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TECHNICAL NOTE

No. 1712

ANALYTICAL METHOD OF DETERMINING TRANSMISSION OF PARTICLES AND RADIATION THROUGH GREAT THICKNESS OF MATTER

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ANALYTICAL METHOD OF DETERMINING TRANSMISSION OF PARTICLES AND RADIATION THROUGH GREAT THICKNESSES OF MATTER

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SUMMARY

A matrix method of determining transmission, scattering, and absorption of radiation, such as gamma rays and neutrons, in large thicknesses of matter is presented. Experimental data as yet unavailable are required for complete utilization of the method. Material radioactivity whose intensity may be considered to be independent of time is also treated.

INTRODUCTION

An analytical method, developed at the NACA Cleveland laboratory, is presented that can be used to determine the relative efficiency of a number of composite shields intended for use with one or more types of radiation. The procedure described is also useful in designing such shields. Application of the method is limited by the unavailability of data on the scattering and absorption of radiation by particular chemical elements in certain ranges of the energy spectrum.

The matrix methods employed in handling a number of simultaneous equations were used (reference 1) to solve a thermal-shielding problem analogous to the first case of this analysis. In reference 1, the efficiency of a number of thin shields, used with a source of thermal radiation, such that all radiation is either reflected at the surfaces of the shields or transmitted without absorption is determined; monochromatic radiation only is considered, and this radiation is assumed always normal to the shield.

Shielding involving interactions between radiation and matter within the body of the shield, such as occurs with gamma rays and neutrons, is calculated herein. The following conditions, to which no experimental exceptions have been found for these types of radiation, were placed on the analysis:

(1) The amounts of radiation transmitted, absorbed, and scattered in a given shield are directly proportional to the amount of radiation entering the shield for a given type of energy spectrum. Two energy spectrums are defined to be of the same type when the ratios of corresponding intensities at each energy are equal. This condition may be alternatively stated as postulating zero saturation effect over the complete range of intensities and energies considered.

(2) The fractions of radiation of any energy that are transmitted, absorbed, and scattered in a given shield are independent of the presence of radiation of another energy. This condition enables the analysis to be made with quantities that obey laws of linear combination.

(3) An elementary thickness of material cannot offer infinite resistance to the transmission of incident radiation in its original direction at its original energy.

When these conditions were established, analysis proved feasible for cases in which polychromatic radiation was partly absorbed, partly degraded in energy, and partly transmitted unchanged in energy. Extensions of the concepts employed permitted consideration also of scattering at an angle to the direction of the incident beam, upgrading of energy as by nuclear reaction, and excitation of neutron emission by gamma-ray bombardment and of gamma rays by neutron bombardment. The scattered radiation from great thicknesses of material must be particularly considered not only because the material gives rise to much scattered radiation and secondary radiation, but because the shielding is required to cover a source that radiates spherically, so that the power of radiation at any point outside of the shielding is the integrated power from all points on the surface of the shield visible at the point of measurement.

The method of calculation presented requires complete information on scattering and absorption in order to yield a true answer. If information on only the primary beam is available, the shielding calculation based on it must be in error by the secondary and scattered radiations that are being neglected. If the energy spectrums of scattered radiation are unknown, the effect of energy on absorption must be neglected. Information is therefore desirable on the absorption and energy-intensity distribution for radiation of all wavelengths to be encountered for all materials that will be handled in order to obtain the truest possible shielding analysis. If appreciable quantities of gamma radiation are converted to other radiation such as neutrons, these quantities must also be known.

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A simplification in the data requirements is that data need be available for only one elementary thickness of absorber and not a number of thicknesses. An elementary thickness is any given thickness for which absorption and scattering data are known or can conveniently be obtained as, for example, 1 centimeter. Calculations can be made by the method presented using currently available data, but the results obtained from the shielding analysis will be somewhat inaccurate.

The data required for neutron absorption are those concerned with inelastic scattering, elastic scattering, and the products of neutron-induced reactions. The data required here are similar to those required for gamma rays except that they include not only the intensities of neutrons of different energies and directions but also the intensity and the direction of the energy spectrum of gamma radiation produced if appreciable. A number of parameters, such as grain size of metals, that have no effect on absorption of gamma radiation appear to influence neutron absorption (reference 2) and must be considered.

Different materials are best for absorption at different incident spectrums of energy. Inasmuch as the spectrum varies through the shield, the shield of optimum absorption characteristics would contain different materials through its thickness. A suitable method of analysis consequently must be capable of determining the spectrum at a station within the shield, selecting an optimum absorber, and ascertaining the new emergent spectrum for the next station.

SYMBOLS

The following symbols are used in this method:

S_i	i^{th} element of shield
I_i	power of radiation from $(i-1)^{th}$ element incident on i^{th} element
I_{i+1}	power of radiation from i^{th} element incident on $(i+1)^{th}$ element
R_i	power of radiation from i^{th} element incident on $(i-1)^{th}$ element

R_{i+1}	power of radiation from $(i+1)^{th}$ element incident on i^{th} element
P	one-half of power of radiation from radioactivity in shield
Q	power of radiation from source of radiation to shield
A	power-transmission matrix for complete shield
T	power-transmission matrix for one element of shield
z	matrix element
t	power-transmission coefficient, or power-scattering coefficient
r	power back-scattering (reflection) coefficient
c	power-conversion coefficient, radiation to mass and heat
h	power-conversion coefficient, radiation to heat
m	power-conversion coefficient, radiation to mass
C	power of radiation converted to mass and heat
H	power of radiation converted to heat
M	power of radiation converted to mass
$\bar{t}$, $\bar{r}$, $\bar{h}$, $\bar{m}$, $\bar{c}$	power-transmission, back-scattering (reflection), heat-conversion, mass-conversion, and total-conversion operators, respectively
F	arbitrary single-valued function of independent variable
a	arbitrary numerical coefficient

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The following subscripts are used to designate positions and locations:

$\alpha, \beta, \ldots, \nu$	zones for radiation scattered at angles to primary direction
$1, 2, \ldots, i, \ldots, n$	successive elements of set (shielding or energies)

The following superscripts are used to designate type of radiation:

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γ	gamma radiation
n	neutron radiation
γn	conversion from gamma radiation to neutron radiation
$n\gamma$	conversion from neutron radiation to gamma radiation

ANALYSIS

The analysis of transmission of particles and radiation through matter may be conveniently broken down into a number of separate cases. Each succeeding case contains an additional consideration of some phenomenon encountered in the transmission problem.

