Eugene E. Lemmon Jr.

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TECHNICAL NOTE

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EFFECT OF ROTOR-BLADE TWIST AND PLAN-FORM TAPER ON HELICOPTER HOVERING PERFORMANCE

By Alfred Gessow

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

A theoretical analysis of the hovering performance of rotors having various combinations of twist and plan-form taper was made in order to estimate the effects of blade twist and taper on rotor efficiency in hovering. The calculations were made with the aid of a strip-analysis procedure similar to that used in propeller analysis, the comparisons being made for rotors of equal solidity. The solidity of the tapered blades was computed by means of an equivalent chord which was derived by weighting the values of chord at each blade element according to the square of its radial distance.

The results indicated that at typical operating thrust coefficients, the addition of linear twist of the order of -8° to -12° to rectangular blades having 0.060 solidity may be expected to increase the rotor thrust at fixed power by approximately 3 to 4 percent or, if the blades are initially tapered with a ratio of root chord to tip chord of 3, by approximately 2 to 3 percent. An increase in thrust of about 5 percent is indicated for a combination of -8° or -12° twist and a taper ratio of 3 to 1. An additional 2-percent gain is indicated if a nonlinear optimum combination of twist and taper is used.

Examination of the effects of a moderate change in solidity and of the use of partial rather than full taper indicated that these factors did not significantly affect the preceding results, when the same amounts of twist and taper were investigated over the same thrust-coefficient range.

INTRODUCTION

Methods for the improvement of rotor hovering performance are being sought, especially in relation to the current interest in large, slow-moving, load-carrying helicopters. The selection of the blade-pitch distribution and plan form that would yield the maximum rotor thrust for a given power input appears to be an effective and relatively inexpensive

way of achieving this end. The importance of a percentage increase in thrust is more apparent when it is realized that, with the current ratio of pay load to gross weight (approx. 20 percent), the percentage increase in pay load is approximately five times the percentage increase in thrust.

Because the improvements in rotor hovering efficiency due to blade twist and taper appear to be of the order of 5 percent, careful experimental investigations are necessary to measure these effects. It was therefore considered desirable to investigate the influence of blade geometry by theoretical methods in order to aid in the planning and interpreting of experimental investigations. Standard methods of propeller strip analysis were used in determining the effect of combinations of twist and taper applicable to helicopter rotors, and the results of the investigation are presented herein. It is important to note that the comparisons were made for rotors of equal solidity, the solidity of the tapered blades being computed by means of an equivalent chord which was derived by weighting the values of chord at each blade element according to the square of its radial distance. Different results would be shown for the effects of taper if the rotor solidity were computed on a different basis, for example, as the ratio of blade area to rotor-disk area.

SYMBOLS

T	rotor thrust, pounds
T_1	thrust of one blade, pounds
R	blade radius, feet
Ω	rotor angular velocity, radians per second
ρ	mass density of air, slugs per cubic foot
C_T	thrust coefficient $\left(T/\pi R^2\rho(\Omega R)^2\right)$
Q_i	induced rotor torque, pound-feet
Q_o	rotor profile-drag torque, pound-feet
Q	total rotor torque, pound-feet $\left(Q_i + Q_o\right)$
C_{Q_i}	induced torque coefficient $\left(Q_i/\pi R^2\rho(\Omega R)^2R\right)$ (C_{Q_o} and C_Q are likewise defined as Q_o or Q divided by $\pi R^2\rho(\Omega R)^2R$)
r	radial distance to a blade element, feet
x	ratio of blade-element radius to rotor-blade radius (r/R)

c	blade-section chord at radius r, feet
c_e	equivalent blade chord, feet $\left(\frac{\int_0^R cr^2\,dr}{\int_0^R r^2\,dr}\right)$
b	number of blades per rotor
σ	rotor solidity $\left(\frac{bc_e}{\pi R}\right)$
σ_x	solidity of a blade element at a radial distance x $\left(\frac{bc}{\pi R}\right)$
v	induced inflow velocity at rotor, feet per second
a	slope of curve of section lift coefficient against section angle of attack (radian measure), assumed to be 5.73
α_r	blade-section angle of attack measured from zero-lift line, radians $(\theta - \phi)$
c_l	section lift coefficient
c_{d_o}	section profile-drag coefficient
δ_0, δ_1, δ_2	coefficients in power series expressing c_{d_o} as a function of α_r $(c_{d_o} = \delta_0 + \delta_1\alpha_r + \delta_2\alpha_r^2)$
θ	blade-section pitch angle measured from line of zero lift, radians
ϕ	inflow angle at blade element, radians $\left(\tan^{-1}\frac{v}{\Omega r}\right)$

Subscript:

T	values at the blade tip

METHOD OF ANALYSIS

The strip-analysis method used herein is similar to the method given in reference 1 for use in propeller analysis. A summary of a simplification of the analysis as applied to a helicopter rotor follows.

The thrust per unit of blade span may be expressed with the usual assumptions prevailing in rotor theory in coefficient form as

$$\frac{dC_T}{dx} = \frac{1}{2}\sigma_x a \alpha_r x^2 \tag{1}$$

The rotor torque is composed of the induced and profile-drag contributions. The induced part, or the torque due to the components of the lift vectors in the plane of rotation, is written as

$$\frac{dC_{Q_i}}{dx} = \frac{1}{2}\sigma_x a \phi \alpha_r x^3 \tag{2}$$

and the profile-drag contribution is

$$\frac{dC_{Q_o}}{dx} = \frac{1}{2}\sigma_x c_{d_o} x^3 \tag{3}$$

The three unknowns in equations (1) to (3), namely, σ_x, ϕ, and α_r, can be determined from the known geometric and aerodynamic properties of the blade by means of the following relationships:

$$\sigma_x = \frac{bc}{\pi R} \tag{4}$$

$$\phi = \frac{\sigma_x a}{16x}\left(\sqrt{1 + \frac{32x\theta}{\sigma_x a}} - 1\right) \tag{5}$$

$$\alpha_r = \theta - \phi \tag{6}$$

A more general expression for ϕ, covering the vertical-climb conditions, was derived in reference 2.

For the calculations that follow, the blade section characteristics are assumed to be represented by the following formulas:

$$\begin{aligned} c_l &= a\alpha_r \\ &= 5.73\alpha_r \end{aligned} \tag{7}$$

$$\begin{aligned} c_{d_o} &= \delta_0 + \delta_1\alpha_r + \delta_2\alpha_r^2 \\ &= 0.0087 - 0.0216\alpha_r + 0.400\alpha_r^2 \end{aligned} \tag{8}$$

Equation (8) is taken from reference 2 and represents the section profile-drag coefficient for conventional semismooth airfoils (drag of smooth airfoils increased by a roughness factor of 17 percent). The equation

yields a minimum profile-drag coefficient of 0.0084 and is representative of well-built plywood blades. The use of a different drag polar in the analysis would not be expected to affect the order of magnitude of the effects of twist and taper.

Equations (1) to (3) were graphically integrated to obtain performance curves of C_T against C_Q for each of the blades investigated.

