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RESEARCH MEMORANDUM

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THEORETICAL EVALUATION OF METHODS OF COOLING THE BLADES OF GAS TURBINES

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RESEARCH MEMORANDUM

THEORETICAL EVALUATION OF METHODS OF COOLING THE BLADES OF GAS TURBINES

By J. C. Sanders and Alexander Mendelson

SUMMARY

A theoretical analysis of the effects of various methods of cooling turbine blades with consequent increase in permissible gas temperature on turbine-cycle efficiency is presented. A study was made of the heat-transfer process in turbine blades and the effects on blade temperatures of cooling the blade root and tip, changing the dimensions of the blade, raising the cycle temperatures, insulating with ceramics, and cooling by circulation of air or water through hollow blades were determined. It was found that cooling the root of the blade, shortening the blade, and cooling hollow blades internally with air or liquid offer possibilities of substantial increases in permissible gas temperatures and correspondingly in cycle efficiency. Cooling the tip of the blade and coating the blade with ceramic were relatively ineffective.

INTRODUCTION

The maximum permissible gas temperature imposes limits on both the efficiency and the power of gas turbines as has been shown by the calculations of Stodola (reference 1) and Piening (reference 2). Currently used gas turbines can be made more compact and powerful than the conventional reciprocating gasoline engine, but because of the temperature limitations, the efficiency is much lower. It is particularly desirable therefore to increase the cycle temperatures.

Cycle temperatures are limited by the destructive effects of high temperatures on turbine blades, nozzles, and combustion chambers. In reference 3 it was pointed out that the turbine blades operate in gas at a lower temperature than the gas in the combustion chambers and the nozzles but that the blades of the turbine operate under much higher stress as a result of the high rotative speed. In fact,

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the rotative speed is limited by the high-temperature-failure properties of the blades and wheel. Consequently, the most important problem encountered in increasing cycle temperatures is providing means of preventing turbine failures due to overheating.

The tendency of turbine blades to fail in the presence of high-temperature gases may be reduced by selecting blade materials more resistant to creep and rupture and by so cooling the blades that they operate at safe temperatures. The following means of cooling turbine blades have been proposed or used: conduction of heat to cooled root or tip; protection of the blades with layers of ceramic heat insulator; internal cooling of hollow blades with air, water, or other fluid; cooling by radiation to cold radiation shields; and alternate flow of hot and cold gas through the turbine.

An analysis was made at the NACA Cleveland Laboratory in the spring of 1945 to determine whether turbine-blade cooling can provide sufficient increase in permissible cycle temperatures to justify a program of research on turbine cooling. The heat-transfer processes in turbine blades are analyzed and an equation for the computation of the approximate blade temperatures is derived. In one specific case, the approximate one-dimensional solution is compared with a more informative two-dimensional solution to illustrate the magnitude of the error in the one-dimensional approximation. Surface heat-transfer coefficients, creep characteristics, and rupture characteristics were obtained from literature. The approximate one-dimensional solution is used to compute the effects on blade temperatures of cooling the blade root and tip, changing the dimensions of the blade, raising the cycle temperatures, insulating with ceramics, and cooling by circulation of air or water through hollow blades. In each case the effectiveness of the cooling in reducing creep and increasing the time to rupture was estimated and the equivalent increase in permissible cycle temperatures was determined. The results of these computations are used as a basis for recommending research on turbine cooling.

LIST OF SYMBOLS

The following symbols are used in the analysis:

A	cross-sectional area of blade
a, b, c, m	experimental constants
C	perimeter of turbine blade
C'	internal perimeter of hollow blade

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c_p specific heat at constant pressure

D inside diameter based on hydraulic radius

E activitation energy

G mass velocity of gas

h_a coefficient of heat transfer between turbine blade and internal coolant

h_g coefficient of heat transfer between hot gas and turbine blade

h_r equivalent radiation heat-transfer coefficient

k thermal conductivity of turbine blade

k_a thermal conductivity of coolant

L length of turbine blade

M parameter defined by $M = h_a C'/kA$

P parameter defined by $P = \frac{h_g C}{kA}\left(1 + \frac{h_r}{h_g} + \frac{C'}{C}\frac{h_a}{h_g}\right)$

q_c heat flow per unit time along the blade

q_{rg} heat radiated per unit time from turbine blade

q_{sa} heat transferred per unit time from turbine blade to internal coolant

q_{sg} heat absorbed per unit time by surface of turbine blade

R parameter defined by $\frac{h_g C}{kA}\left(T_g + \frac{h_r}{h_g}T_o + \frac{C'}{C}\frac{h_a}{h_g}T_{a_i}\right)$

r_1 radius of turbine wheel at blade root

r_2 radius of turbine wheel at blade tip

S applied tensile stress

T temperature at any point on turbine blade

T_a temperature of internal coolant of turbine blade

T_{a_i} initial temperature of internal coolant of turbine blade

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T_b temperature at base of blade

T_e equivalent temperature defined by R/P

T_g temperature of hot gas

T_o temperature of cold body

V velocity of gas

w width of turbine blade

x distance from leading edge to point under consideration

y distance from base of blade to any point on blade

Δs increase in entropy

Δy increment along length of blade

δ rate of elongation

μ absolute viscosity

ν kinematic viscosity of gas

ρ density of gas

σ Stefan-Boltzmann constant

ANALYSIS

Heat-Transfer Processes in Turbine Blades

Equation for temperature distribution in turbine blades. - The heat-transfer processes are first analyzed for a hollow blade cooled by internal circulation of coolant and by conduction and radiation. By proper substitution of coefficients, the resultant equation reduces to a simpler one for a solid blade. A one-dimensional analysis similar to that given by Carslaw (reference 4, p. 66) for a thin rod radiating heat is used. In the present analysis, the heat transfers by convection and radiation are separately accounted for and the temperature of only one end of the blade is assumed known.

The following sketch shows the significant dimensions and environmental temperatures:

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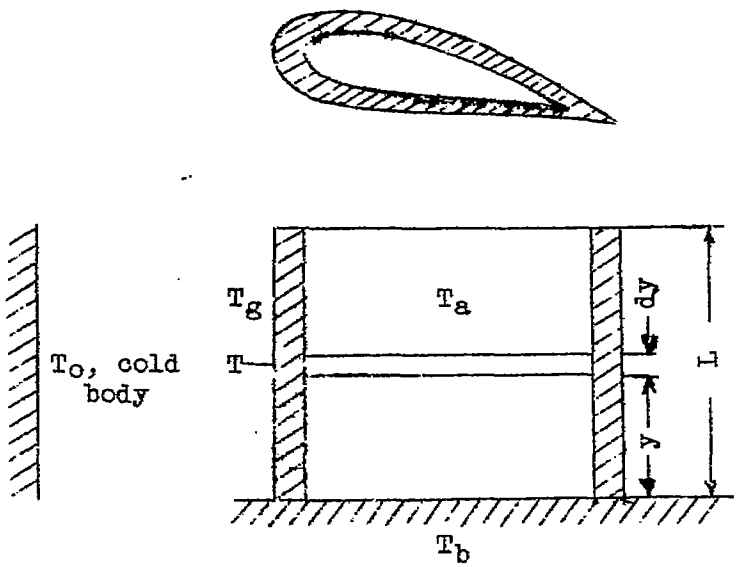

In the one-dimensional solution, the temperatures at all points in a section perpendicular to the axis of the blade are assumed equal and the surface coefficient of heat transfer is constant over the entire surface of the blade. The final equation for the temperature distribution in the blade, which is derived in the appendix, as equation (25) is

$$T - T_e = \frac{\cosh\left[\sqrt{P}\,(L - y)\right]}{\cosh\left(\sqrt{P}\,L\right)}\,(T_b - T_e) - \frac{Mc}{P^{3/2}}\,\frac{\sinh\left(\sqrt{P}\,y\right)}{\cosh\left(\sqrt{P}\,L\right)} + \frac{M_c}{P}\,y$$

In the case of a solid blade, M reduces to zero and the equation becomes

$$\frac{T - T_e}{T_b - T_e} = \frac{\cosh\left[\sqrt{P}\,(L - y)\right]}{\cosh\left(\sqrt{P}\,L\right)} \tag{1}$$

where P and T_e become

$$P = \frac{h_g C}{kA}\left(1 + \frac{h_r}{h_g}\right) \tag{2}$$

$$T_e = \frac{T_g h_g + T_o h_r}{h_g + h_r} \tag{3}$$

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Surface coefficient of heat transfer from hot gas. - Lelchuk (reference 5) found by experiment with flow inside a pipe that the equation developed by Nikuradse for computing the coefficient of friction and heat transfer and the numerical values of the constants usually used with this equation applied equally well for a range of airspeeds up to a Mach number of 1 provided that the stagnation temperature of the gas is used rather than the free-stream temperature. This principle was assumed by Crocco (reference 6) to apply to the case of heat transfer from a flat plate to a stream of high-velocity gas. By substitution of the symbols used herein, his equation for the average coefficient is

$$\frac{h_g}{w} = 0.0356\ \rho c_p V \left(\frac{\nu}{Vw}\right)^{1/5} \tag{4}$$

The computed value of h_g is shown in figure 1 at the following conditions assumed for the calculations in this report:

Nozzle pressure ratio	3.0
Pressure in tail pipe, pounds per square inch	14.7
Mean peripheral velocity of blade, feet per second	1200

Actually the coefficient of heat transfer varies from a high value at the leading edge of the blade to a low value at the trailing edge, as has been shown by Squire (reference 7) in the case of an airfoil. Equation (4) for the average coefficient was derived by integrating the heat dissipation from the leading edge to the trailing edge and dividing by the chord. The following equation for the local coefficient of heat transfer $h_{g_{loc}}$ is thereby obtained by the proper differentiation of the equation for the average coefficient

$$h_{g_{loc}} = \frac{4}{5}\left[0.0356\ \rho c_p V \left(\frac{\nu}{V}\right)^{1/5}\right] x^{-1/5} \tag{5}$$

The local coefficient of heat transfer is shown in figure 2 when the temperature of the gas in the nozzle box is 1600° F and the other conditions are the same as for figure 1. These local coefficients were used in the two-dimensional analysis made to determine the error in the simple one-dimensional solution.

Coefficient of heat transfer to cooling fluid. - The coefficient of heat transfer between the blade and the coolant circulating inside was assumed to be the same as in a circular pipe of the same hydraulic radius. The coefficient of heat transfer in a circular pipe was shown in reference 8 (p. 168) to be

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$$\frac{h_a D}{k_a} = 0.023 \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{c_p \mu}{k_a}\right)^{0.4} \tag{6}$$

Thermal conductivity. - A survey of alloys of high nickel-chromium content (reference 9) led to a selection of 0.674 Btu/(hr)(sq in.)(°F/in.) for the thermal conductivity in this analysis.

Comparison of temperature distributions in a solid blade obtained with one-dimensional and with two-dimensional analysis. - The temperature distribution throughout the solid blade was computed by a two-dimensional network. The method consists in selecting a convenient number of points throughout the blade, assuming them to be connected by heat-conducting rods, and solving simultaneously the equations obtained by setting the total heat flow into each junction of the network equal to zero. This method permits the proper recognition of the effects of the actual variation in thickness of the blade section and the variation in surface coefficient of heat transfer from the leading edge to the trailing edge. This method, however, is too tedious and cumbersome for general use in the preliminary survey of methods of cooling turbine blades. A more simple and approximate relaxation method as formulated in reference 10 can be used.

The results of these computations for one specific case are compared in figure 3 with the one-dimensional solution for the same conditions. The one-dimensional solution agrees very well with the two-dimensional solution for the center of the blade but the two-dimensional solution shows both the leading edge and the trailing edge to be hotter. The higher temperature of the leading and trailing edges is expected because the surface coefficient is highest over the forward surfaces and the tapering of the trailing edge makes heat removal by conduction more difficult.

