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FORMULAS AND TABLES OF COEFFICIENTS FOR NUMERICAL DIFFERENTIATION WITH FUNCTION VALUES GIVEN AT UNEQUALLY SPACED POINTS AND APPLICATION TO SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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FORMULAS AND TABLES OF COEFFICIENTS FOR NUMERICAL DIFFERENTIATION WITH FUNCTION VALUES GIVEN AT UNEQUALLY SPACED POINTS AND APPLICATION TO SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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SUMMARY

General differentiation formulas for successive derivatives of a function are obtained in terms of the values of the function at unequally spaced arguments and the corresponding distances between the successive arguments using Lagrangian polynomials of various degrees. The remainder term is also obtained. Tables of coefficients in the formulas for the first four derivatives are given in intervals of 0.01 for the special case where only one spacing at either end is different from the others, as is often encountered near a curved boundary, for different ratios of this spacing to the others.

A general discussion of applying these formulas to the numerical solution of partial differential equations is made. In particular, the application to the equations of the elliptic type is illustrated with a problem involving derivatives of both first and second order and with the value of the function given on a circular boundary. After replacing the derivatives by the formulas based on fourth-degree polynomials, the resulting set of equations is solved by both relaxation and matrix methods. Similar application to problems of compressible flow past isolated and cascade airfoils and through turbomachines, and temperature distribution and thermal stress in cooled turbine blades, is indicated.

INTRODUCTION

In recent years, use of numerical methods in the solution of partial differential equations that are intractable by exact analytical methods increased. In these methods, a network of pivotal points over the domain of the problem is considered and the differential equation is replaced by its finite-difference expressions that involve the values of the "wanted function" at these points.

In general, the accuracy of the solution depends on the fineness of the network and on the accuracy of the finite-difference expressions used. For the case where the spacing between the points is everywhere the same, differentiation formulas based on polynomials up to the ninth degree are available (reference 1). In most practical problems, however, a part of or the whole boundary line of the domain is often curved, which renders the spacing near the boundary different from that elsewhere. In the past, a linear or higher-order extrapolation formula was usually used to obtain, from the given boundary value, the value at a fictitious point outside the boundary at a distance from the first point inside the boundary equal to that between all the other points (references 2 and 3). In reference 4, a differentiation formula based on unequal distances between the points is obtained for the second-order derivative using a second-degree polynomial, but the error term is not given. It is felt that a need exists for general formulas for computing different orders of derivatives by using the values of functions at nonequidistant points based on polynomials of various degrees. Such formulas will not only help to solve more conveniently those problems with curved boundaries, especially for computation on large-scale digital computing machines, but will also help to reduce the number of points involved in a problem by using relatively small spacings only where needed. The formulas will also help to determine more accurately the non-homogeneous terms in the differential equation that are not given but are to be obtained through differentiation during the calculation.

In connection with the theoretical investigation being conducted at the NACA Lewis laboratory on fluid flow past cascades of arbitrary airfoils and through turbomachines with arbitrary hub and casing shapes, general differentiation formulas are obtained in terms of the values of the function at non-equidistant points and the corresponding distances between the successive points by using Lagrangian polynomials of various degrees. The error term is also obtained. Tables of coefficients for the first four derivatives for the special case where only one spacing at one end is different from the others, as often encountered near a curved boundary, are computed for different ratios of the unequal distance to the equal distances from 0.1 to 1.29 in intervals of 0.01.

A general discussion of the application of these formulas to the numerical solutions of partial differential equations is presented, including the advantages of using a higher-degree polynomial. The method is illustrated with a problem involving a second-order partial differential equation with a circular boundary, which is treated in reference 3 by the use of a difference-correction method. After the

derivatives are replaced by fourth-degree differentiation formulas, the resulting equations are solved by both relaxation and matrix methods. The application to fluid flow past isolated and cascade airfoils and through turbomachines and to temperature distribution and thermal stress in cooled turbine blades are indicated.

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SYMBOLS

The following symbols are used in this report:

${}^{m}_{n}A^{i}_{j}$	differentiation coefficients in equation (15) used to multiply function value at point x_j to give m^{th} derivative at x_i using n^{th}-degree polynomial
a,b,c,d	successive distances between x_0, x_1, x_2, x_3, and x_4
${}^{m}_{n}B^{i}_{j}$	${}^{m}_{n}A^{i}_{j} \Big/ \Pi'_{n+1}(x_j)$
${}^{m}_{n}C^{i}_{j}$	${}^{m}_{n}B^{i}_{j} \Big/ b^{m}$
$D^{m}y$	m^{th} derivative of y with respect to x
$[d]$	diagonal matrix
E	first term in R
F,G,H,J,K	functions of η and ζ
$[I]$	unit matrix
J_0	Bessel function of first kind of order zero
$[L]$	lower triangular matrix
$[l]$	lower triangular matrix with unit diagonal elements
$[M]$	coefficient matrix
m	order of derivative
N	number of grid points

n	degree of polynomial
$\{P\},\{Q\},\{S\},$ $\{T\},\{W\}$	column matrices
q	velocity
${}^{m}R_{n}^{i}$	remainder term of mth derivative at point x_i obtained by using n^{th}-degree polynomial
r,z	polar coordinates
$[u]$	upper triangular matrix with unit diagonal elements
$[V]$	composite matrix
x	independent variable
y	dependent variable
$[\alpha]$	submatrix
β	order of $[\alpha]$
γ	ratio of specific heats
η,ζ	rectangular coordinates
λ	constant
ξ	value of x lying between greatest and least of x
$\Pi_{n+1}(x)$	product of $(n+1)$ factors, $(x-x_0)$, $(x-x_1)$, . . . $(x-x_n)$
ρ	density
φ	dependable variable
ψ	stream function

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Superscripts:

1,2, . . . N	interior points
a,b,c,d, . . .	boundary points
', ", "', iv	first to fourth-order derivatives
n	n^{th} derivative

Subscripts:

i,j,k,l,p,q	denote value at given arguments
0	stagnation value

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GENERAL DIFFERENTIATION FORMULAS

If $y = f(x)$ is a function of x and the value of y at $n+1$ arguments or points $x_0, x_1, \ldots x_i, \ldots x_n$ are known, the general Lagrangian interpolation polynomial of degree n is (reference 5)

$$y = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \ldots +$$

$$\frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n + \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)(x-x_1)\ldots(x-x_n) \quad (1)$$

in which the last term is the remainder term with the value of ξ lying between the least and the greatest of the numbers $x, x_0, x_1, \ldots x_n$. By using

$$\Pi_{n+1}(x) = (x-x_0)(x-x_1)\ldots(x-x_n) \quad (2)$$

and its first derivative

$$\Pi'_{n+1}(x) = \Pi_{n+1}(x)\sum_{k=0}^{n}\frac{1}{x-x_k} \quad (3)$$

equation (1) can be written as

$$y = \sum_{j=0}^{n} \frac{\Pi_{n+1}(x)}{x-x_j} \frac{y_j}{\Pi'_{n+1}(x_j)} + {}_n^0R \tag{4}$$

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where

$${}_n^0R = \frac{\Pi_{n+1}(x)}{(n+1)!} f^{(n+1)}(\xi) \tag{5}$$

and

$$\Pi'_{n+1}(x_j) = (x_j-x_0)(x_j-x_1) \ . \ . \ . \ (x_j-x_{j-1})(x_j-x_{j+1}) \ . \ . \ . \ (x_j-x_n) \tag{6}$$

From equations (3), (4), and (5), the formulas for the successive derivatives of y with respect to x at any point x can be obtained as follows:

$$D^1y = \sum_{j=0}^{n} \left[\frac{y_j}{\Pi'_{n+1}(x_j)} \frac{\Pi_{n+1}(x)}{x-x_j} \sum_{\substack{k=0 \\ k\neq j}}^{n} \frac{1}{x-x_k} \right] + {}_n^1R \tag{7}$$

where

$${}_n^1R = \frac{\Pi'_{n+1}(x)}{(n+1)!} f^{(n+1)}(\xi) + \frac{\Pi_{n+1}(x)}{(n+1)!} f^{(n+2)}(\xi) \frac{d\xi}{dx} + \ . \ . \ . \tag{8}$$

$$D^2y = 2! \sum_{j=0}^{n} \left[\frac{y_j}{\Pi'_{n+1}(x_j)} \frac{\Pi_{n+1}(x)}{x-x_j} \sum_{\substack{k=0 \\ k\neq j}}^{n} \left(\frac{1}{x-x_k} \sum_{\substack{l=k+1 \\ l\neq j}}^{n} \frac{1}{x-x_l} \right) \right] + {}_n^2R \tag{9}$$

where

$${}_n^2R = \frac{\Pi''_{n+1}(x)}{(n+1)!} f^{(n+1)}(\xi) + 2 \frac{\Pi'_{n+1}(x)}{(n+1)!} f^{(n+2)}(\xi) \frac{d\xi}{dx} + \ . \ . \ . \tag{10}$$

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$$D^3y = 3!\sum_{j=0}^{n}\left\{\frac{y_j}{\Pi'_{n+1}(x_j)}\frac{\Pi_{n+1}(x)}{x-x_j}\sum_{\substack{k=0\\k\neq j}}^{n}\left[\frac{1}{x-x_k}\sum_{\substack{l=k+1\\l\neq j}}^{n}\left(\frac{1}{x-x_l}\sum_{\substack{p=l+1\\p\neq j}}^{n}\frac{1}{x-x_p}\right)\right]\right\} + {}^{3}_{n}R \tag{11}$$

where

$${}^{3}_{n}R = \frac{\Pi'''_{n+1}(x)}{(n+1)!}f^{(n+1)}(\xi) + 3\,\frac{\Pi''_{n+1}(x)}{(n+1)!}\,f^{(n+2)}(\xi)\,\frac{d\xi}{dx} + \ldots \tag{12}$$

$$D^4y = 4!\sum_{j=0}^{n}\left(\frac{y_j}{\Pi'_{n+1}(x_j)}\frac{\Pi_{n+1}(x)}{x-x_j}\right.$$

$$\left.\sum_{\substack{k=0\\k\neq j}}^{n}\left\{\frac{1}{x-x_k}\sum_{\substack{l=k+1\\l\neq j}}^{n}\left[\frac{1}{x-x_l}\sum_{\substack{p=l+1\\p\neq j}}^{n}\left(\frac{1}{x-x_p}\sum_{\substack{q=p+1\\q\neq j}}^{n}\frac{1}{x-x_q}\right)\right]\right\}\right) + {}^{4}_{n}R \tag{13}$$

where

$${}^{4}_{n}R = \frac{\Pi^{iv}_{n+1}(x)}{(n+1)!}f^{(n+1)}(\xi) + 4\,\frac{\Pi'''_{n+1}(x)}{(n+1)!}\,f^{(n+2)}(\xi)\,\frac{d\xi}{dx} + \ldots \tag{14}$$

and so forth.

In the preceding formulas of the remainder terms, only the first two terms are given. The value of ξ is unknown. For the case where $f^{(n+1)}(x)$ and $f^{(n+2)}(x)$ do not vary much over the range of x, they may be estimated by the values at the point x in question by using appropriate differentiation formulas.

For the case where the derivatives are required only at the given arguments x_i, the fact that most of the terms containing $\Pi_{n+1}(x_i)$ vanish greatly simplifies these formulas. The resulting expression for the first derivative agrees with that given in reference 5. For the present work, it is found to be more convenient to keep the general form as previously given, which enables the same operation at all points. Thus, the mth derivative of y at x_i based on a polynomial of nth degree $(n \geq m)$ may be written as 1411

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{n} \frac{{}^m_n A^i_j \; y_j}{\Pi'_{n+1}(x_j)} + {}^m_n R^i \tag{15}$$

$$= \sum_{j=0}^{n} {}^m_n B^i_j \; y_j + {}^m_n R^i \tag{16}$$

For $m = 1$

$${}^1_n A^i_j = \frac{\Pi_{n+1}(x_i)}{x_i - x_j} \sum_{\substack{k=0 \\ k \neq j}}^{n} \frac{1}{x_i - x_k} \tag{17}$$

$${}^1_n R^i = \frac{\Pi'_{n+1}(x_i)}{(n+1)!} f^{(n+1)}(\xi) + \frac{\Pi_{n+1}(x_i)}{(n+1)!} f^{(n+2)}(\xi) \frac{d\xi}{dx} + \ldots \tag{18}$$

For $m = 2$,

$${}^2_n A^i_j = \frac{\Pi_{n+1}(x_i)}{x_i - x_j} \sum_{\substack{k=0 \\ k \neq j}}^{n} \left(\frac{1}{x_i - x_k} \sum_{\substack{l=k+1 \\ l \neq j}}^{n} \frac{1}{x_i - x_l} \right) \tag{19}$$

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$$ {}^{2}_{n}R^{i} = \frac{\Pi''_{n+1}(x_i)}{(n+1)!} f^{(n+1)}(\xi) + 2 \frac{\Pi'_{n+1}(x_i)}{(n+1)!} f^{(n+2)}(\xi) \frac{d\xi}{dx} + \ldots \tag{20} $$

and so forth. With this setup, the expression of the coefficients A in terms of the given unequal distances between the points can be obtained very easily; for example, if the successive spacings between five points x_0, x_1, x_2, x_3, and x_4 are a, b, c, and d, resively, the coefficients A can be obtained in the following manner: First, the expressions of $\Pi_5(x_i)$, including the factor zero for $i = 0,1,2,3,4$, are written. For example, at $i = 1$, $\Pi_5(x_1)$ is $a(0)(-b)(-b-c)(-b-c-d)$. Coefficients ${}^{1}_{4}A^{1}_{j}$ for $j = 0,1,2,3,4$ are then obtained simply by taking out the appropriate x_1-x_j factor from $\Pi_5(x_1)$ and applying the summation operation. At all of the other points except at $j = i = 1$, all of the terms except one vanish because of the factor zero in $\Pi_5(x_1)$, with the result ${}^{1}_{4}A^{1}_{j}$ equal to $-b(b+c)(b+c+d)$; $ab(b+c) + ab(b+c+d) + a(b+c)(b+c+d) - b(b+c)(b+c+d)$; $a(b+c)(b+c+d)$; $ab(b+c+d)$; and $ab(b+c)$ at $j = 0,1,2,3,4$, respectively. The other coefficients can be obtained in a similar manner.

The expressions of the various coefficients in formula (15) in terms of the unequal spacings a,b, . . . between successive arguments, using three, four, and five points, respectively, are given in the following formulas:

General Three-Point Differentiation Formula

$$ (D^m y)_{x=x_i} = m! \sum_{j=0}^{2} \frac{{}^{m}_{2}A^{i}_{j}\, y_j}{\Pi'_3(x_j)} + {}^{m}_{2}R^{i} $$

m	i	A_0	A_1	A_2	$E/f^{(3)}(\xi)$
1	0	$-(2a+b)$	$-(a+b)$	$-a$	$a(a+b)/6$
	1	$-b$	$a-b$	a	$-ab/6$
	2	b	$a+b$	$a+2b$	$(a+b)b/6$
2	0	1	1	1	$-(2a+b)/3$
	1	1	1	1	$(a-b)/3$
	2	1	1	1	$(a+2b)/3$

$\Pi'_3(x_0)$	$\Pi'_3(x_1)$	$\Pi'_3(x_2)$
$a(a+b)$	$-ab$	$(a+b)b$

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General Four-Point Differentiation Formula

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{3} \frac{{}_3A_j^{\,m}\, y_j}{\Pi_4'(x_j)} + {}_mR_3^{\,i}$$

m	i	A_0	A_1	A_2	A_3	$E/f^{(4)}(\xi)$
1	0	$a(a+b)+a(a+b+c)+(a+b)(a+b+c)$	$(a+b)(a+b+c)$	$a(a+b+c)$	$a(a+b)$	$-\frac{a(a+b)(a+b+c)}{24}$
	1	$b(b+c)$	$-ab-a(b+c)+b(b+c)$	$-a(b+c)$	$-ab$	$\frac{ab(b+c)}{24}$
	2	$-bc$	$-(a+b)c$	$(a+b)b-(a+b)c-bc$	$(a+b)b$	$-\frac{(a+b)bc}{24}$
	3	$(b+c)c$	$(a+b+c)c$	$(a+b+c)(b+c)$	$(a+b+c)(b+c)+(a+b+c)c+(b+c)c$	$\frac{(a+b+c)(b+c)c}{24}$
2	0	$-a-(a+b)-(a+b+c)$	$-(a+b)-(a+b+c)$	$-a-(a+b+c)$	$-a-(a+b)$	$\frac{1}{12}\left[a(a+b)+a(a+b+c)+(a+b)(a+b+c)\right]$
	1	$-b-(b+c)$	$a-b-(b+c)$	$a-(b+c)$	$a-b$	$\frac{1}{12}\left[-ab-a(b+c)+b(b+c)\right]$
	2	$b-c$	$(a+b)-c$	$(a+b)+b-c$	$(a+b)+b$	$\frac{1}{12}\left[(a+b)b-(a+b)c-bc\right]$
	3	$(b+c)+c$	$(a+b+c)+c$	$(a+b+c)+(b+c)$	$(a+b+c)+(b+c)+c$	$\frac{1}{12}\left[(a+b+c)(b+c)+(a+b+c)c+(b+c)c\right]$
3	0	1	1	1	1	$-\frac{a+(a+b)+(a+b+c)}{4}$
	1	1	1	1	1	$\frac{a-b-(b+c)}{4}$
	2	1	1	1	1	$\frac{(a+b)+b-c}{4}$
	3	1	1	1	1	$\frac{(a+b+c)+(b+c)+c}{4}$

$\Pi_4'(x_0)$	$\Pi_4'(x_1)$	$\Pi_4'(x_2)$	$\Pi_4'(x_3)$
$-a(a+b)(a+b+c)$	$ab(b+c)$	$-(a+b)bc$	$(a+b+c)(b+c)c$

General Five-Point

$$(\Pi^m y)_{x=x_i} = m! \sum_{j=0}^{4} \frac{{}^{m}_{4}A_j^{\,i}\, y_j}{\Pi_5'(x_j)} + {}^{m}_{4}R^{i}$$

m	i	A_0	A_1	A_2
1	0	$\left[\begin{array}{l} a(a+b)(a+b+c)+ \\ a(a+b)(a+b+c+d)+ \\ a(a+b+c)(a+b+c+d)+ \\ (a+b)(a+b+c)(a+b+c+d) \end{array}\right]$	$-(a+b)(a+b+c)(a+b+c+d)$	$-a(a+b+c)(a+b+c+d)$
	1	$-b(b+c)(b+c+d)$	$\left[\begin{array}{l} ab(b+c)+ab(b+c+d)+ \\ a(b+c)(b+c+d)- \\ b(b+c)(b+c+d) \end{array}\right]$	$a(b+c)(b+c+d)$
	2	$bc(c+d)$	$(a+b)c(c+d)$	$\left[\begin{array}{l} -(a+b)bc-(a+b)b(c+d)+ \\ (a+b)c(c+d)+bc(c+d) \end{array}\right]$
	3	$-(b+c)cd$	$-(a+b+c)cd$	$-(a+b+c)(b+c)d$
	4	$(b+c+d)(c+d)d$	$(a+b+c+d)(c+d)d$	$(a+b+c+d)(b+c+d)d$
2	0	$\left[\begin{array}{l} a(a+b)+a(a+b+c)+a(a+b+c+d)+ \\ (a+b)(a+b+c)+(a+b)(a+b+c+d)+ \\ (a+b+c)(a+b+c+d) \end{array}\right]$	$\left[\begin{array}{l} (a+b)(a+b+c)+ \\ (a+b)(a+b+c+d)+ \\ (a+b+c)(a+b+c+d) \end{array}\right]$	$\left[\begin{array}{l} a(a+b+c)+a(a+b+c+d)+ \\ (a+b+c)(a+b+c+d) \end{array}\right]$
	1	$\left[\begin{array}{l} b(b+c)+b(b+c+d)+ \\ (b+c)(b+c+d) \end{array}\right]$	$\left[\begin{array}{l} -ab-a(b+c)-a(b+c+d)+ \\ b(b+c)+b(b+c+d)+ \\ (b+c)(b+c+d) \end{array}\right]$	$\left[\begin{array}{l} -a(b+c)-a(b+c+d)+ \\ (b+c)(b+c+d) \end{array}\right]$
	2	$-bc-b(c+d)+c(c+d)$	$\left[\begin{array}{l} -(a+b)c-(a+b)(c+d)+ \\ c(c+d) \end{array}\right]$	$\left[\begin{array}{l} (a+b)b-(a+b)c-(a+b)\times \\ (c+d)-bc-b(c+d)+c(c+d) \end{array}\right]$
	3	$(b+c)c-(b+c)d-cd$	$(a+b+c)c-(a+b+c)d-cd$	$\left[\begin{array}{l} (a+b+c)(b+c)-(a+b+c)d- \\ (b+c)d \end{array}\right]$
	4	$\left[\begin{array}{l} (b+c+d)(c+d)+(b+c+d)d+ \\ (c+d)d \end{array}\right]$	$\left[\begin{array}{l} (a+b+c+d)(c+d)+ \\ (a+b+c+d)d+(c+d)d \end{array}\right]$	$\left[\begin{array}{l} (a+b+c+d)(b+c+d)+ \\ (a+b+c+d)d+(b+c+d)d \end{array}\right]$
3	0	$-a-(a+b)-(a+b+c)-(a+b+c+d)$	$\left[\begin{array}{l} -(a+b)-(a+b+c)- \\ (a+b+c+d) \end{array}\right]$	$-a-(a+b+c)-(a+b+c+d)$
	1	$-b-(b+c)-(b+c+d)$	$a-b-(b+c)-(b+c+d)$	$a-(b+c)-(b+c+d)$
	2	$b-c-(c+d)$	$(a+b)-c-(c+d)$	$(a+b)+b-c-(c+d)$
	3	$(b+c)+c-d$	$(a+b+c)+c-d$	$(a+b+c)+(b+c)-d$
	4	$(b+c+d)+(c+d)+d$	$(a+b+c+d)+(c+d)+d$	$(a+b+c+d)+(b+c+d)+d$
4	0	1	1	1
	1	1	1	1
	2	1	1	1
	3	1	1	1
	4	1	1	1