Ideally twisted blade.—Ideal twist is defined as the twist that would produce uniform inflow and, consequently, minimum induced losses if the rotational losses are ignored. Being very small (of the order of 1 percent for rotors), rotational losses are not considered in the analysis. The special case of a rectangular blade having ideal twist is such that its performance can be explicitly expressed in a simple fashion. The expression obtained from equation (A18) of reference 2 is

$$C_Q = \frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma}{8}\delta_0 + \frac{2}{3}\frac{\delta_1}{a}C_T + \frac{4\delta_2}{\sigma a^2}C_T^2 \tag{9}$$

Optimum blade.— The optimum blade may be defined as a blade that will produce a given thrust for the least total power required. In practice, a blade that approaches the optimum is one that has uniform inflow over the disk and each section of which is operating at the angle of attack that results in the least profile-drag losses. Such a configuration would be expected to be close to the optimum over the normal operating range of thrust coefficients because in this range the induced losses are a large percentage of the total losses. It is shown in the appendix that the twist and taper of a rotor having both minimum induced and profile-drag losses are as follows

$$\left.\begin{aligned} \theta &= \alpha_r + \frac{v}{x\Omega R} \\ c &= \frac{C_T}{x} \end{aligned}\right\} \tag{10}$$

where α_r and v are constant by definition. The performance of the optimum blade (derived in the appendix) can be expressed by

$$C_Q = \frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma}{9}c_{d_0} \tag{11}$$

Tip-loss effects.— The theory used, like the ordinary vortex theory, is based on the assumption of an infinite number of blades and, therefore, neglects blade interference or "tip-loss" effects. The effect of the tip-loss on the comparative hovering performance of the various designs investigated in this paper was studied by recalculating the rotor performance

of several representative blade configurations by the method of reference 1, which includes the Goldstein correction to the vortex theory. The sample studies indicated that, though the assumption of an infinite number of blades overestimated the calculated thrust at fixed power by a maximum of approximately 3 percent, no effect could be noted on the comparative performance of the blades investigated. This conclusion would probably be invalid if the thrust loading of the designs investigated differed considerably from the optimum loading, which is the loading that will yield minimum induced losses. The thrust loadings of the relatively lightly loaded helicopter rotors investigated are not considered to differ sufficiently from the optimum to affect the analysis. The designs investigated fell well within the limits of applicability of Goldstein's correction factor as discussed in reference 3.

SAMPLE CONFIGURATIONS

The combinations of twist and plan-form taper of the blades investigated are tabulated as follows:

Blade solidity	Blade twist (deg)	Ratio of root chord to tip chord
0.060	0	1 and 3
	-8	1 and 3
	-12	1 and 3
	Ideal	1
	Ideal	Optimum plan form
.042	0	1 and 3
	-8	1 and 3
	-12	1 and 3
	Ideal	1

The amount of linear twist is indicated by the washout, whereas linear taper is indicated by the ratio of root chord to tip chord.

RESULTS AND DISCUSSION

Rotor induced losses, which are of the order of 80 percent of the total power expended in hovering, are a minimum for a given thrust when the thrust distribution is such that the rotor induced velocity is uniform across the disk. The induced losses of a rotor for which the induced

velocity increases with the radius, as is the case with rotors having untwisted rectangular blades, may be minimized by increasing the load carried by the inner part of the blade. This effect can be accomplished by twisting the blade so that the root end operates at higher pitch angles than does the tip (washout) or tapering the blade in plan form so that the root chord is greater than the tip chord.

Although profile-drag losses are of less importance than induced losses over the greater part of the normal operating range of thrust coefficients, the same expedients of twist and taper are beneficial with respect to the reduction of profile-drag losses. This condition results from the fact that the profile-drag losses are dependent on the cube of the blade-section velocity, whereas the thrust varies as the square of the velocity; thus, it is advantageous for the thrust to be produced by the low-velocity sections.

The discussion of the individual effects of twist and taper that follow are limited to a solidity of 0.060. The extent to which the conclusions are affected by a change in solidity is discussed hereinafter.

Effect of twist.- The manner in which twist affects the inflow distribution and the blade loading (as represented by the section angles of attack) of an untapered blade operating at a representative thrust coefficient of 0.0056 is shown in figure 1. The induced velocities and section angles of attack are plotted against the square of the radius in order to put proper emphasis on each blade element inasmuch as the thrust contribution of each element varies as the square of the section velocity. The figure indicates that the application of increasing amounts of linear twist approach the effects produced by ideal twist, in that both the induced velocity and the blade loading are increased near the inboard end of the blade. Although the effects of twist are such that the maximum section angle of attack occurring on the blade is increased, the effects of blade stall are minimized inasmuch as the radial distance of the maximum angle of attack is decreased.

The reduction in induced and profile-drag losses that may be realized with a more uniform-inflow distribution may be seen from figure 2, in which both induced and profile torque coefficients are plotted over a range of thrust coefficient for untapered blades having various amounts of twist. The figure indicates that twist is beneficial in reducing rotor losses over the range of thrust coefficient above the conventional minimum (that is, above $C_T \approx 0.0030$) although the net change in profile-drag losses appears insignificant. As might be expected, twist is detrimental at thrust coefficients near zero inasmuch as losses are incurred in producing negative thrust over the outer part of the blades. The optimum amount of linear twist would consequently decrease with lower operating thrust coefficients.

It should be noted in figure 2 that a linear twist of -12° accomplishes most of the reduction of induced loss made possible by ideal twist, and

yields, at the same time, an almost identical profile-drag loss over the normal operating range. A few degrees more twist would probably produce slightly better results, but twist very much above 12° would be excessive. Larger amounts of twist would also be undesirable in other flight conditions.

The effects of twist illustrated thus far result in the comparative performance shown in figure 3 of the twisted untapered rotors. The curves are compared in the figure with a curve representing the performance of a rotor having minimum induced loss (uniform inflow) and zero profile drag in order to determine the extent to which the ideal rotor can be approached by practical designs. The significant results from the figure are summarized in the following table, which gives the percentage increase in thrust due to blade twist at fixed power:

Blade twist (deg)	Blade taper (ratio of root chord to tip chord)	Increase in thrust from untapered blade (percent)	
		$C_Q = 0.00026$	$C_Q = 0.00044$
0	1	--	--
-8	1	2	3
-12	1	3	4
Ideal	1	5	5

The torque coefficients were chosen to correspond to $C_T = 0.0040$ and $C_T = 0.0060$ for the untwisted blade.

Effect of taper.- The changes in rotor performance brought about by blade taper are similar to the changes effected by twist, in that the larger chord at the inner part of the blade causes a more uniform-inflow distribution. The separate beneficial effects of taper on the induced and profile-drag losses of untwisted blades are shown in figure 4 and the over-all effect is shown in figure 5. The conclusions to be drawn from these figures are given in the following table:

Blade twist (deg)	Blade taper (ratio of root chord to tip chord)	Increase in thrust from untapered blade (percent)	
		$C_Q = 0.00026$	$C_Q = 0.00044$
0	1	--	--
0	3	2	3

An increase of several percent in hovering performance may not justify the additional production costs of tapering the blades used on

small helicopters, especially when the benefits of twist can be had at no additional production costs. Even small efficiency gains, however, are significant with large helicopters, and the tapering of large-diameter blades appears highly desirable for structural reasons.

Effects of combined twist and taper.- Design considerations make it interesting to determine whether the full performance gains due to twist or taper are realized when both are present. The performance of blades having both twist and taper were calculated and plotted in figures 6 and 7 for comparison with figures 3 and 5, which show the effects of twist and taper acting separately. The comparison is summarized in the following table:

Blade twist (deg)	Blade taper (ratio of root chord to tip chord)	Thrust increase (percent)	
		$C_Q = 0.00026$	$C_Q = 0.0044$
Effect of twist; without taper			
0	1	--	--
-8	1	2	3
-12	1	3	4
Effect of twist; with taper			
0	3	--	--
-8 and -12	3	3	2
Effect of taper; without twist			
0	1	--	--
0	3	2	3
Effect of taper; with twist			
-12	1	--	--
-8 and -12	3	1	1

The increase in thrust obtained with various combinations of blade twist and taper as compared with the thrust of an untwisted rectangular blade is shown in the following table:

Blade twist (deg)	Blade taper (ratio of root chord to tip chord)	Increase in thrust from untwisted untapered blade (percent)	
		$C_Q = 0.00026$	$C_Q = 0.00044$
0	1	---	---
-8	3	5	5
-12	3	5	5
Ideal	Optimum	7	7

Effect of solidity on the benefits of twist and taper.- In order to determine the extent to which the benefits of twist and taper can be obtained with rotors of lower solidity than the 0.060 solidity treated, a similar analysis, covering equal amounts of twist and taper as well as the same thrust-coefficient range, was made for a rotor having a solitity equal to 0.042. The results are summarized by the hovering-performance curves of figures 8 and 9. A comparison of these figures with the equivalent figures for the rotor of 0.060 solidity shows that, in general, the same results were obtained with both rotors.