Time-rupture and creep characteristics of turbine-blade materials. - High blade temperatures may cause blade failure in several ways, such as tensile rupture and creep, oxidation, and chipping and distortion of thin blade edges. The tensile rupture and the creep are the most important and are therefore considered in this analysis. The time-rupture properties probably have more effect on turbine failure than the creep characteristics but the equations relating creep to stress and temperature have been more fully developed than those for time rupture. Most of the analysis of the effectiveness of blade cooling is therefore based upon creep and in one case the results obtained by creep analysis are compared with an analysis using time-rupture characteristics.

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Creep characteristics. - The following empirical equations relating second-stage creep to stress and temperature were proposed in reference 11:

$$\delta = 2m \sinh (aS) \tag{7}$$

$$\log_{10}\left(\frac{m}{T}\right) = 10.319 + \frac{\Delta s}{4.577\ T} - \frac{E}{4.577\ T} \tag{8}$$

$$\log_e a = -\left(c + \frac{b}{T}\right) \tag{9}$$

Sufficient data on the second-stage creep of alloys used in gas turbines to evaluate the constants for these equations are unavailable. The best alloy for which adequate data are available was 18:8 stainless steel (reference 12, p. 218). The values of the constants for this metal are as follows:

$$\Delta s = -10.66$$

$$E = -2778$$

$$c = -1.695$$

$$b = 1135$$

Time-rupture characteristics. - Reliable time-rupture data were obtained from investigations carried out at the University of Michigan for materials considered for blades of gas turbines. The alloy Vitallium having the following percentage composition was chosen for this analysis:

C	Mn	Si	Cr	Ni	Co	Mo	Fe
0.24	0.98	0.63	27.6	3.06	Balance	5.13	1.76

The same type of empirical equation was used for the correlation of the time-rupture data as for the creep data.

Stresses in the turbine blades. - Only centrifugal loads were considered in the computation of blade stresses. The shearing stress resulting from the force of the gas on the blade is small and deformation in bending proceeds until a neutral configuration with no bending stress is achieved. With high peripheral speed, the deflection to this equilibrium configuration is small. Stress concentration at the root of the blade is also neglected. This simplification is justified by the agreement between the indicated point of failure shown in the following sections of this report and the examination of actual blade failures in accelerated destruction tests.

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The following equation for the stress in the turbine blade was derived from simple kinematic principles:

$$S = \frac{\rho}{2}\left(\frac{V_2}{r_2}\right)^2 \left\{ r_2^2 - \left[r_2 - (L - y)\right]^2 \right\} \tag{10}$$

The outer radius was assumed to be 11.48 inches and the peripheral velocity at 9 inches was assumed to be 1200 feet per second.

EVALUATION OF METHODS OF COOLING TURBINE BLADES

The methods of cooling turbine blades and reducing their creep are considered in this section and are listed below in the order in which they are discussed:

1. Cooling root of blade
2. Cooling tip of blade
3. Reducing blade length
4. Decreasing value of C/kA
5. Thermal insulation with ceramic coating
6. Circulation of air through hollow blades
7. Circulation of water through hollow blades

Cooling root of blade. - The effects of root temperature on blade temperatures were computed by the one-dimensional solution for a blade whose dimensions and operating conditions were as follows:

Blade length L, inches	2.48
Blade perimeter C, inches	3.4
Cross-sectional area of blade A, square inch	0.18
Temperature of surface absorbing radiation, $^{\circ}F$	1000

When the root of the blade is cooled to 400° F, blade temperatures in the vicinity of the root are reduced, but half the length of the blade is unaffected (fig. 4). The stress is a maximum at the root, however, and diminishes to zero at the tip. Computation of the local creep rates and rupture time along the blade show that points of highest creep and shortest time to rupture lie in the zone influenced by cooling of the root. (See figs. 5 and 6.) Cooling of

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the root progressively shifted the weakest point from the root in the case of no cooling to a point 0.6 inch from the root when the temperature was reduced to 400° F. A more detailed analysis of the effect of root cooling is presented in reference 13.

A digression on the distribution of creep and rupture time along the blade would be desirable to show the nature of blade failures in gas turbines. The tenth root of the rate of creep has been plotted in figure 5 to make possible the reading of the creep from the curve. The creep rate plotted directly in figure 7 shows a narrow region of high creep 0.6 inch from the root and practically no creep either nearer to or farther from the root. This narrow region of high creep is a weak zone where the blade would be expected to fail in tensile creep. An analysis using the rupture time yields approximately the same result. Examination of turbines that have been tested to destruction in accelerated torque-stand tests clearly show such a zone of temperature demarcation and rupture.

Figures similar to figure 5 were prepared for cases where the temperature of the gas in the nozzle box is 1800° and 2000° F and the maximum values of creep along the blade were read from the curves and plotted in figure 8. From figure 8, an estimate of the increase in permissible nozzle-box temperature accompanying reduction of blade-root temperature can be obtained. Thus, if a nozzle-box temperature of 1600° F is permitted when the root temperature is 1192° F, then cooling the root to 800° F will permit a nozzle-box temperature of 1700° F.

Cooling tip of blade. - Cooling of the tip of the blade to 400° F has no effect on the temperature of the critical region of the blade (fig. 9). Consequently, cooling of the tip of the blade is ineffective in raising the permissible temperature of the gas in the nozzle box when blades of this length are used. Cooling of the tip would be beneficial with very short blades.

Reducing blade length. - Reducing the length of the turbine blade had little effect on temperature distribution because blade temperatures are uniform at distances greater than 1.0 inch from the base. The shortening of the blade, however, reduces the tensile stress and consequently the rate of creep as shown in figure 10. The reduction in maximum creep with decrease in blade length is so great that reducing the blade length from 2.5 inches to 1.5 inches permits an increase in nozzle-box temperature of more than 400° F.

Decreasing value of C/kA. - Increasing the conduction of heat to the root of the blade could be achieved by decreasing the ratio of the perimeter to the area of the cross section of the blade C/kA and by increasing the thermal conductivity of the blade material.

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These factors influence temperature and creep through their effect on the parameter

$$P = \frac{h_g C}{kA}\left(1 + \frac{h_r}{h_g}\right)$$

The effects of this parameter P on the maximum rate of creep are shown in figure 11. When P is reduced from 25 to 15 inches (which is equivalent to a 40-percent reduction in the ratio of perimeter to area or a 67-percent increase in thermal conductivity), the permissible nozzle-box temperature is increased 40° F.

Thermal insulation with ceramic coating. - Computations were made of the effectiveness of a layer of a ceramic material on the surface of the blade in insulating the blade from the hot gas and thereby reducing its temperature and creep. A layer of aluminum silicate 0.02 inch thick was assumed for a blade whose maximum thickness was 0.20 inch. The ceramic was assumed to be selfsupporting. The creep of the blade (fig. 12) is not significantly reduced. This conclusion does not apply to the solid ceramic blade nor does it evaluate the effectiveness of the ceramic coating in preventing corrosion and chipping of the blade edges.

Circulation of air through hollow turbine blades. - Circulation of air through hollow turbine blades as a means of cooling the blades has been used by Lorenzen (reference 14). The cooling air is introduced at the prevailing atmospheric pressure through an opening at the center of the turbine wheel and flows radially through the hollow wheel and blades where it is discharged at high velocity to a stationary diffuser, which recovers the excess kinetic energy of the air by converting it to pressure energy. This arrangement is shown in figure 13.

For purposes of this analysis, the following assumptions were made: (1) The external shape of the hollow blade was the same as that used in the computation of figure 3 for the solid blade; (2) the air passage was assumed to be 75 percent of the total cross-sectional area of the blade; (3) the turbine wheel had 70 blades and operated with a flow of 30 pounds of working fluid per second thereby permitting the quantity of cooling air to be expressed as a percentage of the working fluid; and (4) the blade-root temperature was maintained at 600° F.

The effectiveness of the cooling air in reducing blade temperatures is shown in figure 14. Even a small flow of cooling air amounting to 2.5 percent of the working fluid reduced the maximum blade temperature from 1192° F in the case of the solid blade to

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990° F with air cooling. The corresponding reductions in maximum rate of creep are shown in figure 15. If a solid blade with a root temperature of 600° F can operate with a temperature in the nozzle box of 1600° F, the hollow blade with an air circulation of 2.5 percent of the working fluid should be able to operate with a nozzle-box temperature of 1940° F.

A comparison of the over-all cycle efficiencies of a gas turbine with solid blades and one with air-cooled blades is shown in the following table. A nozzle-box temperature of 1600° F was assumed for the turbine with the solid blades and 2000° F for the one with the air-cooled blades; efficiencies of 0.9 were assumed for the turbine and the compressor; and optimum pressure ratio, as shown in figure 16, was used in each case.

	Pressure recovery for cooling air	Over-all cycle efficiency
Uncooled turbine	---	0.338
Air-cooled turbine	1.0	.395
Do----------	.5	.393
Do----------	0	.391

This comparison shows that even in the extreme case where none of the energy of the cooling air is recovered, a net improvement in cycle efficiency results from the higher permissible temperature in the nozzle box. Cooling of the turbine blades by circulation of air through hollow blades therefore appears to be practicable in spite of the fact that the air-cooled blades which have been tried have met with little success. The probable difficulty with these turbines is that all of the working fluid first passed through the hollow blades as coolant and received the natural compression. Because this compression efficiency is low as a result of limitations on the proportions of the blades and the cycle efficiency is very sensitive to compressor efficiency, the efficiency of such a gas-turbine engine is necessarily low. The advantage of air cooling can be realized only in engines using a small flow of cooling air with respect to working fluid flow.

Circulation of water through hollow blades. - Computations were made on the temperatures in the water-cooled blade shown in figure 17, which compares the two-dimensional temperature distribution in the water-cooled blade with that of the solid blade (fig. 3). The small coolant flow of 0.0222 pound per second was very effective in reducing blade temperatures. Computation of the power required to circulate the water by the common method of computing friction in pipes shows

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a very small power dissipation of 0.0012 horsepower per blade. The statement was made in reference 15 (pp. 158-161), however, that the fluid friction in a pipe in the field of high acceleration induces a superturbulence, which may increase the friction loss as much as a thousand times. Aside from this possibility of very high pumping loss, water cooling of turbine blades appears to be effective and practicable.

SUMMARY OF RESULTS

From a theoretical analysis of the various methods of cooling turbine blades, it was found that sufficient improvements in the efficiency of gas-turbine engines may be obtained through cooling the turbine blades to justify extensive research in turbine cooling. Cooling of the root of the blade, shortening the blade, and cooling hollow blades internally with air or liquid offer possibilities of substantial increases in permissible gas temperatures. Cooling of the tip of the blade and coating the blade with a ceramic were relatively ineffective.

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APPENDIX - ONE-DIMENSIONAL TEMPERATURE DISTRIBUTION

The heat absorbed per unit time over the surface of an element of the blade length Δy from the hot working fluid is

$$\Delta q_{sg} = h_g (T_g - T) C\Delta y \tag{11}$$

Only a negligible amount of heat is directly transferred from the gas to the blade by radiation but heat is radiated directly from the blade to the cooler tail pipe and perhaps to the nozzle box. The entire blade was assumed to be radiating to a region at 1000° F. The heat radiated per unit time from the element of the blade is

$$\Delta q_{rg} = \sigma (T^4 - T_o^4) C\Delta y \tag{12}$$

where σ is the Stefan-Boltzmann constant. The use of this relation results in a nonlinear differential equation, which is difficult to solve. However, McAdams (reference 8, fig. 27) has computed the equivalent heat-transfer coefficients based upon the first power of the temperature difference. The radiation is therefore assumed to follow the law

$$\Delta q_{rg} = h_r (T - T_o) C\Delta y \tag{13}$$

where h_r is, in this case, the coefficient equivalent for radiation. A mean value taken from reference 8, figure 27, was used.