$\Pi_5'(x_0)$	$\Pi_5'(x_1)$	$\Pi_5'(x_2)$
$a(a+b)(a+b+c)(a+b+c+d)$	$-ab(b+c)(b+c+d)$	$(a+b)bc(c+d)$
1, 2, 3, 4	1, 2, 3, 4	2, 3, 3, 4

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Differentiation Formula

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{4} \frac{{}_m A_j^i \, y_j}{\Pi_5'(x_j)} + {}_m R^i$$

A_3	A_4	$E/f^{(5)}(\xi)$
$-a(a+b)(a+b+c+d)$	$-a(a+b)(a+b+c)$	$\frac{1}{120}\, a(a+b)(a+b+c)(a+b+c+d)$
$ab(b+c+d)$	$ab(b+c)$	$-\frac{1}{120}\, ab(b+c)(b+c+d)$
$-(a+b)b(c+d)$	$-(a+b)bc$	$\frac{1}{120}\, (a+b)(bc(c+d)$
$\left[(a+b+c)(b+c)c-(a+b+c)\times (b+c)d-(a+b+c)cd-(b+c)cd\right]$	$(a+b+c)(b+c)c$	$-\frac{1}{120}\, (a+b+c)(b+c)cd$
$(a+b+c+d)(b+c+d)(c+d)$	$\left[(a+b+c+d)(b+c+d)(c+d)+ (a+b+c+d)(b+c+d)d+ (a+b+c+d)(c+d)d+(b+c+d)(c+d)d\right]$	$\frac{1}{120}\, (a+b+c+d)(b+c+d)(c+d)d$
$\left[a(a+b)+a(a+b+c+d)+ (a+b)(a+b+c+d)\right]$	$a(a+b)+a(a+b+c)+ (a+b)(a+b+c)$	$-\frac{1}{60}\left[a(a+b)(a+b+c)+a(a+b)(a+b+c+d)+ a(a+b+c)(a+b+c+d)+(a+b)(a+b+c)(a+b+c+d)\right]$
$\left[-ab-a(b+c+d)+ b(b+c+d)\right]$	$-ab-a(b+c)+b(b+c)$	$\frac{1}{60}\left[ab(b+c)+ab(b+c+d)+a(b+c)(b+c+d)- b(b+c)(b+c+d)\right]$
$\left[(a+b)b-(a+b)(c+d)- b(c+d)\right]$	$(a+b)b-(a+b)c-bc$	$\frac{1}{60}\left[-(a+b)bc-(a+b)b(c+d)+ (a+b)c(c+d)+bc(c+d)\right]$
$\left[(a+b+c)(b+c)+(a+b+c)c- (a+b+c)d+(b+c)c-(b+c)d-cd\right]$	$(a+b+c)(b+c)+(a+b+c)c+(b+c)c$	$\frac{1}{60}\left[(a+b+c)(b+c)c-(a+b+c)(b+c)d- (a+b+c)cd-(b+c)cd\right]$
$\left[(a+b+c+d)(b+c+d)+(a+b+c+d)\times (c+d)+(b+c+d)(c+d)\right]$	$\left[(a+b+c+d)(b+c+d)+(b+c+d)(c+d)+ (a+b+c+d)(c+d)+(b+c+d)d+ (a+b+c+d)d+(c+d)d\right]$	$\frac{1}{60}\left[(a+b+c+d)(b+c+d)(c+d)+(a+b+c+d)(b+c+d)d+ (a+b+c+d)(c+d)d+(b+c+d)(c+d)d\right]$
$-a-(a+b)-(a+b+c+d)$	$-a-(a+b)-(a+b+c)$	$\frac{1}{20}\left[a(a+b)+a(a+b+c)+a(a+b+c+d)+(a+b)(a+b+c)+ (a+b)(a+b+c+d)+(a+b+c)(a+b+c+d)\right]$
$a-b-(b+c+d)$	$a-b-(b+c)$	$\frac{1}{20}\left[-ab-a(b+c)-a(b+c+d)+b(b+c)+ b(b+c+d)+(b+c)(b+c+d)\right]$
$(a+b)+b-(c+d)$	$(a+b)+b-c$	$\frac{1}{20}\left[(a+b)b-(a+b)c-(a+b)(c+d)- bc-b(c+d)+c(c+d)\right]$
$(a+b+c)+(b+c)+c-d$	$(a+b+c)+(b+c)+c$	$\frac{1}{20}\left[(a+b+c)(b+c)+(a+b+c)c -(a+b+c)d+(b+c)c-(b+c)d-cd\right]$
$(a+b+c+d)+(b+c+d)+(c+d)$	$(a+b+c+d)+(b+c+d)+(c+d)+d$	$\frac{1}{20}\left[(a+b+c+d)(b+c+d)+(b+c+d)(c+d)+(a+b+c+d)(c+d)+ (b+c+d)d+(a+b+c+d)d+(c+d)d\right]$
1	1	$-\frac{1}{5}\left[a+(a+b)+(a+b+c)+(a+b+c+d)\right]$
1	1	$\frac{1}{5}\left[a-b-(b+c)-(b+c+d)\right]$
1	1	$\frac{1}{5}\left[(a+b)+b-c-(c+d)\right]$
1	1	$\frac{1}{5}\left[(a+b+c)+(b+c)+c-d\right]$
1	1	$\frac{1}{5}\left[(a+b+c+d)+(b+c+d)+(c+d)+d\right]$

$\Pi_5'(x_3)$	$\Pi_5'(x_4)$
$-(a+b+c)(b+c)cd$	$(a+b+c+d)(b+c+d)(c+d)d$

$3, 3, 3_2 4_2$ $\quad 4_0 \quad 4_1 \quad 4_2$

The corresponding simplified expressions for the special case where only the first spacing is different from the rest, as often encountered near the boundary of a problem, are given in the following formulas: In these formulas, r is equal to the ratio of a to the other uniform spacings $b = c = d \ldots$. For ready applications, the differentiation coefficients and the coefficient of the first remainder term of this special case are computed for a range of r from 0.10 to 1.29, in intervals of 0.01 and are given in the attached tables. 1411

Three-Point Differentiation Formula for a=rb

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{2} \frac{{}_m A_j^i \, y_j}{\Pi_3'(x_j)} + {}_m R^i$$

m	i	A_0	A_1	A_2	$E/f^{(3)}(\xi)$
1	0	$-(1+2r)b$	$-(1+r)b$	$-rb$	$\frac{1}{6} r(1+r)b^2$
	1	$-b$	$-(1-r)b$	rb	$-\frac{1}{6} rb^2$
	2	b	$(1+r)b$	$(2+r)b$	$\frac{1}{6} (1+r)b^2$
2	0	1	1	1	$-\frac{1}{3} (1+2r)b$
	1	1	1	1	$-\frac{1}{3} (1-r)b$
	2	1	1	1	$+\frac{1}{3} (2+r)b$

$\Pi_3'(x_0)$	$\Pi_3'(x_1)$	$\Pi_3'(x_2)$
$r(1+r)b^2$	$-rb^2$	$(1+r)b^2$

Four-Point Differentiation Formula for $a=rb=rc$

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{3} \frac{{}^{m}_{3}A_j^{\,i}\, y_j}{\Pi_4'(x_j)} + {}^{m}_{3}R^{i}$$

m	i	A_0	A_1	A_2	A_3	$E/f^{(4)}(\xi)$
1	0	$\left[r(2+r)+2(1+r)^2\right]b^2$	$(1+r)(2+r)b^2$	$r(2+r)b^2$	$r(1+r)b^2$	$-\frac{r(1+r)(2+r)b^3}{24}$
	1	$2b^2$	$(2-3r)b^2$	$-2rb^2$	$-rb^2$	$\frac{2rb^3}{24}$
	2	$-b^2$	$-(1+r)b^2$	$-b^2$	$(1+r)b^2$	$-\frac{(1+r)b^3}{24}$
	3	$2b^2$	$(2+r)b^2$	$2(2+r)b^2$	$(8+3r)b^2$	$\frac{2(2+r)b^3}{24}$
2	0	$-3(1+r)b$	$-(3+2r)b$	$-2(1+r)b$	$-(1+2r)b$	$\frac{\left[r(2+r)+2(1+r)^2\right]b^2}{12}$
	1	$-3b$	$-(3-r)b$	$-(2-r)b$	$-(1-r)b$	$\frac{(2-3r)b^2}{12}$
	2	0	rb	$(1+r)b$	$(2+r)b$	$-\frac{b^2}{12}$
	3	$3b$	$(3+r)b$	$(4+r)b$	$(5+r)b$	$\frac{(8+3r)b^2}{12}$
3	0	1	1	1	1	$-\frac{3(1+r)b}{4}$
	1	1	1	1	1	$-\frac{(3-r)b}{4}$
	2	1	1	1	1	$\frac{(1+r)b}{4}$
	3	1	1	1	1	$\frac{(5+r)b}{4}$

$\Pi_4'(x_0)$	$\Pi_4'(x_1)$	$\Pi_4'(x_2)$	$\Pi_4'(x_3)$
$-r(1+r)(2+r)b^3$	$2rb^3$	$-(1+r)b^3$	$2(2+r)b^3$

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Five-Point Differentiation Formula for aerbarcard

$$(D^m y)_{x=x_i} = m! \sum_{j=0}^{4} \frac{{}^m_4 A_j^i \, y_j}{\Pi_5'(x_j)} + {}^m_4 R^i$$

m	i	A_0	A_1	A_2	A_3	A_4	$E/f^{(5)}(\xi)$
1	0	$\begin{bmatrix}(1+r)(2+r)(3+r)+\\ r(2+r)(3+r)+\\ r(1+r)(3+r)+\\ r(1+r)(2+r)\end{bmatrix}b^3$	$-(1+r)(2+r)(3+r)b^3$	$-r(2+r)(3+r)b^3$	$-r(1+r)(3+r)b^3$	$-r(1+r)(2+r)b^3$	$\frac{1}{120}\,r(1+r)(2+r)(3+r)b^4$
	1	$-6b^3$	$(-6+11r)b^3$	$(6r)b^3$	$(3r)b^3$	$2rb^3$	$-\frac{1}{20}\,rb^4$
	2	$2b^3$	$2(1+r)b^3$	$(1-r)b^3$	$-2(1+r)b^3$	$-(1+r)b^3$	$\frac{1}{60}\,(1+r)b^4$
	3	$-2b^3$	$-(2+r)b^3$	$-2(2+r)b^3$	$-(4+r)b^3$	$2(2+r)b^3$	$-\frac{1}{60}\,(2+r)b^4$
	4	$6b^3$	$2(3+r)b^3$	$3(3+r)b^3$	$6(3+r)b^3$	$(39+11r)b^3$	$\frac{1}{20}\,(3+r)b^4$
2	0	$\begin{bmatrix}(1+r)(5+2r)+\\ (2+r)(3+4r)\end{bmatrix}b^2$	$\begin{bmatrix}(1+r)(5+2r)+\\ (2+r)(3+r)\end{bmatrix}b^2$	$\begin{bmatrix}r(5+2r)+\\ (2+r)(3+r)\end{bmatrix}b^2$	$\begin{bmatrix}2r(2+r)+\\ (1+r)(3+r)\end{bmatrix}b^2$	$\begin{bmatrix}r(3+2r)+\\ (1+r)(2+r)\end{bmatrix}b^2$	$-\frac{1}{60}\begin{bmatrix}(1+r)(2+r)(3+r)+\\ r(2+r)(3+r)+\\ r(1+r)(3+r)+\\ r(1+r)(2+r)\end{bmatrix}b^3$
	1	$11b^2$	$(11-6r)b^2$	$(6-5r)b^2$	$(3-4r)b^2$	$(2-3r)b^2$	$\frac{1}{60}\,(-6+11r)b^3$
	2	$-b^2$	$-(1+3r)b^2$	$-(3+2r)b^2$	$-(3+r)b^2$	$-b^2$	$\frac{1}{60}(1-r)b^3$
	3	$-b^2$	$-b^2$	rb^2	$(3+2r)b^2$	$(8+3r)b^2$	$-\frac{1}{60}\,(4+r)b^3$
	4	$11b^2$	$(11+3r)b^2$	$(15+4r)b^2$	$(21+5r)b^2$	$(29+6r)b^2$	$\frac{1}{60}\,(39+11r)b^3$
3	0	$-(6+4r)b$	$-(6+3r)b$	$-(5+3r)b$	$-(4+3r)b$	$-3(1+r)b$	$\frac{1}{20}\begin{bmatrix}(1+r)(5+2r)+\\ (2+r)(3+4r)\end{bmatrix}b^2$
	1	$-6b$	$-(6-r)b$	$-(5-r)b$	$-(4-r)b$	$-(3-r)b$	$\frac{1}{20}\,(11-6r)b^2$
	2	$-2b$	$-(2-r)b$	$-(1-r)b$	rb	$(1+r)b$	$-\frac{1}{20}\,(3+2r)b^2$
	3	$2b$	$(2+r)b$	$(3+r)b$	$(4+r)b$	$(5+r)b$	$\frac{1}{20}\,(3+2r)b^2$
	4	$6b$	$(6+r)b$	$(7+r)b$	$(8+r)b$	$(9+r)b$	$\frac{1}{20}\,(29+6r)b^2$
4	0	1	1	1	1	1	$-\frac{1}{5}\,(6+4r)b$
	1	1	1	1	1	1	$-\frac{1}{5}\,(6-r)b$
	2	1	1	1	1	1	$-\frac{1}{5}\,(1-r)b$
	3	1	1	1	1	1	$\frac{1}{5}\,(4+r)b$
	4	1	1	1	1	1	$\frac{1}{5}\,(9+r)b$

$\Pi_5'(x_0)$	$\Pi_5'(x_1)$	$\Pi_5'(x_2)$	$\Pi_5'(x_3)$	$\Pi_5'(x_4)$
$r(1+r)(2+r)(3+r)b^4$	$-6rb^4$	$2(1+r)b^4$	$-2(2+r)b^4$	$6(3+r)b^4$

In all formulas and tables, E denotes the first term in the remainder series. If the value of r of a given problem lies between computed intervals, either linear or higher-order interpolation, depending upon the value of r and the number of significant figures required, may be used to obtain the desired value. In the coefficient tables where $r = 1$, the first term in the remainder series of some derivatives is equal to zero, therefore the second remainder term is required. Because of the unknown nature of $d\xi/dx$ in the second remainder term, the error term listed in the coefficient tables is obtained by using a Taylor series expansion at the point x_i in question.

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Although the tables of coefficients are computed for the case where the first interval is different from the others, they may also be used for the case where the last interval is different from the others by taking the coefficient in the reversed order and multiplying by $(-1)^m$, as indicated by the headings at the bottom of these tables. These two headings could be used together to get a short interval for interpolation in the case of three-point formulas.

It may be noted that in the formulas, the coefficients A satisfy the following relation:

$$m\,{}^{m}_{n}A^{i}_{i} = \sum_{\substack{j=0\\ j\neq i}}^{n} {}^{m}_{n}A^{i}_{j} \tag{21}$$

and that in the attached tables, the coefficients ${}^{m}_{n}B^{i}_{j}$ satisfy the following relations

$$\sum_{j=0}^{n} {}^{m}_{n}B^{i}_{j} = 0 \tag{22}$$

and

$$\sum_{j=0}^{n} (x_j - \lambda)^k\, {}^{m}_{n}B^{i}_{j} = \begin{matrix} 1 & (m = k) \\ 0 & (m > k) \end{matrix} \tag{23}$$

where λ is any convenient constant.

APPLICATION TO SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

The numerical solution of partial differential equations consists of two main steps; namely, replacing the differential equation by its finite-difference expression at a number of chosen nodal points and solving the unknowns from the resulting set of algebraic equations. The differentiation formulas are presented herein in order to obtain more accurate and convenient expressions of the finite-difference equations near a curved boundary and also to enable the use of a network of varying spacings, which may be very desirable in the machine computation of problems involving a large number of points, such as encountered in fluid-flow problems. 1411

Consider the following type of partial differential equation of the second order frequently encountered in engineering problems:

$$F \frac{\partial^2\varphi}{\partial\eta^2} + G \frac{\partial\varphi}{\partial\eta} + H \frac{\partial^2\varphi}{\partial\zeta^2} + J \frac{\partial\varphi}{\partial\zeta} + K = 0 \tag{24}$$

where F, G, H, J, and K are functions of the two independent variables η and ζ. In order to use the differentiation formula (16), it is convenient to choose a rectangular grid over the whole region. (See fig. 1.) Substituting equation (16) (without the remainder terms) into equation (24) in each direction at a point, the φ value of which is φ^i, results in the following algebraic equation:

$$\sum_{j=0}^{n} \left(F^i \, {}^{2}_{n}B^i_j + G^i \, {}^{1}_{n}B^i_j\right) \varphi^j + \sum_{k=0}^{n} \left(H^i \, {}^{2}_{n}B^i_k + J^i \, {}^{1}_{n}B^i_k\right) \varphi^k + K^i = 0 \tag{25}$$

where φ^j and φ^k denote φ values along the η and ζ directions, respectively. (See fig. 1 for a typical point near boundary with $n = 4$.)

Obviously, only when the remainder terms ${}^{2}_{n}R^i$ and ${}^{1}_{n}R^i$ are negligible, compared with the main terms in equation (16), will equation (25) represent the original equation (24) at these points with sufficient accuracy, and the solution obtained by solving the set of equations (25) covering the whole region will give a good approximate solution of the original differential equation (24). When formulas (18) and (20) and the E terms listed in the tables of coefficients are referred to, it is seen that the magnitude of the coefficient of the remainder term is controlled entirely by the degree of the polynomial used and the grid spacing chosen.

For n^{th}-degree polynomial representation, derivatives of orders higher than n are involved in the remainder terms; $\Pi'_{n+1}(x)$, $\Pi''_{n+1}(x)$, . . . are of the order of the spacing raised to the n, $(n-1)$ . . . power, respectively; and the denominator $(n+1)!$ increases rapidly with
1411 increasing values of n. Because numerical calculation is usually performed with dimensionless quantities, with the distance expressed in terms of a principal dimension of the problem, the spacing involved in actual calculation is usually less than 1. Hence the higher power on the spacing resulting from a higher-degree polynomial representation reduces the magnitude of the remainder term. Consequently, a higher-degree polynomial representation will always permit a smaller number of grid points necessary for a given accuracy, if the successive derivatives do not increase faster than the combined effect of a decreasing $\Pi'_{n+1}(x)$ . . . in the numerator and an increasing $(n+1)!$ in the denominator of the coefficient. For example, a comparison of the remainder term of the equally spaced fourth-degree or five-point formula at the off-center points x_1 and x_3 with the second-degree or three-point formula at the central point x_1 indicates that, for the same accuracy, the five-point formula requires a grid spacing not smaller than that required by the three-point formula raised to two-thirds power if the fifth derivative is not greater than the fourth derivative. Under such a condition, if the grid spacing required for a given desired accuracy by using the three-point formula is 0.1, the spacing required by using five-point formula is 0.215, which means a reduction in the number of grid points of four and eight times can be expected by going from three-point formula to five-point formula in a two-dimensional and three-dimensional problem, respectively. This saving certainly justifies the use of slightly more complicated formulas and is especially important for fluid-flow problems where a very large region must be included in the calculation. This situation is amply demonstrated in reference 3, where it is found in various problems that the inclusion of differences up to the fourth order with a relatively coarse grid gives more accurate results than using only second-degree polynomial representation with a network four times as fine.

In most practical problems, the successive derivatives decrease in magnitude. There are, however, problems that include regions where the successive derivatives increase rapidly in magnitude. The only way to make the remainder term negligible then is to make the spacing small enough. One interesting example of this nature is given in reference 6, where the function $y = (1+x^2)^{-1}$ has a successive even-order derivative that increases rapidly at $x = 0$. For a spacing of 1, it was shown that at $x = 0$ the accuracy of the computed derivative decreases with the increasing degree of polynomial representation. This decrease is due to

the fact that the increase of the derivatives is greater than the increase in $(n+1)!$. With the spacing of 1, however, none of the three polynomials used is good enough for actual calculation. When the spacing is reduced to one-half, the fourth-degree polynomial gives better results in several places, but still none of them gives acceptable accuracy. (See table I.) When the interval is further reduced to one-fourth, the fourth-degree polynomial gives best results everywhere and is the only one that gives acceptable accuracy over the whole range. When the interval is still further reduced to one-eighth, the fourth-degree polynomial gives values accurate to within 1 percent everywhere except for y'' at $x = -1$. The second-degree polynomials, however, give results not quite as good as those given by the fourth-degree polynomial with twice that spacing. Thus, when a small enough spacing (0.25 or 0.125) is used, the coefficient of the error term decreases with increasing order of polynomial representation (because of the higher power on the spacing term and the larger value of the factorial term in the denominator) at a rate sufficiently fast to overcome the increase of the successive derivatives at $x = 0$.