Effect of partial taper.- In conventionally tapered rotor blades, the taper usually extends from the tip to approximately one-half of the radius, the remainder or inboard part being rectangular in plan form. In order to determine whether the use of partial, rather than full, taper resulted in a loss of hovering efficiency, the performance of a blade tapered over its outer half and having a ratio of root chord to tip chord of 3 to 1 (the root chord being calculated by extending the leading and trailing edges of the tapered part to the root) was calculated and compared with the performance of a fully tapered blade having the same taper ratio and solidity. Little difference was found to exist between the two rotors, probably because the outer and more important part of each rotor was tapered in the same manner. Reference to figure 10 shows the inflow distribution and the power loss of both the partially and fully tapered rotors operating at the same thrust coefficient to be similar, which further substantiates this conclusion.

CONCLUDING REMARKS

The following remarks are based on a theoretical analysis made of the effects of blade twist and taper on the hovering performance of rotors having solidities of 0.060 and 0.042, the solidities of the tapered blades being computed by means of an equivalent chord.

The addition of linear twist of the order of -8° to -12° to rectangular blades may be expected to increase the thrust produced by the rotor at fixed power by approximately 3 to 4 percent or, if the blades are initially tapered with a ratio of root chord to tip chord of 3, by approximately 2 to 3 percent. An increase in thrust of about 5 percent is indicated for a combination of -8° or -12° twist and a taper ratio of 3 to 1. An additional 2-percent gain might be realized by the use of a nonlinear optimum combination of twist and taper. The use of a moderately lower solidity and partial instead of full taper would not be expected to affect significantly the preceeding conclusions if similar amounts of twist and taper are applied over the same thrust-coefficient range.

In connection with the use of twist in blade design, it should be remembered that negative twist has been found to improve the efficiency of the rotor in forward flight as well as in hovering and may be expected to delay blade stalling at high forward speeds and adverse compressibility effects at high tip speeds. Blade stalling is delayed when twist is employed because twist unloads the tips by reducing the tip angles of attack. Compressibility losses are minimized for the same reason, in that the critical Mach number of the blade section is increased at reduced lift coefficients.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., October 9, 1947

APPENDIX

DERIVATION OF PERFORMANCE EQUATION FOR OPTIMUM ROTOR

The optimum rotor for hovering is closely approximated by one which has a uniform inflow over the disk and which has all its sections operating at a constant angle of attack.

In hovering, the blade-element pitch is

$$\theta = \alpha_r + \phi$$

$$= \alpha_r + \frac{v}{\Omega r} \tag{1}$$

The pitch distribution for the optimum rotor is thus composed of two parts: a constant part, α_r, and a variable part, ϕ, which varies inversely as the radius for a constant inflow v. The differential thrust on a blade element is

$$dT_1 = \frac{1}{2} \rho(\Omega r)^2 a\alpha_r c \, dr \tag{2}$$

In order to maintain α_r independent of r, the chord must now be adjusted so that uniform downwash can be obtained. This latter condition exists when dT varies linearly with r, that is, when the thrust varies linearly from zero at the blade root to a maximum at the tip. (Such loading is associated with uniform downwash inasmuch as the mass of air influenced by a blade element depends on the radial position of the element. Thus dT varies linearly with r when v is constant along the span.)

It can be seen from equation (2) that dT can be made a linear function of r by letting

$$c = c_T \frac{R}{r} \tag{3}$$

Substituting equation (3) in equation (2) gives

$$dT_1 = \frac{1}{2} \rho\Omega^2 r a\alpha_r c_T R \, dr \tag{4}$$

The form of equation (4) indicates that the blade is producing uniform downwash.

Integrating equation (4) and multiplying the resulting expression by the number of blades b to give the total thrust produced by the rotor results in

$$T = \frac{b}{2} \rho\Omega^2 \frac{R^3}{2} a\alpha_r c_T \tag{5}$$

Expressed in nondimensional form, equation (5) reduces to

$$\begin{aligned} C_T &= \frac{\sigma_T}{4} a\alpha_r \\ &= \frac{\sigma_T}{4} c_l \end{aligned} \tag{6}$$

where

$$\sigma_T = \frac{bc_T}{\pi R} \tag{7}$$

Induced Torque

The induced torque may be expressed as

$$Q_i = \int_0^R \frac{b}{2} \rho\Omega^2 r^3 c_l \phi c \, dr \tag{8}$$

With the appropriate substitution for c and ϕ, equation (8) after integration becomes

$$Q_i = \frac{b}{4} \rho(\Omega R) R^2 c_l c_T v \tag{9}$$

In coefficient form, equation (9) reduces to

$$C_{Q_i} = \frac{\sigma_T}{4} c_l \frac{v}{\Omega R} \tag{10}$$

From momentum considerations

$$\begin{aligned} v &= \sqrt{\frac{T}{2\pi R^2 \rho}} \\ &= \Omega R \sqrt{\frac{C_T}{2}} \end{aligned}$$

or

$$\frac{v}{\Omega R} = \sqrt{\frac{C_T}{2}} \tag{11}$$

Substituting equations (6) and (11) into equation (10) gives the expression for C_{Q_i} finally as

$$C_{Q_i} = \frac{C_T^{3/2}}{\sqrt{2}} \tag{12}$$

Profile-Drag Torque

In a similar manner, the profile-drag torque may be expressed as

$$Q_o = \int_0^R \frac{b}{2} \rho(\Omega r)^2 c_{d_o} c \, dr \, r$$

$$= \frac{b}{6} \rho(\Omega R)^2 R^2 c_T c_{d_o} \tag{13}$$

or, in nondimensional form, as

$$C_{Q_o} = \frac{1}{6} \sigma_T c_{d_o} \tag{14}$$

With the addition of equations (12) and (14), the performance of the the optimum rotor is

$$C_Q = \frac{C_T^{3/2}}{\sqrt{2}} + \frac{1}{6} \sigma_T c_{d_o} \tag{15}$$

In order to compare the optimum rotor with other rotors of the same solidity, it is necessary to determine the conventional weighted solidity of such a rotor. The equivalent chord of the optimum rotor, as calculated from its definition and equation (3), is