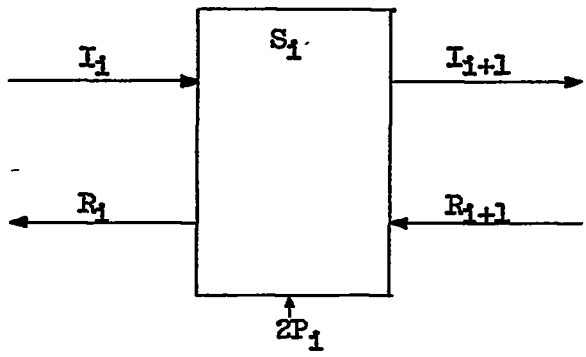
Case I - Radioactive Shield

The following hypothesis is made:

Monochromatic radiation
Radiation normal to shield
Source of radiation both outside shield and from long-lived radioactivity of shield material

Method. - The shield may be considered as consisting of elements orthogonal to the radiation. The ith element in a shield is S_i. Radiations from S_{i-1} and S_{i+1}, namely I_i and R_{i+1}, respectively,

are incident on S_i. From S_i emerge radiations I_{i+1} and R_i to S_{i+1} and S_{i-1}, respectively. Radiation is assumed to originate from sources outside the shield as well as from an independent source of radiation $2P_i$ within the shield. In practice, such an independent source might exist in the form of long-lived radioisotopes in the shielding of a cyclotron that had operated over a period of time.



Let t_i, the power-transmission coefficient, be a number that determines the amount of power transmitted through the element; for a given thickness, it is a function of the wavelength of the radiation and of the material constituting the element.

$$t_i = \left(\frac{I_{i+1}}{I_i}\right)_{R_{i+1}=0} = \left(\frac{R_i}{R_{i+1}}\right)_{I_i=0} \tag{1}$$

Similarly the back-scattering, or reflection, coefficient is

$$r_i = \left(\frac{R_i}{I_i}\right)_{R_{i+1}=0} = \left(\frac{I_{i+1}}{R_{i+1}}\right)_{I_i=0} \tag{2}$$

If the shield power $2P_i$ is distributed equally between the two faces of the element, then

$$I_{i+1} = t_i I_i + r_i R_{i+1} + P_i \tag{3}$$

$$R_i = t_i R_{i+1} + r_i I_i + P_i \tag{4}$$

If equation (3) is rewritten,

$$I_i = \frac{1}{t_i} I_{i+1} - \frac{r_i}{t_i} R_{i+1} - \frac{1}{t_i} P_i \tag{5}$$

and, if equation (5) is substituted in equation (4),

$$R_i = t_i R_{i+1} + \frac{r_i}{t_i} I_{i+1} - \frac{r_i^2}{t_i} R_{i+1} - \frac{r_i}{t_i} P_i + P_i$$

$$= \frac{r_i}{t_i} I_{i+1} + \left(t_i - \frac{r_i^2}{t_i}\right) R_{i+1} + \left(1 - \frac{r_i}{t_i}\right) P_i \tag{6}$$

In addition, the following identity can be set up between the arbitrary radioactive powers of two elements:

$$P_{i-1} \equiv \frac{P_{i-1}}{P_i} P_i \tag{7}$$

with the following additional definitions:

Condition	Definition
$i = 1$	$P_0 = 0$
$P_i = 0,\ P_{i-1} \neq 0$	$\left(\frac{P_{i-1}}{P_i}\right) P_i = P_{i-1}$
$P_i = 0,\ P_{i-1} = 0$	$\frac{P_{i-1}}{P_i} = 0$

In matrix notation, equations (5) to (7) may be expressed as

$$\begin{bmatrix} I_i \\ R_i \\ P_{i-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{t_i} & -\frac{r_i}{t_i} & -\frac{1}{t_i} \\ \frac{r_i}{t_i} & t_i - \frac{r_i^2}{t_i} & 1 - \frac{r_i}{t_i} \\ 0 & 0 & \frac{P_{i-1}}{P_i} \end{bmatrix} \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ P_i \end{bmatrix} \tag{8}$$

The transmission matrix may be abbreviated for conciseness of notation. Equation (8) then reads

$$\begin{bmatrix} I_i \\ R_i \\ P_{i-1} \end{bmatrix} = \begin{bmatrix} T_i \end{bmatrix} \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ P_i \end{bmatrix} \tag{9}$$

For shield element S_{i-1}, similarly,

$$\begin{bmatrix} I_{i-1} \\ R_{i-1} \\ P_{i-2} \end{bmatrix} = \begin{bmatrix} T_{i-1} \end{bmatrix} \begin{bmatrix} I_i \\ R_i \\ P_{i-1} \end{bmatrix} \tag{10}$$

The over-all effect of n elements is obtained by matrix multiplication of transmission matrices:

$$\begin{bmatrix} I_1 \\ R_1 \\ 0 \end{bmatrix} = \begin{bmatrix} T_1 \end{bmatrix} \begin{bmatrix} T_2 \end{bmatrix} \cdots \begin{bmatrix} T_n \end{bmatrix} \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ P_n \end{bmatrix} \tag{11}$$

where $\left(\frac{P_{i-1}}{P_i}\right)\left(\frac{P_i}{P_{i+1}}\right) = \frac{P_{i-1}}{P_{i+1}}$, or

$$\begin{bmatrix} I_1 \\ R_1 \\ 0 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ P_n \end{bmatrix} \tag{12}$$

where $I_{n+1} + P_n$ is the power of radiation emitted from the shield to the surroundings and R_{n+1} is the power of radiation emitted from the surroundings to the shield. From a practical aspect, $I_{n+1} + P_n + R_{n+1}$ might be the tolerance of a human being to radiation, and R_{n+1} the background radiation in the vicinity of the individual from sources other than I_{n+1} and P_n.

If the source absorbs no radiation and reflects $R_1 + P_1$ back to the shield, then

$$I_1 = R_1 + P_1 + Q$$

where Q is that portion of the power of radiation from the source intercepted by the area of the shield. If the source absorbs all back-scattered radiation, then I_1 is Q alone. The practical case lies somewhere between these extremes with the source absorbing some of the back-scattered power.

Inversely, with R_i and I_i known and R_{i+1} and I_{i+1} desired, equations (5), (6), and (7) are rewritten as

$$R_{i+1} = \frac{1}{t_i} R_i - \frac{r_i}{t_i} I_i - \frac{1}{t_i} P_i \tag{13}$$

$$I_{i+1} = \frac{r_i}{t_i} R_i + \left(t_i - \frac{r_i^2}{t_i}\right) I_i + \left(1 - \frac{r_i}{t_i}\right) P_i \tag{14}$$

$$P_{i+1} = \frac{P_{i+1}}{P_i} P_i \tag{15}$$

where the definitions under equation (7) are now given with P_{i+1} replacing P_{i-1}. In matrix notation,

$$\begin{bmatrix} R_{i+1} \\ I_{i+1} \\ P_{i+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{t_i} & -\frac{r_i}{t_i} & -\frac{1}{t_i} \\ \frac{r_i}{t_i} & t_i - \frac{r_i^2}{t_i} & 1 - \frac{r_i}{t_i} \\ 0 & 0 & \frac{P_{i+1}}{P_i} \end{bmatrix} \begin{bmatrix} R_i \\ I_i \\ P_i \end{bmatrix} \tag{16}$$

For the entire shield,

$$\begin{bmatrix} R_{n+1} \\ I_{n+1} \\ 0 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} R_1 \\ I_1 \\ P_1 \end{bmatrix} \tag{17}$$

The over-all transmission matrix A is the same in this inverse circumstance as that previously given except that $\frac{P_{i+1}}{P_i}$ replaces $\frac{P_{i-1}}{P_i}$.(provided that the order of R and I is reversed in the column matrices), P_1 is entered in the column matrix of the radiation entering the first shielding element, and P_n is removed from the column matrix of the radiation leaving the last element in the shield. The definitions given under equation (7) hold when P_{i+1} replaces P_i.