Before starting computation of a new problem, it is therefore always desirable to make a difference analysis of a known solution of a similar problem or of an approximate solution of the given problem to see approximately how the wanted function varies over the entire domain of the problem. In most practical problems, the successive derivatives usually decrease over most of the regions and it is always advantageous to use higher-degree polynomial representation with relatively coarse spacing. If successive derivatives are expected to increase rapidly in certain regions, however, it is essential to use very small spacings in those regions. In fluid-flow problems, where it is necessary to extend the domain of calculation far away from the object to take care of the boundary conditions, it would be desirable to use an increasingly large spacing away from the object.

After the number of nodal points of a given problem and the value of n at these points are chosen, the appropriate method of solving the resulting set of equations (25) is determined by the type of the given differential equation. If the original equation is of the elliptic type with the value of the function or its normal derivatives specified on a closed boundary, the set of equations (25) covering the whole region is to be solved simultaneously by either indirect or direct methods. If the original differential equation is of the parabolic or hyperbolic type where initial values of the function are given on a part of the boundary, the step-by-step forward integration procedure is appropriate (along characteristic lines in the case of hyperbolic equations).

In problems of the boundary-value type, the relaxation method developed by Southwell (reference 2) has proved to be much superior to other iterative methods and has been used to solve many important engineering problems (references 2, 3, and 7 to 10). The differentiation formula obtained herein will help to obtain equation (25) more accurately and conveniently at the boundary and to reduce the labor of computation by adopting a network of varying spacing in problems involving large domains.

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In problems such as fluid flow over an isolated airfoil or a cascade of airfoils including a large number of nodal points, the work involved is still great even with an increasingly large spacing away from the airfoils. Although this tremendous amount of hand computing may be avoided by solving the problems on a modern high-speed large-scale digital computing machine, the machine can only be set up in a relatively simple way either with a straight iteration method of Liebmann or the faster relaxation method and many passes have to be made over the network to reduce error everywhere to an allowable limit. Because of the recent finding that the previous error estimate of the direct solution of a set of algebraic equations by the elimination technique is far too pessimistic (references 11 to 14), and because the coefficient matrix of equation (25), although of very large order, contains only a few nonzero elements in each row, thus giving a much smaller effective order of the matrix, it is quite conceivable that two- and three-dimensional-flow problems involving grid points up to a few hundred can be solved very rapidly with acceptable accuracy by this technique on the digital machines.

The simplest way of directly solving equations (25) seems to be as follows: Equations (25) are first written in the following matrix form: All the unknown φ's at the N interior points are denoted by superscripts 1, 2, . . . and the given boundary values are denoted with superscripts a, b, . . . w, as shown in figure 2. (The order of numbering is from top to bottom and from left to right.) Then at each interior point, there is one equation (25). When the combined coefficients of unknown φ's in the equation at φ^i point are denoted by M^i_j, and the sum of K^i and the product of the known boundary φ's and the corresponding coefficient are denoted as P^i, the N sets of equation can be written in the compact matrix form

$$M^i_j \ \varphi^j = P^i$$

or

$$\left[M\right] \ \left\{\varphi\right\} = \left\{P\right\} \tag{26}$$

where $[M]$, $\{\varphi\}$, and $\{P\}$ are square and column matrices of order N.

The nature of the coefficient matrix M, from the problem involving 29 unknown φ's shown in figure 2, is indicated in figure 3. (The central-point five-point formula is used everywhere except at the points next to the boundary.) In general, the largest number in each row is located at the diagonal line. It is clearly indicated in figure 3 that the matrix M can be considered as composed of $6 \times 6 = 36$ submatrices; the ones on the diagonal line are square matrices of order five and contains only four zero elements and the remaining ones are diagonal matrices also of order five. These submatrices are designated $[\alpha]$ in figure 3. Wherever a curved boundary exists, a part of these submatrices is cut off, as is α_j^6. In case the three-point formula is used throughout, the order of matrix M necessarily becomes several times larger and there will be only one line of diagonal submatrices running along each side of the main ones on the main diagonal. 1411

In equation (26), if the coefficient matrix M is nonsingular, a unique solution for φ always exists. Although there are a number of methods to solve this matrix equation (references 12 and 13), the following method is found to be the best in taking advantage of the large number of zero elements in the given coefficient matrix M to reduce to a minimum the number of operations and recordings of the present problem. This method involves the resolution of the coefficient matrix M into the product of an upper triangular and a lower triangular matrix as follows: From the theorem of triangular resolution of a matrix (references 11 and 12),

$$[M] = [l][d][u]$$

where $[l]$ and $[u]$ are lower and upper triangular matrices both with unit elements along the diagonal, and $[d]$ is a diagonal matrix. It is found convenient to combine $[l]$ and $[d]$ into a lower triangular matrice $[L]$, which yields

$$[M] = [L][u] \tag{28}$$

The elements of matrices $[L]$ and $[u]$ can be obtained by the ordinary matrix multiplication rule as follows (superscripts and subscripts indicate the number of the row and the column, respectively):

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(a) $L_1^1 = M_1^1 \quad L_1^2 = M_1^2 \; . \; . \; . \quad \text{or} \quad L_1^i = M_1^i$

(b) $u_2^1 = \dfrac{M_2^1}{L_1^1} \quad u_3^1 = \dfrac{M_3^1}{L_1^1} \; . \; . \; . \quad \text{or} \quad u_j^1 = \dfrac{M_j^1}{L_1^1}$

(c) $L_2^2 = M_2^2 - L_1^2 u_2^1 \quad L_2^3 = M_2^3 - L_1^3 u_2^1 \; . \; . \; . \quad \text{or} \quad L_2^i = M_2^i - L_1^i u_2^1$

(d) $u_3^2 = \dfrac{M_3^2 - L_1^2 u_3^1}{L_2^2} \quad u_4^2 = \dfrac{M_4^2 - L_1^2 u_4^1}{L_2^2} \; . \; . \; . \quad \text{or} \quad u_j^2 \quad \dfrac{M_j^2 - L_1^2 u_j^1}{L_2^2}$

(e) $L_3^3 = M_3^3 - L_1^3 u_3^1 - L_2^3 u_3^2 \quad L_3^4 = M_3^4 - L_1^4 u_3^1 - L_2^4 u_3^2 \; . \; . \; . \quad \text{or}$

$$L_3^i = M_3^i - L_1^i u_3^1 - L_2^i u_3^2$$

(f) $u_4^3 = \dfrac{M_4^3 - L_1^3 u_4^1 - L_2^3 u_4^2}{L_3^3} \quad u_5^3 = \dfrac{M_5^3 - L_1^3 u_5^1 - L_2^3 u_5^2}{L_3^3} \; . \; . \; . \quad \text{or}$

$$u_j^3 = \frac{M_j^3 - L_1^3 u_j^1 - L_2^3 u_j^2}{L_3^3}$$

and so forth. The sequence of operation is always (1) obtain L along a column, and (2) obtain u along a row. The general operation can simply be written as

(1) Operation along a column j:

$$L_j^i = M_j^i - \sum_{k=1}^{j-1} L_k^i u_j^k \quad (i \geq j) \tag{29}$$

(2) Operation along a row i:

$$u_j^i = \frac{M_j^i - \sum_{k=1}^{i-1} L_k^i u_j^k}{L_i^i} \qquad (j > i) \tag{30}$$

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The nature of $[L]$ and $[u]$ are indicated in figure 4. In fact, a composite matrix $[V] = [L] + [u] - [I]$ can be used in place of separate matrices $[L]$ and $[u]$ to facilitate recording. Then both L and u in equations (29) and (30) are replaced by V. The composite matrix is called auxiliary matrix in reference 15, where formulas similar to equations (29) and (30) were obtained by an induction process. The composite matrix V is shown in figure 5. It may be noted that in the case of problems where there are more unknowns in the central region than shown in figure 2, matrix V will have more rows like those from row 11 to row 25. The difference between these typical rows from those above and below them is due to the unsymmetrical formulas at points near to the left and right boundaries.

When $[L]$ and $[u]$ are obtained, the solution of the set of equations involves the following forward and backward substitutions: Defining a column matrix Q of the same order N as P yields

$$[L]\ \{Q\} = \{P\} \tag{31}$$

giving

$$Q^1 = \frac{P^1}{L_1^1}$$

$$Q^2 = \frac{P^2 - L_1^2 Q^1}{L_2^2}$$

or, in general,

$$Q^i = \frac{P^i - \sum_{j=1}^{i-1} L_j^i Q^j}{L_i^i} \tag{32}$$

The unknown matrix φ is then obtained by

$$[u] \{\varphi\} = \{Q\} \tag{33}$$

that is,

$$\varphi^N = Q^N$$

$$\varphi^{N-1} = Q^{N-1} - u_N^{N-1} Q^N$$

or, in general,

$$\varphi^i = Q^i - \sum_{j=N}^{i+1} u_j^i Q^j \tag{34}$$

When equation (32) is compared with equation (30), it is seen that Q^i is obtained in precisely the same manner as u^i. The column matrix P can therefore be added to the right of matrix M and treated in the same manner as the last column of M (fig. 6). In addition, it is advisable to add a checking column S to M defined by

$$S^i = \sum_{j=1}^{N} M_j^i + P^i \tag{35}$$

Then one more column T is obtained behind Q, and W behind φ. (See fig. 6.) The formulas for checking are

$$T^i = 1 + \sum_{j=i+1}^{N+1} V_j^i \tag{36}$$

and

$$W^i = 1 + \varphi^i \tag{37}$$

If several sets of values of $\{P\}$ are given at the same time, they can be placed behind $[M]$ in the beginning of computation, thus obtaining several sets of φ at the same time. This situation arises when solutions are required for different boundary values or different values

of $K(\eta, \zeta)$ in the original differential equation (25). If solution is required for a different value of $\{P\}$ any time after the first solution is obtained, the matrix V is always available and the work required is only the forward and backward substitution processes. In all cases, the principal work involved is the transformation of matrix M to matrix V. Once this transformation is done for a given problem, solutions for different $\{P\}$'s can be obtained very quickly. 1411

This method can be extended to nonlinear differential equations, where the coefficients F, G, H, J, and K depend on φ. One way of extending the method is to recompute the values of F, G, H, J, and K after a solution of φ is obtained. Recomputing means, however, repetition of the whole process as shown in figure 6. For some types of elliptic differential equation such as the one encountered in subsonic flow, a much simpler method is possible. For example, the equations defining the irrotational motion of a compressible nonviscous fluid are

$$\frac{\partial}{\partial \eta}\left(\frac{1}{\rho}\frac{\partial \psi}{\partial \eta}\right) + \frac{\partial}{\partial \zeta}\left(\frac{1}{\rho}\frac{\partial \psi}{\partial \zeta}\right) = 0 \tag{38}$$

and

$$\frac{\rho}{\rho_0} = \left[1 - \frac{\gamma-1}{2}\left(\frac{q}{a_0}\right)^2\right]^{\frac{1}{\gamma-1}} \tag{39}$$

where ψ is the stream function defined by

$$q_\eta = \frac{1}{\rho}\frac{\partial \psi}{\partial \zeta} \qquad q_\zeta = -\frac{1}{\rho}\frac{\partial \psi}{\partial \eta} \tag{40}$$

Equation (38) can be written as (reference 8)

$$\frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial^2 \psi}{\partial \zeta^2} - \frac{1}{\rho}\left(\frac{\partial \rho}{\partial \eta}\frac{\partial \varphi}{\partial \eta} + \frac{\partial \rho}{\partial \zeta}\frac{\partial \psi}{\partial \zeta}\right) = 0 \tag{41}$$

By taking the term involving density as constant for each calculation and as the K term in equation (25), a matrix M is obtained that is determined only by the geometric shape of the problem. The matrix V can then be used to compute successively improved values of $\{\psi\}$ using successively improved values of $\{P\}$, which contains the successively improved values of K (or ρ). For subsonic flow, the variation of density with velocity is limited, thus this process of treating the nonlinearity converges rapidly (references 9 and 10). This method is

especially convenient when a large number of cases for a given isolated or cascade airfoil is required, such as with various stream Mach numbers in which case the matrices M and V remain fixed.

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It is believed that the method described herein will find increasing use in problems of fluid flow past isolated and cascade airfoils, fluid flow through turbomachines; temperature distribution and thermal stress in a cooled turbine blade, and many other boundary-value problems. For an ordinary completely filled matrix of order N, the number of multiplication and division processes involved in the matrix solution is of the order of $\left(\frac{N^3}{3} + N^2\right)$ for obtaining $[V]$, and N^2 for obtaining $\{Q\}$ and $\{\varphi\}$. Because of the large number of zero elements in $[M]$, the corresponding number of multiplication and division processes involved using the five-point differentiation formula is of the order of $(4\beta^2 N+4\beta N)$ for $[V]$, and $4\beta N$ for $\{Q\}$ and $\{\varphi\}$, where β is of the order of the submatrix α. (If the three-point differentiation formula is used, there are $(\beta^2 N+4\beta N)$ and $2\beta N$ operations, respectively, with N necessarily several times larger.) This reduction in the effective order of the matrix is very helpful in reducing rounding-off errors. If the present digital computing machine is designed to handle an ordinary matrix of order of 100 (reference 11), it may therefore be expected to handle the special matrix of the present problems of the order of several hundred, which is sufficient for most fluid-flow problems.

Numerical Example

In reference 3, the function φ, which satisfies the differential equation

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{\partial^2\varphi}{\partial z^2} = 0 \tag{42}$$

and takes the values

$$\varphi = 10{,}000\ J_0\ (3r)\ e^{3z} \tag{43}$$

on the boundary of a circle with center at $r = 1.5$, $z = 0$, and radius of 0.5 (fig. 7), is obtained by an improved relaxation method with difference correction. Accurate results were obtained in reference 3 by the inclusion of differences up to the fourth order, even with a relatively coarse network. Because of the curved boundary all around, it was felt that it

would be more convenient to use the differentiation coefficients obtained herein in order to avoid the extrapolation for both φ and high-order differences near the boundary. In order to compare the results on the same basis, the same network involving 16 interior points and the five-point differentiation formula (corresponding to the inclusion of differences up to the fourth order) are used. The solution is obtained by both relaxation and matrix methods. 1411

Relaxation solution. - Basically, this problem involves only two values of a, (0.1 and 0.18990), differing from the uniform spacing $b = 0.2$ (fig. 7). The spacing ratio is therefore 0.5 and 0.94950, respectively. The differentiation coefficients for 0.5 can be obtained directly from the coefficient tables, and the coefficients for 0.94950 can be taken, for engineering accuracy, as equal to those for 0.95 given in the coefficient tables. For the present calculation, however, it is better to compute these coefficients more accurately from the formulas so that any difference in the result will not be attributed to this approximation. After the coefficients are obtained, they are combined according to equation (42) and are given in figure 8. The corresponding relaxation pattern is given in figure 9.

It is convenient to obtain the solution in two steps. In the first step only the five or seven larger coefficients of the nine coefficients at each point are used to compute the residual and to improve the φ value at each point. After the required change of φ at all points is reduced to the order of 10, new residuals are computed and φ values are improved using all nine coefficients. The computation is stopped when the required change of φ is less than 1.

Matrix solution. - Because of the possible accumulation of rounding-off error in the matrix calculation, the boundary values are computed to five significant figures and the elements of coefficient matrix M are given to five decimal places. Matrix P is given to two decimal places, giving at least six significant numbers. It was originally intended to keep five decimal places in the matrix V calculation also, but it later developed that some elements in u become very small and the computation was extended to seven decimal places, giving a minimum of three significant figures in one place. In spite of this decrease in significant figures, the substitution of the φ values obtained into the original equation gives a new $\{P\}$, $\{P'\}$, which checks with the original $\{P\}$ to within two on the fifth significant number. The composite matrix V and the column matrices Q, φ, and P' are all given in table II. Calculations were also made keeping five significant figures everywhere, which still gave φ accurate to within 1.3 percent.

A comparison of solutions obtained by relaxation and matrix solutions using the differentiation formula (16), the relaxation solution with difference correction (reference 3), and the exact values are given in table III. The first two solutions agree very well and can be taken as the values the fourth-degree piece-wise polynomial approximation and the grid size used will give for this problem. Although these values are, in general, accurate enough for engineering purposes, some of them do not agree with the exact value as well as the solution obtained in reference 3. This difference is caused by the fact that in reference 3 the boundary values are obtained by extrapolation based on the approximate interior values and a constant fifth-order difference obtained from them is used in subsequent calculations. Also at points next to the boundary, the relaxation coefficients seem to be based on a linear variation from those points to the boundary. Because of these variations, the procedure given in reference 3 will not always give more accurate answers than a strictly fourth-degree polynomial approximation as used herein.

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CONCLUDING REMARKS

General differentiation formulas are obtained in terms of the values of the function at unequally spaced arguments and the corresponding distances between the successive arguments using Lagrangian polynomials of any degree. The remainder term is also obtained. The coefficients in the formulas for the first four derivatives at the given arguments are explicitly given. For the special case where there is only one spacing at either end of the arguments different from the others, as often encountered in many practical problems, the differentiation coefficients are computed for different ratios of this spacing to the others, varying from 0.1 to 1.29 and in intervals of 0.01, and are given in the coefficient tables.

The formulas and the coefficients obtained can be used to obtain the approximate values of the various derivatives at any point within the range of given arguments when the values of the function are given at a number of points unequally spaced. They can also be used in the numerical integration of partial differential equations where the starting value is given on a curve unequally spaced from the regular grid lines, or when the interval must be changed during calculation. In particular, in the numerical solutions of elliptic-type partial differential equations, they can be used to obtain the finite-difference expressions more accurately and conveniently at points near a curved boundary, to enable the use of a gradually increasing grid spacing away from the object if the boundary condition is given at infinity, and to render the solution of these problems practical on large-scale digital machines by either iterative or direct methods.

In most practical problems, use of differentiation formulas based on polynomials of higher degree than second is possible and advantageous to enable the use of a relatively coarse grid over most of the domain of the problem to reduce the amount of work. 1411

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, July 5, 1950.

REFERENCES

1. Bickley, W. G.: Formulae for Numerical Differentiation. The Math. Gazette (London), vol. 25, 1941, pp. 19-27.

2. Southwell, R. V.: Relaxation Methods in Theoretical Physics. Claredon Press (Oxford), 1946.

3. Fox, L.: Some Improvements in the Use of Relaxation Methods for the Solution of Ordinary and Partial Differential Equations. Proc. Roy. Soc., London, vol. 190, no. A1020, ser. A, June 17, 1947, pp. 31-59.

4. Shortley, G. H., and Weller, R.: The Numerical Solution of Laplace's Equation. Jour. Appl. Phys., vol. 9, no. 5, May 1938, pp. 334-348.

5. Milne, William Edmund: Numerical Calculus. Princeton Univ. Press, 1949.

6. Southwell, R. V.: The Quest for Accuracy in Computations Using Finite Differences. Chap. 4 of Numerical Methods of Analysis in Engineering. Macmillan Co., 1949, pp. 66-74.

7. Emmons, Howard W.: The Numerical Solution of Partial Differential Equations. Quarterly Appl. Math., vol. II, no. 3, Oct. 1944, pp. 173-195.

8. Emmons, Howard W.: The Numerical Solution of Compressible Fluid Flow Problems. NACA TN 932, 1944.

9. Emmons, Howard W.: The Theoretical Flow of a Frictionless, Adiabatic, Perfect Gas Inside of a Two-Dimensional Hyperbolic Nozzle. NACA TN 1003, 1946.

10. Emmons, Howard W.: Flow of a Compressible Fluid past a Symmetrical Airfoil in a Wind Tunnel and in Free Air. NACA TN 1746, 1948.

11. von Neumann, John, and Goldstine, H. H.: Numerical Inverting of Matrices of High Order. Bull. Amer. Math. Soc., vol. 53, no. 11, Nov. 1947, pp. 1021-1099.

12. Turing, A. M.: Rounding-Off Errors in Matrix Processes. Quarterly Jour. Mech. and Appl. Math., vol. 1, 1948, pp. 287-308.

13. Fox, L., Huskey, H. D., and Wilkinson, J. H.: Notes on the Solution of Algebraic Linear Simultaneous Equations. Quarterly Jour. Mech. and Appl. Math., vol. 1, 1948, pp. 149-173.

14. Mitchell, Herbert F., Jr.: Inversion of a Matrix of Order 38. MTAC, vol. III, no. 23, July 1948, pp. 161-166.

15. Crout, Prescott D.: A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients. AIEE Trans. (Suppl.), vol. 60, 1941, pp. 1235-1241.