$$c_e = \frac{3}{2} c_T \tag{16}$$

so that

$$\sigma = \frac{3}{2} \sigma_T \tag{17}$$

Thus, in terms of weighted solidity, equation (15) becomes

$$c_Q = \frac{c_T^{3/2}}{\sqrt{2}} + \frac{\sigma}{9} c_{d_o} \tag{18}$$

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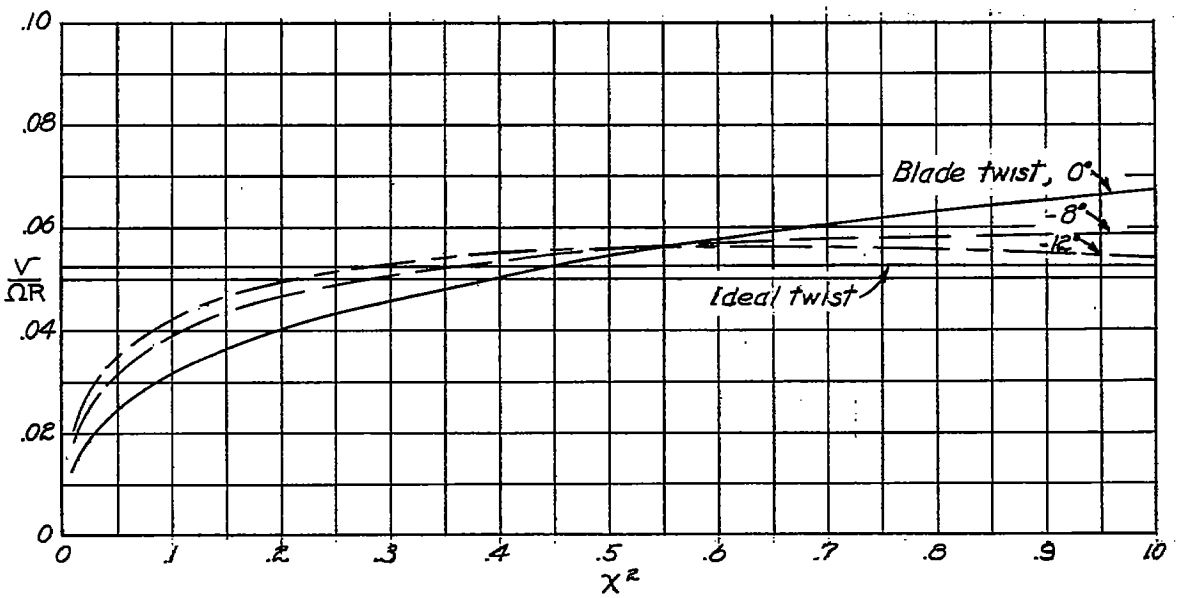
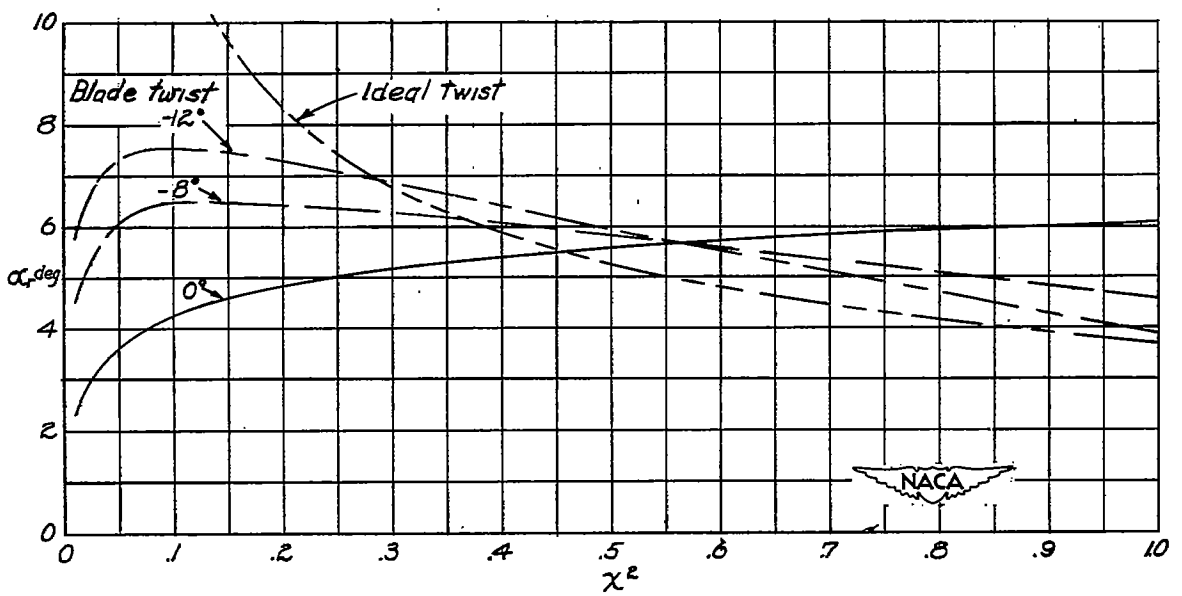
3. Crigler, John L.: Comparison of Calculated and Experimental Propeller Characteristics for Four-, Six-, and Eight-Blade Single-Rotating Propellers. NACA ACR No. 4B04, 1944.



(a) Inflow distribution.



(b) Blade-section angle-of-attack distribution.

Figure 1.- Effect of twist on spanwise inflow and angle-of-attack distribution of an untapered blade. $\sigma = 0.060$; $C_T = 0.0056$.

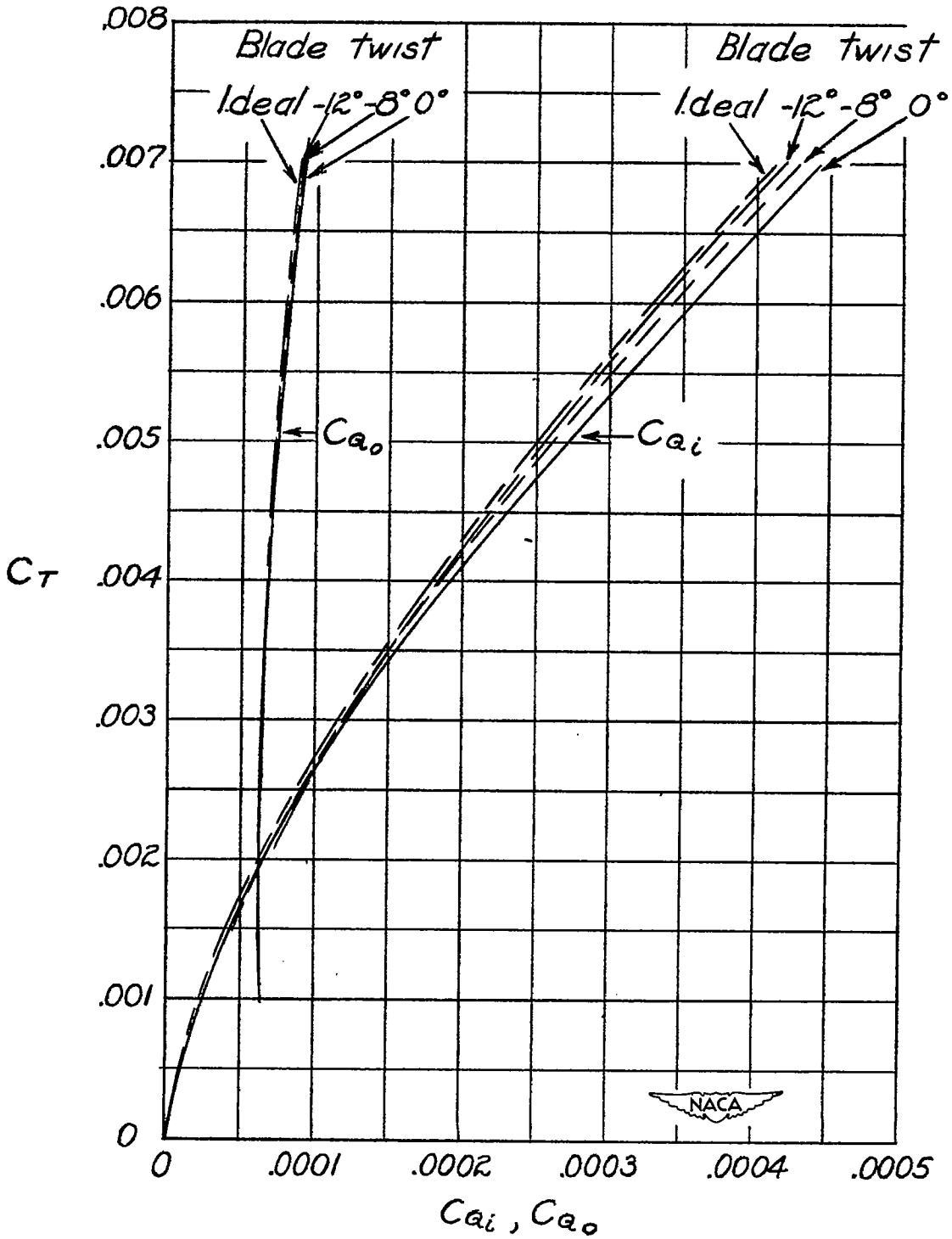


Figure 2.- Effect of twist on the induced and profile-drag losses of a rotor having untapered blades. σ = 0.060.