For a shield with no radioactivity, $P_i = 0$ and a second-order transmission matrix may be set up from equation (8):

$$\begin{bmatrix} I_i \\ R_i \end{bmatrix} = \begin{bmatrix} \frac{1}{t_i} & -\frac{r_i}{t_i} \\ \frac{r_i}{t_i} & t_i - \frac{r_i^2}{t_i} \end{bmatrix} \begin{bmatrix} I_{i+1} \\ R_{i+1} \end{bmatrix} \tag{18}$$

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The calculation of the over-all transmission matrix may be performed by matrix computation as indicated for the third-order matrix. Inasmuch as the reciprocal of this transmission matrix is the transmission matrix with the order of the rows and the order of the columns reversed, the inverse problem may be set up with an identical transmission matrix provided only that the order of I and R is reversed:

$$\begin{bmatrix} R_{i+1} \\ I_{i+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{t_i} & -\frac{r_i}{t_i} \\ \frac{r_i}{t_i} & t_i - \frac{r_i}{t_i} \end{bmatrix} \begin{bmatrix} R_i \\ I_i \end{bmatrix} \tag{19}$$

If a complete shield is to be analyzed, the first step is to break down the thickness into a number of elements, the materials of which are known, for whose thicknesses transmission and reflection data are available. The radiation from the source may be assumed to be monochromatic or to consist of a number of monochromatic components. This radiation will be assumed not to be degraded by passage through the shield but only to be absorbed and back-scattered. Inasmuch as degradation of energy is more common for most applications of thick shielding than either upgrading of energy by nuclear reaction or transmission with energy unchanged, and inasmuch as neutron absorption generally increases with decreasing energy (neglecting resonance) as does gamma-ray absorption (in the range of energy above the K-absorption edges and below the point at which pair formation causes the absorption to rise with increasing energy), the assumption of no degradation of energy leads to the design of a safe shield. The shield may be less safe than indicated by analysis if the energy of radiation is above the energy at which a minimum exists in the absorption characteristic. If the power transmitted and back-scattered by an element is known, application of the analysis will produce a very safe shield, in general, because the analysis not only neglects the effects of degradation of energy by scattering, but assumes a path for traverse of radiation normal to the shield, whereas actually the radiation traverses a longer path at some angle to the normal.

Example 1. - Assume (1) that $t_i = 0.8$ and $r_i = 0.1$ for all values of i at some energy for some material of a certain elementary thickness and that t and r are constants; (2) that the shield is 3 elements in thickness; (3) that

the power of radioactivity from each element is 0.0020 roentgen per hour and that the spectrum is of the same type as that of the source; (4) that background radiation is negligible; and (5) that human tolerance is 0.01 roentgen per hour. What size source can safely be shielded?

These data are substituted in equation (11) to yield

$$\begin{bmatrix} I_1 \\ R_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{0.8} & -\frac{0.1}{0.8} & -\frac{1}{0.8} \\ \frac{0.1}{0.8} & 0.8-\frac{0.01}{0.8} & 1-\frac{0.1}{0.8} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{0.8} & -\frac{0.1}{0.8} & -\frac{1}{0.8} \\ \frac{0.1}{0.8} & 0.8-\frac{0.01}{0.8} & 1-\frac{0.1}{0.8} \\ 0 & 0 & 1 \end{bmatrix}^2 \begin{bmatrix} 0.0100 \\ 0.0000 \\ 0.0020 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9737}{5120} & -\frac{20,169}{51,200} & -\frac{26,001}{5120} \\ \frac{20,169}{51,200} & \frac{227,447}{512,000} & \frac{82,863}{51,200} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.0100 \\ 0.0000 \\ 0.0020 \end{bmatrix} = \begin{bmatrix} 0.0089 \\ 0.0072 \\ 0 \end{bmatrix}$$

Therefore

$$I_1 = 0.0089 \text{ roentgen per hour}$$
$$R_1 = 0.0072 \text{ roentgen per hour}$$

and

$$I_1 - R_1 = 0.0017 \text{ roentgen per hour}$$

The source power Q equals I_1 if the source absorbs all radiation; $Q = I_1 - R_1$ if the source reflects all radiation from the shield back to the shield. The result may consequently be expressed as

$$0.0089 \geqq Q \geqq 0.0017$$

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Example 2. - Assume (1) that $t_i = 0.8$ and $r_i = 0.1$ for all values of i at some energy for some material of a certain elementary thickness; (2) that the source power is 2 roentgens per hour; (3) that human tolerance is 0.0099 roentgen per hour; (4) that background radiation is negligible; and (5) that the material of the shield is not radioactive. How many elementary thicknesses are required to shield the source safely?

Two alternatives may be set up, one in which the source absorbs all back-scattered radiation and one in which it absorbs none.

When the source absorbs all back-scattered radiation,

$$\begin{bmatrix} 2 \\ \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{0.8} & -\frac{0.1}{0.8} \\ \\ \frac{0.1}{0.8} & 0.8-\frac{0.01}{0.8} \end{bmatrix}^x \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = [T]^x \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix}$$

By calculation,

$$[T]^{26} \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 1.6554 \\ \\ 0.4859 \end{bmatrix}$$

and

$$[T]^{27} \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 2.0085 \\ \\ 0.5896 \end{bmatrix}$$

The result indicates that 27 elements of shielding are needed to shield a source with an intensity of 2 roentgens per hour if the source absorbs all back-scattered radiation from the shielding.

When the source absorbs no back-scattered radiation,

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$$\begin{bmatrix} 2 + R_1 \\ \\ R_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{0.8} & -\frac{0.1}{0.8} \\ \\ \frac{0.1}{0.8} & 0.8 - \frac{0.01}{0.8} \end{bmatrix}^x \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = [T]^x \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix}$$

By calculation,

$$[T]^{28} \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 2.4369 \\ \\ 0.7154 \end{bmatrix} = \begin{bmatrix} 1.7215 + 0.7154 \\ \\ 0.7154 \end{bmatrix}$$

$$[T]^{29} \begin{bmatrix} 0.0099 \\ \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 2.9567 \\ \\ 0.8680 \end{bmatrix} = \begin{bmatrix} 2.0887 + 0.8680 \\ \\ 0.8680 \end{bmatrix}$$

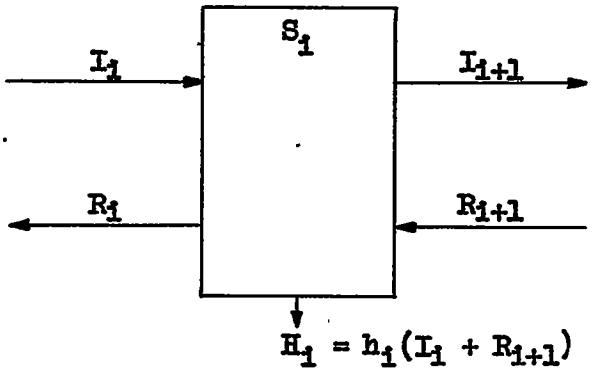
The result therefore indicates that 29 elementary thicknesses of shielding are required to shield a source whose intensity is 2 roentgens per hour if the source absorbs no radiation back-scattered from the shielding.