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TABLE I - COMPARISON OF 3-, 4-, AND 5-POINT DIFFERENTIATION FORMULA WITH $y = 1/1+x^2$

x	y	y' Spacing 1.000 3 point	1.000 4 point	1.000 5 point	0.500 3 point	0.500 4 point	0.500 5 point	0.250 3 point	0.250 4 point	0.250 5 point	0.125 3 point	0.125 4 point	0.125 5 point	Exact values
-1.000	0.50000	1.00	1.40	1.70	0.700	0.500	0.200	0.520	0.468	0.493	0.50208	0.49741	0.50133	0.50000
- .875	.56638										.56000	.56233	.56103	.56090
- .750	.64000							.600	.626	.618	.61092	.61587	.61456	.61475
- .625	.71911										.64000	.64792	.64643	.64767
- .500	.80000				.500	.600	.700	.602	.628	.636	.63048	.63840	.63985	.64103
- .375	.87673										.56472	.57555	.57594	.57692
- .250	.94118							.400	.442	.433	.43164	.44325	.44219	.44248
- .125	.98464										.23528	.24476	.24177	.24272
0	1.00000	0	- .200	- .300	0	.100	0	0	.024	0	0	.00349	0	0
.125	.98464										- .23528	- .23877	- .24177	- .24272
.250	.94118							- .400	- .424	- .433	- .43164	- .44325	- .44219	- .44248
.375	.87673										- .56472	- .57555	- .57594	- .57692
.500	.80000				- .500	- .600	- .585	- .602	- .628	- .636	- .63048	- .63840	- .63985	- .64103
.625	.71911										- .64000	- .64495	- .64643	- .64767
.750	.64000							- .600	- .607	- .616	- .61092	- .61325	- .61456	- .61475
.875	.56638										- .56000	- .56069	- .56151	- .56090
1.000	.50000	- .400	- .400	- .500	- .492	- .485	- .523	- .4995	- .4975	- .5019	- .50000	- .49963	- .50016	- .50000
1.125	.44138										- .43904	- .43820	- .43843	- .43860
1.250	.39024							- .3846	- .3805	- .3815	- .38172	- .38063	- .38075	- .38052
1.375	.34595										- .33020	- .32916	- .32913	- .28409
1.500	.30769				- .300	- .287	- .290	- .2882	- .2844	- .2842	- .28508	- .28405	- .28405	- .28409
1.625	.27468										- .24616	- .24529	- .24521	- .24528
1.750	.24615							- .2154	- .2125	- .2121	- .21292	- .21215	- .21210	- .21212
1.875	.22145										- .18460	- .18393	- .18388	- .18391
2.000	.20000	- .200	- .200	- .100	- .1698	- .1626	- .1597	- .1624	- .1604	- .1599	- .16060	- .16007	- .16000	- .16000
2.125	.18130										- .14020	- .13972	- .13969	- .13971
2.250	.16495							- .1241	- .1227	- .1224	- .12284	- .12245	- .12241	- .12245
2.375	.15059										- .10808	- .10769	- .10773	- .10771
2.500	.13793				- .1000	- .0928	- .0958	- .09632	- .09530	- .09509	- .09544	- .09512	- .09515	- .09513
2.625	.12673										- .08456	- .08432	- .08431	- .08432
2.750	.11679							- .07586	- .07484	- .07505	- .07520	- .07500	- .07498	- .07501
2.875	.10793										- .06716	- .06696	- .06698	- .06698
3.000	.10000	0	0	.300	- .0517	- .0660	- .0573	- .05846	- .06050	- .05988	- .05972	- .06012	- .06006	- .06000
		y"												
-1.000	0.50000	-1.00	-2.20	-3.30	-0.400	0.800	3.00	0.320	0.941	0.579	0.46410	0.57505	0.46037	0.50000
- .875	.56638										.46410	.46311	.47381	.47083
- .750	.64000							.320	.320	.353	.35192	.35117	.36181	.36066
- .625	.71911										.11410	.11386	.12581	.12798
- .500	.80000				- .400	- .400	- .600	- .3011	- .3011	- .2682	- .26667	- .26610	- .25461	- .25641
- .375	.87673										- .78718	- .78550	- .78277	- .77985
- .250	.94118							-1.318	-1.318	-1.355	-1.34551	-1.34264	-1.35189	-1.35345
- .125	.98464										-1.80128	-1.79744	-1.82235	-1.82252
0	1.00000	-1.00	-1.00	- .900	-1.60	-1.60	-1.80	-1.882	-1.882	-1.976	-1.96923	-1.96503	-1.99403	-2.00000
.125	.98464										-1.80128	-1.79744	-1.82235	-1.82252
.250	.94118							-1.318	-1.318	-1.355	-1.34551	-1.34264	-1.35189	-1.35345
.375	.87673										- .78718	- .78550	- .78277	- .77985
.500	.80000				- .400	- .400	- .369	- .3011	- .3011	- .2682	- .26667	- .26610	- .25461	- .25641
.625	.71911										.11410	.11386	.12580	.12798
.750	.64000							.3200	.3200	.3581	.35192	.35117	.36181	.36066
.875	.56638										.46410	.46311	.46992	.47083
1.000	.50000	.200	.200	.300	.431	.431	.508	.4838	.4838	.5015	.49744	.49638	.50091	.50000
1.125	.44138										.47949	.47846	.48059	.48129
1.250	.39024							.4354	.4354	.4396	.43910	.43817	.43941	.43805
1.375	.34595										.38654	.38571	.38571	.38665
1.500	.30769				.338	.338	.344	.3362	.3362	.3354	.33654	.33582	.33595	.33508
1.625	.27468										.28718	.28657	.28608	.28699
1.750	.24615							.2462	.2462	.2445	.24551	.24499	.24475	.24429
1.875	.22145										.20833	.20789	.20757	.20738
2.000	.20000	.200	.200	.300	.182	.182	.177	.17760	.17760	.17597	.17628	.17591	.17547	.17600
2.125	.18130										.15064	.15032	.15019	.14956
2.250	.16495							.12848	.12848	.12725	.12756	.12729	.12699	.12735
2.375	.15059										.10897	.10874	.10853	.10873
2.500	.13793				.0966	.0966	.0907	.09408	.09408	.09325	.09359	.09339	.09323	.09316
2.625	.12673										.08077	.08060	.08053	.08009
2.750	.11679							.06960	.06960	.06877	.06923	.06908	.06896	.06909
2.875	.10793										.05962	.05949	.05936	.05983
3.000	.10000	.200	.200	- .900	.0966	.0106	.0749	.06960	.04512	.05421	.05962	.04989	.05168	.05200

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TABLE II - MATRICES

[M]

-267.12962	55.55556	-9.16667	1.05820	58.33333	0	0	0	-10.00000	0
44.79167	-113.72955	33.12500	-2.15773	0	16.06181	0	0	0	6.76386
-2.63605	37.14286	-112.61347	38.09525	0	0	16.06181	0	0	0
1.38890	-11.25000	62.50000	-265.97222	0	0	0	58.33333	0	0
41.66667	0	0	0	-111.04163	12.00307	8.10520	-2.01294	35.00000	0
0	33.77655	0	0	35.91529	-125.68808	31.41051	-1.85290	0	33.47601
0	0	33.77655	0	-2.40380	35.83659	-125.60134	31.33227	0	0
0	0	0	41.66667	-1.45640	4.75186	22.14991	-116.45913	0	0
-2.38095	0	0	0	35.00000	0	0	0	-111.04163	12.00307
0	-2.10998	0	0	0	33.47601	0	0	35.91529	-125.68808
0	0	-2.10998	0	0	0	33.47601	0	-2.40380	35.83659
0	0	0	-2.38095	0	0	0	35.00000	-1.45640	4.75186
1.19048	0	0	0	-10.00000	0	0	0	58.33333	0
0	-1.79032	0	0	0	6.76386	0	0	0	16.06181
0	0	-1.79032	0	0	0	6.76386	0	0	0
0	0	0	1.19048	0	0	0	-10.00000	0	0

[V]

-267.12962	-0.2079723	0.0343154	-0.0039614	-0.2183709	0	0	0	0.0374350
44.79167	-104.414234	-.3025257	.0189658	-.0936770	-.1538279	0	0	.0160589
-2.63605	36.5946346	-101.4521944	-.3685555	-.0281161	-.0554870	-.1583190	0	.0048199
1.38890	-10.9611473	59.1363106	-243.9638186	-.0038496	-.0065385	-.0383762	-.2391065	.0006599
41.66667	8.6655132	1.1917320	.4399293	-101.0958821	-.1325976	-.0822067	.0188707	-.3293410
0	33.77655	10.2182744	3.1254019	39.3787054	-114.6833645	-.3172683	.0161200	-.1079084
0	0	33.77655	12.4485333	-1.4062133	37.6056837	-107.9606518	-.3124206	-.0317137
0	0	0	41.66667	-1.2960000	4.8524510	25.1819075	-98.6827768	-.0087949
-2.38095	-.4951716	-.0680989	-.0251388	34.4316722	4.4854429	4.2418569	.5971775	-98.9806470
0	-2.10998	-.6383232	-.1952401	-.2163554	33.0860528	10.3708186	2.6641098	39.8037154
0	0	-2.10998	-.7776447	-.0623180	-.1304243	33.0656146	10.1477177	-1.2898335
0	0	0	-2.38095	-.0091657	-.0167832	-.0974501	34.4006975	-1.1601983
1.19048	.2475869	.0340496	.0125695	-9.7158349	-1.2482391	-1.1888604	-.1649528	54.9109438
0	-1.79032	-.5416178	-.1656614	-.1835777	6.4329809	1.9337838	.4643078	.7306407
0	0	-1.79032	-.6598323	-.0528769	-.1106651	6.4156391	1.8493894	.1994372
0	0	0	1.19048	.0045829	.0083916	.0487252	-9.7003475	-.0821391

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M, P, P', V, Q, AND φ

						{P}	{P'}
0	0	1.19048	0	0	0	-10,633.95	-10,633.90
0	0	0	-1.79032	0	0	16,192.25	16,192.24
6.76386	0	0	0	-1.79032	0	12,948.37	12,948.81
0	-10.00000	0	0	0	1.19048	210,879.21	210,879.16
0	0	-2.38095	0	0	0	-28,623.17	-28,623.26
0	0	0	-2.10998	0	0	1,465.22	1,465.38
33.47601	0	0	0	-2.10998	0	-5,985.97	-5,986.01
0	35.00000	0	0	0	-2.38095	39,737.87	39,737.77
8.10520	-2.01294	41.66667	0	0	0	-60,857.24	-60,857.31
31.41051	-1.85290	0	33.77655	0	0	-19,234.20	-19,234.19
-125.60134	31.33227	0	0	33.77655	0	-45,213.27	-45,213.12
22.14991	-116.45913	0	0	0	41.66667	-10,311.87	-10,311.84
0	0	-267.12962	55.55556	-9.16667	1.05820	-12,593.27	-12,593.24
0	0	44.79167	-113.72955	33.12500	-2.15773	279,277.63	279,277.65
16.06181	0	-2.63605	37.14286	-112.61347	38.09525	362,314.27	362,314.29
0	58.33333	1.38890	-11.25000	62.50000	-265.97222	1,767,582.26	1,767,582.19

							{Q}	{φ}
0	0	0	-0.0044566	0	0	0	39.8082	-165.778
-.0647792	0	0	-.0019118	.0171463	0	0	-138.0002	-973.204
-.0233664	-.0666704	0	-.0005738	.0061848	.0176469	0	-178.4462	-1526.881
-.0027535	-.0161608	.0409897	-.0000786	.0007288	.0042776	-.0048797	-901.2153	-1592.452
-.0058400	-.0008562	.0001784	.0215436	.0015458	.0002266	-.0000212	281.6818	-299.186
-.3151405	-.0066747	.0011783	.0067811	.0245499	.0017667	-.0001403	2.8409	-1778.430
-.1173240	-.3351118	.0051345	.0018929	.0105503	.0261707	-.0006113	-106.9779	-2789.769
-.0465208	-.0926545	-.3359990	.0005004	.0041868	.0085683	.0219044	-814.0599	-2904.401
-.1425472	-.0973574	.0186346	-.4129549	.0020374	.0013189	.0000935	703.5440	-549.735
-108.4001334	-.3614651	.0164546	-.1493731	-.3026124	.0036258	.0004802	411.1965	-3247.006
39.9112882	-99.4336322	-.3416497	-.0539286	-.1177253	-.3290843	.0022621	502.6303	-5089.370
6.1634932	27.3809686	-95.5275121	-.0199028	-.0518187	-.0911553	-.4274860	-.6088	-5294.207
7.2470032	7.5376636	1.3861196	-242.7113753	-.2416465	.0272843	-.0067061	228.7089	-1014.156
18.3115734	7.3851759	2.3535394	48.2264553	-95.6904192	-.3587721	.0090153	-2681.7607	-5923.307
.7883387	18.5566855	6.9388359	-1.3093557	39.5476886	-91.8021580	-.4424078	-5012.7002	-9277.338
-.4513107	-1.0342833	54.6805588	2.3247851	-8.0738959	64.2624144	-213.8575898	-9639.6072	-9639.607

TABLE III - COMPARISON OF RESULTS

At	Relaxation solution with difference correction (reference 3)	Relaxation solution with fourth-degree differentiation formula (16)	Matrix solution with fourth-degree differentiation formula (16)	Exact value
	φ			
1	-169	-165	-166	-168
2	-981	-972	-973	-978
3	-1536	-1526	-1527	-1531
4	-1596	-1593	-1592	-1593
5	-307	-298	-299	-305
6	-1783	-1777	-1778	-1781
7	-2791	-2790	-2790	-2790
8	-2901	-2905	-2904	-2902
9	-560	-549	-550	-556
10	-3248	-3247	-3247	-3245
11	-5083	-5090	-5089	-5083
12	-5283	-5295	-5294	-5288
13	-1019	-1014	-1014	-1014
14	-5918	-5923	-5923	-5914
15	-9264	-9278	-9277	-9262
16	-9628	-9640	-9640	-9636

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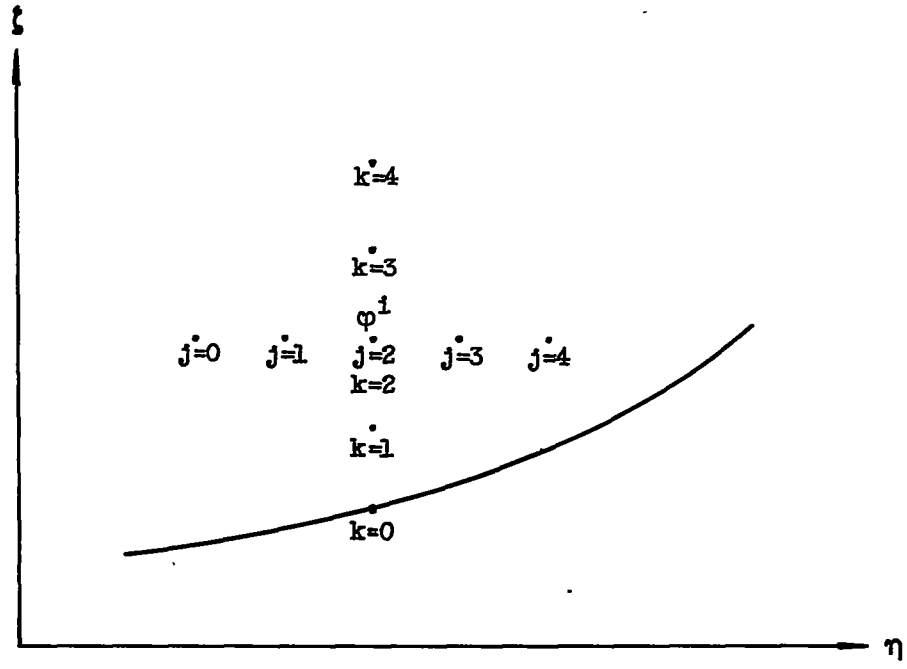

Figure 1. - Notation i, j, and k.

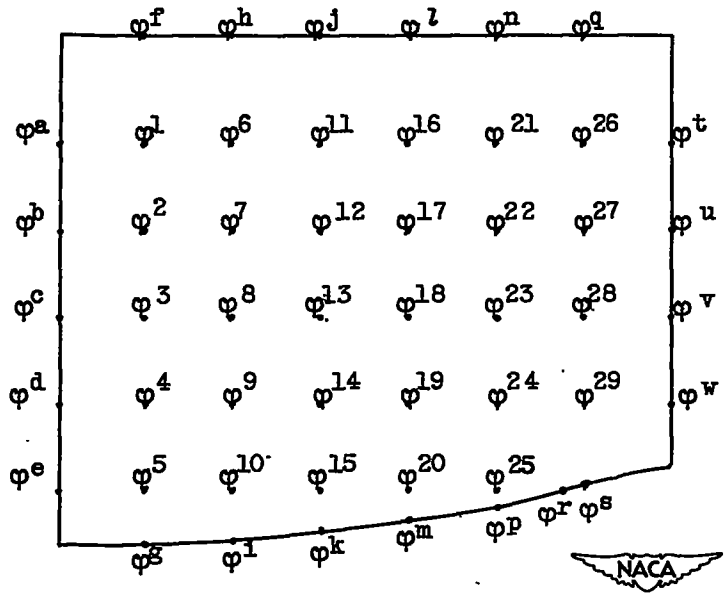

Figure 2. - Numbering system.

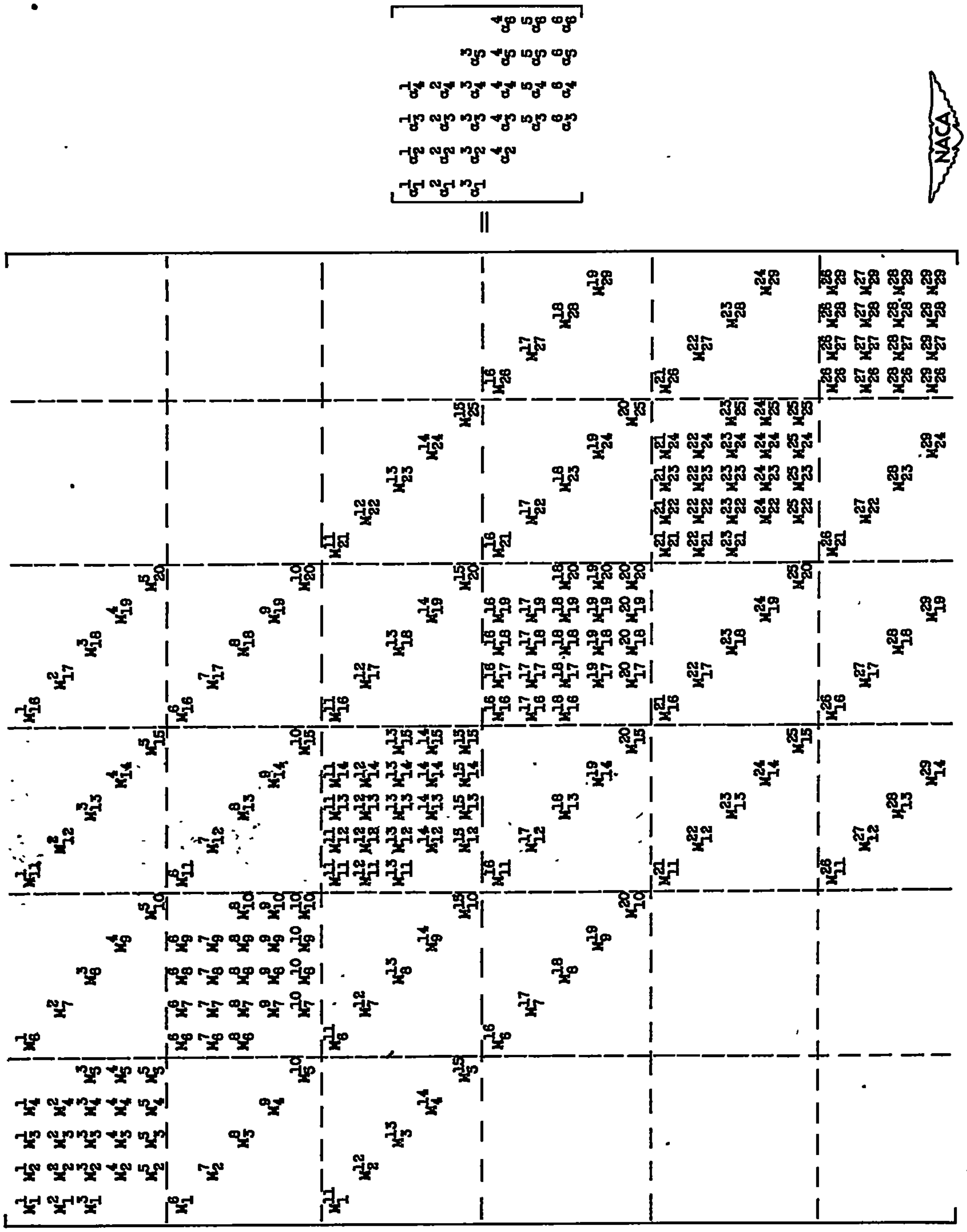

Figure 3. - Matrix M.