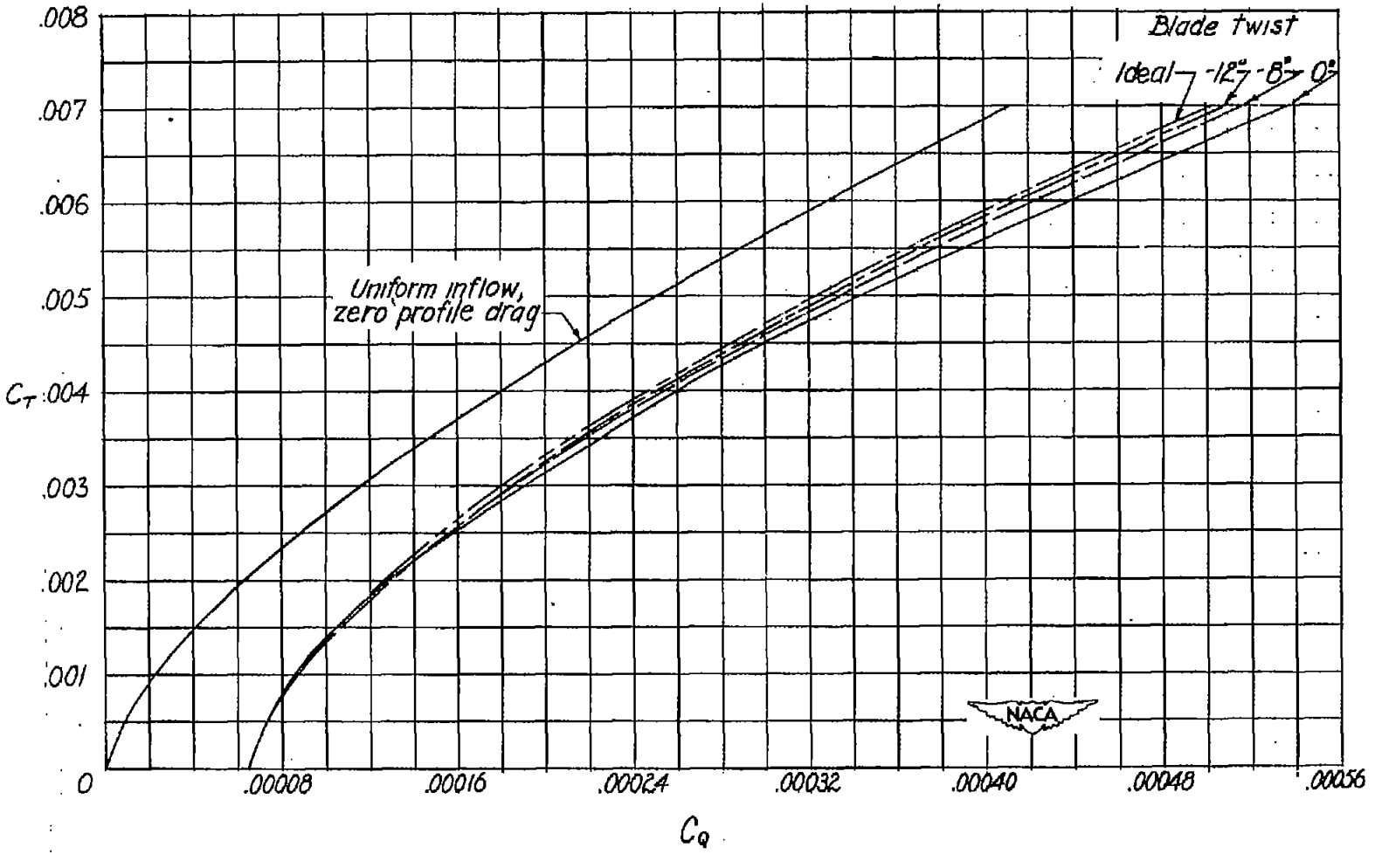


Figure 3.- Effect of twist on the performance of rotors having untapered blades. $\sigma = 0.060$.

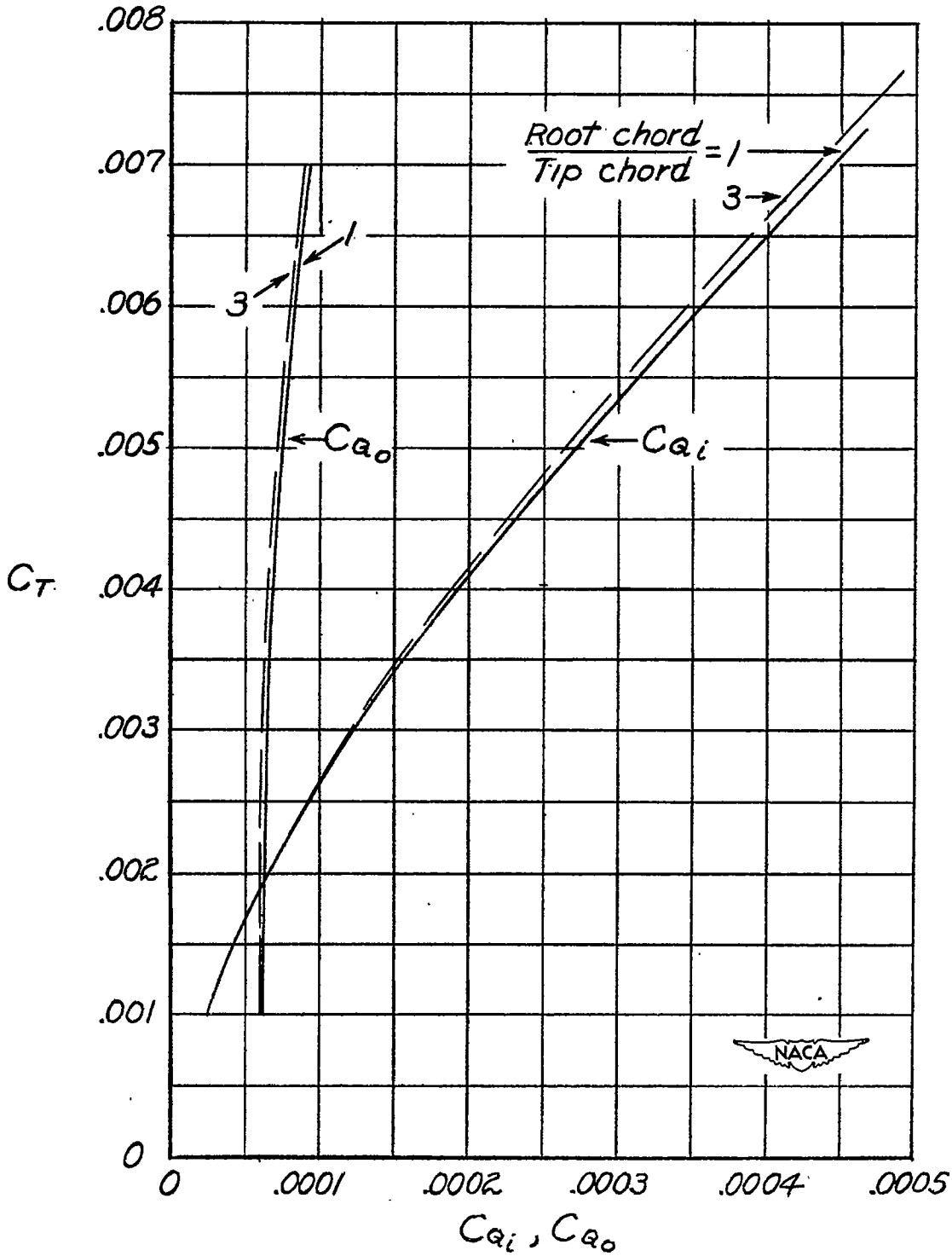


Figure 4.- Effect of taper on the induced and profile-drag losses of rotors having untwisted blades. σ = 0.060.