Case II - Conversion to Heat

The following hypothesis is made:

Monochromatic radiation
Radiation normal to shield
Source of radiation outside shield
Radiation absorbed converted to heat

Method. - The conversion of radiation to heat is included; the amount of radiation absorbed is proportional to the amount incident upon the element. Radioactivity of the shielding is not considered.



The equations relating the powers of radiation incident on and scattered from the ith element are

$$\begin{aligned} I_{i+1} &= t_i I_i + r_i R_{i+1} \\ R_i &= t_i R_{i+1} + r_i I_i \\ H_i &= h_i I_i + h_i R_{i+1} \end{aligned} \tag{20}$$

Solving for I_i, R_i, and H_i explicitly and setting in matrix form gives

$$\begin{bmatrix} I_i \\ R_i \\ H_i \end{bmatrix} = \begin{bmatrix} \frac{1}{t_i} & -\frac{r_i}{t_i} & 0 \\ \frac{r_i}{t_i} & t_i - \frac{r_i^2}{t_i} & 0 \\ \frac{h_i}{t_i} & -\frac{h_i r_i}{t_i} & 0 \end{bmatrix} \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ H_{i+1} \end{bmatrix}$$

$$= \left[T_i\right] \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ H_{i+1} \end{bmatrix} \tag{21}$$

For the complete shield,

$$\begin{bmatrix} I_1 \\ R_1 \\ H_1 \end{bmatrix} = \left[T_1\right] \left[T_2\right] \cdots \left[T_n\right] \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ 0 \end{bmatrix}$$

$$= \left[A\right] \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ 0 \end{bmatrix} \tag{22}$$

An alternate method of determining H_1 consists in determining only I_1 and R_1 by matrix methods and then computing H_1 from the third of equations (20).

Example 1. - Assume that an element of a uniform shield transmits 0.8 of incident radiation, back-scatters 0.1 of it, and absorbs the rest; the tolerance level is 0.0099 roentgen per hour. If the source power is 0.1626 roentgen per hour, how much heat does a 14-element shield absorb? How much heat does the source absorb?

The values of the power coefficients are

$$t = 0.8$$
$$r = 0.1$$
$$h = 1 - t - r = 1 - 0.8 - 0.1 = 0.1$$

Application of equation (22) yields the following data:

$$\begin{bmatrix} I_1 \\ R_1 \\ H_1 \end{bmatrix} = \left[T\right]^{14} \begin{bmatrix} 0.0099 \\ 0.0000 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1626 \\ 0.0476 \\ 0.0202 \end{bmatrix} \tag{22a}$$

Successive applications of equation (21) yield the following table:

Element	Heat absorbed, H_n (equivalent roentgens/hr)
14	0.0012
13	.0017
12	.0021
11	.0027
10	.0034
9	.0041
8	.0051
7	.0062
6	.0076
5	.0093
4	.0113
3	.0137
2	.0166
1	.0202
	0.1052 total heat absorbed in shield

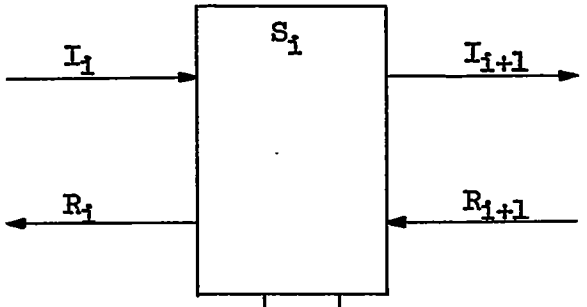
The amount of radiation absorbed as heat in the source is the amount back-scattered from the first element of the shield R_1; $R_1 = 0.0476$ roentgen per hour by calculation from equation (22a).

Case III - Polychromatic Radiation and Degradation of Energy

The following hypothesis is made:

Polychromatic radiation
Radiation normal to shield
Source of radiation outside shield
Radiation absorbed converted to heat and mass
Radiation degraded or upgraded in energy

Method. - The radiation incident on and back-scattered from an element of shielding is assumed to consist of a spectrum of radiation.



$$H_i = \overline{h}_i(I_i + R_{i+1})$$
$$M_i = \overline{m}_i(I_i + R_{i+1})$$

The power coefficients t_i and r_i take on different values for each portion of the spectrum incident on the element of the shield. For example, at an energy of 2.0 Mev, t_i might be 0.82 for S_i, whereas, at 1.8 Mev, t_i might be 0.81. In the same way, the proportions of radiation converted to heat and mass depend on the energy of radiation incident on the element. At 0.9 Mev, for example, $M_i = 0$ for gamma rays because no pair formation occurs.

The power coefficients $\overline{t_i}$, $\overline{r_i}$, $\overline{h_i}$, and $\overline{m_i}$ must consequently, be regarded as operators that convert one spectrum of radiation to another, such that the coefficients at each portion of the spectrum alter the corresponding intensities differently.

Some of the energy also must be degraded (neglecting nuclear reactions) after encountering an element of the shield. Thus, if a monochromatic beam of gamma radiation of 1 Mev encounters an element of a shield, much of the 1-Mev radiation passes through undisturbed in energy, but some radiation is degraded to the range of, perhaps, 0.9 to 1 Mev, some to the range 0.8 to 0.9 Mev, and so forth. For these calculations, it is convenient to regard an incident spectrum of radiation as consisting of a number of discrete energy ranges, each of a certain intensity. The power operators then convert each range to a number of other ranges of energy besides permitting some of the original range to pass through.

These operators may be set up as matrices. Let the incident spectrum of radiation be represented as

$$F(x) = a_1x_1 + a_2x_2 + \cdot \cdot \cdot + a_ix_i + \cdot \cdot \cdot + a_nx_n$$

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where x_i represents unit power at energy i. Then the array of intensity coefficients may be considered to be a matrix:

$$F = \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_i \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} \tag{23}$$

The transmission matrix may be written correspondingly in expanded matrix form:

$$\bar{t} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1i} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2i} & \cdots & t_{2n} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ t_{i1} & t_{i2} & \cdots & t_{ii} & \cdots & t_{in} \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ t_{n1} & t_{n2} & \cdots & t_{ni} & \cdots & t_{nn} \end{bmatrix} \tag{24}$$

where t_{ji} is that part of the operation that converts incident energy x_i to emergent energy x_j. The operation $\bar{t}F$ may be performed by matrix multiplication.