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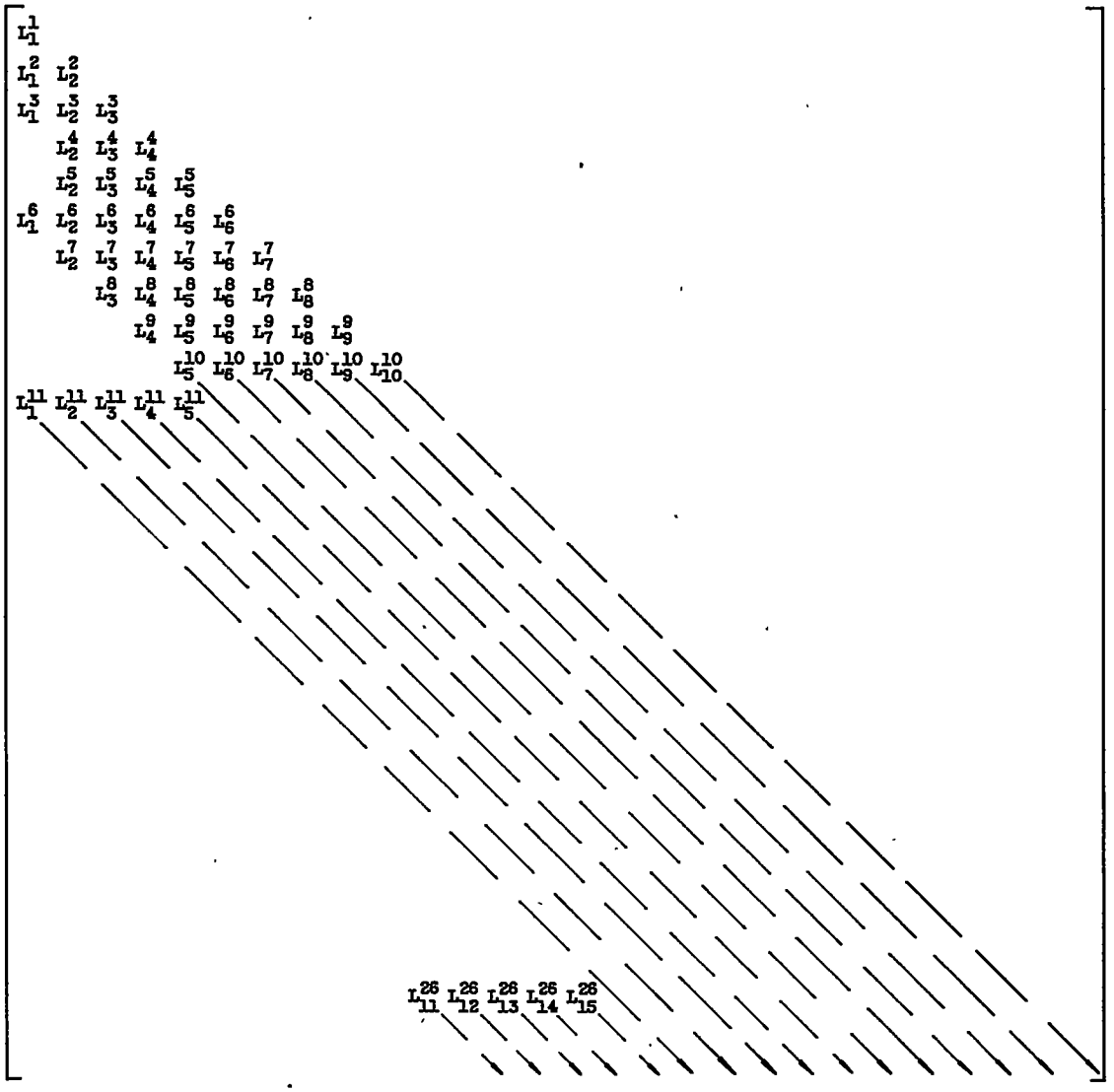
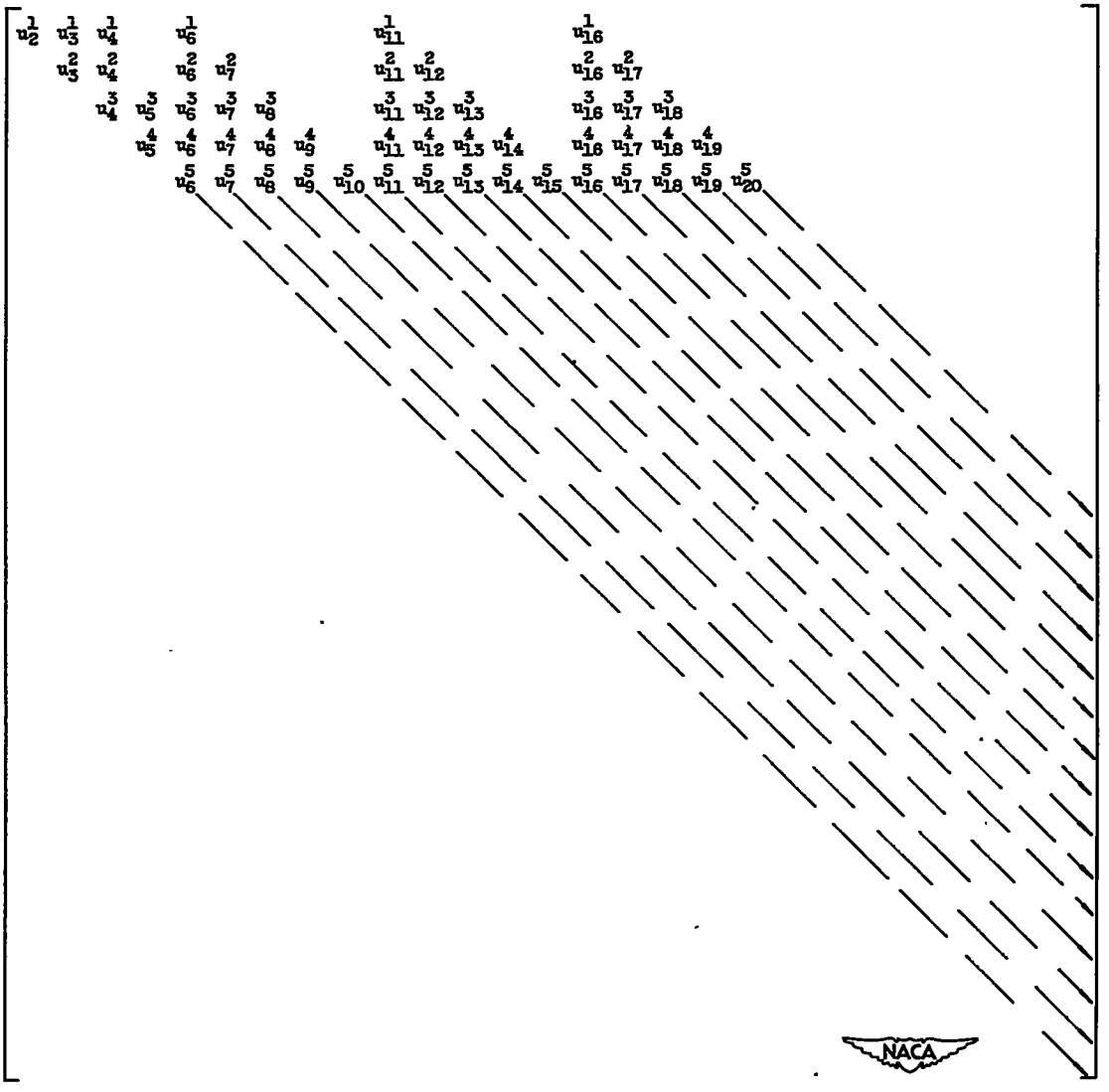


(a) [L]

Figure 4. - Matrices L and u.



(b) [u]

Figure 4. - Concluded. Matrices L and u.

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$$[V] = [L] + [u] - [I]$$

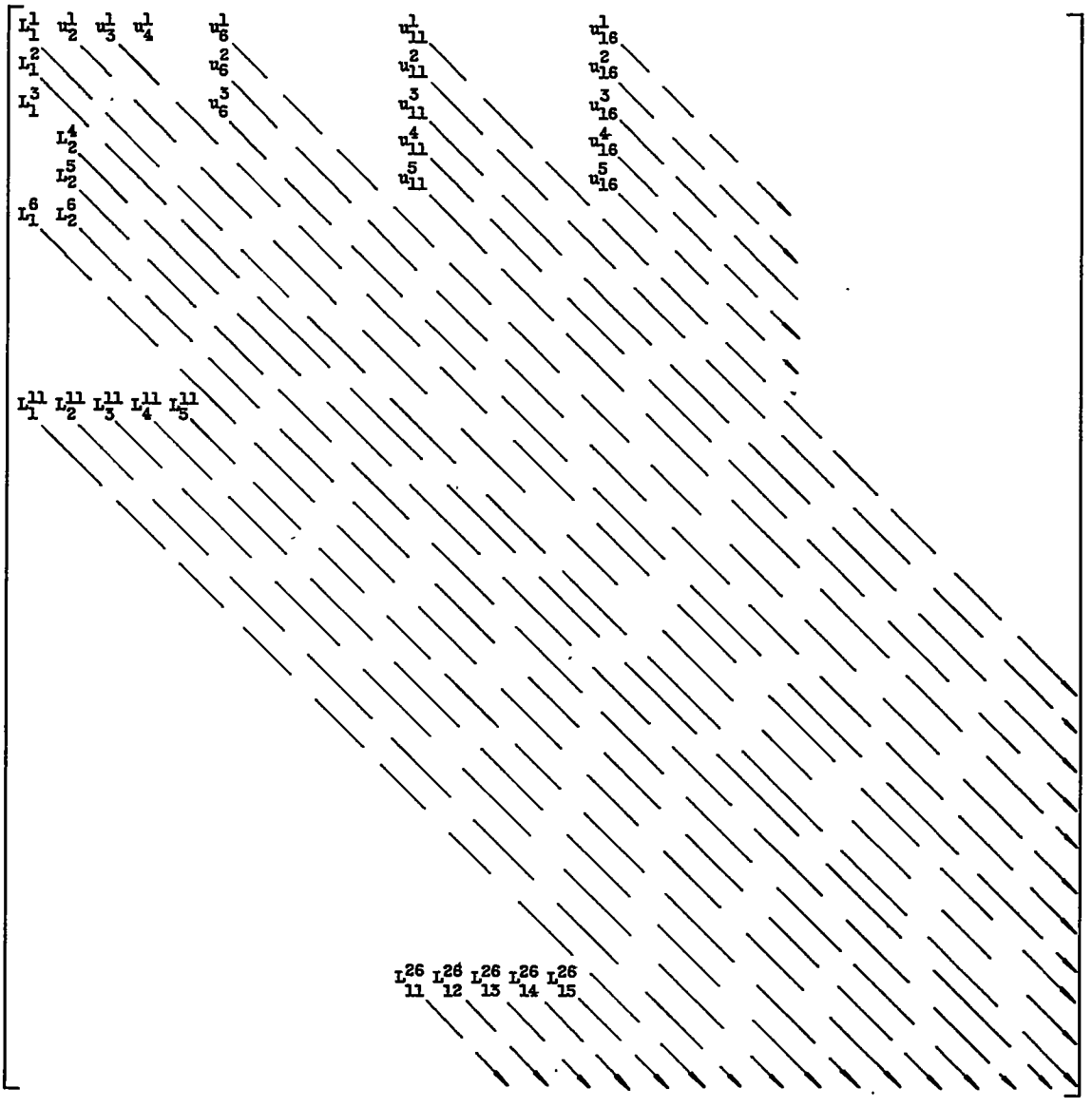
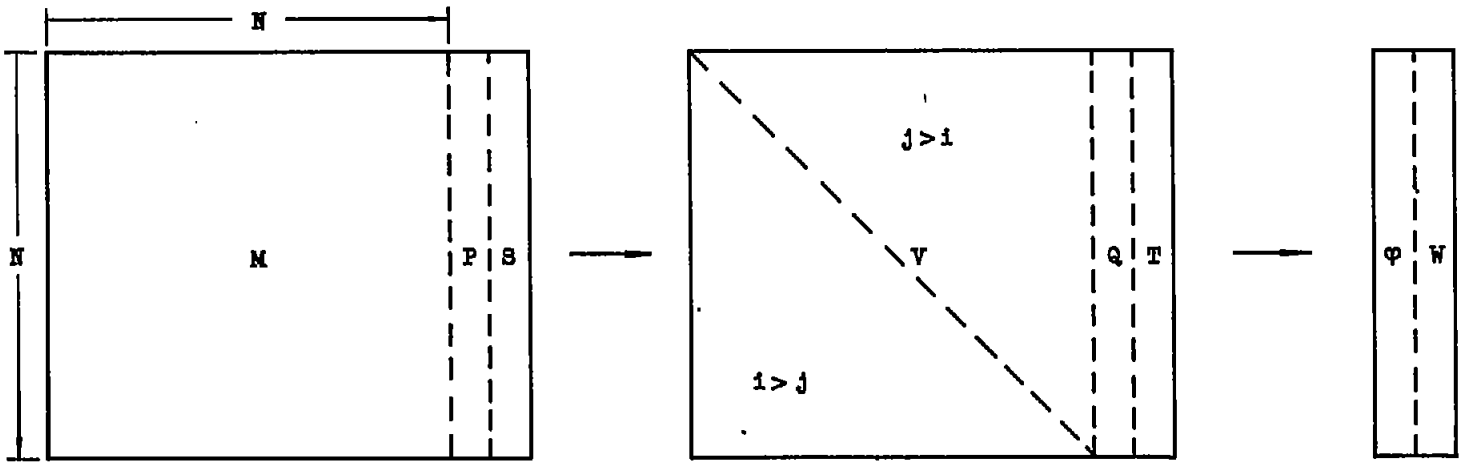


Figure 5. - Matrix V.

$$[M]\,[\varphi] = [P]$$



$$S^i = \sum_{j=1}^{N} M_j^i + P^i$$

$$i \geq j, \quad V_j^i = M_j^i - \sum_{k=1}^{j-1} V_k^i V_j^k$$

$$j > i, \quad V_j^i = \frac{M_j^i - \sum_{k=1}^{i-1} V_k^i V_j^k}{V_i^i}$$

$$\varphi^i = Q^i - \sum_{j=N}^{i+1} V_j^i Q^j$$

$$W^i = T^i - \sum_{j=N}^{i+1} V_j^i T^j$$

For checking: $T^i = 1 + \sum_{j=i+1}^{N+1} V_j^i \qquad W^i = 1 + \varphi^i$

Figure 6. - Operation diagram and operating formulas.

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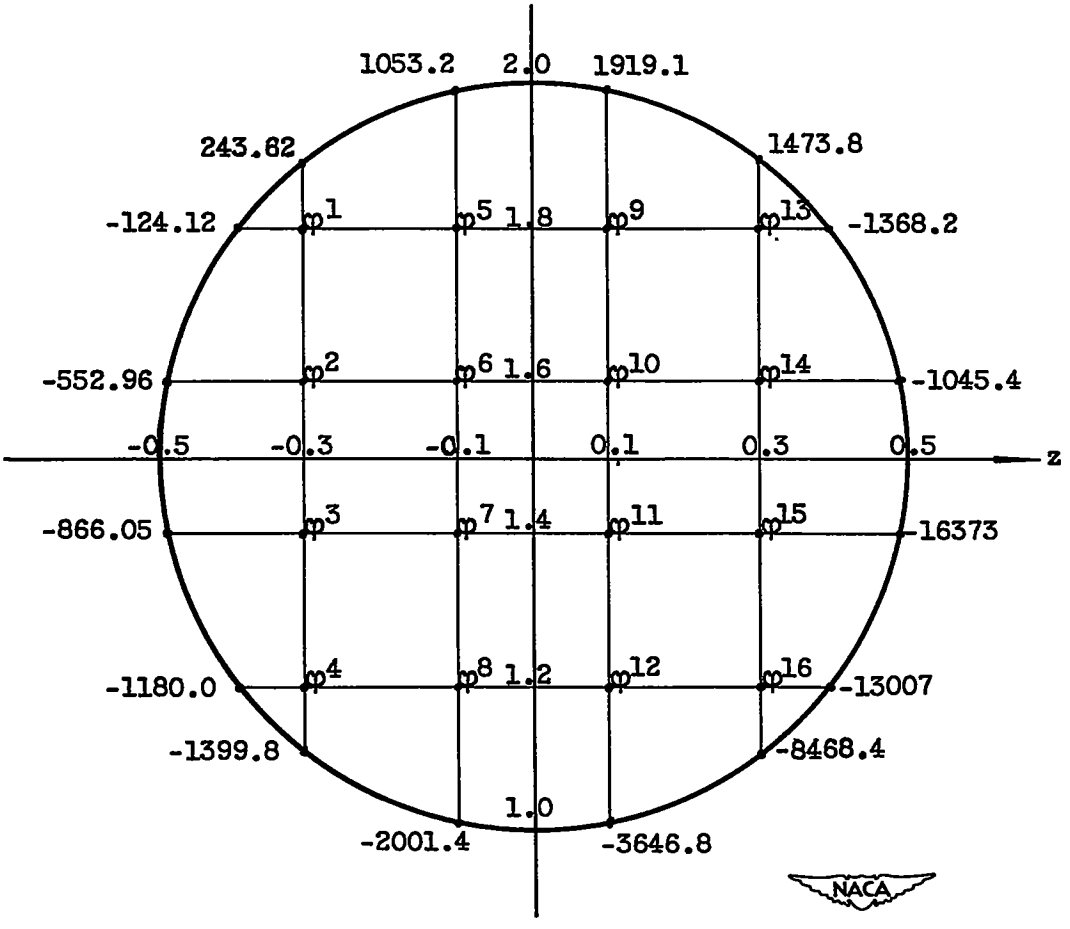

Figure 7. - Boundary values and order in which φ is numbered.

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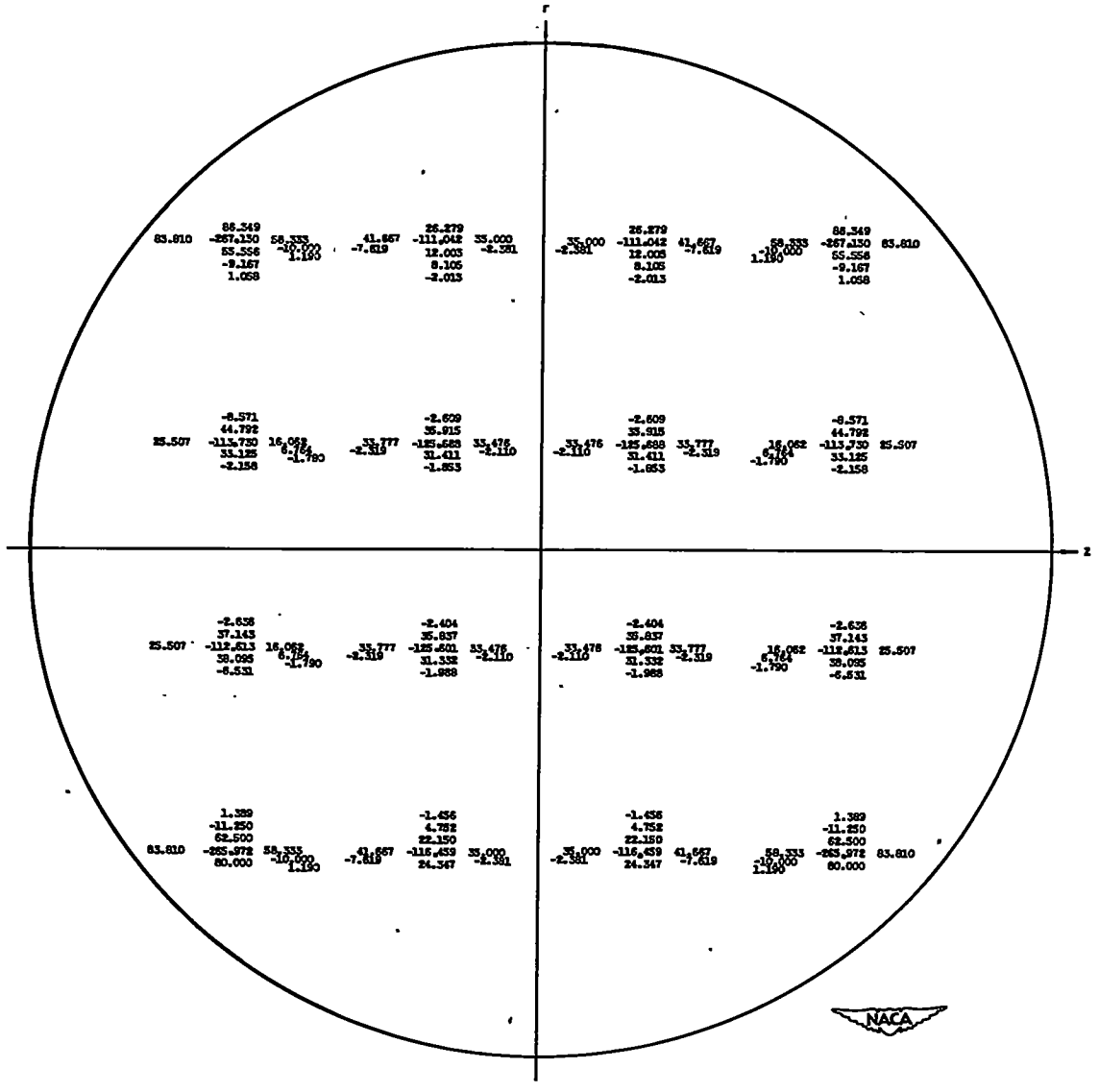

Figure 8. - Coefficients used to compute residuals.