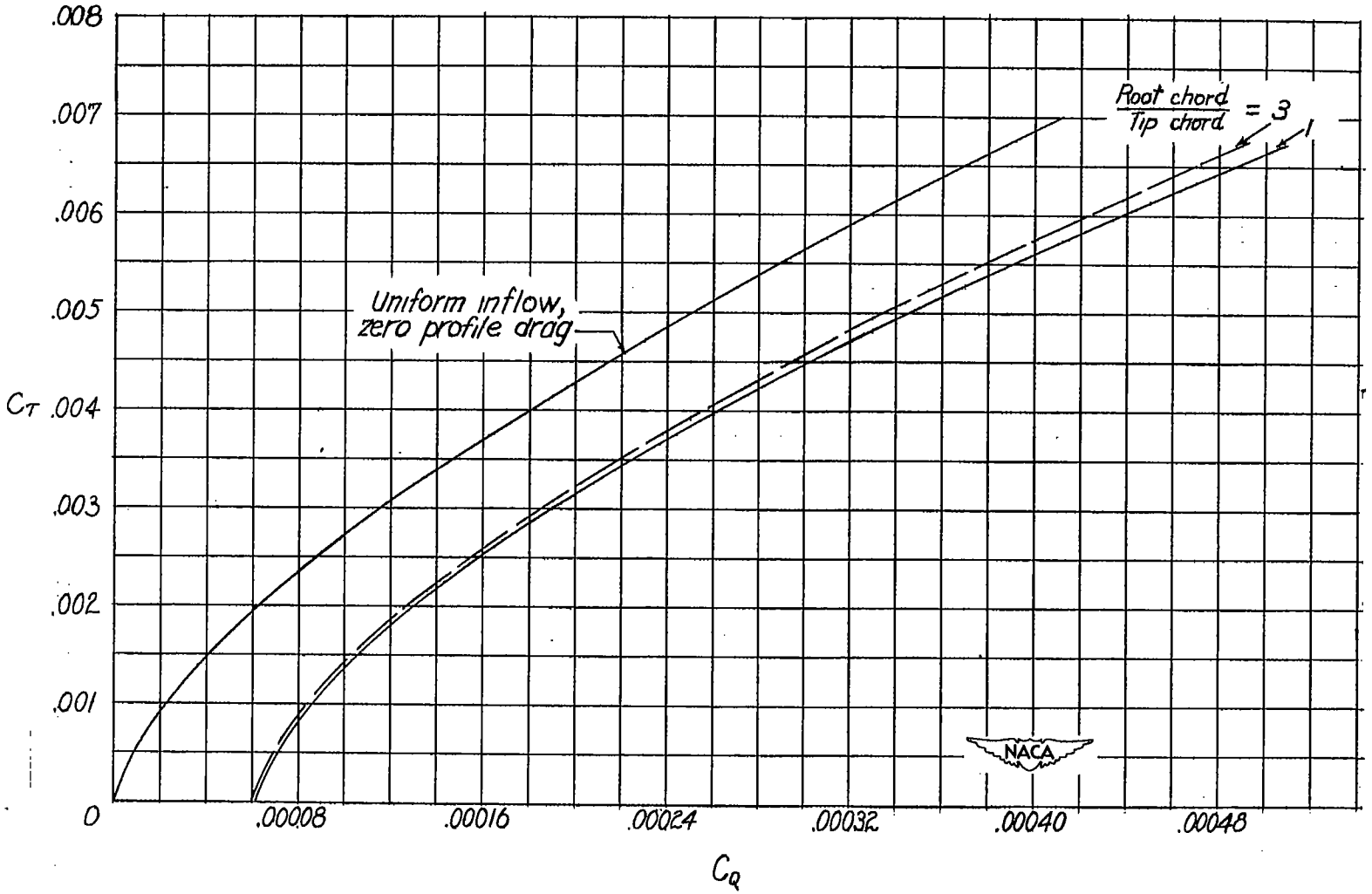


Figure 5.- Effect of taper on the performance of rotors having untwisted blades. σ = 0.060.

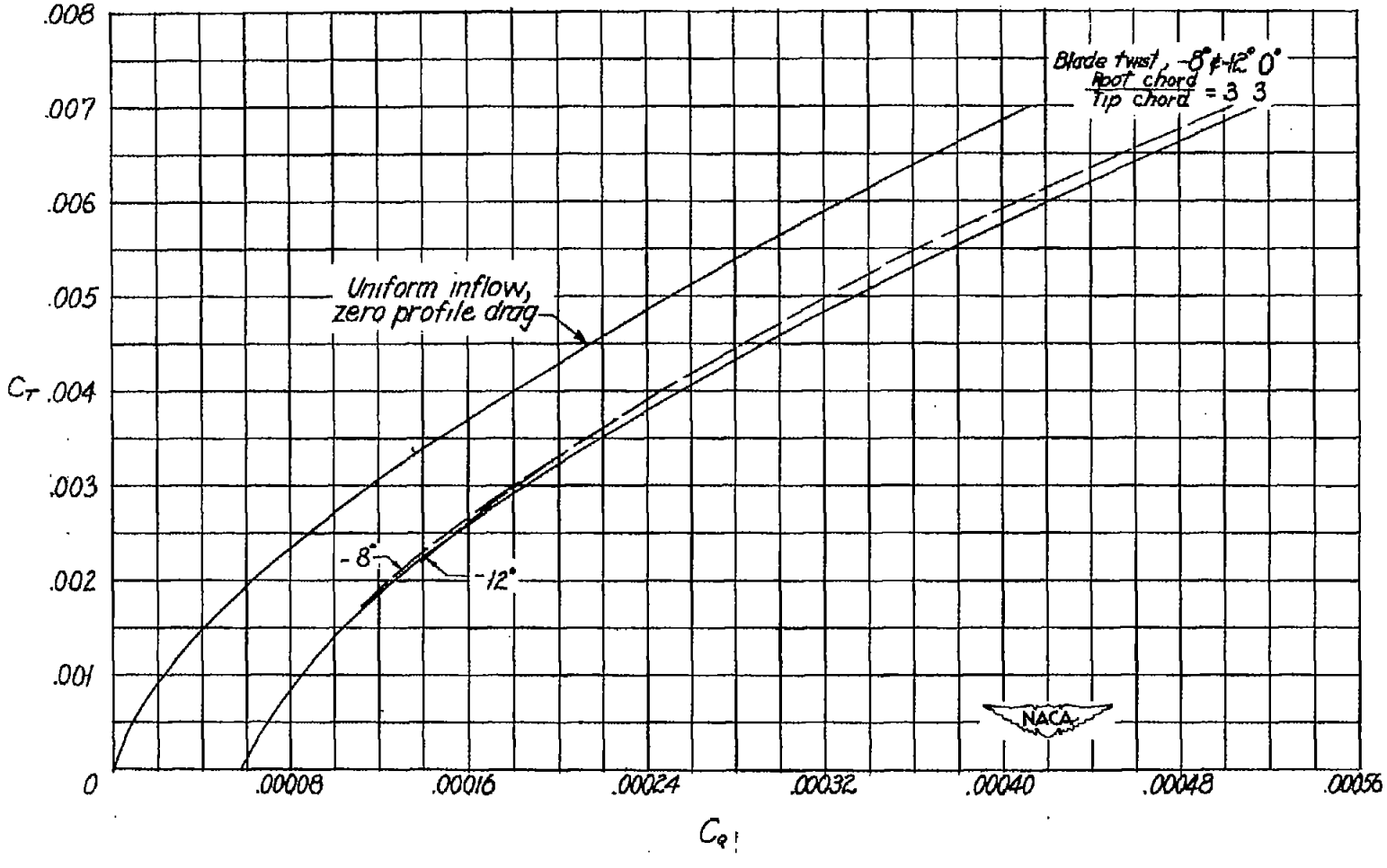


Figure 6.- Effect of twist on the performance of rotors having blades that are tapered so that the ratio of root chord to tip chord is 3. $\sigma = 0.060$.