The heat flow through the element Δy in the direction along the blade is

$$\Delta q_c = -kA \frac{d^2T}{dy^2} \Delta y \tag{14}$$

The heat removed from the blade section of length Δy by the circulation of the internal coolant is

$$\Delta q_{sa} = h_a (T - T_a) C'\Delta y \tag{15}$$

The temperature T_a of the cooling air rises as the air passes through the blade. The assumption that the temperature increases in proportion to its distance of travel in the blade is reasonably accurate and produces equations simple enough to solve

$$T_a = T_{a_1} + cy \tag{16}$$

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Then

$$\Delta q_{sa} = h_a \left(T - T_{a_i} - cy\right) C'\Delta y \tag{17}$$

The equation for the temperature distribution along the blade is derived by equating the sum of the heat flows through the blade portion Δy to zero and solving the resultant linear differential equation for T.

$$\Delta q_{sg} - \Delta q_{rg} - \Delta q_c - \Delta q_{sa} = 0 \tag{18}$$

Substituting the expressions for these terms gives

$$h_g(T_g - T)C\Delta y - h_r(T - T_o)C\Delta y + kA\frac{d^2T}{dy^2}\Delta y - h_a\left(T - T_{a_i} - cy\right)C'\Delta y = 0 \tag{19}$$

Letting

$$P = \frac{h_g C}{kA}\left(1 + \frac{h_r}{h_g} + \frac{C'}{C}\frac{h_a}{h_g}\right)$$

$$R = \frac{h_g C}{kA}\left(T_g + \frac{h_r}{h_g}T_o + \frac{C'}{C}\frac{h_a}{h_g}T_{a_i}\right)$$

$$M = \frac{h_a C'}{kA}$$

When the limit is taken as Δy approaches zero, the following differential equation is obtained:

$$\frac{d^2T}{dy^2} - PT + R + Mcy = 0 \tag{20}$$

The general solution of the reduced equation is

$$T = a_1 e^{y\sqrt{P}} + a_2 e^{-y\sqrt{P}} \tag{21}$$

where a_1 and a_2 are constants of integration. A particular solution of the equation is

$$T = \frac{Mc}{P}y + \frac{R}{P} \tag{22}$$

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The general solution is the sum of these two solutions

$$T = a_1 e^{y\sqrt{P}} + a_2 e^{-y\sqrt{P}} + \frac{Mc}{P} y + \frac{R}{P} \tag{23}$$

Let $R/P = T_e$

$$T - T_e = a_1 e^{y\sqrt{P}} + a_2 e^{-y\sqrt{P}} + \frac{Mc}{P} y \tag{24}$$

The constants are evaluated by letting $T = T_b$ when $y = 0$, and $\frac{dT}{dy} = 0$ when $y = L$.

The final equation then becomes

$$T - T_e = \frac{\cosh\left[\sqrt{P}(L-y)\right]}{\cosh\left(\sqrt{P}\,L\right)} (T_b - T_e) - \frac{Mc}{P^{3/2}} \frac{\sinh\left(\sqrt{P}\,L\right)}{\cosh\left(\sqrt{P}\,L\right)} + \frac{Mc}{P} y \tag{25}$$

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REFERENCES

1. Stodola, A.: Steam and Gas Turbines. Vol. II. McGraw-Hill Book Co., Inc., 1927.

2. Piening, Werner: The Efficiency of Combustion Turbines with Constant-Pressure Combustion. NACA TM No. 975, 1941.

3. Haverstick, J. S., and Moody, A. M. G.: High-Temperature Gas-Turbine Power Plants. Mech. Eng., vol. 67, no. 4, April 1945, pp. 229-233.

4. Carslaw, H. S.: Introduction to the Mathematical Theory of the Conduction of Heat in Solids. Dover Pub. (New York), 1945.

5. Lelchuk, V. L.: Heat Transfer and Hydraulic Flow Resistance for Streams of High Velocity. NACA TM No. 1054, 1943.

6. Crocco, Luigi: Transmission of Heat from a Flat Plate to a Fluid Flowing at a High Velocity. NACA TM No. 690, 1932.

7. Squire, H. B.: Heat Transfer Calculation for Aerofoils. R. & M. No. 1986, British A.R.C., Nov. 1942.

8. McAdams, William H.: Heat Transmission. McGraw-Hill Co., Inc., 2d ed., 1942.

9. Barratt, T., and Nettleton, H. R.: Thermal Conductivity of Liquids and Solids. Vol. V of International Critical Tables of Numerical Data, Physics, Chemistry and Technology, Edward W. Washburn, ed., McGraw-Hill Book Co., Inc., 1929, pp. 218-233.

10. Emmons, H. W.: The Numerical Solution of Heat-Conduction Problems. A.S.M.E. Trans., vol. 65, no. 6, Aug. 1943, pp. 607-612.

11. Dushman, Saul, Dunbar, L. W., and Huthsteiner, H.: Creep of Metals. Jour. Appl. Phys., vol. 15, no. 2, Feb. 1944, pp. 108-124.

12. Anon.: Compilation of Available High-Temperature Creep Characteristics of Metals and Alloys. A.S.T.M., A.S.M.E., March 1938.

Security Classification of This Report Has Been Cancelled

13. Wolfenstein, Lincoln, Meyer, Gene L., and McCarthy, John S.: Cooling of Gas Turbines. II - Effectiveness of Rim Cooling of Blades. NACA RM No. E7B11b, 1947.

14. Lorenzen, Christian: The Lorenzen Gas Turbine and Supercharger for Gasoline and Diesel Engines. Mech. Eng., vol. 52, no. 7, July 1930, pp. 665-672.

15. Sinclair, Harold: Recent Developments in Hydraulic Couplings. Proc. Inst. Mech. Eng., vol. 130, April 1935, pp. 75-157; discussion by H. Föttinger, pp. 158-161.

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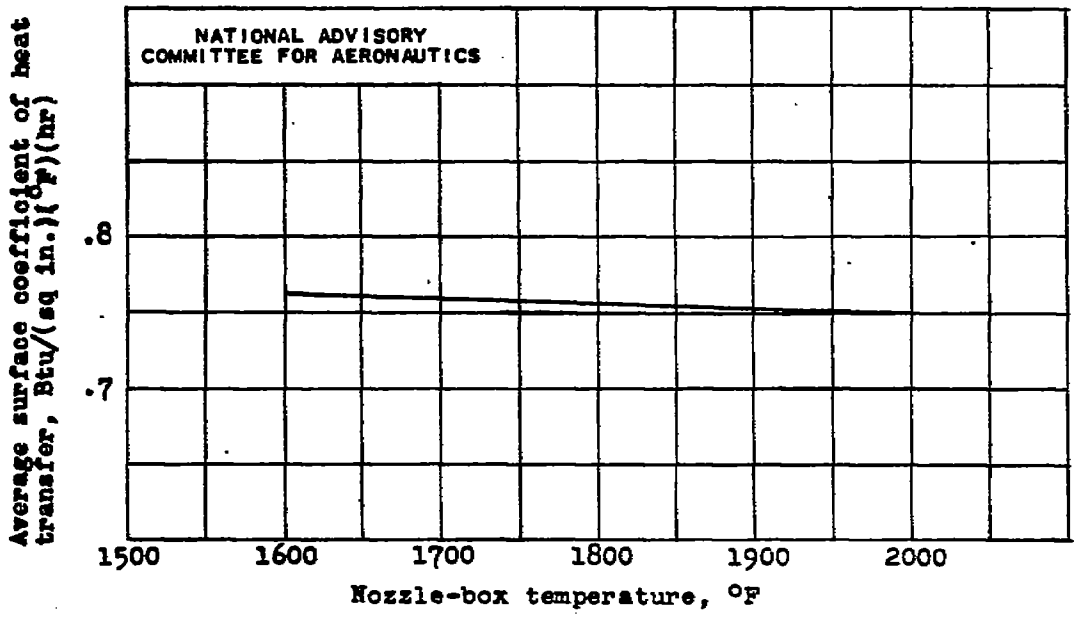

Figure 1. - Effect of nozzle-box temperature on surface coefficient of heat transfer at turbine blades when pressure ratio and blade velocity are maintained constant. Pressure ratio, 3.0; mean velocity of blades, 1200 feet per second.

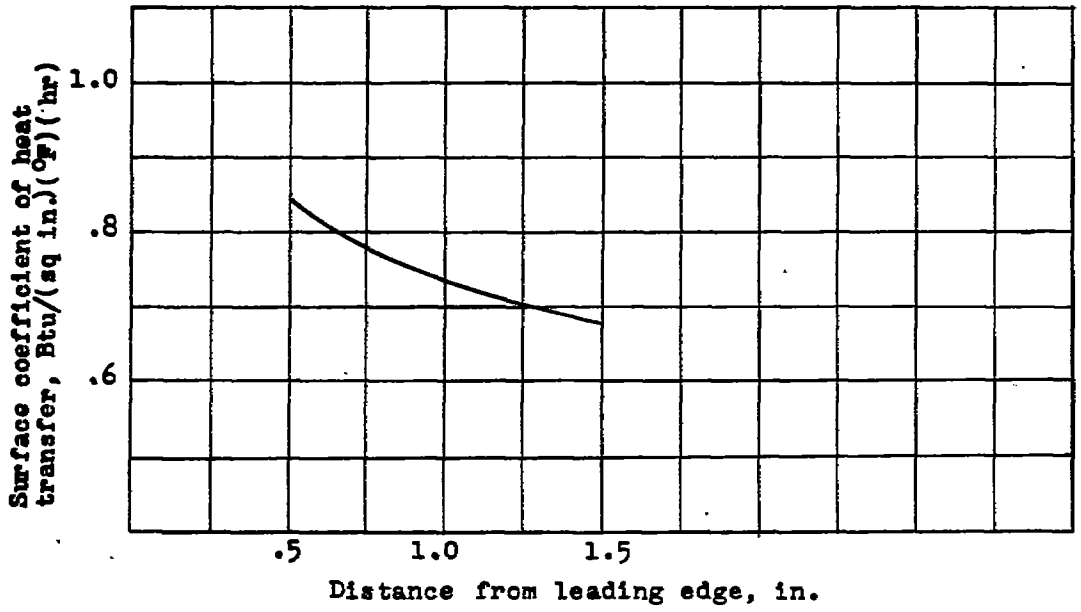

Figure 2. - Variation of surface coefficient of heat transfer from stream of gas at high velocity to flat plate parallel to the wind stream. Velocity of gas stream, 1425 feet per second; free-stream temperature, 1090° F; stream pressure, 14.7 pounds per square inch.