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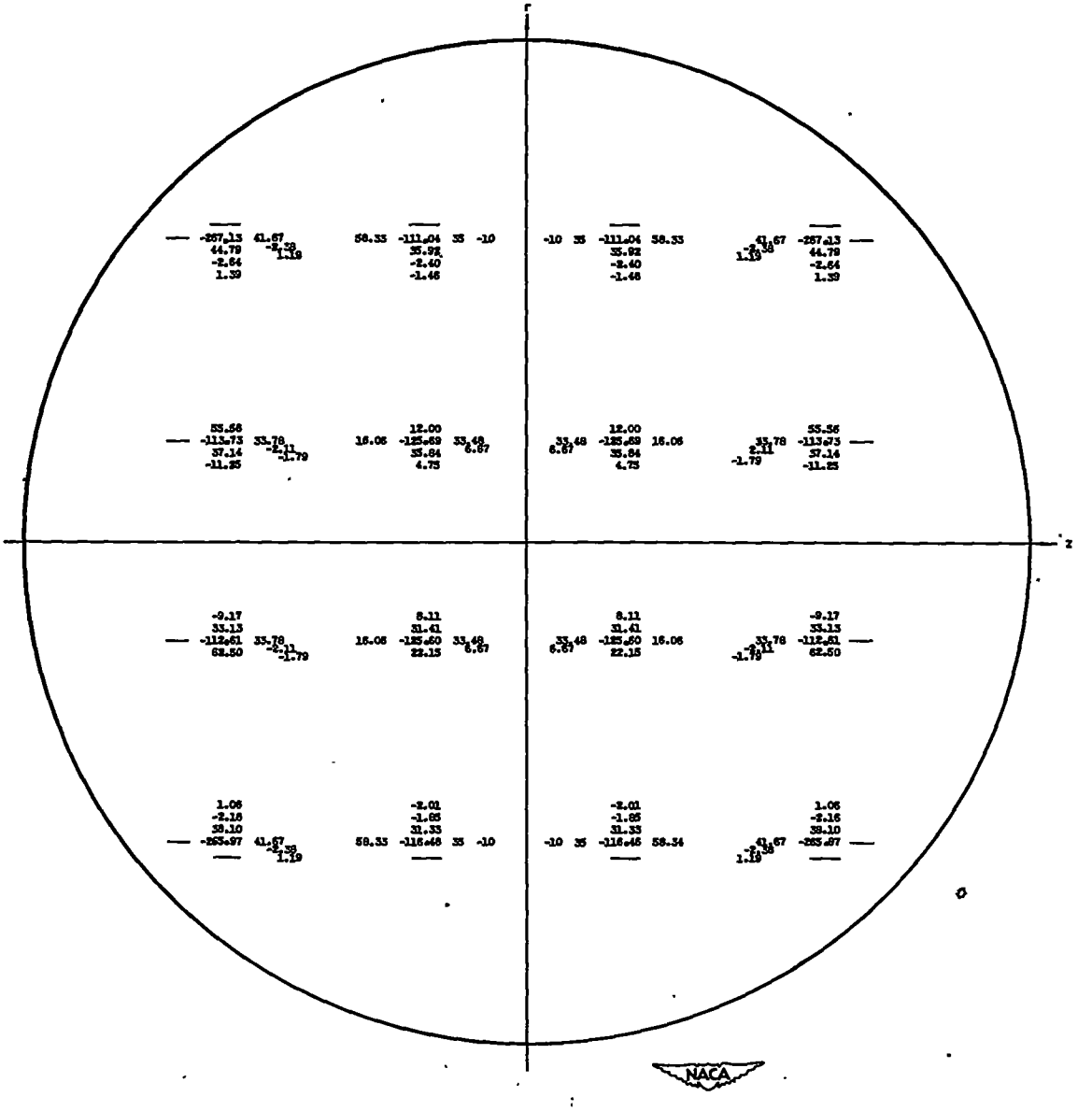

Figure 9. - Improvement pattern.

THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \sum_{j=0}^{2} {}_2^iB_j^i \, y_j + {}_2^iR^i \qquad (i=0,2)$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2^1B_0^0\, b$	${}_2^1B_1^0\, b$	${}_2^1B_2^0\, b$	$\frac{{}_2^1C^0}{b^2 f^{(3)}(\xi)}$	
0.10	-10.90909091	11.00000000	-0.090909091	0.0183	0.10
.11	- 9.99180999	10.09090909	- .099099099	.0204	.11
.12	- 9.22619048	9.33333333	- .107142857	.0224	.12
.13	- 8.57726344	8.69230769	- .115044248	.0248	.13
.14	- 8.02005012	8.14285714	- .122807018	.0266	.14
.15	- 7.53623188	7.66666667	- .130434783	.0288	.15
.16	- 7.11206896	7.25000000	- .137931034	.0309	.16
.17	- 6.73705380	6.88235294	- .145299145	.0332	.17
.18	- 6.40301318	6.55555556	- .152542373	.0354	.18
.19	- 6.10349403	6.26315789	- .159663866	.0377	.19
.20	- 5.83333333	6.00000000	- .166666667	.0400	.20
.21	- 5.58835104	5.76190476	- .173553719	.0424	.21
.22	- 5.36512668	5.54545455	- .180327869	.0447	.22
.23	- 5.16083422	5.34782609	- .186991870	.0472	.23
.24	- 4.97311828	5.16666667	- .193548387	.0496	.24
.25	- 4.80000000	5.00000000	- .200000000	.0521	.25
.26	- 4.63980464	4.84615385	- .206349206	.0546	.26
.27	- 4.49110528	4.70370370	- .212598425	.0572	.27
.28	- 4.35267857	4.57142857	- .218750000	.0597	.28
.29	- 4.22346966	4.44827586	- .224806202	.0624	.29
.30	- 4.10256410	4.33333333	- .230769231	.0650	.30
.31	- 3.98916523	4.22580645	- .236641221	.0677	.31
.32	- 3.88257576	4.12500000	- .242424242	.0704	.32
.33	- 3.78218273	4.03030303	- .248120301	.0732	.33
.34	- 3.68744513	3.94117647	- .253731343	.0759	.34
.35	- 3.59788360	3.85714286	- .259259259	.0788	.35
.36	- 3.51307190	3.77777778	- .264705882	.0816	.36
.37	- 3.43262971	3.70270270	- .270072993	.0845	.37
.38	- 3.35621663	3.63157895	- .275362319	.0874	.38
.39	- 3.28352702	3.56410256	- .280575540	.0904	.39
.40	- 3.21428571	3.50000000	- .285714286	.0933	.40
.41	- 3.14824425	3.43902439	- .290780142	.0964	.41
.42	- 3.08517773	3.38095238	- .295774648	.0994	.42
.43	- 3.02488209	3.32558140	- .300699301	.1025	.43
.44	- 2.96717172	3.27272727	- .305555556	.1056	.44
.45	- 2.91187739	3.22222222	- .310344828	.1088	.45
.46	- 2.85884455	3.17391304	- .315068493	.1119	.46
.47	- 2.80793168	3.12765957	- .319727891	.1152	.47
.48	- 2.75900901	3.08333333	- .324324324	.1184	.48
.49	- 2.71195727	3.04081633	- .328859060	.1217	.49
.50	- 2.66666667	3.00000000	- .333333333	.1250	.50
.51	- 2.62303597	2.96078431	- .337748344	.1284	.51
.52	- 2.58097166	2.92307692	- .342105263	.1317	.52
.53	- 2.54038722	2.88679245	- .346405229	.1352	.53
.54	- 2.50120250	2.85185185	- .350649351	.1386	.54
.55	- 2.46334311	2.81818182	- .354838710	.1421	.55
.56	- 2.42673993	2.78571429	- .358974359	.1456	.56
.57	- 2.39132864	2.75438596	- .363057325	.1492	.57
.58	- 2.35704932	2.72413793	- .367088608	.1527	.58
.59	- 2.32384607	2.69491525	- .371069182	.1564	.59
.60	- 2.29166667	2.66666667	- .375000000	.1600	.60
.61	- 2.26046227	2.63934426	- .378881988	.1637	.61
.62	- 2.23018718	2.61290323	- .382716049	.1674	.62
.63	- 2.20079852	2.58730159	- .386503067	.1712	.63
.64	- 2.17225610	2.56250000	- .390243902	.1749	.64
.65	- 2.14452214	2.53846154	- .393939394	.1788	.65
.66	- 2.11756115	2.51515152	- .397590361	.1826	.66
.67	- 2.09133971	2.49253731	- .401197605	.1865	.67
.68	- 2.06582633	2.47058824	- .404761905	.1904	.68
.69	- 2.04099134	2.44927536	- .408284024	.1944	.69
	$-\,{}_2^1B_2^2\, a$	$-\,{}_2^1B_1^2\, a$	$-\,{}_2^1B_0^2\, a$	$\frac{{}_2^1C^2}{a^2 f^{(3)}(\xi)}$	$\rho=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

1(a)

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THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \sum_{j=0}^{2} {}_2^iB_j^i \, y_j + {}_2^iR^i \qquad (i=0,2)$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2^1B_0^0\ b$	${}_2^1B_1^0\ b$	${}_2^1B_2^0\ b$	$\frac{{}_2^1E^0}{b^2f^{(3)}(\xi)}$	
0.70	- 2.01680672	2.42857143	-0.411764706	0.1983	0.70
.71	- 1.99324603	2.40845070	- .415204678	.2024	.71
.72	- 1.97028424	2.38888889	- .418604651	.2064	.72
.73	- 1.94789770	2.36986301	- .421965318	.2105	.73
.74	- 1.92606400	2.35135135	- .425287356	.2146	.74
.75	- 1.90476190	2.33333333	- .428571429	.2188	.75
.76	- 1.88397129	2.31578947	- .431818182	.2229	.76
.77	- 1.86367305	2.29870130	- .435028249	.2272	.77
.78	- 1.84384903	2.28205128	- .438202247	.2314	.78
.79	- 1.82448200	2.26582278	- .441340782	.2357	.79
.80	- 1.80555556	2.25000000	- .444444444	.2400	.80
.81	- 1.78705409	2.23456790	- .447513812	.2444	.81
.82	- 1.76896274	2.21951220	- .450549451	.2487	.82
.83	- 1.75126736	2.20481928	- .453551913	.2532	.83
.84	- 1.73395445	2.19047619	- .456521739	.2576	.84
.85	- 1.71701113	2.17647059	- .459459459	.2621	.85
.86	- 1.70042511	2.16279070	- .462365591	.2666	.86
.87	- 1.68418465	2.14942529	- .465240642	.2712	.87
.88	- 1.66827853	2.13636364	- .468085106	.2757	.88
.89	- 1.65269603	2.12359551	- .470899471	.2804	.89
.90	- 1.63742690	2.11111111	- .473684211	.2850	.90
.91	- 1.62246131	2.09890110	- .476439791	.2896	.91
.92	- 1.60778986	2.08695652	- .479166667	.2944	.92
.93	- 1.59340353	2.07526882	- .481865285	.2992	.93
.94	- 1.57929370	2.06382979	- .484536082	.3039	.94
.95	- 1.56545209	2.05263158	- .487179487	.3088	.95
.96	- 1.55187075	2.04166667	- .489795918	.3136	.96
.97	- 1.53854205	2.03092784	- .492385787	.3185	.97
.98	- 1.52545867	2.02040816	- .494949495	.3234	.98
.99	- 1.51261357	2.01010101	- .497487437	.3284	.99
1.00	- 1.50000000	2.00000000	- .500000000	.3333	1.00
1.01	- 1.48761145	1.99009901	- .502487562	.3384	1.01
1.02	- 1.47544166	1.98039216	- .504950495	.3434	1.02
1.03	- 1.46348462	1.97087379	- .507389163	.3485	1.03
1.04	- 1.45173454	1.96153846	- .509803922	.3536	1.04
1.05	- 1.44018583	1.95238095	- .512195122	.3588	1.05
1.06	- 1.42883912	1.94339623	- .514563107	.3639	1.06
1.07	- 1.41767123	1.93457944	- .516908213	.3692	1.07
1.08	- 1.40669516	1.92592593	- .519230769	.3744	1.08
1.09	- 1.39590009	1.91743119	- .521531100	.3797	1.09
1.10	- 1.38528139	1.90909091	- .523809524	.3850	1.10
1.11	- 1.37483455	1.90090090	- .526066351	.3904	1.11
1.12	- 1.36455526	1.89285714	- .528301887	.3957	1.12
1.13	- 1.35443932	1.88495575	- .530516432	.4012	1.13
1.14	- 1.34448270	1.87719298	- .532710280	.4066	1.14
1.15	- 1.33468150	1.86956522	- .534883721	.4121	1.15
1.16	- 1.32503193	1.86206897	- .537037037	.4176	1.16
1.17	- 1.31553035	1.85470085	- .539170507	.4232	1.17
1.18	- 1.30617322	1.84745763	- .541284404	.4287	1.18
1.19	- 1.29695714	1.84033613	- .543378995	.4344	1.19
1.20	- 1.28787879	1.83333333	- .545454545	.4400	1.20
1.21	- 1.27893497	1.82644628	- .547511312	.4457	1.21
1.22	- 1.27012258	1.81967213	- .549549549	.4514	1.22
1.23	- 1.26143862	1.81300813	- .551569507	.4572	1.23
1.24	- 1.25288018	1.80645161	- .553571429	.4629	1.24
1.25	- 1.24444444	1.80000000	- .555555556	.4688	1.25
1.26	- 1.23612867	1.79365079	- .557522124	.4746	1.26
1.27	- 1.22793021	1.78740157	- .559471366	.4805	1.27
1.28	- 1.21984649	1.78125000	- .561403509	.4864	1.28
1.29	- 1.21187502	1.77519380	- .563318777	.4924	1.29
	$-\ {}_2^1B_2^2\ a$	$-\ {}_2^1B_1^2\ a$	$-\ {}_2^1B_0^2\ a$	$\frac{{}_2^1E^2}{a^2f^{(3)}(\xi)}$	$p=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

1(b)

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THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_1} = \sum_{j=0}^{2} {}_2B_j^i \, y_j + {}_2R^i$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2B_0^i\, b$	${}_2B_1^i\, b$	${}_2B_2^i\, b$	$\frac{{}_2E^i}{b^2 f^{(3)}(\xi)}$	
0.10	-9.09090909	9.00000000	0.090909091	-0.0167	0.10
.11	-8.19000819	8.09090909	.099099099	- .0183	.11
.12	-7.44047619	7.33333333	.107142857	- .0200	.12
.13	-6.80735194	6.69230769	.115044248	- .0217	.13
.14	-6.26566416	6.14285714	.122807018	- .0233	.14
.15	-5.79710145	5.66666666	.130434783	- .0250	.15
.16	-5.38793103	5.25000000	.137931034	- .0267	.16
.17	-5.02765209	4.88235294	.145299145	- .0283	.17
.18	-4.70809793	4.55555556	.152542373	- .0300	.18
.19	-4.42282176	4.26315789	.159663866	- .0317	.19
.20	-4.16666667	4.00000000	.166666667	- .0333	.20
.21	-3.93545848	3.76190476	.173553719	- .0350	.21
.22	-3.72578241	3.54545454	.180327869	- .0367	.22
.23	-3.53481796	3.34782609	.186991870	- .0383	.23
.24	-3.36021905	3.16666667	.193548387	- .0400	.24
.25	-3.20000000	3.00000000	.200000000	- .0417	.25
.26	-3.05250305	2.84615385	.206349206	- .0433	.26
.27	-2.91630213	2.70370370	.212598425	- .0450	.27
.28	-2.79017857	2.57142857	.218750000	- .0467	.28
.29	-2.67308206	2.44827586	.224806202	- .0483	.29
.30	-2.56410256	2.33333333	.230769231	- .0500	.30
.31	-2.46244767	2.22580645	.236641221	- .0517	.31
.32	-2.36742424	2.12500000	.242424242	- .0533	.32
.33	-2.27842333	2.03030303	.248120301	- .0550	.33
.34	-2.19490781	1.94117647	.253731343	- .0567	.34
.35	-2.11640212	1.85714286	.259259259	- .0583	.35
.36	-2.04248366	1.77777778	.264705882	- .0600	.36
.37	-1.97277570	1.70270270	.270072993	- .0617	.37
.38	-1.90694127	1.63157895	.275362319	- .0633	.38
.39	-1.84467810	1.56410256	.280575540	- .0650	.39
.40	-1.78571429	1.50000000	.285714286	- .0667	.40
.41	-1.72980453	1.43902439	.290780142	- .0683	.41
.42	-1.67672703	1.38095238	.295774648	- .0700	.42
.43	-1.62628070	1.32558140	.300699301	- .0717	.43
.44	-1.57828283	1.27272727	.305555556	- .0733	.44
.45	-1.53256705	1.22222222	.310344828	- .0750	.45
.46	-1.48898154	1.17391304	.315068493	- .0767	.46
.47	-1.44738747	1.12765957	.319727891	- .0783	.47
.48	-1.40765766	1.08333333	.324324324	- .0800	.48
.49	-1.36967539	1.04081633	.328859060	- .0817	.49
.50	-1.33333333	1.00000000	.333333333	- .0833	.50
.51	-1.29853266	.960784314	.337748344	- .0850	.51
.52	-1.26518219	.923076923	.342105263	- .0867	.52
.53	-1.23319768	.886792453	.346405229	- .0883	.53
.54	-1.20250120	.851851852	.350649351	- .0900	.54
.55	-1.17302053	.818181818	.354838710	- .0917	.55
.56	-1.14468864	.785714286	.358974359	- .0933	.56
.57	-1.11744329	.754385965	.363057325	- .0950	.57
.58	-1.09122654	.724137931	.367088608	- .0967	.58
.59	-1.06598444	.694915254	.371069182	- .0983	.59
.60	-1.04166667	.666666667	.375000000	- .1000	.60
.61	-1.01822625	.639344262	.378881988	- .1017	.61
.62	-0.995619275	.612903226	.382716049	- .1033	.62
.63	- .973804655	.587301587	.386503067	- .1050	.63
.64	- .952743902	.562500000	.390243902	- .1067	.64
.65	- .932400932	.538461538	.393939394	- .1083	.65
.66	- .912741877	.515151515	.397590361	- .1100	.66
.67	- .893734918	.492537313	.401197605	- .1117	.67
.68	- .875350140	.470588235	.404761905	- .1133	.68
.69	- .857559386	.449275362	.408284024	- .1150	.69
	$-{}_2B_2^i\, a$	$-{}_2B_1^i\, a$	$-{}_2B_0^i\, a$	$\frac{{}_2E^i}{a^2 f^{(3)}(\xi)}$	$p=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

2(a)

1411

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THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^1y)_{x=x_1} = \sum_{j=0}^{2} {}_2^1B_j^1\, y_j + {}_2^1R^1$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2^1B_0^1\, b$	${}_2^1B_1^1\, b$	${}_2^1B_2^1\, b$	$\frac{{}_2^1E^1}{b^2f^{(3)}(\xi)}$	
0.70	-0.840336134	0.428571429	0.411764706	-0.1167	0.70
.71	- .823655383	.408450704	.415204678	- .1183	.71
.72	- .807493540	.388888889	.418604651	- .1200	.72
.73	- .791828332	.369863014	.421965318	- .1217	.73
.74	- .776638708	.351351351	.425287356	- .1233	.74
.75	- .761904762	.333333333	.428571429	- .1250	.75
.76	- .747607656	.315789474	.431818182	- .1267	.76
.77	- .733729547	.298701299	.435028249	- .1283	.77
.78	- .720253529	.282051282	.438202247	- .1300	.78
.79	- .707163567	.265822785	.441340782	- .1317	.79
.80	- .694444444	.250000000	.444444444	- .1333	.80
.81	- .682081713	.234567901	.447513812	- .1350	.81
.82	- .670061646	.219512195	.450549451	- .1367	.82
.83	- .658371190	.204819277	.453551913	- .1383	.83
.84	- .646997930	.190476190	.456521739	- .1400	.84
.85	- .635930048	.176470588	.459459459	- .1417	.85
.86	- .625156289	.162790698	.462365591	- .1433	.86
.87	- .614665929	.149425287	.465240642	- .1450	.87
.88	- .604448743	.136363636	.468085106	- .1467	.88
.89	- .594494977	.123595506	.470899471	- .1483	.89
.90	- .584795322	.111111111	.473684211	- .1500	.90
.91	- .575340889	.0989010989	.476439791	- .1517	.91
.92	- .566123188	.0869565217	.479166667	- .1533	.92
.93	- .557134102	.0752688172	.481865285	- .1550	.93
.94	- .548365876	.0638297872	.484536082	- .1567	.94
.95	- .539811066	.0526315789	.487179487	- .1583	.95
.96	- .531462585	.0416666667	.489795918	- .1600	.96
.97	- .523313622	.0309278351	.492385787	- .1617	.97
.98	- .515357658	.0204081633	.494949495	- .1633	.98
.99	- .507588447	.0101010101	.497487437	- .1650	.99
1.00	- .500000000	.0000000000	.500000000	- .1667	1.00
1.01	- .492586572	- .0099009901	.502487562	- .1683	1.01
1.02	- .485342652	- .0196078431	.504950495	- .1700	1.02
1.03	- .478262949	- .0291262136	.507389163	- .1717	1.03
1.04	- .471342383	- .0384615385	.509803922	- .1733	1.04
1.05	- .464576074	- .0476190476	.512195122	- .1750	1.05
1.06	- .457959333	- .0566037736	.514563107	- .1767	1.06
1.07	- .451487692	- .0654205607	.516908213	- .1783	1.07
1.08	- .445156695	- .0740740741	.519230769	- .1800	1.08
1.09	- .438962293	- .0825688073	.521531100	- .1817	1.09
1.10	- .432900433	- .0909090909	.523809524	- .1833	1.10
1.11	- .426967252	- .0990990991	.526066351	- .1850	1.11
1.12	- .421159030	- .107142857	.528301887	- .1867	1.12
1.13	- .415472184	- .115044248	.530516432	- .1883	1.13
1.14	- .409903263	- .122807018	.532710280	- .1900	1.14
1.15	- .404448938	- .130434783	.534883721	- .1917	1.15
1.16	- .399106003	- .137931034	.537037037	- .1933	1.16
1.17	- .393871362	- .145299145	.539170507	- .1950	1.17
1.18	- .388742031	- .152542373	.541284404	- .1967	1.18
1.19	- .383715130	- .159663866	.543378995	- .1983	1.19
1.20	- .378787879	- .166666666	.545454545	- .2000	1.20
1.21	- .373957593	- .173553719	.547511312	- .2017	1.21
1.22	- .369221681	- .180327869	.549549549	- .2033	1.22
1.23	- .364577637	- .186991870	.551569507	- .2050	1.23
1.24	- .360023041	- .193548387	.553571429	- .2067	1.24
1.25	- .355555556	- .200000000	.555555556	- .2083	1.25
1.26	- .351172918	- .206349206	.557522124	- .2100	1.26
1.27	- .346872940	- .212598425	.559471366	- .2117	1.27
1.28	- .342653509	- .218750000	.561403509	- .2133	1.28
1.29	- .338512576	- .224806202	.563318777	- .2150	1.29
	$-{}_2^1B_2^1\, a$	$-{}_2^1B_1^1\, a$	$-{}_2^1B_0^1\, a$	$\frac{{}_2^1E^1}{a^2f^{(3)}(\xi)}$	$p=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