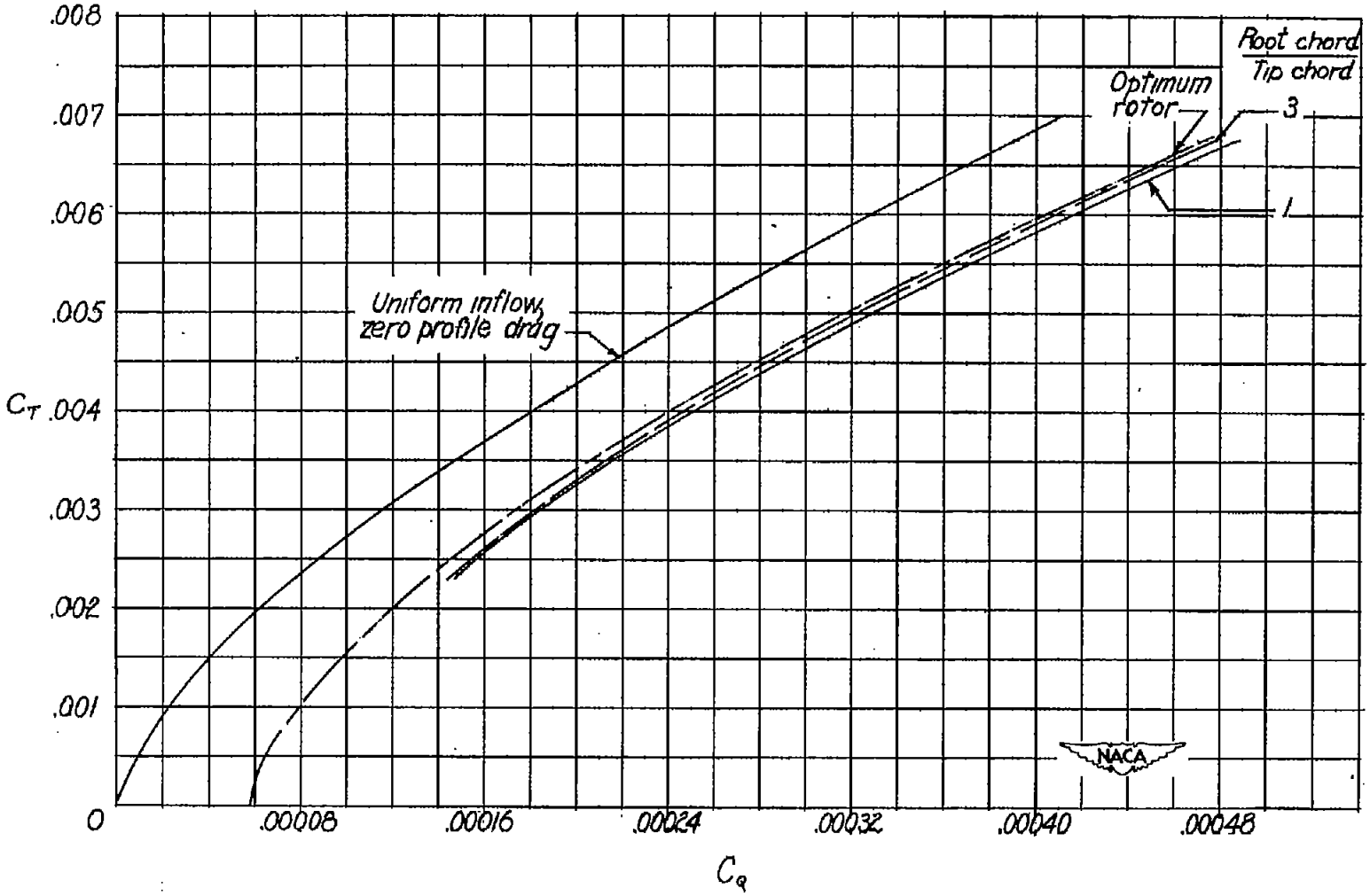


Figure 7.- Comparison of the performance of the optimum rotor with the performance of rotors having blades with -12° twist. σ = 0.060.

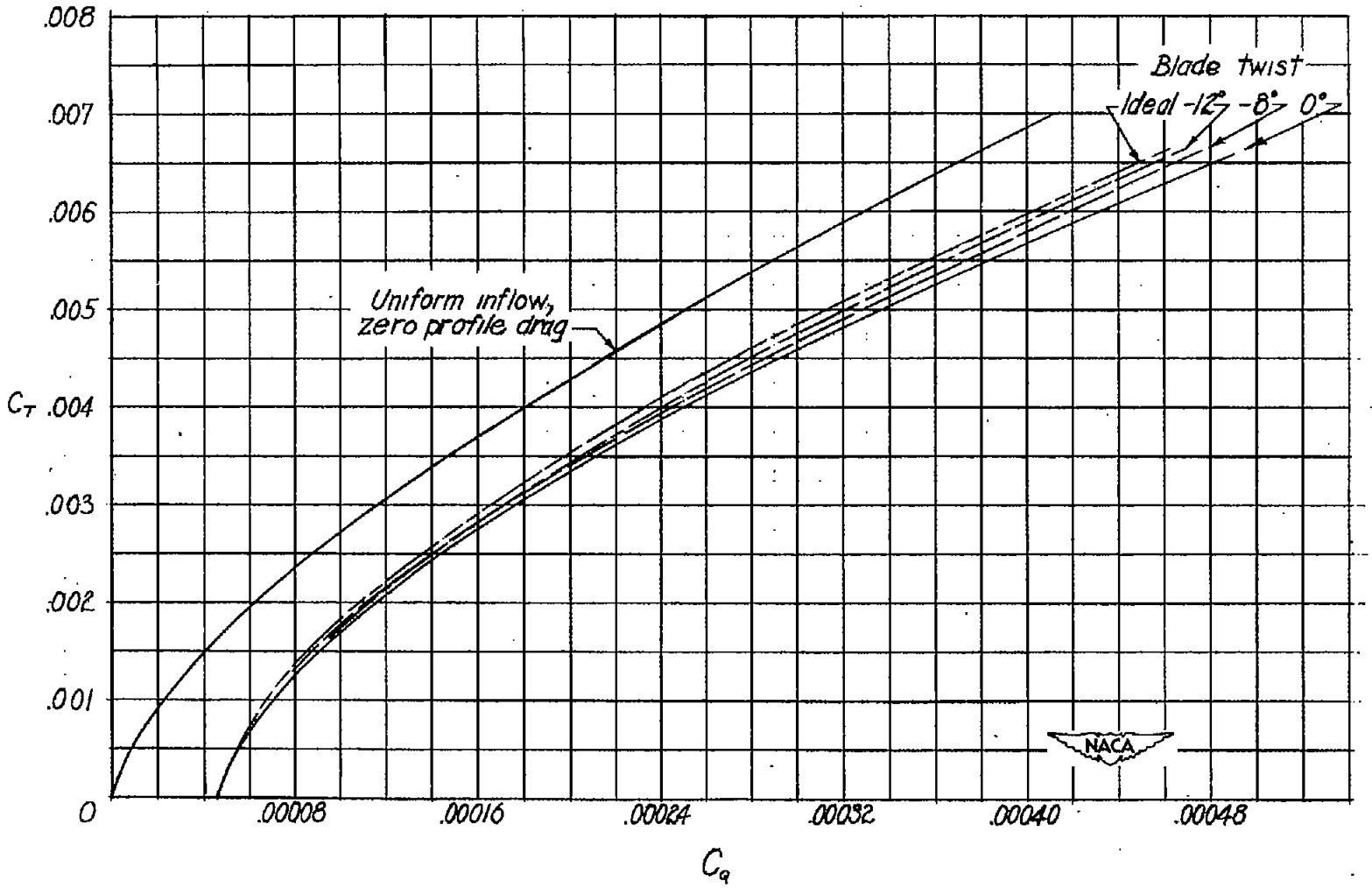


Figure 8.- Effect of twist on the performance of rotors having untapered blades. $\sigma = 0.042$.