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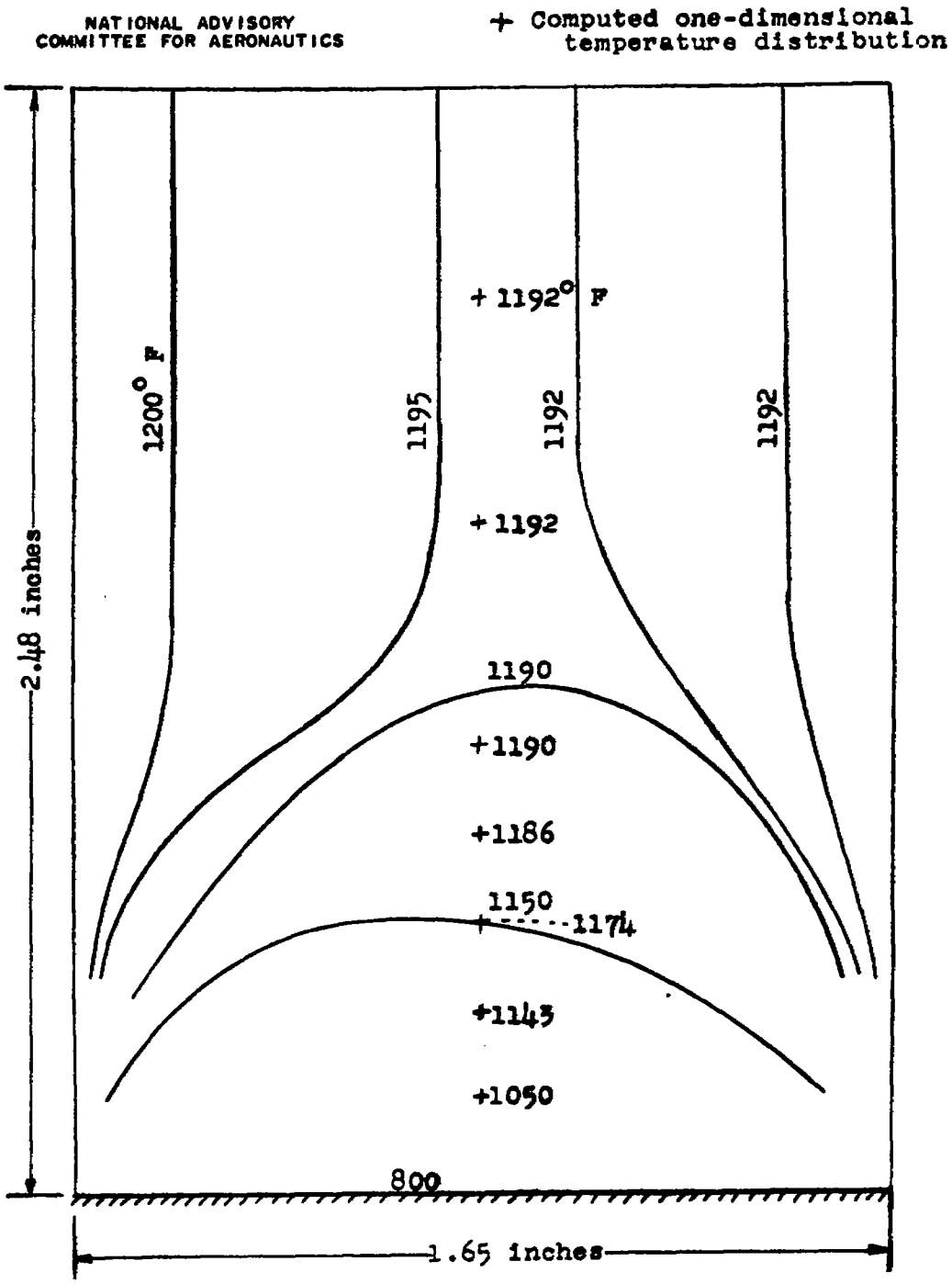

Figure 3.- Computed two-dimensional temperature distribution in turbine blade. Nozzle-box temperature, 1600° F; pressure ratio, 3.0; mean blade velocity, 1200 feet per second; blade-root temperature, 800° F; radiation to cold body at 1000° F.

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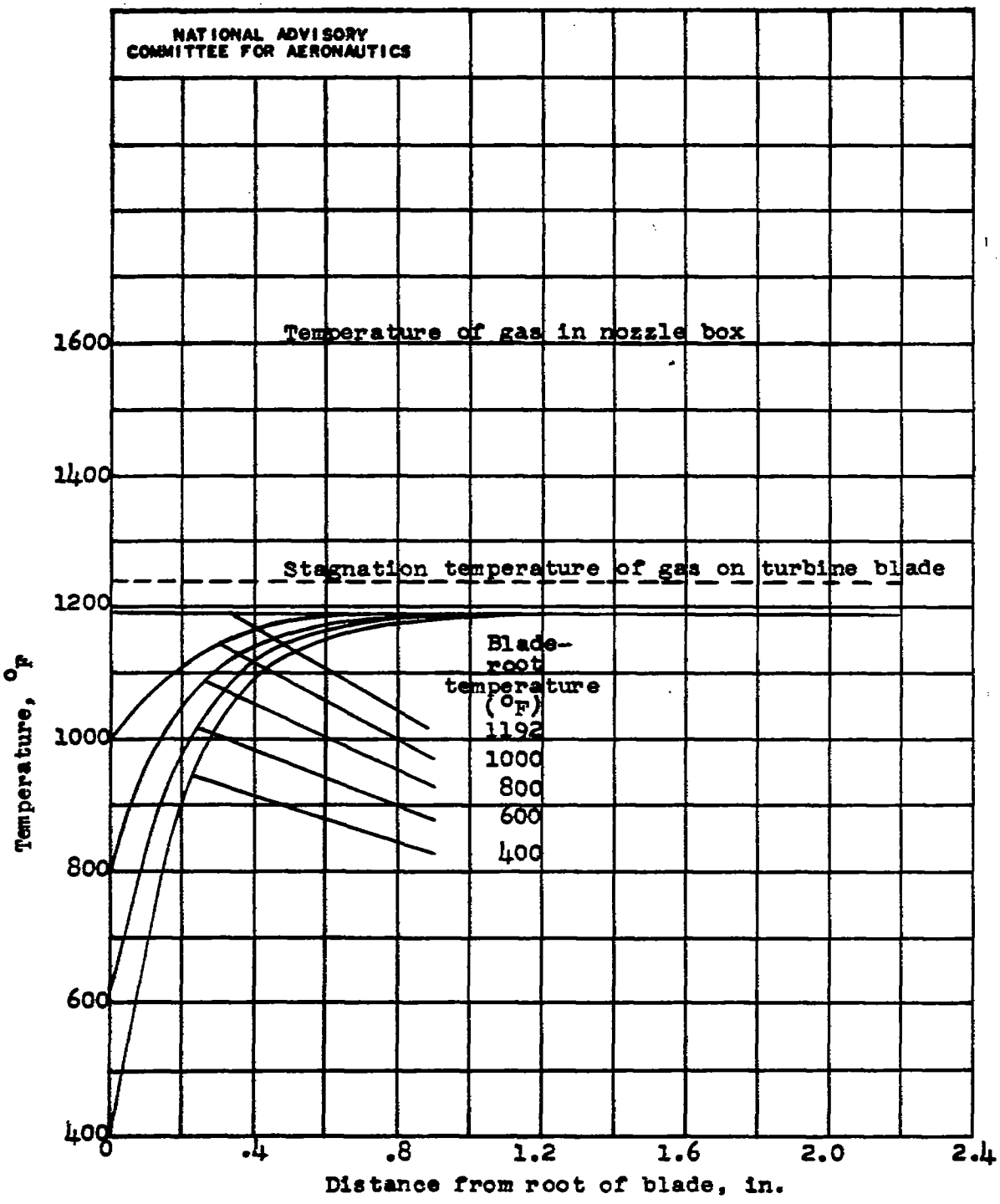

Figure 4. - Effect of blade-root temperature on temperature distribution in gas-turbine blade. (hC/kA), 21.4; radiation to cold body at 1000° F; blade length, 2.48 inches.

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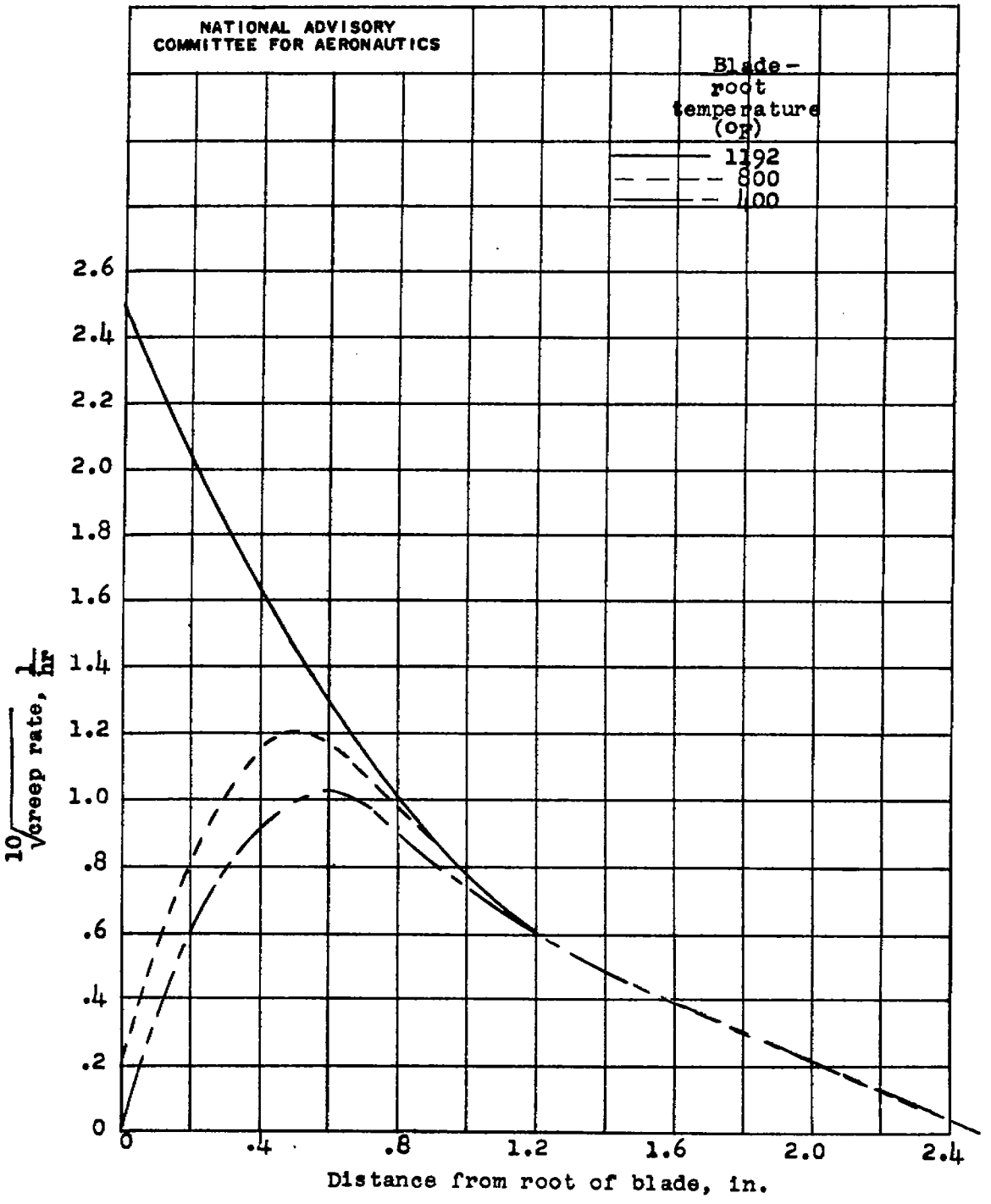

Figure 5. - Creep distribution along blade of gas turbine when the blade root is cooled to specified temperatures. Pressure ratio, 3.0; mean blade velocity, 1200 feet per second; blade material, 18:8 stainless steel; nozzle-box temperature, 1600° F.