2(b)

THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_1} = \sum_{j=0}^{2} {}^{1}_{2}B^{i}_{j}\, y_j + {}^{1}_{2}R^{i} \qquad (i=2,0)$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}^{1}_{2}B^{2}_{0}\,b$	${}^{1}_{2}B^{2}_{1}\,b$	${}^{1}_{2}B^{2}_{2}\,b$	$\frac{{}^{1}_{2}E^{2}}{b^2 f^{(3)}(\xi)}$	
0.10	9.09090909	-11.00000000	1.90909091	0.1833	0.10
.11	8.19000819	-10.09090909	1.90090090	.1850	.11
.12	7.44047619	- 9.33333333	1.89285714	.1867	.12
.13	6.80735194	- 8.69230769	1.88495575	.1883	.13
.14	6.26566416	- 8.14285714	1.87719298	.1900	.14
.15	5.79710145	- 7.66666667	1.86956522	.1917	.15
.16	5.38793103	- 7.25000000	1.86206897	.1933	.16
.17	5.02765209	- 6.88235294	1.85470085	.1950	.17
.18	4.70809793	- 6.55555556	1.84745763	.1967	.18
.19	4.42282176	- 6.26315789	1.84033613	.1983	.19
.20	4.16666667	- 6.00000000	1.83333333	.2000	.20
.21	3.93545848	- 5.76190476	1.82644628	.2017	.21
.22	3.72578241	- 5.54545455	1.81967213	.2033	.22
.23	3.53481796	- 5.34782609	1.81300813	.2050	.23
.24	3.36021505	- 5.16666667	1.80645161	.2067	.24
.25	3.20000000	- 5.00000000	1.80000000	.2083	.25
.26	3.05250505	- 4.84615385	1.79365079	.2100	.26
.27	2.91630213	- 4.70370370	1.78740157	.2117	.27
.28	2.79017857	- 4.57142857	1.78125000	.2133	.28
.29	2.67308206	- 4.44827586	1.77519380	.2150	.29
.30	2.56410256	- 4.33333333	1.76923077	.2167	.30
.31	2.46244767	- 4.22580645	1.76335878	.2183	.31
.32	2.36742424	- 4.12500000	1.75757576	.2200	.32
.33	2.27842333	- 4.03030303	1.75187970	.2217	.33
.34	2.19490781	- 3.94117647	1.74626866	.2233	.34
.35	2.11640212	- 3.85714286	1.74074074	.2250	.35
.36	2.04248366	- 3.77777778	1.73529412	.2267	.36
.37	1.97277570	- 3.70270270	1.72992701	.2283	.37
.38	1.90694127	- 3.63157895	1.72463768	.2300	.38
.39	1.84467810	- 3.56410256	1.71942446	.2317	.39
.40	1.78571429	- 3.50000000	1.71428571	.2333	.40
.41	1.72980453	- 3.43902439	1.70921986	.2350	.41
.42	1.67672703	- 3.38095238	1.70422535	.2367	.42
.43	1.62628070	- 3.32558140	1.69930070	.2383	.43
.44	1.57828283	- 3.27272727	1.69444444	.2400	.44
.45	1.53256705	- 3.22222222	1.68965517	.2417	.45
.46	1.48898154	- 3.17391304	1.68493151	.2433	.46
.47	1.44738747	- 3.12765957	1.68027211	.2450	.47
.48	1.40765766	- 3.08333333	1.67567568	.2467	.48
.49	1.36967539	- 3.04081633	1.67114094	.2483	.49
.50	1.33333333	- 3.00000000	1.66666667	.2500	.50
.51	1.29853266	- 2.96078431	1.66225166	.2517	.51
.52	1.26518219	- 2.92307692	1.65789474	.2533	.52
.53	1.23319768	- 2.88679245	1.65359477	.2550	.53
.54	1.20250120	- 2.85185185	1.64935065	.2567	.54
.55	1.17302053	- 2.81818182	1.64516129	.2583	.55
.56	1.14468864	- 2.78571429	1.64102564	.2600	.56
.57	1.11744329	- 2.75438596	1.63694268	.2617	.57
.58	1.09122654	- 2.72413793	1.63291139	.2633	.58
.59	1.06598444	- 2.69491525	1.62893082	.2650	.59
.60	1.04166667	- 2.66666667	1.62500000	.2667	.60
.61	1.01822625	- 2.63934426	1.62111801	.2683	.61
.62	0.995618275	- 2.61290323	1.61728395	.2700	.62
.63	.973804655	- 2.58730159	1.61349693	.2717	.63
.64	.952743902	- 2.56250000	1.60975610	.2733	.64
.65	.932400932	- 2.53846154	1.60606061	.2750	.65
.66	.912741877	- 2.51515152	1.60240964	.2767	.66
.67	.893734918	- 2.49253731	1.59880240	.2783	.67
.68	.875350140	- 2.47058824	1.59523810	.2800	.68
.69	.857558586	- 2.44927536	1.59171598	.2817	.69
	$-{}^{1}_{2}B^{0}_{2}\,a$	$-{}^{1}_{2}B^{0}_{1}\,a$	$-{}^{1}_{2}B^{0}_{0}\,a$	$\frac{{}^{1}_{2}E^{0}}{a^2 f^{(3)}(\xi)}$	$p=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

3(a)

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THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \sum_{j=0}^{2} {}_2^iB_j^i\, y_j + {}_2^iR^i \qquad (i=2,0)$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2^1B_0^2\, b$	${}_2^1B_1^2\, b$	${}_2^1B_2^2\, b$	$\frac{{}_2^1E^2}{b^2 f^{(3)}(\xi)}$	
0.70	0.840336134	- 2.42857145	1.58823529	0.2833	0.70
.71	.823655383	- 2.40845070	1.58479532	.2850	.71
.72	.807493540	- 2.38888889	1.58139535	.2867	.72
.73	.791828332	- 2.36986301	1.57803468	.2883	.73
.74	.776658708	- 2.35135135	1.57471264	.2900	.74
.75	.761904762	- 2.33333333	1.57142857	.2917	.75
.76	.747607656	- 2.31578947	1.56818182	.2933	.76
.77	.733729547	- 2.29870130	1.56497175	.2950	.77
.78	.720253529	- 2.28205128	1.56179775	.2967	.78
.79	.707163567	- 2.26582278	1.55865922	.2983	.79
.80	.694444444	- 2.25000000	1.55555556	.3000	.80
.81	.682081713	- 2.23456790	1.55248619	.3017	.81
.82	.670061646	- 2.21951220	1.54945055	.3033	.82
.83	.658371180	- 2.20481928	1.54644809	.3050	.83
.84	.646997930	- 2.19047619	1.54347826	.3067	.84
.85	.635930048	- 2.17647059	1.54054054	.3083	.85
.86	.625156289	- 2.16279070	1.53763441	.3100	.86
.87	.614665929	- 2.14942529	1.53475936	.3117	.87
.88	.604446743	- 2.13636364	1.53191489	.3133	.88
.89	.594494977	- 2.12359551	1.52910053	.3150	.89
.90	.584795322	- 2.11111111	1.52631579	.3167	.90
.91	.575340889	- 2.09890110	1.52356021	.3183	.91
.92	.566123188	- 2.08695652	1.52083333	.3200	.92
.93	.557134102	- 2.07526882	1.51813472	.3217	.93
.94	.548365876	- 2.06382979	1.51546392	.3233	.94
.95	.539811066	- 2.05263158	1.51282051	.3250	.95
.96	.531462585	- 2.04166667	1.51020408	.3267	.96
.97	.523313622	- 2.03092784	1.50761421	.3283	.97
.98	.515357658	- 2.02040816	1.50505051	.3300	.98
.99	.507588447	- 2.01010101	1.50251256	.3317	.99
1.00	.500000000	- 2.00000000	1.50000000	.3333	1.00
1.01	.492586572	- 1.99009901	1.49751244	.3350	1.01
1.02	.485342852	- 1.98039216	1.49504950	.3367	1.02
1.03	.478262949	- 1.97087379	1.49261084	.3383	1.03
1.04	.471342383	- 1.96153846	1.49019608	.3400	1.04
1.05	.464576074	- 1.95238095	1.48780488	.3417	1.05
1.06	.457959333	- 1.94339623	1.48543689	.3433	1.06
1.07	.451487652	- 1.93457944	1.48309179	.3450	1.07
1.08	.445158695	- 1.92592593	1.48076923	.3467	1.08
1.09	.438962293	- 1.91743119	1.47846890	.3483	1.09
1.10	.432900433	- 1.90909091	1.47619048	.3500	1.10
1.11	.426967252	- 1.90090090	1.47393365	.3517	1.11
1.12	.421159050	- 1.89285714	1.47169811	.3533	1.12
1.13	.415472184	- 1.88495575	1.46948357	.3550	1.13
1.14	.409903263	- 1.87719298	1.46728972	.3567	1.14
1.15	.404448858	- 1.86956522	1.46511628	.3583	1.15
1.16	.399106003	- 1.86206897	1.46296296	.3600	1.16
1.17	.393871562	- 1.85470085	1.46082949	.3617	1.17
1.18	.388742031	- 1.84745763	1.45871560	.3633	1.18
1.19	.383715130	- 1.84033613	1.45662100	.3650	1.19
1.20	.378787879	- 1.83333333	1.45454545	.3667	1.20
1.21	.373957593	- 1.82644628	1.45248869	.3683	1.21
1.22	.369221681	- 1.81967213	1.45045045	.3700	1.22
1.23	.364577657	- 1.81300813	1.44843049	.3717	1.23
1.24	.360023041	- 1.80645161	1.44642857	.3733	1.24
1.25	.355555556	- 1.80000000	1.44444444	.3750	1.25
1.26	.351172818	- 1.79365079	1.44247788	.3767	1.26
1.27	.346872940	- 1.78740157	1.44052863	.3783	1.27
1.28	.342653509	- 1.78125000	1.43859649	.3800	1.28
1.29	.338512576	- 1.77519380	1.43668122	.3817	1.29
	$-{}_2^1B_2^0\, a$	$-{}_2^1B_1^0\, a$	$-{}_2^1B_0^0\, a$	$\frac{{}_2^1E^0}{a^2 f^{(3)}(\xi)}$	$\rho=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

3(b)

L4TT

THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \sum_{j=0}^{2} {}^{2}_{2}B^{i}_{j}\, y'_j + {}^{2}_{2}R^{i} \qquad (i=0,1,2)$$

$\frac{x_i-x_0}{x_2-x_1}\frac{a}{b}$	${}^{2}_{2}B^{i}_{0}\ b^2$	${}^{2}_{2}B^{i}_{1}\ b^2$	${}^{2}_{2}B^{i}_{2}\ b^2$	$\frac{{}^{2}_{2}E^{0}}{bf^{(3)}(\xi)}$	$\frac{{}^{2}_{2}E^{1}}{bf^{(3)}(\xi)}$	$\frac{{}^{2}_{2}E^{2}}{bf^{(3)}(\xi)}$	
0.10	18.1818182	-20.0000000	1.81818182	-0.4000	-0.3000	0.7000	0.10
.11	16.3800164	-18.1818182	1.80180180	- .4067	- .2967	.7033	.11
.12	14.8809524	-16.6666667	1.78571429	- .4133	- .2933	.7067	.12
.13	13.6147039	-15.3846154	1.76991150	- .4200	- .2900	.7100	.13
.14	12.5313283	-14.2857143	1.75438596	- .4267	- .2867	.7133	.14
.15	11.5942029	-13.3333333	1.73913043	- .4333	- .2833	.7167	.15
.16	10.7758621	-12.5000000	1.72413793	- .4400	- .2800	.7200	.16
.17	10.0553042	-11.7647059	1.70940171	- .4467	- .2667	.7233	.17
.18	9.41619586	-11.1111111	1.69491525	- .4533	- .2733	.7267	.18
.19	8.84564352	-10.5263158	1.68067227	- .4600	- .2700	.7300	.19
.20	8.33333333	-10.00000000	1.66666667	- .4666	- .2667	.7333	.20
.21	7.87091696	- 9.52380952	1.65289256	- .4733	- .2633	.7367	.21
.22	7.45156482	- 9.09090909	1.63934426	- .4800	- .2600	.7400	.22
.23	7.06963592	- 8.69565217	1.62601626	- .4867	- .2567	.7433	.23
.24	6.72043010	- 8.33333333	1.61290323	- .4933	- .2533	.7467	.24
.25	6.40000000	- 8.00000000	1.60000000	- .5000	- .2500	.7500	.25
.26	6.10500610	- 7.69230769	1.58730159	- .5067	- .2467	.7533	.26
.27	5.83260426	- 7.40740741	1.57480315	- .5133	- .2433	.7567	.27
.28	5.58035714	- 7.14285714	1.56250000	- .5200	- .2400	.7600	.28
.29	5.34616412	- 6.89655172	1.55038760	- .5267	- .2367	.7633	.29
.30	5.12820512	- 6.66666667	1.53846154	- .5333	- .2333	.7667	.30
.31	4.92489534	- 6.45161290	1.52671756	- .5400	- .2300	.7700	.31
.32	4.73484848	- 6.25000000	1.51515152	- .5467	- .2267	.7733	.32
.33	4.55684666	- 6.06060606	1.50375940	- .5533	- .2233	.7766	.33
.34	4.38981562	- 5.88235294	1.49253731	- .5600	- .2200	.7800	.34
.35	4.23280424	- 5.71428571	1.48148148	- .5667	- .2167	.7833	.35
.36	4.08496732	- 5.55555556	1.47058824	- .5733	- .2133	.7867	.36
.37	3.94555140	- 5.40540541	1.45985401	- .5800	- .2100	.7900	.37
.38	3.81388254	- 5.26315789	1.44927536	- .5867	- .2067	.7933	.38
.39	3.68935620	- 5.12820513	1.43884892	- .5933	- .2033	.7967	.39
.40	3.57142858	- 5.00000000	1.42857143	- .6000	- .2000	.8000	.40
.41	3.45960906	- 4.87804878	1.41843972	- .6067	- .1967	.8033	.41
.42	3.35345406	- 4.76190476	1.40845070	- .6133	- .1933	.8067	.42
.43	3.25256140	- 4.65116279	1.39860140	- .6200	- .1900	.8100	.43
.44	3.15656566	- 4.54545455	1.38888889	- .6267	- .1867	.8133	.44
.45	3.06513410	- 4.44444444	1.37931034	- .6333	- .1833	.8167	.45
.46	2.97796307	- 4.34782609	1.36986301	- .6400	- .1800	.8200	.46
.47	2.89477493	- 4.25531915	1.36054422	- .6467	- .1767	.8233	.47
.48	2.81531532	- 4.16666667	1.35135135	- .6533	- .1733	.8267	.48
.49	2.73935077	- 4.08163265	1.34228188	- .6600	- .1700	.8300	.49
.50	2.66666667	- 4.00000000	1.33333333	- .6667	- .1667	.8333	.50
.51	2.59706532	- 3.92156863	1.32450331	- .6733	- .1633	.8367	.51
.52	2.53036437	- 3.84615385	1.31578947	- .6800	- .1600	.8400	.52
.53	2.46639536	- 3.77358491	1.30718954	- .6867	- .1567	.8433	.53
.54	2.40500241	- 3.70370370	1.29870130	- .6933	- .1533	.8467	.54
.55	2.34604106	- 3.63636364	1.29032258	- .7000	- .1500	.8500	.55
.56	2.28937729	- 3.57142857	1.28205128	- .7067	- .1467	.8533	.56
.57	2.23488658	- 3.50877193	1.27388535	- .7133	- .1433	.8567	.57
.58	2.18245308	- 3.44827586	1.26582278	- .7200	- .1400	.8600	.58
.59	2.13196887	- 3.38983051	1.25786164	- .7267	- .1367	.8633	.59
.60	2.08333333	- 3.33333333	1.25000000	- .7333	- .1333	.8667	.60
.61	2.03645250	- 3.27868852	1.24223602	- .7400	- .1300	.8700	.61
.62	1.99123855	- 3.22580645	1.23456790	- .7467	- .1267	.8733	.62
.63	1.94760931	- 3.17460317	1.22699387	- .7533	- .1233	.8767	.63
.64	1.90548780	- 3.12500000	1.21951220	- .7600	- .1200	.8800	.64
.65	1.86480186	- 3.07692308	1.21212121	- .7667	- .1167	.8833	.65
.66	1.82548375	- 3.03030303	1.20481928	- .7733	- .1133	.8867	.66
.67	1.78746984	- 2.98507463	1.19760479	- .7800	- .1100	.8900	.67
.68	1.75070028	- 2.94117647	1.19047619	- .7867	- .1067	.8933	.68
.69	1.71511677	- 2.89855072	1.18343195	- .7933	- .1033	.8967	.69
	${}^{2}_{2}B^{i}_{2}\ a^2$	${}^{2}_{2}B^{i}_{1}\ a^2$	${}^{2}_{2}B^{i}_{0}\ a^2$	$\frac{{}^{2}_{2}E^{2}}{af^{(3)}(\xi)}$	$\frac{{}^{2}_{2}E^{1}}{af^{(3)}(\xi)}$	$\frac{{}^{2}_{2}E^{0}}{af^{(3)}(\xi)}$	$\rho=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

4(a)

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THREE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \sum_{j=0}^{2} {}_2^2B_j^i \, y_j + {}_2^2R^i \qquad (i=0,1,2)$$

