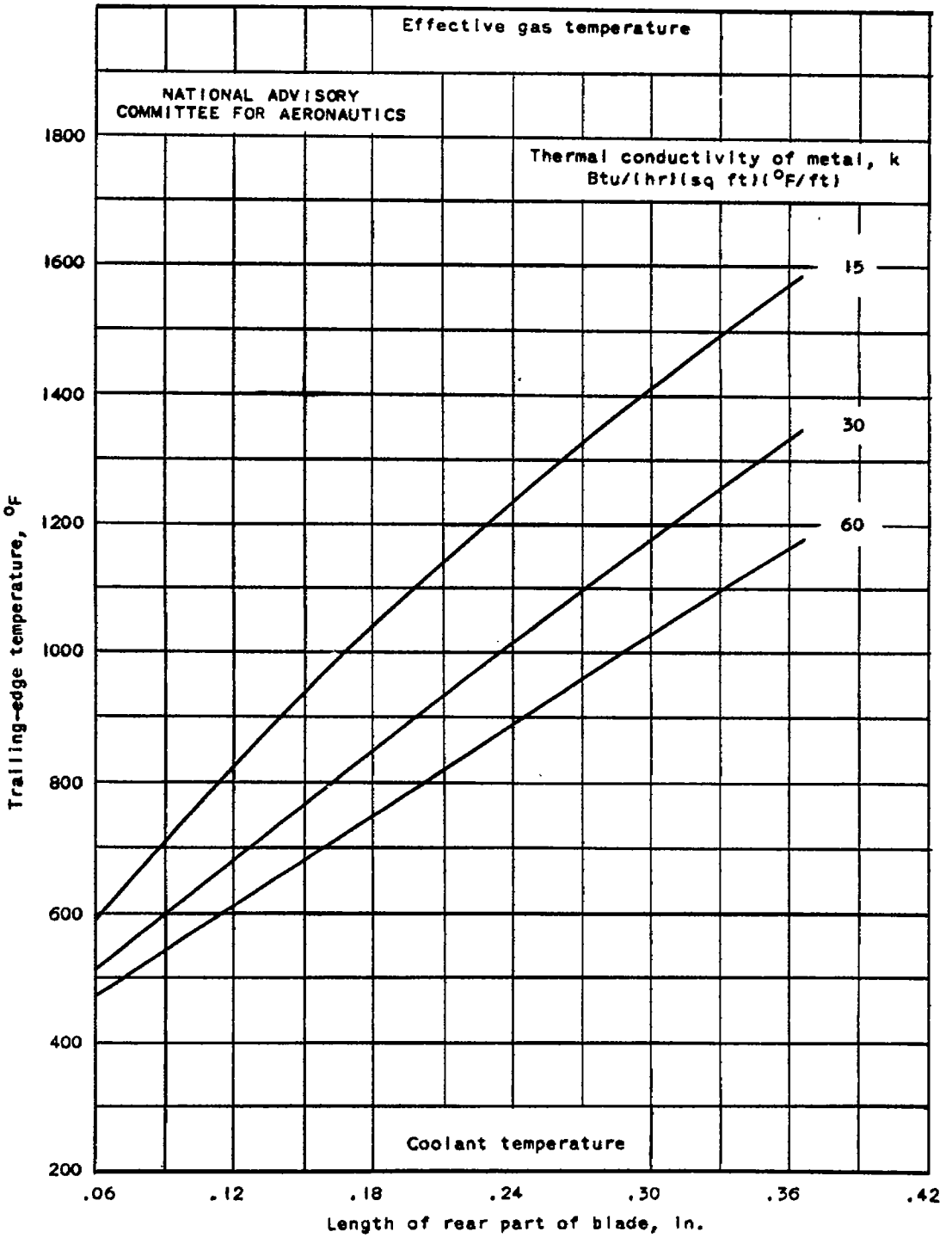

Figure 8. - Trailing-edge temperature as function of length of rear part of blade for water-cooled turbine blades with seven coolant passages and various thermal conductivities.