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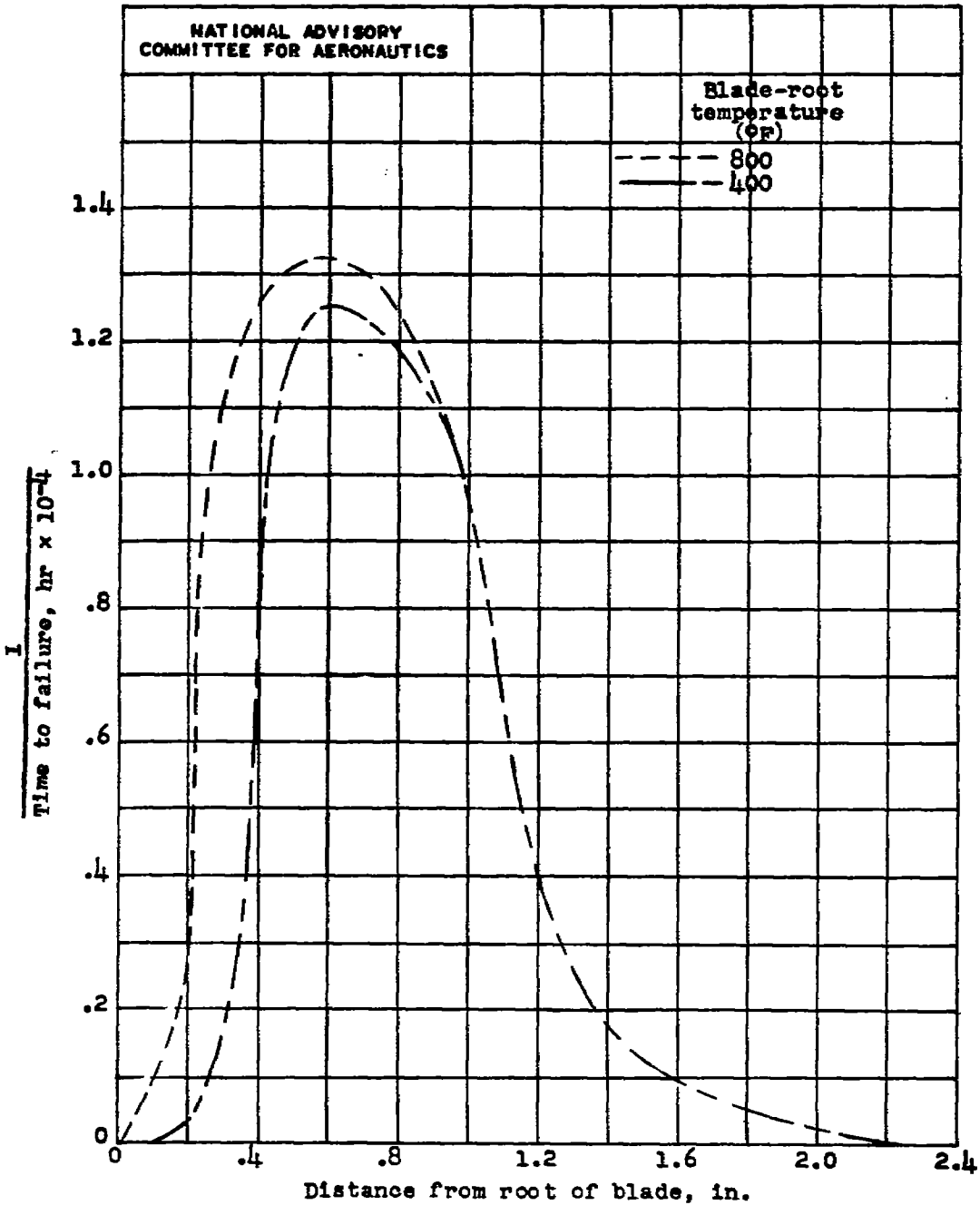

Figure 6. - Distribution of rate of failure by high-temperature rupture along gas-turbine blade when blade root is cooled to specified temperatures. Mean blade velocity, 1200 feet per second; material, Vitallium; nozzle-box temperature, 1600° F.

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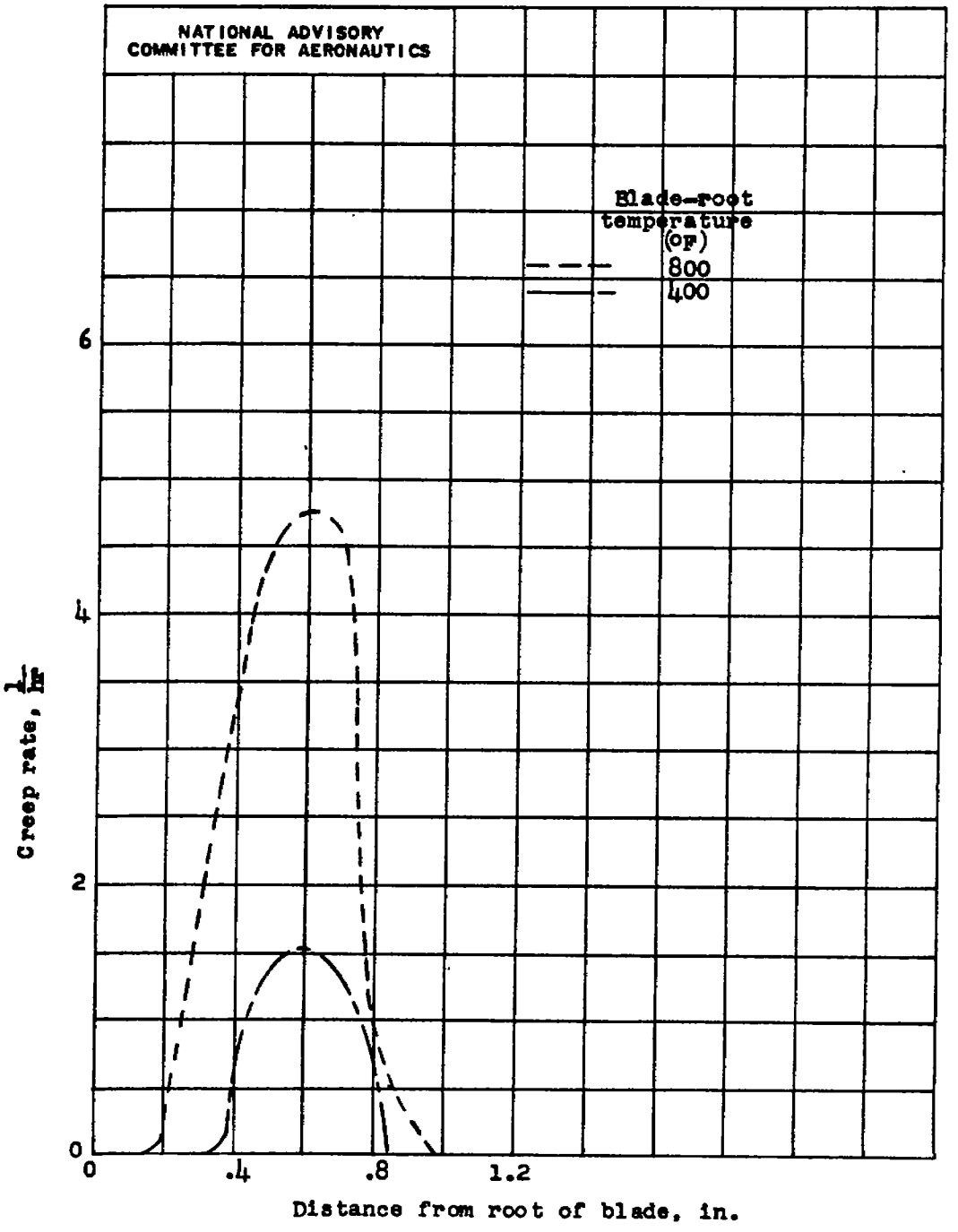

Figure 7. - Illustration of critical zone of failure in turbine blade. Mean blade velocity, 1200 feet per second; blade material, 18:8 stainless steel; nozzle-box temperature, 1600° F.

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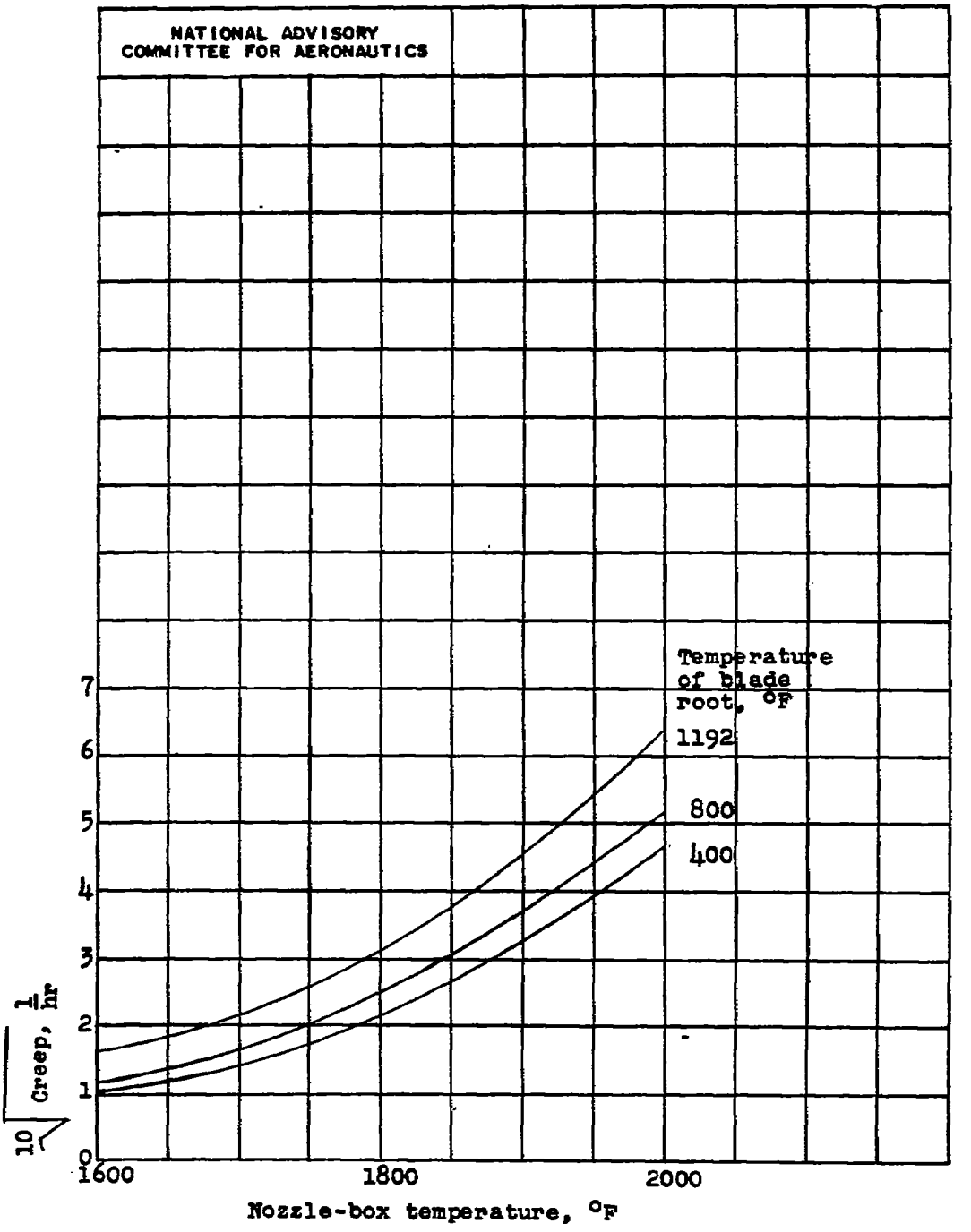

Figure 8. - Effect of nozzle-box temperature on maximum creep rate in turbine blade with blade-root temperature maintained at 1192°, 800°, and 400° F.