An examination of the matrix $\bar{t}$ reveals that the terms above the principal diagonal represent upgrading of energy as by nuclear reaction; the terms below the main diagonal represent degrading of energy; and the terms on the main diagonal represent transmission of radiation of unchanged energy. A common case is that for which no upgrading occurs. The matrix $\bar{t}$ takes on triangular form for that case. The operator $\bar{h}$ contains terms only on the principal diagonal.

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Example. - Let a hypothetical spectrum of radiation of the following energy distribution be incident on a shield element:

Range of incident energy (Mev)	Relative power of incident radiation
1.0-0.9	10
.9- .8	9
.8- .7	8
.7- .6	8
.6- .5	7
.5- .4	6
.4- .3	6
.3- .2	7
.2- .1	8
.1- .01	9
Total	78

This distribution appears in matrix form from equation (23):

$$F = \begin{bmatrix} 10 \\ 9 \\ 8 \\ 8 \\ 7 \\ 6 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Also given are the transmission, reflection, and absorption characteristics of a hypothetical shield element as shown in the following table:

Incident energy (Mev)	Power coefficients	Power factors of radiation emitted										
		Range of energy emitted (Mev)										
		1.0-0.9	0.9-0.8	0.8-0.7	0.7-0.6	0.6-0.5	0.5-0.4	0.4-0.3	0.3-0.2	0.2-0.1	0.1-0.01	Total
1.0-0.9	t	0.70	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.80
	r	.01	.07	.01	.01	.00	.00	.00	.00	.00	.00	.10
	h											.10
0.9-0.8	t		0.65	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.75
	r		.01	.07	.01	.01	.01	.01	.01	.01	.01	.15
	h											.10
0.8-0.7	t			0.60	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.70
	r			.01	.08	.01	.01	.01	.01	.02	.00	.15
	h											.15
0.7-0.6	t				0.50	0.01	0.02	0.02	0.02	0.02	0.01	0.80
	r				.02	.10	.03	.02	.02	.01	.00	.20
	h											.20
0.6-0.5	t					0.40	0.03	0.02	0.02	0.02	0.01	0.50
	r					.03	.10	.02	.02	.02	.01	.20
	h											.30
0.5-0.4	t						0.30	0.03	0.03	0.02	0.02	0.40
	r						.05	.15	.03	.01	.01	.25
	h											.35
0.4-0.3	t							0.20	0.04	0.03	0.03	0.30
	r							.10	.30	.05	.05	.50
	h											.20
0.3-0.2	t								0.15	0.05	0.05	0.25
	r								.10	.25	.05	.40
	h											.35
0.2-0.1	t									0.15	0.05	0.20
	r									.10	.20	.30
	h											.50
0.1-0.01	t										0.10	0.10
	r										.20	.20
	h											.70

The problem is to find the intensity distribution of the radiation transmitted and back-scattered, and to determine the power of radiation absorbed as heat.

From the data given,

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$$\overline{t}F = \begin{bmatrix} 0.70 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.65 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & 0.60 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & 0.01 & 0.50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.40 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.02 & 0.02 & 0.02 & 0.03 & 0.30 & 0 & 0 & 0 & 0 \\ 0.01 & 0.02 & 0.02 & 0.02 & 0.02 & 0.03 & 0.20 & 0 & 0 & 0 \\ 0.01 & 0.01 & 0.02 & 0.02 & 0.02 & 0.03 & 0.04 & 0.15 & 0 & 0 \\ 0.01 & 0.01 & 0.01 & 0.02 & 0.02 & 0.02 & 0.03 & 0.05 & 0.15 & 0 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.02 & 0.03 & 0.05 & 0.05 & 0.10 \end{bmatrix} \begin{bmatrix} 10 \\ 9 \\ 8 \\ 8 \\ 7 \\ 6 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 7.00+0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \\ 0.10+5.85+0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 \\ 0.10+0.09+4.80+0 + 0 + 0 + 0 + 0 + 0 + 0 \\ 0.10+0.09+0.08+4.00+0 + 0 + 0 + 0 + 0 + 0 \\ 0.10+0.09+0.08+0.08+2.80+0 + 0 + 0 + 0 + 0 \\ 0.20+0.18+0.16+0.16+0.21+1.80+0 + 0 + 0 + 0 \\ 0.10+0.18+0.16+0.16+0.14+0.18+1.20+0 + 0 + 0 \\ 0.10+0.09+0.16+0.16+0.14+0.18+0.24+1.05+0 + 0 \\ 0.10+0.09+0.08+0.16+0.14+0.12+0.18+0.35+1.20+0 \\ 0.10+0.09+0.08+0.08+0.07+0.12+0.18+0.35+0.40+0.90 \end{bmatrix}$$

The operations $\overline{r}F$ and $\overline{h}F$ may be performed in a similar manner. The results are summarized in the following table:

Incident energy (Mev)	Power emitted	Relative power of radiation transmitted, back-scattered, and absorbed										
		Range of energy (Mev)										
		1.0-0.9	0.9-0.8	0.8-0.7	0.7-0.6	0.6-0.5	0.5-0.4	0.4-0.3	0.3-0.2	0.2-0.1	0.1-0.01	Total
1.0-0.9	I	7.00	0.10	0.10	0.10	0.10	0.20	0.10	0.10	0.10	0.10	8.00
	R	.10	.70	.10	.10	.00	.00	.00	.00	.00	.00	1.00
	H											1.00
0.9-0.8	I		5.85	0.09	0.09	0.09	0.18	0.18	0.09	0.09	0.09	6.75
	R		.09	.63	.09	.09	.09	.09	.09	.09	.09	1.35
	H											.90
0.8-0.7	I			4.80	0.08	0.08	0.16	0.16	0.16	0.08	0.08	5.60
	R			.08	.64	.08	.08	.08	.08	.16	.00	1.20
	H											1.20
0.7-0.6	I				4.00	0.08	0.16	0.16	0.16	0.16	0.08	4.80
	R				.16	.80	.24	.16	.16	.08	.00	1.60
	H											1.60
0.6-0.5	I					2.80	0.21	0.14	0.14	0.14	0.07	3.50
	R					.21	.70	.14	.14	.14	.07	1.40
	H											2.10
0.5-0.4	I						1.80	0.18	0.18	0.12	0.12	2.40
	R						.30	.90	.18	.06	.06	1.50
	H											2.10
0.4-0.3	I							1.20	0.24	0.18	0.18	1.80
	R							.60	1.80	.30	.30	3.00
	H											1.20
0.3-0.2	I								1.05	0.35	0.35	1.75
	R								.70	1.75	.35	2.80
	H											2.45
0.2-0.1	I									1.20	0.40	1.60
	R									.80	1.60	2.40
	H											4.00
0.1-0.01	I										0.90	0.90
	R										1.80	1.80
	H											6.30
Total transmitted		7.00	5.95	4.99	4.27	3.15	2.71	2.12	2.12	2.42	2.57	37.10
Total back-scattered		.10	.79	.81	.99	1.18	1.41	1.97	3.15	3.38	4.27	18.06
Total absorbed												22.85