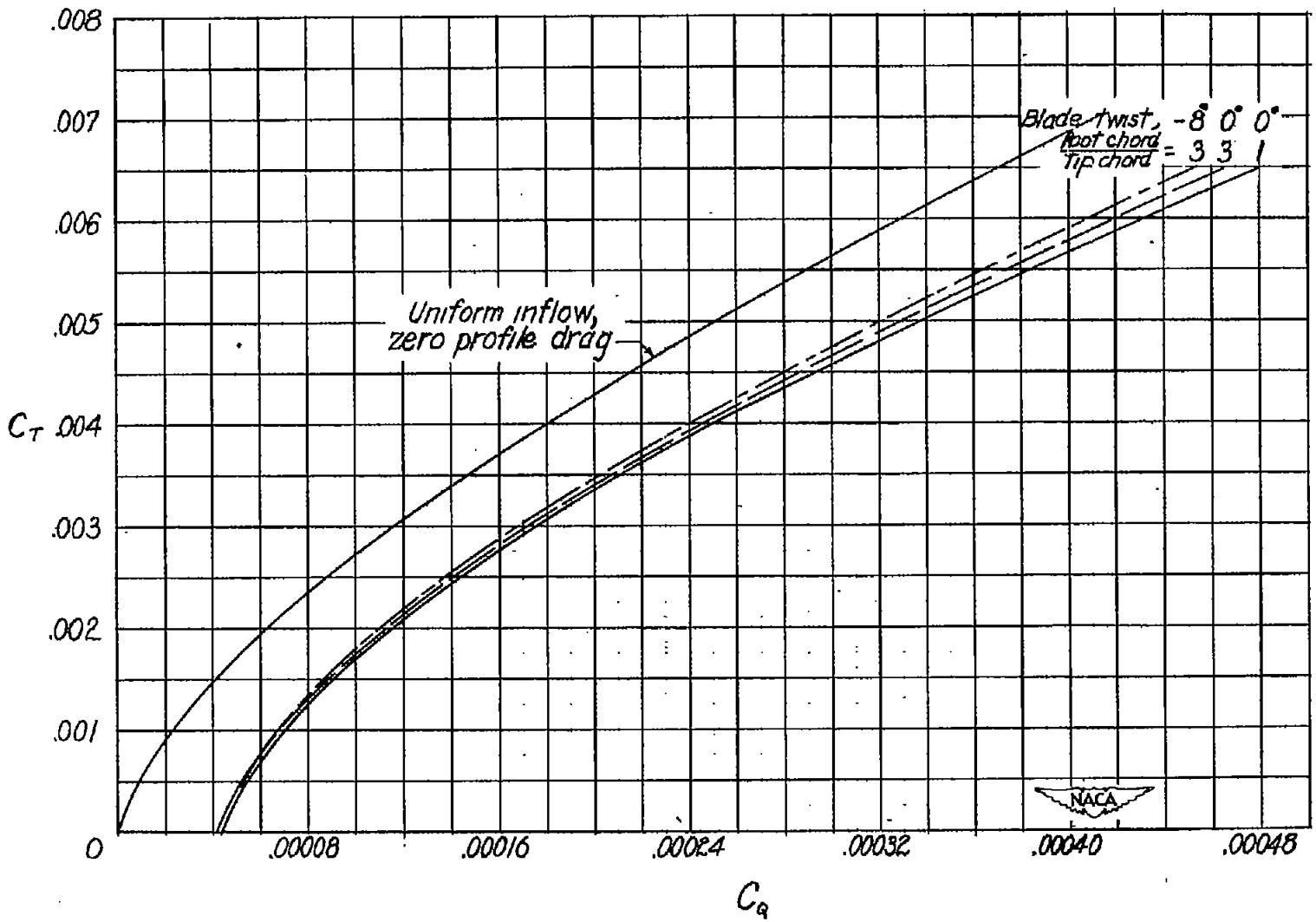


Figure 9.- Effect of both twist and taper on rotor performance. $\sigma = 0.042$.

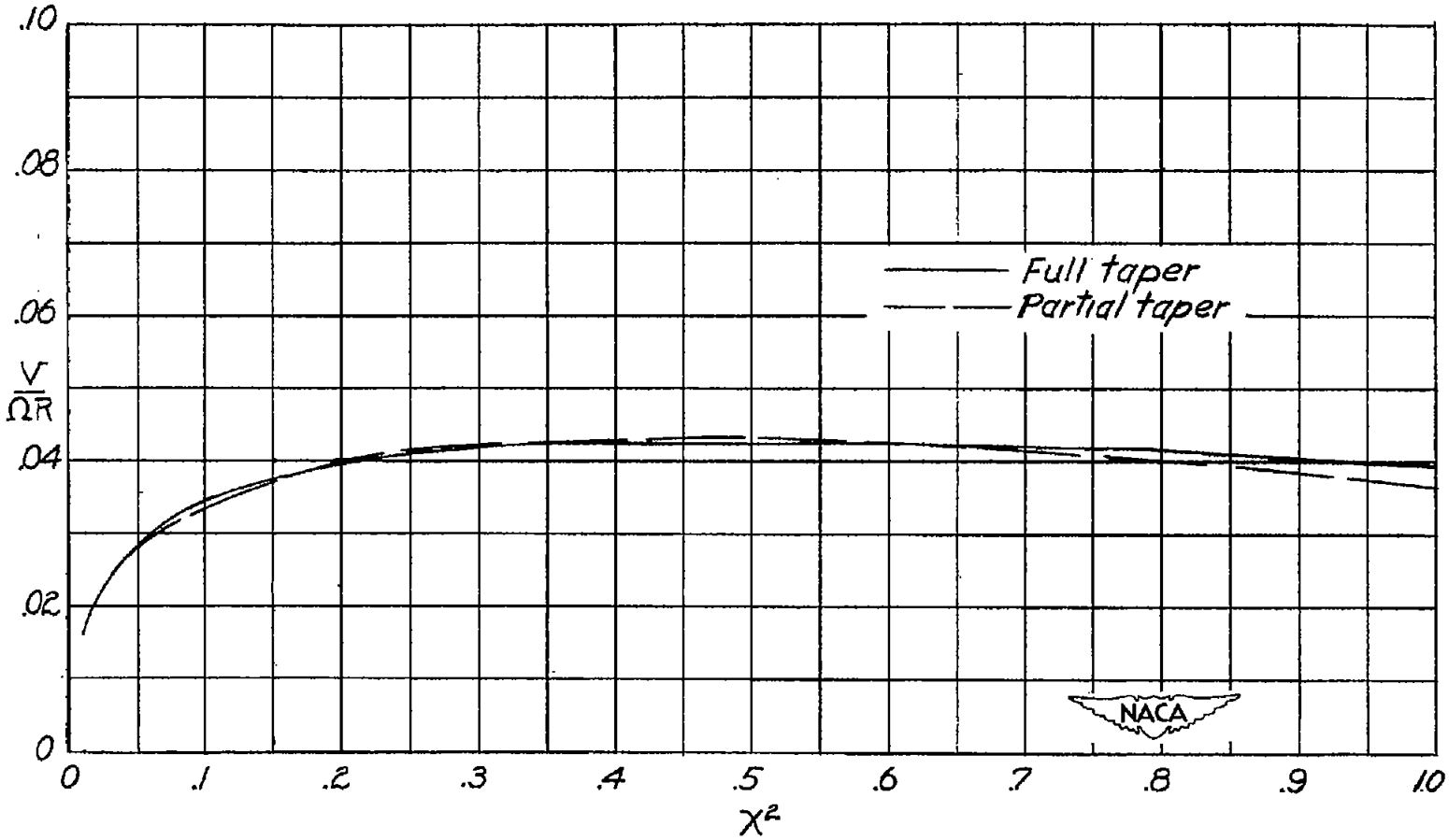


Figure 10.- Effect of partial taper and full taper on rotor inflow distribution. σ = 0.042; blade twist, - 8°; ratio of root chord to tip chord, 3; C_T = 0.0032.