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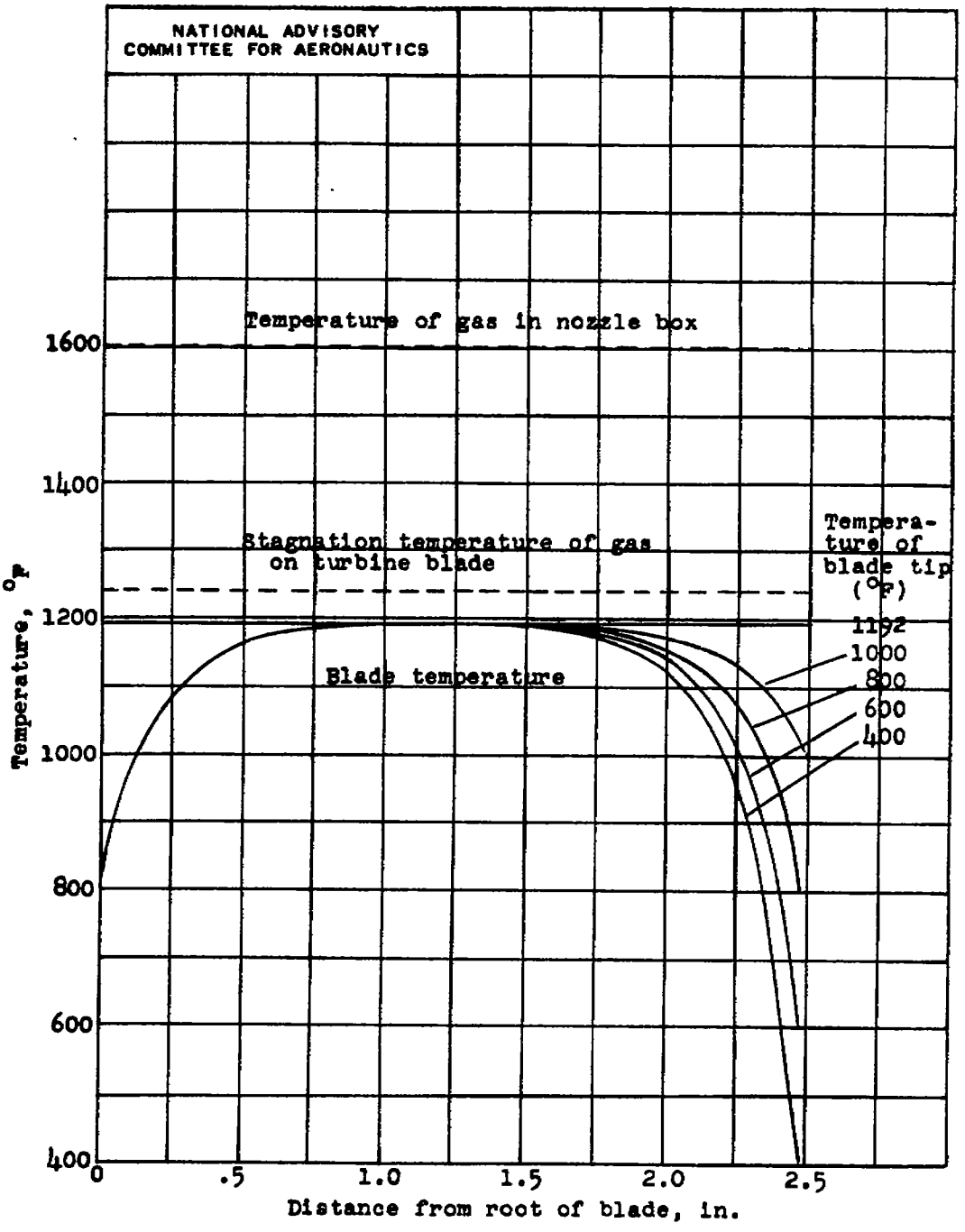

Figure 9. - Effect of temperature of blade tip on temperature distribution in gas-turbine blade. (hC/kA), 21.4; radiation to a cold body at 1000° F; blade length, 2.48 inches.

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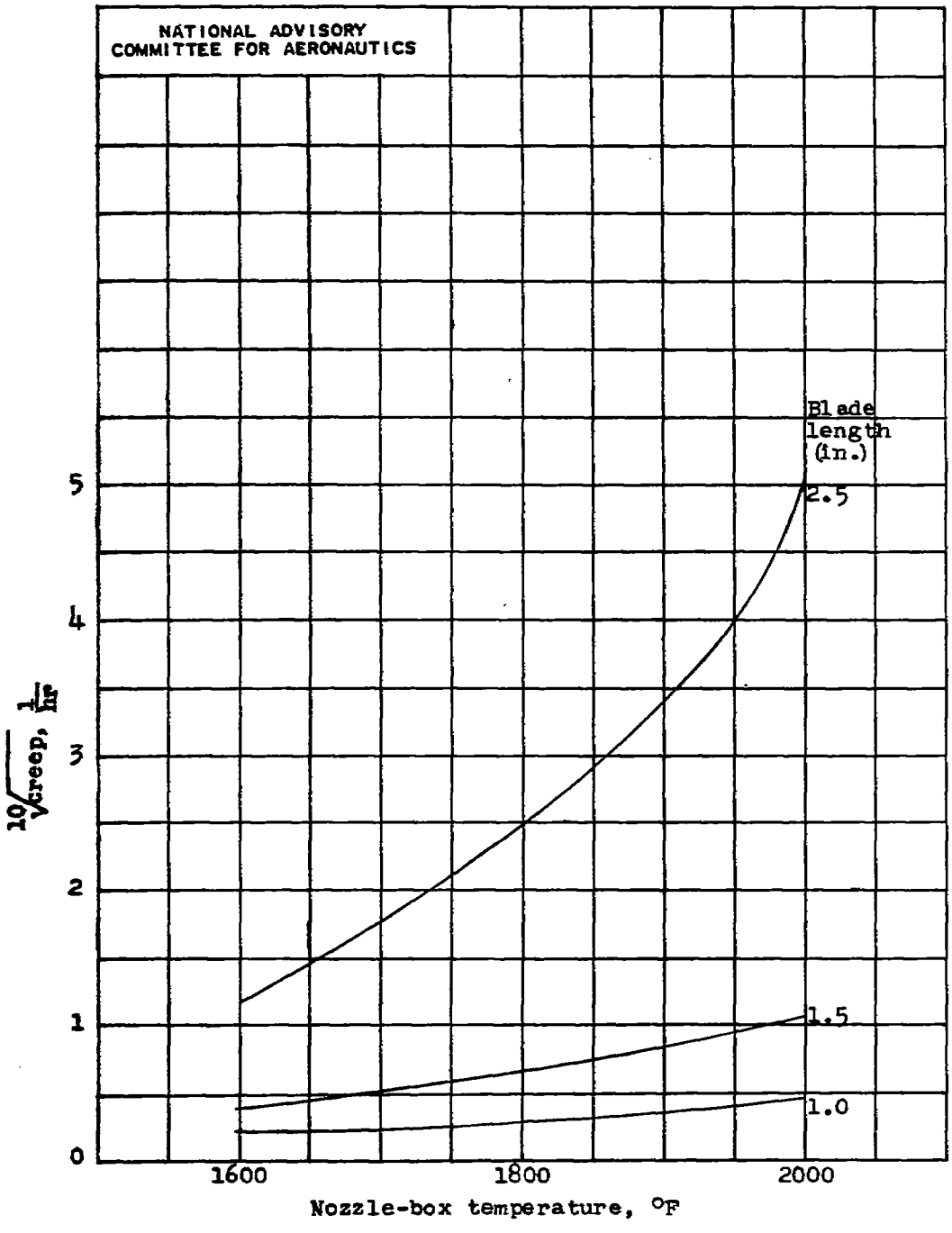

Figure 10. - Effect of blade length on maximum rate of blade creep. P, 25; pressure ratio, 3.0; mean blade velocity, 1200 feet per second; blade material, 18:8 stainless steel.

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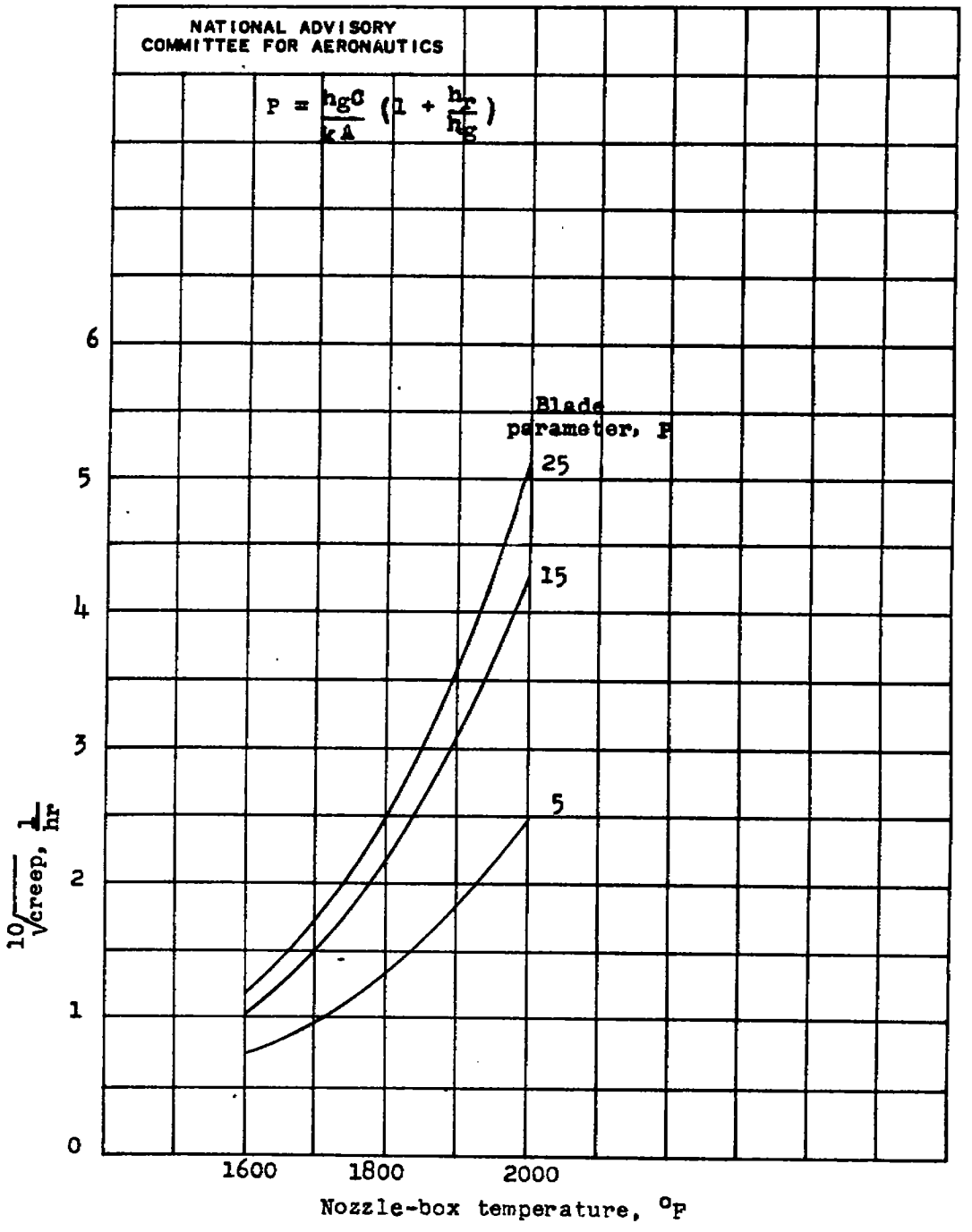

Figure 11. - Effect of blade parameter P on maximum rate of creep in blade. Mean blade velocity, 1200 feet per second; blade length, 2.48 inches; blade material, 18:8 stainless steel.