Equations may be written for the disposition of radiation incident on a shield element, as in previous cases. The figure illustrating case III gives

$$\left.\begin{aligned} I_{i+1} &= \bar{t}_i I_i + \bar{r}_i R_{i+1} \\ R_i &= \bar{t}_i R_{i+1} + \bar{r}_i I_i \\ M_i &= \bar{m}_i I_i + \bar{m}_i R_{i+1} \\ H_i &= \bar{h}_i I_i + \bar{h}_i R_{i+1} \end{aligned}\right\} \tag{25}$$

In the interpretation of these equations, the fact that $\bar{t}$, $\bar{r}$, $\bar{m}$, $\bar{h}$, I_i, I_{i+1}, R_i, R_{i+1}, M_i, and H_i are matrices must be kept in mind. In general, two matrices are noncommutative in multiplication; $\bar{t}_{i-1}\ \bar{t}_i I_i \neq \bar{t}_i \bar{t}_{i-1} I_i$ except for certain types of matrix if $\bar{t}_i$ and $\bar{t}_{i-1}$ are different. Practically, this noncommutativity means that an element of carbon in front of an element of lead, in general, would act differently on radiation from an element of lead in front of an element of carbon.

Before transposition of terms in equations (25), an examination into the existence of the reciprocal matrix and into the uniqueness of solution for the inverse operation when $\bar{t}$ is a singular matrix must be made. The reciprocal matrix exists if, and only if, the determinant of the original matrix is nonzero. The conditions of the problem state that no energy level is completely extinguished and consequently the diagonal elements of the original matrix are nonzero. A frequently occurring transmission matrix is that corresponding to the case of no upgrading of energy; the elements of the matrix above the principal diagonal are all zero so that the matrix is triangular. Inasmuch as a triangular matrix with all nonzero elements on the diagonal is never singular, the reciprocal matrix in this instance always exists.

A transmission matrix may be singular, then, only when upgrading of energy takes place in the transmission process. There are two practical methods by which a solution may be obtained when the transmission matrix is singular. The first method (reference 3) yields a solution in terms of a number of arbitrary parameters equal to the degeneracy of the matrix. These parameters, together with the boundary condition imposed by a knowledge of the spectrum incident on the first element, may yield a unique solution to the problem. The second method consists of the arbitrary adjustment of the elements of the transmission matrix so that it is no longer singular. The

number of elements to be adjusted is, in general, equal to the degeneracy of the transmission matrix. This distortion of data is insignificant if the adjustments are smaller than the experimental errors in determining the data.

Multiplication from the left of equations (25) by $\bar{t}_i^{-1}$ (the reciprocal of the matrix $\bar{t}_i$) and transposing yields

$$\left.\begin{aligned}
I_i &= \bar{t}_i^{-1} I_{i+1} - \bar{t}_i^{-1} \bar{r}_i R_{i+1} \\
R_i &= \bar{r}_i \bar{t}_i^{-1} I_{i+1} + (\bar{t}_i - \bar{r}_i \bar{t}_i^{-1} \bar{r}_i) R_{i+1} \\
M_i &= \bar{m}_i \bar{t}_i^{-1} I_{i+1} + (\bar{m}_i - \bar{m}_i \bar{t}_i^{-1} \bar{r}_i) R_{i+1} \\
H_i &= \bar{h}_i \bar{t}_i^{-1} I_{i+1} + (\bar{h}_i - \bar{h}_i \bar{t}_i^{-1} \bar{r}_i) R_{i+1}
\end{aligned}\right\} \tag{26}$$

If equations (26) are rewritten in matrix notation and if the bars over the symbols for the operators are dropped for convenience of notation,

$$\begin{bmatrix} I_i \\ R_i \\ M_i \\ H_i \end{bmatrix} = \begin{bmatrix} t_i^{-1} & -t_i^{-1} r_i & 0 & 0 \\ r_i t_i^{-1} & t_i - r_i t_i^{-1} r_i & 0 & 0 \\ m_i t_i^{-1} & m_i - m_i t_i^{-1} r_i & 0 & 0 \\ h_i t_i^{-1} & h_i - h_i t_i^{-1} r_i & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ M_{i+1} \\ H_{i+1} \end{bmatrix}$$

$$= \left[T_i\right] \begin{bmatrix} I_{i+1} \\ R_{i+1} \\ M_{i+1} \\ H_{i+1} \end{bmatrix} \tag{27}$$

and

$$\begin{bmatrix} I_1 \\ R_1 \\ \\ M_1 \\ H_1 \end{bmatrix} = \begin{bmatrix} T_1 \end{bmatrix} \begin{bmatrix} T_2 \end{bmatrix} \cdots \begin{bmatrix} T_n \end{bmatrix} \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

As usual, the sum of the elements in the matrices

$$\sum_{i=1}^{i=n} H_i$$

is the total heat absorbed by the shield.

It may be noted that a spectrum can be chosen for I_{i+1} that could not possibly have been emitted by a given shield upon which a spectrum I_i was incident. The following example will be examined for illustration: Let $R_{i+1} = 0$. Let I_{i+1} and I_i each consist of two energy ranges. Then

$$\bar{t}_i I_i = I_{i+1} \tag{28a}$$

Let

$$\bar{t} = \begin{bmatrix} 0.4 & 0 \\ 0.4 & 0.9 \end{bmatrix}$$

By substitution in equation (28a)

$$\begin{bmatrix} 0.4 & 0 \\ \\ 0.4 & 0.9 \end{bmatrix} \begin{bmatrix} z_{11} \\ \\ z_{12} \end{bmatrix} = \begin{bmatrix} 0.4x_{11} \\ \\ 0.4z_{11} + 0.9z_{12} \end{bmatrix} = I_{i+1}$$

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Obviously, I_{i+1} can never be chosen to be a spectrum whose upper-energy region is of higher power than its lower-energy region, because such a choice leads to the absurdity of a negative power for some range of energy in I_i. The problem of shielding polychromatic radiation must be solved by starting from only the incident radiation instead of from the emitted radiation in order to avoid such absurd results.

Let equation (28) be rewritten:

$$\begin{bmatrix} I_1 \\ R_1 \\ C_1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix} \begin{bmatrix} I_{n+1} \\ R_{n+1} \\ 0 \end{bmatrix} \tag{29}$$

where the over-all transmission matrix A has been partitioned into submatrices U_{ij}. Then

$$I_1 = U_{11}I_{n+1} + U_{12}R_{n+1}$$
$$R_1 = U_{21}I_{n+1} + U_{22}R_{n+1}$$

or

$$\left.\begin{aligned} I_{n+1} &= U_{11}^{-1}\, I_1 - U_{11}^{-1}\, U_{12}R_{n+1} \\ R_1 &= U_{21}U_{11}^{-1}\, I_1 + (U_{22} - U_{21}U_{11}^{-1}\, U_{12})R_{n+1} \end{aligned}\right\} \tag{30}$$

which is the required solution provided that the inverse matrices exist, inasmuch as all terms on the right side of equation (30) are positive or zero under the conditions of the problem. When the inverse matrices do not exist because the original matrices were singular, the methods of solution in terms of parameters or adjustment of the original matrices may be used.