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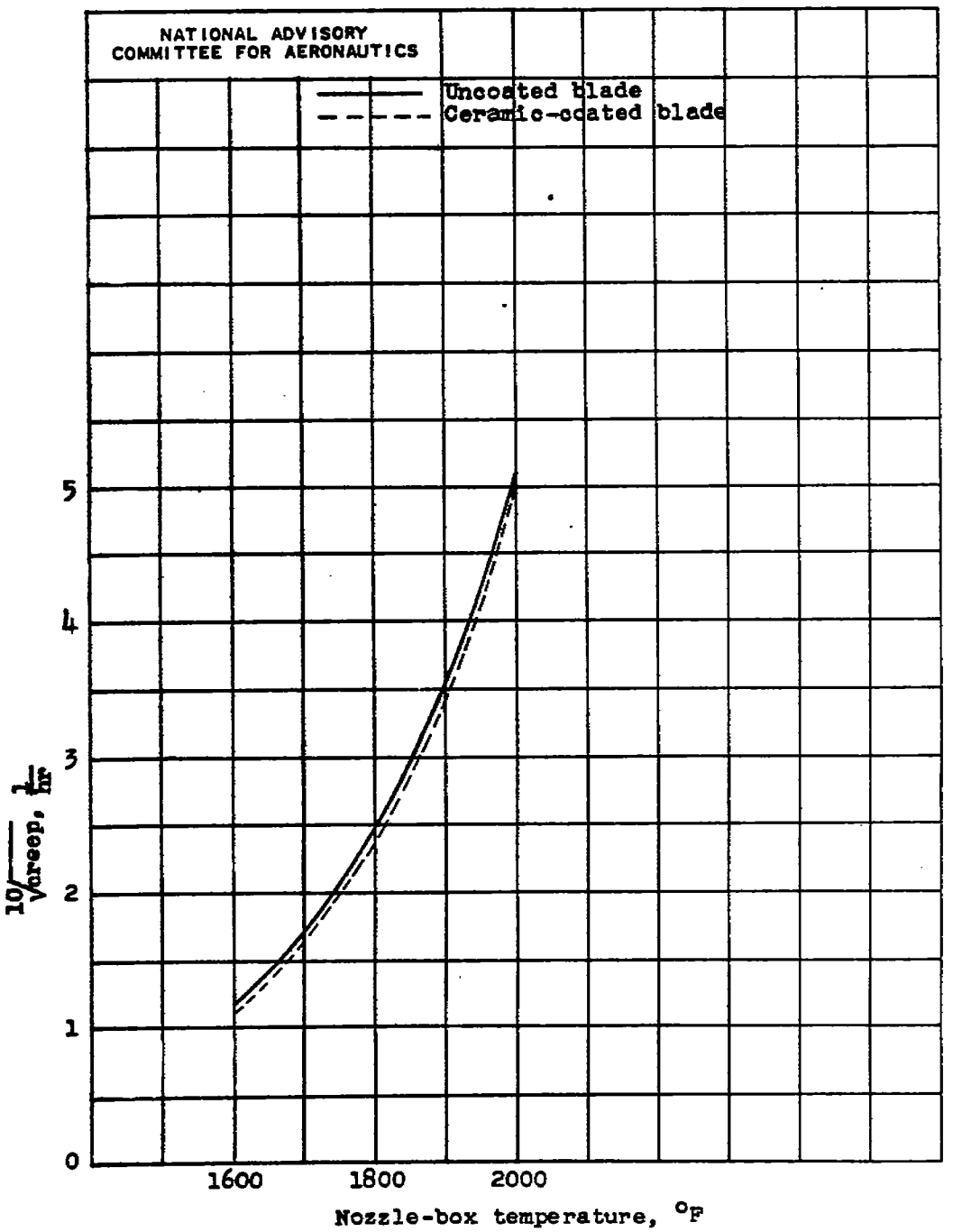

Figure 12. - Effect of ceramic coating on maximum rate of creep in blade. Mean blade velocity, 1200 feet per second; blade length, 2.48 inches; blade material, 18:8 stainless steel.

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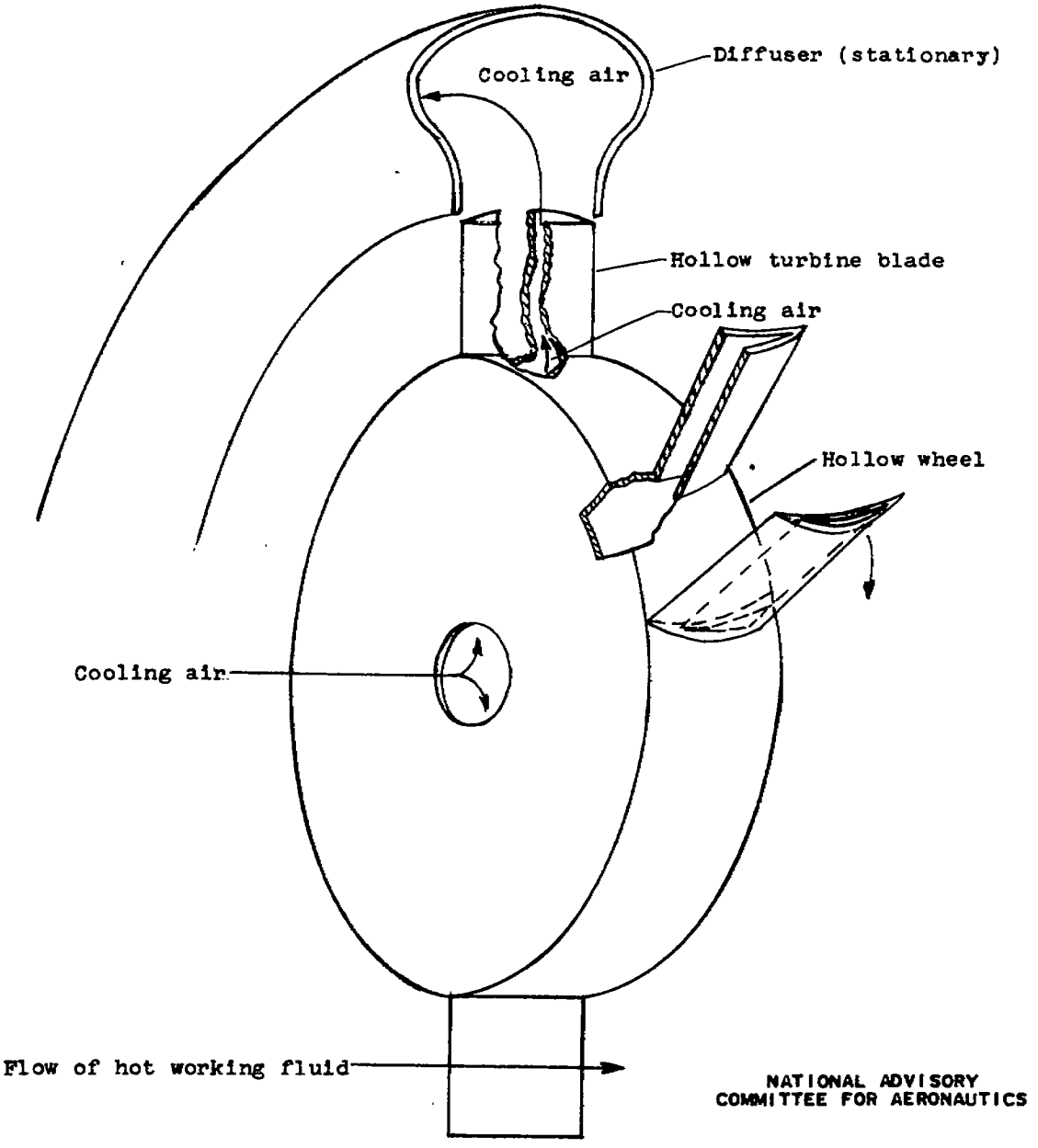

Figure 13. - Method of circulating cooling air through hollow turbine blade.

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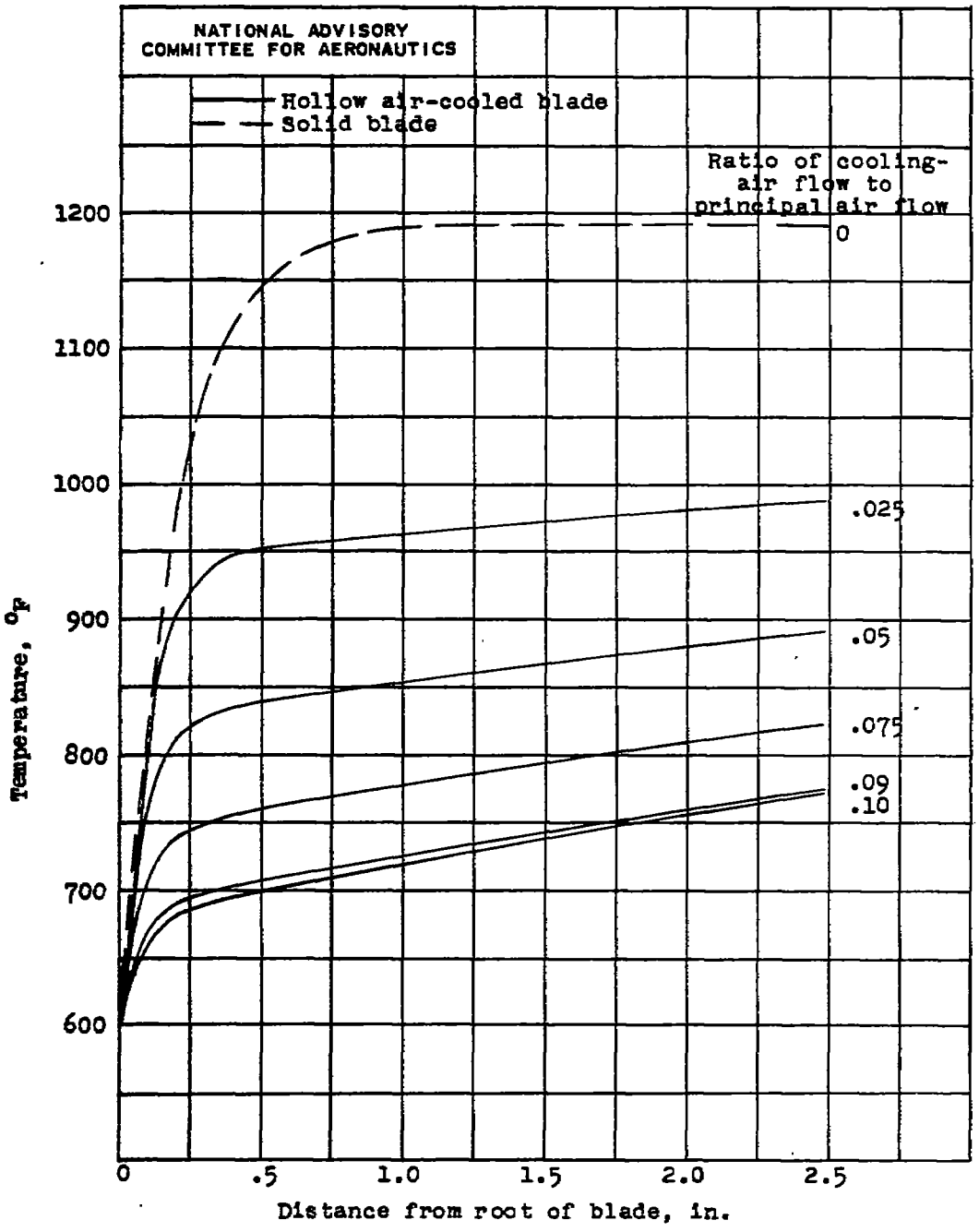

Figure 14. - Effect on blade temperatures of circulating cooling air through hollow blades. Root temperature maintained constant at 600° F; nozzle-box temperature, 1600° F; pressure ratio, 3.0; mean blade speed, 1200 feet per second.