Case IV - Gamma Plus Neutron Radiation

The following hypothesis is made:

Polychromatic gamma plus neutron radiation

Radiation normal to shield
Source of radiation outside shield
Radiation absorbed converted to energy other than gamma rays and neutrons
Gamma radiation gives rise to neutrons and neutrons give rise to gamma rays
Radiation degraded or upgraded in energy

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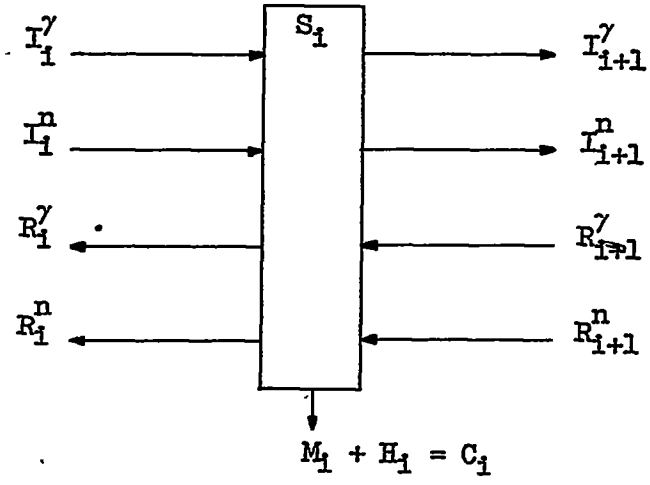
Method. - Incident gamma radiation I_i^γ and neutron radiation I_i^n are considered to be partly transmitted (with and without energy change), partly back-scattered, partly converted to mass and heat, and partly converted one to the other (γ to n and n to γ). In order to minimize the amount of notation, both M_i (radiation power converted to mass other than neutrons) and H_i (radiation power converted to heat) will be combined into one term C_i (converted power).



The following equations may then be set up (the bars over the operators $\bar{t}$, $\bar{r}$, and $\bar{c}$ will be dropped for convenience of notation in this case):

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$$I_{i+1}^{\gamma} = t_i^{\gamma} I_i^{\gamma} + t_i^{n\gamma} I_i^{n} + r_i^{\gamma} R_{i+1}^{\gamma} + r_i^{n\gamma} R_{i+1}^{n} \tag{31}$$

$$I_{i+1}^{n} = t_i^{n} I_i^{n} + t_i^{\gamma n} I_i^{\gamma} + r_i^{n} R_{i+1}^{n} + r_i^{\gamma n} R_{i+1}^{\gamma} \tag{32}$$

$$R_i^{\gamma} = t_i^{\gamma} R_{i+1}^{\gamma} + t_i^{n\gamma} R_{i+1}^{n} + r_i^{\gamma} I_i^{\gamma} + r_i^{n\gamma} I_i^{n} \tag{33}$$

$$R_i^{n} = t_i^{n} R_{i+1}^{n} + t_i^{\gamma n} R_{i+1}^{\gamma} + r_i^{n} I_i^{n} + r_i^{\gamma n} I_i^{\gamma} \tag{34}$$

$$C_i = c_i^{\gamma} \left(I_i^{\gamma} + R_{i+1}^{\gamma}\right) + c_i^{n} \left(I_i^{n} + R_{i+1}^{n}\right) \tag{35}$$

Equations (28) and (29) may be solved simultaneously for I_i^{γ} and I_i^{n}. These values may be substituted in equations (30) to (32) to yield values of R_i^{γ}, R_i^{n}, and C_i in terms of only I_{i+1}^{γ}, R_{i+1}^{γ}, I_{i+1}^{n}, and R_{i+1}^{n}. The resulting matrix has the form:

$$\begin{bmatrix} I_i^{\gamma} \\ I_i^{n} \\ R_i^{\gamma} \\ R_i^{n} \\ C_i \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} \end{bmatrix} \begin{bmatrix} I_{i+1}^{\gamma} \\ I_{i+1}^{n} \\ R_{i+1}^{\gamma} \\ R_{i+1}^{n} \\ C_{i+1} \end{bmatrix} \tag{36}$$

and

$$\begin{bmatrix} I_1^{\gamma} \\ I_1 \\ R_1^{\gamma} \\ R_1^{n} \\ C_1 \end{bmatrix} = \left[T_1\right] \left[T_2\right] \cdots \left[T_n\right] \begin{bmatrix} I_{n+1}^{\gamma} \\ I_{n+1}^{n} \\ R_{n+1}^{\gamma} \\ R_{n+1}^{n} \\ 0 \end{bmatrix} = \left[A\right] \begin{bmatrix} I_{n+1}^{\gamma} \\ I_{n+1}^{n} \\ R_{n+1}^{\gamma} \\ R_{n+1}^{n} \\ 0 \end{bmatrix} \tag{37}$$

which is the required solution.

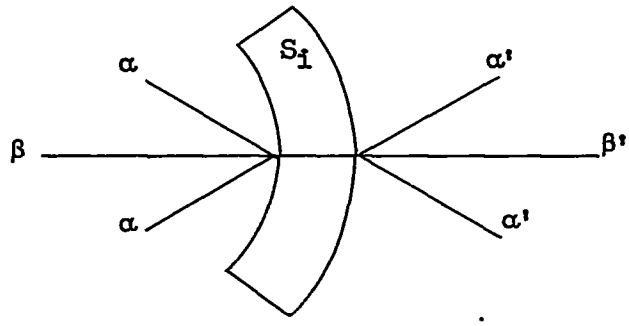
The calculations here are lengthy because of frequent occurrence of inverse operators. Machine methods are necessary for handling a case of this complexity.

Case V - Radiation Not Normal to Shield

The following hypothesis is made: 983

Polychromatic radiation
Radiation not normal to shield (spherical shield)
Source of radiation outside shield
Radiation absorption neglected
Radiation degraded in energy
Inverse-square decrease in power neglected

A spherical shield was chosen because reasons of symmetry allow neglect of the displacement of radiation from the normal to the shield that is caused by scattering at an angle other than 180°. In order to handle directional scattering, the incident radiation may be assumed to arrive at a unit area of the shield S_i in a number of conical zones identified as α and β when two zones are chosen. An equal number of zones of emergence are chosen and designated by primes.