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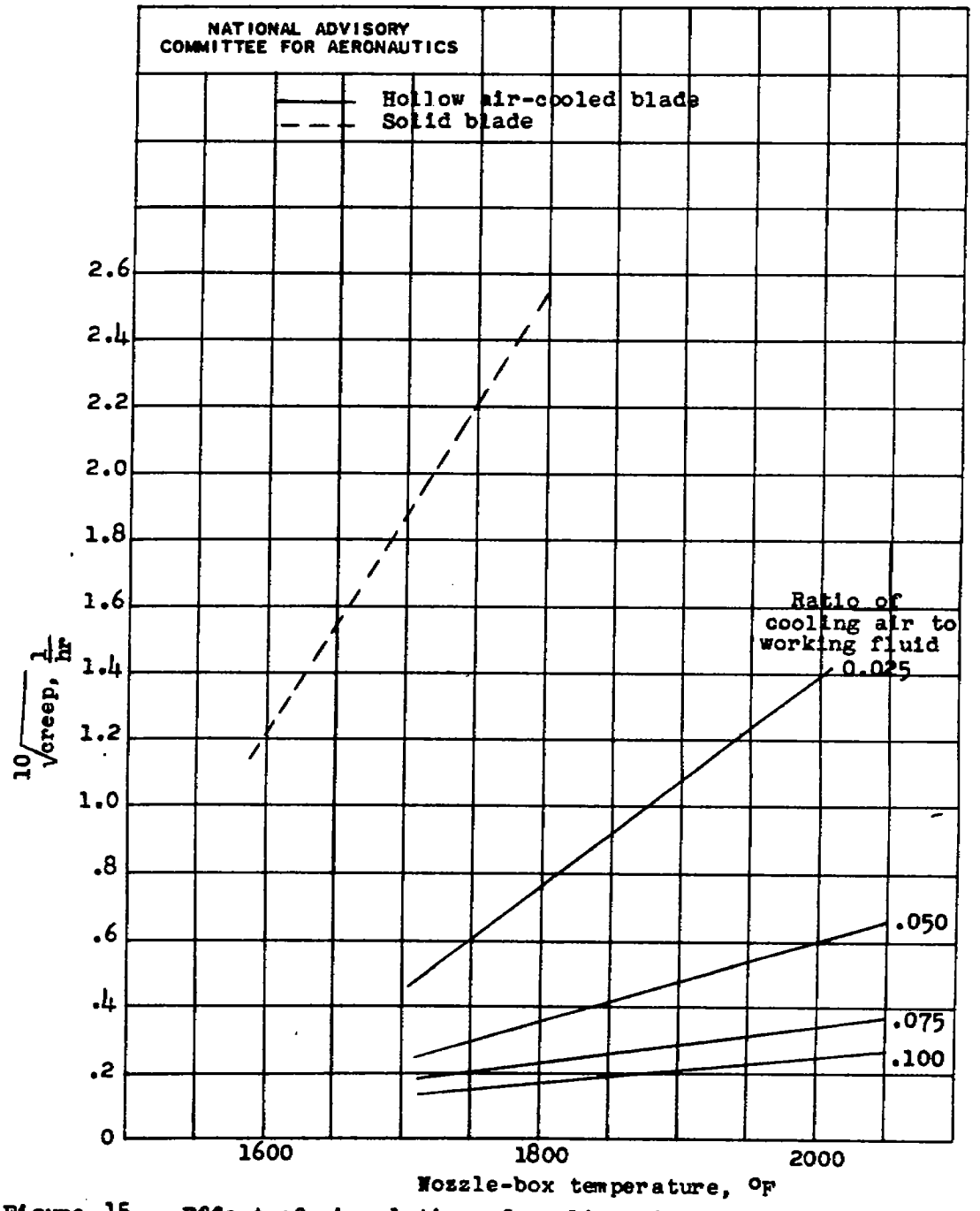

Figure 15. - Effect of circulation of cooling air on maximum rate of creep in hollow air-cooled blades.

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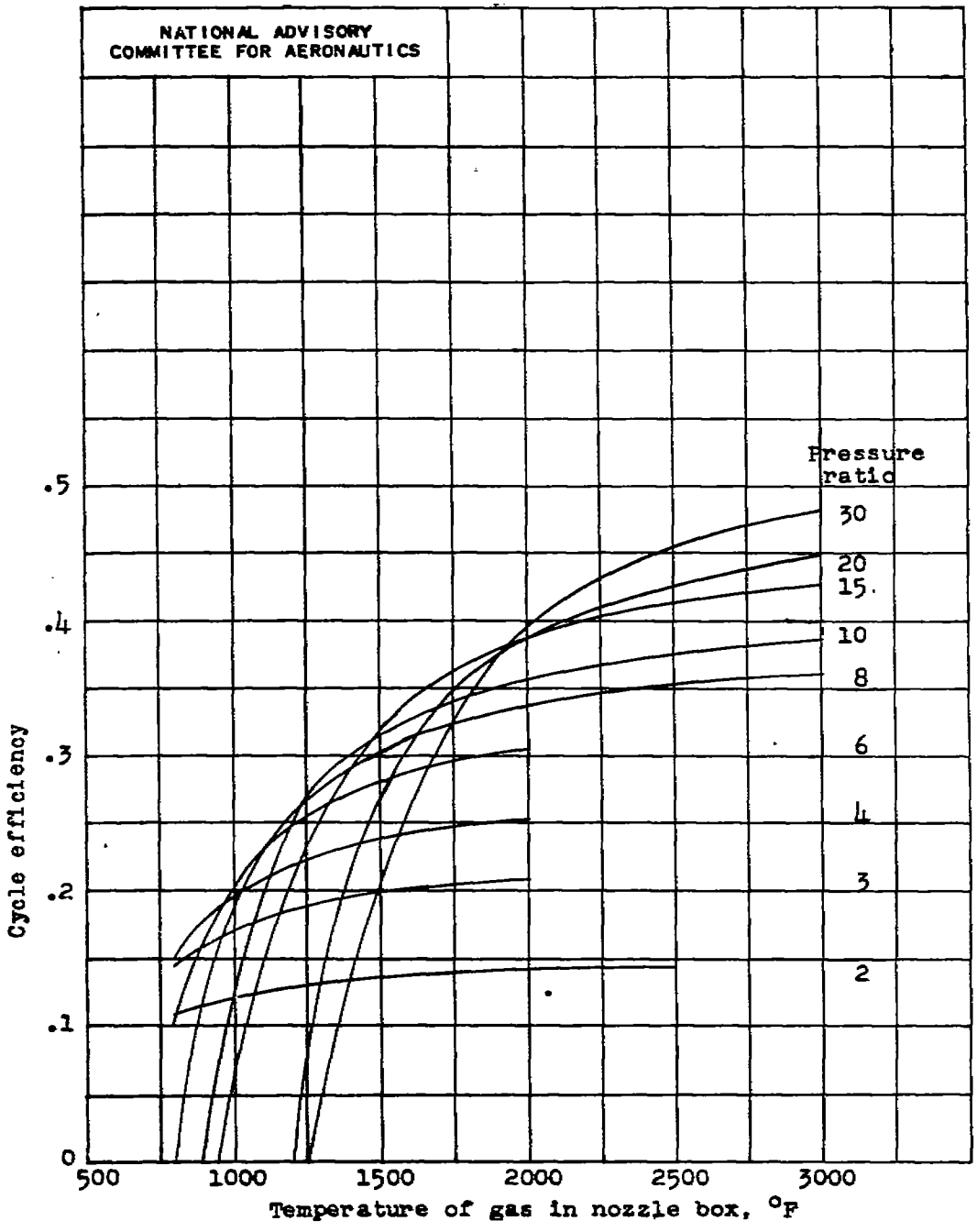

Figure 16. - Effect of gas temperature and pressure ratio on cycle efficiency of gas-turbine engine. Adiabatic efficiency of compressor 0.90; efficiency of turbine, 0.90; intake-air temperature, 100° F.

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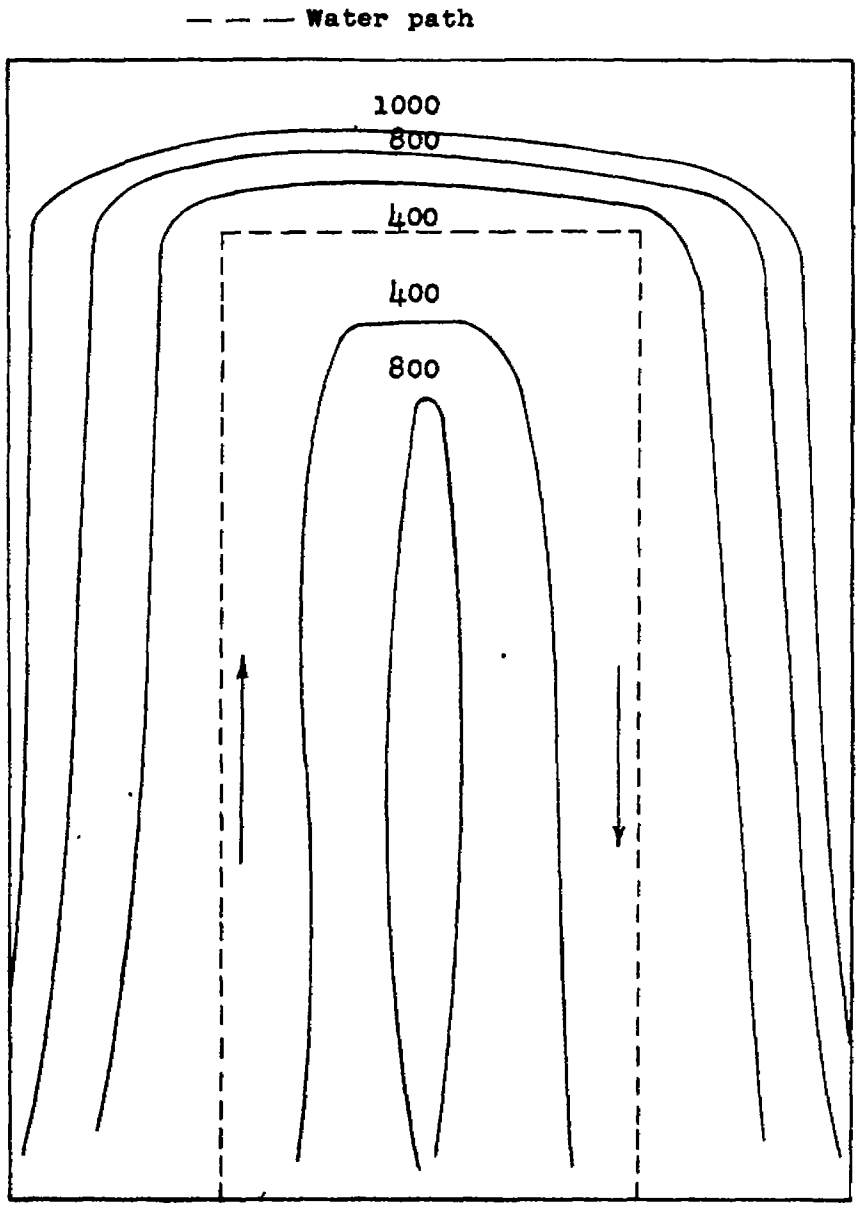

Figure 17. - Computed two-dimensional temperature distribution in water-cooled turbine blade. Nozzle-box temperature, 1600° F; pressure ratio, 3.0; mean blade velocity, 1200 feet per second; blade-root temperature, 800° F; radiation to cold body at 1000° F; inlet-water temperature, 100° F; outlet water temperature, 184° F.

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