The equation for scattered radiation may consequently be set up:

$$I_{i+1,\alpha'} = \bar{t}_{\alpha\alpha'} I_{i,\alpha} + \bar{t}_{\beta\alpha'} I_{i,\beta} + \bar{r}_{\beta'\alpha'} R_{i+1,\beta'} + \bar{r}_{\alpha'\alpha'} R_{i+1,\alpha'}$$

Expressions for $I_{i+1,\beta'}$, $R_{i,\alpha}$, and $R_{i,\beta}$ may be similarly set up in matrix form. The coefficients are not all independent (neglecting Bragg reflections):

$$\bar{t}_{\beta\alpha'} = \bar{t}_{\alpha\beta'}$$

$$\bar{r}_{\beta\alpha} = \bar{r}_{\beta'\alpha'}$$

By consideration of the difference in path length in traversing the element at angles greater than 90°, $\bar{t}_{\beta\beta'}$ and $\bar{t}_{\alpha\alpha'}$ can be related. The coefficients for the path lengths that are not integral multiples of the thickness of the standard element can be obtained by determining a fractional power of the transmission matrix. The question arises as to whether this fractional power of a matrix is unique for shielding applications where all elements of matrices must be positive and real, and for special matrices such as triangular matrices that result when energy is not upgraded. These and other questions that must be considered when analytically determining the transmission matrix for an element that is not an integral multiple of the standard thickness will not be considered because it was assumed that the data could be experimentally obtained.

A question that must be considered is whether degeneracy exists to any greater extent in transmission matrices of the $\bar{t}_{\alpha\beta'}$ and $\bar{r}_{\alpha\beta}$ types than in the $\bar{t}_{\alpha\alpha'}$ type, and what are the consequences of this greater degeneracy, if any. It is physically apparent that inasmuch as radiation in the incident and emergent beams are no longer in the same direction for the $\bar{t}_{\alpha\beta'}$ type of transmission, no condition on the problem prevents complete extinguishing. In two cases, however, this extinguishing will not occur: (1) When adequate amount of unmodified scattering is obtained; and (2) when modified scattering does not degrade energy out of initial energy range (with upgrading of energy neglected). For these cases, the transmission matrix is never singular. In general, however, more singular transmission matrices of the $\bar{t}_{\alpha\beta'}$ type may be expected than of the $\bar{t}_{\alpha\alpha'}$ type.

DISCUSSION

A limited number of cases have been presented to indicate the wide variety of problems that may be handled by matrix methods. The inverse-square decrease in intensity with distance from the source has been neglected. This decrease can be taken into account by multiplying the radiation power at any station in the shield by the ratio of the area of the first element to the area of the element at the given station. Thus, if I_1 and R_1 were determined

at the surface of a sphere 2 feet in diameter and I_n and R_n at a sphere 6 feet in diameter, the calculated values of I_n and R_n would be multiplied by 1/9 to include the inverse-square decrease.

Data on the directional distribution of intensity and energy at high energy are required for gamma rays. Many available theoretical expressions for scattering and absorption may be extrapolated to higher energies to provide these data. Equations such as the Klein-Nishina equations (reference 4) appear to be adequate, at least in the range of energy that has been experimentally checked. Experimental data exist in the form of tables, particularly for wavelengths above 0.005 A. One of the most comprehensive compilations of mass-absorption coefficients for high-energy radiations may be found in reference 5. Such mass-absorption coefficients yield information on the amounts of radiation transmitted in a primary beam through a thin absorber for many elements and a number of wavelengths above 0.005 A. The radiation considered absorbed includes not only such radiation as may be converted to heat, but also radiation diffracted by the crystal lattice of the absorber, radiation deviated from the primary beam by Compton scattering, and some radiation participating in pair formation (a factor almost negligible below the maximum energy of 2.5 Mev tabulated). Combinations of experimental and theoretical data yield truer results than either alone.

Data are required only for scattering and absorption by chemical elements and not by compounds and mixtures because interaction between nuclear radiation and solids is negligibly affected by binding forces between atoms or by the physical state of the element. The elementary thickness of absorber may consequently be regarded as that thickness that contains a weight of element per unit area that is regarded as standard. For example, if 2 grams per square centimeter of element A and 1 gram per square centimeter of element B are standard where the atomic weight of B is, say, twice that of A, then the compound AB in thickness g grams per square centimeter may be regarded as containing alternately standard thickness of A and B for a total thickness of g/3 elements of thickness each of A and B.

The inherent advantage of requiring data for only one thickness of absorber is tempered by the fact that some thicknesses may be inadequate for accurately determining variation of spectral distribution of energy with angle of scattering. Different thicknesses of absorber may be required to obtain experimental data in some instances.

It should be noted that elements of low atomic number do not necessarily possess poorer mass-absorption coefficients than elements

of high atomic number for X-rays and gamma rays despite the fact that the low-number elements do absorb most poorly at wavelengths of 0.02 A and below (neglecting absorption edges). Hydrogen is a notable exception of this sort inasmuch as the data of reference 5 show that hydrogen possesses a mass-absorption coefficient 64 percent greater than that of lead at a radiation of 0.010-A wavelength, and 83 percent greater at 0.005 A. At 0.005 A, the coefficient of carbon is only 10 percent lower than that of lead. At 0.005 A, a long-chain hydrocarbon that might be regarded as possessing essentially twice as many hydrogen atoms as carbon atoms would possess two parts by weight that absorb 83 percent more than lead, and 12 parts by weight that absorb 10 percent less than lead -- a combination that absorbs gamma rays as well as lead, weight for weight.

Machine methods for solution of matrix multiplications are certainly necessary for comprehensive solutions. If, for example, both neutron and gamma radiation are considered, if heat and mass conversion are desired, and if five zones of scattering are chosen, a 30th-order square matrix is set up, each term of which is a matrix of order equal to the number of energy zones into which the spectrum is divided. For a 5-foot thickness of shielding, 60 matrix multiplications are required if the element is 1 inch in thickness; 600 if the element is 0.1 inch in thickness.

The type of composite shield that should be used would be determined by trial-and-error calculations. A single material might be used for a first computation. Upon analysis of that hypothesis, observation might reveal that the spectral distribution of neutrons and gamma radiation is such at any station in the shield that the succeeding elements might well be another material better fitted for absorbing or degrading energy. The calculation would then be repeated and the result reanalyzed. Such calculations, although lengthy for accurate results, might be more economical than experimental verification of the adequacy of complete shields.

A number of simplifications to the exact treatment of the shielding problem can be made by making approximations on some of the shielding processes. The following table lists some changes that readily suggest themselves in order to reduce the amount of computation (all the suggested approximations yield an oversafe shield, that is, thicker or heavier than required):

Exact mechanisms	Approximating assumptions
Radiation is scattered continuously at angles to primary direction of incident radiation	That small number of finite zones exist; that radiation may be allocated only to zones having equal or lesser path length through shield element than in true case; that all radiation scattered passes normally through element.
Spectrum of radiation is scattered	That all radiation is monochromatic, if possible of that energy at which minimum absorption occurs.
Radiation is degraded or acted upon differently according to energy	That radiation is degraded or acted upon by constant factor independent of energy.

CONCLUSION

An analysis of the shielding of radiation indicates that matrix methods appear to be immediately applicable for solution of shielding problems, provided that adequate experimental data on scattering and absorption for standard thickness of absorbers are available.

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