$r=\frac{x_1-x_0}{x_2-x_1}=\frac{a}{b}$	${}_2^2B_0^i\ b^2$	${}_2^2B_1^i\ b^2$	${}_2^2B_2^i\ b^2$	${}_2^2E^0 / bf^{(3)}(\xi)$	${}_2^2E^1 / bf^{(3)}(\xi)$	${}_2^2E^2 / bf^{(3)}(\xi)$	
0.70	1.68067227	- 2.85714286	1.17647059	-0.8000	-0.1000	0.9000	0.70
.71	1.64731077	- 2.81690141	1.16959064	- .8067	- .0967	.9033	.71
.72	1.61498708	- 2.77777778	1.16279070	- .8133	- .0933	.9067	.72
.73	1.58365666	- 2.73972603	1.15606936	- .8200	- .0900	.9100	.73
.74	1.55327742	- 2.70270270	1.14942529	- .8267	- .0867	.9133	.74
.75	1.52380952	- 2.66666667	1.14285714	- .8333	- .0833	.9167	.75
.76	1.49521531	- 2.63157895	1.13636364	- .8400	- .0800	.9200	.76
.77	1.46745909	- 2.59740260	1.12994350	- .8467	- .0767	.9233	.77
.78	1.44050706	- 2.56410256	1.12359551	- .8533	- .0733	.9267	.78
.79	1.41432713	- 2.53164557	1.11731844	- .8600	- .0700	.9300	.79
.80	1.38888889	- 2.50000000	1.11111111	- .8667	- .0667	.9333	.80
.81	1.36416343	- 2.46913580	1.10497238	- .8733	- .0633	.9367	.81
.82	1.34012329	- 2.43902439	1.09890110	- .8800	- .0600	.9400	.82
.83	1.31674238	- 2.40963855	1.09289617	- .8867	- .0567	.9433	.83
.84	1.29399586	- 2.38095238	1.08695652	- .8933	- .0533	.9467	.84
.85	1.27186010	- 2.35294118	1.08108108	- .9000	- .0500	.9500	.85
.86	1.25031258	- 2.32558140	1.07526882	- .9067	- .0467	.9533	.86
.87	1.22933186	- 2.29885058	1.06951872	- .9133	- .0433	.9567	.87
.88	1.20889749	- 2.27272727	1.06382979	- .9200	- .0400	.9600	.88
.89	1.18898945	- 2.24719101	1.05820106	- .9267	- .0367	.9633	.89
.90	1.16959064	- 2.22222222	1.05263158	- .9333	- .0333	.9667	.90
.91	1.15068178	- 2.19780220	1.04712042	- .9400	- .0300	.9700	.91
.92	1.13224638	- 2.17391304	1.04166667	- .9467	- .0267	.9733	.92
.93	1.11426820	- 2.15053763	1.03626943	- .9533	- .0233	.9767	.93
.94	1.09673174	- 2.12765957	1.03092784	- .9600	- .0200	.9800	.94
.95	1.07962213	- 2.10526316	1.02564103	- .9667	- .0167	.9833	.95
.96	1.06292517	- 2.08333333	1.02040816	- .9733	- .0133	.9867	.96
.97	1.04662724	- 2.06185567	1.01522843	- .9800	- .0100	.9900	.97
.98	1.03071532	- 2.04081633	1.01010101	- .9867	- .0067	.9933	.98
.99	1.01517689	- 2.02020202	1.00502513	- .9933	- .0033	.9967	.99
1.00	1.00000000	- 2.00000000	1.00000000	-1.0000	[a]0	1.0000	1.00
1.01	.985173144	- 1.98019802	.995024876	-1.0067	.0033	1.0033	1.01
1.02	.970685304	- 1.96078431	.990099010	-1.0133	.0067	1.0067	1.02
1.03	.956525898	- 1.94174757	.985221675	-1.0200	.0010	1.0100	1.03
1.04	.942684766	- 1.92307692	.980392157	-1.0267	.0133	1.0133	1.04
1.05	.929152148	- 1.90476190	.975609756	-1.0333	.0167	1.0167	1.05
1.06	.915918666	- 1.88679245	.970873786	-1.0400	.0200	1.0200	1.06
1.07	.902975304	- 1.86915888	.966183575	-1.0467	.0233	1.0233	1.07
1.08	.890313390	- 1.85185185	.961538462	-1.0533	.0267	1.0267	1.08
1.09	.877924586	- 1.83486239	.956937799	-1.0600	.0300	1.0300	1.09
1.10	.865800866	- 1.81818182	.952380952	-1.0667	.0333	1.0333	1.10
1.11	.853934504	- 1.80180180	.947867299	-1.0733	.0367	1.0367	1.11
1.12	.842318060	- 1.78571429	.943396226	-1.0800	.0400	1.0400	1.12
1.13	.830944368	- 1.76991150	.938967136	-1.0867	.0433	1.0433	1.13
1.14	.819806526	- 1.75438596	.934579439	-1.0933	.0467	1.0467	1.14
1.15	.808897876	- 1.73913043	.930232558	-1.1000	.0500	1.0500	1.15
1.16	.798212006	- 1.72413793	.925925926	-1.1067	.0533	1.0533	1.16
1.17	.787742724	- 1.70940171	.921658986	-1.1133	.0567	1.0592	1.17
1.18	.777484062	- 1.69491525	.917431193	-1.1200	.0600	1.0600	1.18
1.19	.767430260	- 1.68067227	.913242009	-1.1267	.0633	1.0633	1.19
1.20	.757575758	- 1.66666667	.909090909	-1.1333	.0667	1.0667	1.20
1.21	.747915186	- 1.65289256	.904977375	-1.1400	.0700	1.0700	1.21
1.22	.738443362	- 1.63934426	.900900901	-1.1467	.0733	1.0733	1.22
1.23	.729155274	- 1.62601626	.896860987	-1.1533	.0767	1.0767	1.23
1.24	.720046082	- 1.61290323	.892857143	-1.1600	.0800	1.0800	1.24
1.25	.711111112	- 1.60000000	.888888889	-1.1667	.0833	1.0833	1.25
1.26	.702345836	- 1.58730159	.884955752	-1.1733	.0867	1.0867	1.26
1.27	.693745880	- 1.57480315	.881057269	-1.1800	.0900	1.0900	1.27
1.28	.685307018	- 1.56250000	.877192982	-1.1867	.0933	1.0933	1.28
1.29	.677025152	- 1.55038760	.873362445	-1.1933	.0967	1.0967	1.29
	${}_2^2B_2^i\ a^2$	${}_2^2B_1^i\ a^2$	${}_2^2B_0^i\ a^2$	$-\frac{{}_2^2E^2}{af^{(3)}(\xi)}$	$-\frac{{}_2^2E^1}{af^{(3)}(\xi)}$	$\frac{{}_2^2E^0}{af^{(3)}(\xi)}$	$\rho=\frac{x_2-x_1}{x_1-x_0}=\frac{b}{a}$

[a]The next order remainder term is 0.0833 $\left[f^{(4)}(x)\right]_{x=x_i}$

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}_3^iC_j^i \, y_j + {}_3^iR^i \qquad (i=0,3)$$

$r=\frac{a}{b}\frac{a}{c}$	${}_3^1C_0^0$	${}_3^1C_1^0$	${}_3^1C_2^0$	${}_3^1C_3^0$	$\frac{{}_3^1E^0}{b^3 f^{(4)}(\xi)}$	
0.10	-11.3852814	11.5500000	-0.190909091	0.0261904762	-0.0096	0.10
.11	-10.4657436	10.6459091	- .209099099	.0289336493	- .0107	.11
.12	- 9.69788859	9.89333333	- .227142857	.0316981132	- .0119	.12
.13	- 9.04674701	9.25730769	- .245044248	.0344835681	- .0130	.13
.14	- 8.48733984	8.71285714	- .262807018	.0372897196	- .0142	.14
.15	- 8.00134816	8.24166667	- .280434783	.0401162791	- .0155	.15
.16	- 7.57503193	7.83000000	- .297931034	.0429629630	- .0167	.16
.17	- 7.19788329	7.46735294	- .315299145	.0458294931	- .0180	.17
.18	- 6.86172878	7.14555556	- .332542373	.0487155963	- .0193	.18
.19	- 6.56011503	6.85815789	- .349663866	.0516210046	- .0206	.19
.20	- 6.28787878	6.60000000	- .366666667	.0545454545	- .0220	.20
.21	- 6.04083973	6.36690476	- .383553719	.0574886878	- .0234	.21
.22	- 5.81557713	6.15545455	- .400327869	.0604504505	- .0248	.22
.23	- 5.60926471	5.96282609	- .416991870	.0634304953	- .0263	.23
.24	- 5.41954685	5.78666667	- .433548387	.0664285714	- .0278	.24
.25	- 5.24444444	5.62500000	- .450000000	.0694444444	- .0293	.25
.26	- 5.08228252	5.47615385	- .466349206	.0724778761	- .0308	.26
.27	- 4.93163391	5.33870370	- .482598425	.0755286344	- .0324	.27
.28	- 4.79127506	5.21142857	- .498750000	.0785964912	- .0340	.28
.29	- 4.66015088	5.09327586	- .514806202	.0816812227	- .0357	.29
.30	- 4.53754671	4.98333333	- .530769231	.0847826087	- .0374	.30
.31	- 4.42206566	4.88080645	- .546641221	.0879004329	- .0391	.31
.32	- 4.31361024	4.78500000	- .562424242	.0910344828	- .0408	.32
.33	- 4.21136728	4.69530303	- .578120301	.0941845494	- .0426	.33
.34	- 4.11479555	4.61117647	- .593731343	.0973504274	- .0444	.34
.35	- 4.02341551	4.53214286	- .609259259	.100531915	- .0463	.35
.36	- 3.93680071	4.45777778	- .624705882	.103728814	- .0481	.36
.37	- 3.85457064	4.38770270	- .640072993	.106940928	- .0500	.37
.38	- 3.77638470	4.32157895	- .655362319	.110168067	- .0520	.38
.39	- 3.70193707	4.25910256	- .670575540	.113410042	- .0540	.39
.40	- 3.63095238	4.20000000	- .685714286	.116666667	- .0560	.40
.41	- 3.56318201	4.14402439	- .700780142	.119937759	- .0581	.41
.42	- 3.49840087	4.09095238	- .715774648	.123225140	- .0601	.42
.43	- 3.43640473	4.04058140	- .730699301	.126522634	- .0623	.43
.44	- 3.37700778	3.99272727	- .745555556	.129836066	- .0644	.44
.45	- 3.32004066	3.94722222	- .760344828	.133163265	- .0666	.45
.46	- 3.26534862	3.90391304	- .775068493	.136504065	- .0688	.46
.47	- 3.21278998	3.86265957	- .789727891	.139858300	- .0711	.47
.48	- 3.16223482	3.82333333	- .804324324	.143225806	- .0734	.48
.49	- 3.11356369	3.78581633	- .818859060	.146606426	- .0757	.49
.50	- 3.06666667	3.75000000	- .833333333	.150000000	- .0781	.50
.51	- 3.02144234	3.71578431	- .847748344	.153406375	- .0805	.51
.52	- 2.97779706	3.68307692	- .862105263	.156825397	- .0830	.52
.53	- 2.93564414	3.65179245	- .876405229	.160256917	- .0855	.53
.54	- 2.89490329	3.62185185	- .890649351	.163700787	- .0880	.54
.55	- 2.85549997	3.59318182	- .904838710	.167156863	- .0906	.55
.56	- 2.81736493	3.56571429	- .918974359	.170625000	- .0932	.56
.57	- 2.78043370	3.53938596	- .933057325	.174105058	- .0958	.57
.58	- 2.74464622	3.51413793	- .947088608	.177596899	- .0985	.58
.59	- 2.70994646	3.48991525	- .961069182	.181100386	- .1012	.59
.60	- 2.67628205	3.46666667	- .975000000	.184615385	- .1040	.60
.61	- 2.64360404	3.44434426	- .988881988	.188141762	- .1068	.61
.62	- 2.61186657	3.42290323	-1.00271605	.191679389	- .1097	.62
.63	- 2.58102666	3.40230159	-1.01650307	.195228137	- .1125	.63
.64	- 2.55104398	3.38250000	-1.03024390	.198787879	- .1155	.64
.65	- 2.52188064	3.36346154	-1.04393939	.202358491	- .1184	.65
.66	- 2.49350100	3.34515152	-1.05759036	.205939850	- .1214	.66
.67	- 2.46587154	3.32753731	-1.07119760	.209531835	- .1245	.67
.68	- 2.43896066	3.31058824	-1.08476190	.213134328	- .1276	.68
.69	- 2.41273855	3.29427536	-1.09828402	.216747212	- .1307	.69
	$-{}_3^1C_3^3$	$-{}_3^1C_2^3$	$-{}_3^1C_1^3$	$-{}_3^1C_0^3$	$-\frac{{}_3^1E^3}{b^3 f^{(4)}(\xi)}$	$r=\frac{c}{b}\frac{c}{a}$

5(a)

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}_3^iC_j^i \, y_j + {}_3^iR^i \qquad (i=0,3)$$

$r=\frac{a}{b}\frac{a}{c}$	${}_3^1C_0^0$	${}_3^1C_1^0$	${}_3^1C_2^0$	${}_3^1C_3^0$	$\frac{{}_3^1E^0}{b^3 f^{(4)}(\xi)}$	
0.70	- 2.38717709	3.27857143	-1.11176471	0.220370370	-0.1339	0.70
.71	- 2.36224972	3.26345070	-1.12520468	.224003690	- .1371	.71
.72	- 2.33793130	3.24888889	-1.13860465	.227647059	- .1404	.72
.73	- 2.31419806	3.23486301	-1.15196532	.231300366	- .1437	.73
.74	- 2.29102750	3.22135135	-1.16528736	.234963504	- .1470	.74
.75	- 2.26839827	3.20833333	-1.17857143	.238636364	- .1504	.75
.76	- 2.24629013	3.19578947	-1.19181818	.242318841	- .1538	.76
.77	- 2.22468388	3.18370130	-1.20502825	.246010830	- .1573	.77
.78	- 2.20356127	3.17205128	-1.21820225	.249712230	- .1608	.78
.79	- 2.18290494	3.16082278	-1.23134078	.253422939	- .1644	.79
.80	- 2.16269841	3.15000000	-1.24444444	.257142857	- .1680	.80
.81	- 2.14292598	3.13956790	-1.25751381	.260871886	- .1717	.81
.82	- 2.12357267	3.12951220	-1.27054945	.264609929	- .1754	.82
.83	- 2.10462425	3.11981928	-1.28355191	.268356890	- .1791	.83
.84	- 2.08606713	3.11047619	-1.29652174	.272112676	- .1829	.84
.85	- 2.06788832	3.10147059	-1.30945946	.275877193	- .1867	.85
.86	- 2.05007546	3.09279070	-1.32236559	.279650350	- .1906	.86
.87	- 2.03261670	3.08442529	-1.33524064	.283432056	- .1946	.87
.88	- 2.01550075	3.07636364	-1.34808511	.287222222	- .1985	.88
.89	- 1.99871680	3.06859551	-1.36089947	.291020761	- .2026	.89
.90	- 1.98225449	3.06111111	-1.37368421	.294827586	- .2066	.90
.91	- 1.96610392	3.05390110	-1.38643979	.298642612	- .2107	.91
.92	- 1.95025561	3.04695652	-1.39916667	.302465753	- .2149	.92
.93	- 1.93470046	3.04026882	-1.41186528	.306296928	- .2191	.93
.94	- 1.91942976	3.03382979	-1.42453608	.310136054	- .2234	.94
.95	- 1.90443514	3.02763158	-1.43717949	.313983051	- .2277	.95
.96	- 1.88970859	3.02166667	-1.44979592	.317837838	- .2321	.96
.97	- 1.87524238	3.01592784	-1.46238679	.321700337	- .2365	.97
.98	- 1.86102914	3.01040816	-1.47494949	.325570470	- .2409	.98
.99	- 1.84706173	3.00510101	-1.48748744	.329448161	- .2454	.99
1.00	- 1.83333333	3.00000000	-1.50000000	.333333333	- .2500	1.00
1.01	- 1.81983736	2.99509901	-1.51248756	.337225914	- .2546	1.01
1.02	- 1.80656749	2.99039216	-1.52495050	.341125828	- .2593	1.02
1.03	- 1.79351763	2.98587379	-1.53738916	.345033003	- .2640	1.03
1.04	- 1.78068191	2.98153846	-1.54980392	.348947368	- .2687	1.04
1.05	- 1.76805468	2.97738095	-1.56219512	.352868852	- .2735	1.05
1.06	- 1.75563051	2.97339623	-1.57456311	.356797386	- .2784	1.06
1.07	- 1.74340413	2.96957944	-1.58690821	.360732899	- .2833	1.07
1.08	- 1.73137048	2.96592593	-1.59923077	.364675325	- .2883	1.08
1.09	- 1.71952469	2.96243119	-1.61153110	.368624595	- .2933	1.09
1.10	- 1.70786203	2.95909091	-1.62380952	.372580645	- .2984	1.10
1.11	- 1.69637796	2.95590090	-1.63606635	.376543408	- .3035	1.11
1.12	- 1.68506808	2.95285714	-1.64830189	.380512821	- .3087	1.12
1.13	- 1.67392814	2.94995575	-1.66051643	.384488818	- .3139	1.13
1.14	- 1.66295404	2.94719298	-1.67271028	.388471338	- .3192	1.14
1.15	- 1.65214181	2.94456522	-1.68488372	.392460317	- .3245	1.15
1.16	- 1.64148762	2.94206897	-1.69703704	.396455696	- .3299	1.16
1.17	- 1.63098776	2.93970085	-1.70917051	.400457413	- .3353	1.17
1.18	- 1.62063863	2.93745763	-1.72128440	.404465409	- .3408	1.18
1.19	- 1.61043676	2.93533613	-1.73337900	.408479624	- .3464	1.19
1.20	- 1.60037879	2.93333333	-1.74545455	.412500000	- .3520	1.20
1.21	- 1.59046145	2.93144628	-1.75751131	.416526480	- .3577	1.21
1.22	- 1.58068159	2.92967213	-1.76954955	.420559006	- .3634	1.22
1.23	- 1.57103615	2.92800813	-1.78156951	.424597523	- .3691	1.23
1.24	- 1.56152216	2.92645161	-1.79357143	.428641975	- .3750	1.24
1.25	- 1.55213675	2.92500000	-1.80555556	.432692308	- .3809	1.25
1.26	- 1.54287714	2.92365079	-1.81752212	.436748466	- .3868	1.26
1.27	- 1.53374061	2.92240157	-1.82947137	.440810398	- .3928	1.27
1.28	- 1.52472454	2.92125000	-1.84140351	.444878049	- .3989	1.28
1.29	- 1.51582659	2.92019380	-1.85331878	.448951368	- .4050	1.29
	$-{}_3^1C_3^3$	$-{}_3^1C_2^3$	$-{}_3^1C_1^3$	$-{}_3^1C_0^3$	$-\frac{{}_3^1E^3}{b^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}^{i}_{3}C^{i}_{j}\, y_j + {}^{i}_{3}R^{i} \qquad (i=1,2)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{1}_{3}C^{1}_{0}$	${}^{1}_{3}C^{1}_{1}$	${}^{1}_{3}C^{1}_{2}$	${}^{1}_{3}C^{1}_{3}$	$\frac{{}^{1}_{3}E^{1}}{b^3 f^{(4)}(\xi)}$	
0.10	-8.65800866	8.50000000	0.181818182	-0.0238095238	0.0083	0.10
.11	-7.76304094	7.59090909	.198198198	- .0260663507	.0092	.11
.12	-7.01931716	6.83333333	.214285714	- .0283018868	.0100	.12
.13	-6.39187976	6.19230769	.230088496	- .0305164319	.0108	.13
.14	-5.85576090	5.64285714	.245614035	- .0327102804	.0117	.14
.15	-5.39265251	5.16666667	.260869565	- .0348837209	.0125	.15
.16	-4.98882503	4.75000000	.275862069	- .0370370370	.0133	.16
.17	-4.63378072	4.38235294	.290598291	- .0391705069	.0142	.17
.18	-4.31935590	4.05555556	.305084746	- .0412844037	.0150	.18
.19	-4.03910665	3.76315789	.319327731	- .0433789954	.0158	.19
.20	-3.78787879	3.50000000	.333333333	- .0454545455	.0167	.20
.21	-3.56150089	3.26190476	.347107438	- .0475113112	.0175	.21
.22	-3.35656073	3.04545455	.360655738	- .0495495495	.0183	.22
.23	-3.17024032	2.84782609	.373983740	- .0515695067	.0192	.23
.24	-3.00019201	2.66666667	.387096774	- .0535714286	.0200	.24
.25	-2.84444444	2.50000000	.400000000	- .0555555556	.0208	.25
.26	-2.70133013	2.34615385	.412698413	- .0575221239	.0217	.26
.27	-2.56942919	2.20370370	.425196850	- .0594713656	.0225	.27
.28	-2.44752506	2.07142857	.437500000	- .0614035088	.0233	.28
.29	-2.33456949	1.94827586	.449612403	- .0633187773	.0242	.29
.30	-2.22965440	1.83333333	.461538462	- .0652173913	.0250	.30
.31	-2.13198933	1.72580645	.473282443	- .0670995671	.0258	.31
.32	-2.04088297	1.62500000	.484848485	- .0689655172	.0267	.32
.33	-1.95572818	1.53030303	.496240602	- .0708154506	.0275	.33
.34	-1.87598958	1.44117647	.507462687	- .0726495726	.0283	.34
.35	-1.80119329	1.35714286	.518518519	- .0744680851	.0292	.35
.36	-1.73091836	1.27777778	.529411765	- .0762711864	.0300	.36
.37	-1.66478962	1.20270270	.540145985	- .0780590717	.0308	.37
.38	-1.60247165	1.13157895	.550724638	- .0798319328	.0317	.38
.39	-1.54366369	1.06410256	.561151079	- .0815899582	.0325	.39
.40	-1.48809524	1.00000000	.571428571	- .0833333333	.0333	.40
.41	-1.43552243	.939024390	.581560284	- .0850622407	.0342	.41
.42	-1.38572482	.880952381	.591549296	- .0867768595	.0350	.42
.43	-1.33850263	.825581395	.601398601	- .0884773663	.0358	.43
.44	-1.29367445	.772727273	.611111111	- .0901639344	.0367	.44
.45	-1.25107514	.722222222	.620689655	- .0918367347	.0375	.45
.46	-1.21055409	.673913043	.630136986	- .0934959350	.0383	.46
.47	-1.17197366	.627659574	.639455782	- .0951417004	.0392	.47
.48	-1.13520779	.583333333	.648648649	- .0967741935	.0400	.48
.49	-1.10014087	.540816327	.657718121	- .0983935743	.0408	.49
.50	-1.06666667	.500000000	.666666667	- .100000000	.0417	.50
.51	-1.03468738	.460784314	.675496689	- .101593626	.0425	.51
.52	-1.00411285	.423076923	.684210526	- .103174603	.0433	.52
.53	- .974859827	.386792453	.692810457	- .104743083	.0442	.53
.54	- .946851341	.351851852	.701298701	- .106299213	.0450	.54
.55	- .920016100	.318181818	.709677419	- .107843137	.0458	.55
.56	- .894288004	.285714286	.717948718	- .109375000	.0467	.56
.57	- .869605673	.254385965	.726114650	- .110894942	.0475	.57
.58	- .845912045	.224137931	.734177215	- .112403101	.0483	.58
.59	- .823154005	.194915254	.742138365	- .113899614	.0492	.59
.60	- .801282051	.166666667	.750000000	- .115384615	.0500	.60
.61	- .780250000	.139344262	.757763975	- .116858238	.0508	.61
.62	- .760014714	.112903226	.765432099	- .118320611	.0517	.62
.63	- .740535859	.087301587	.773006135	- .119771863	.0525	.63
.64	- .721775684	.062500000	.780487805	- .121212121	.0533	.64
.65	- .703698817	.038461538	.787878788	- .122641509	.0542	.65
.66	- .686272088	.015151515	.795180723	- .124060150	.0550	.66
.67	- .669464358	- .007462687	.802395210	- .125468165	.0558	.67
.68	- .653246375	- .029411765	.809523810	- .126865672	.0567	.68
.69	- .637590622	- .050724638	.816568047	- .128252788	.0575	.69
	$-{}^{1}_{3}C^{2}_{3}$	$-{}^{1}_{3}C^{2}_{2}$	$-{}^{1}_{3}C^{2}_{1}$	$-{}^{1}_{3}C^{2}_{0}$	$-\frac{{}^{1}_{3}E^{2}}{b^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

6(a)

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}^{i}_{3}C^{i}_{j}\, y_j + {}^{i}_{3}R^{i} \qquad (i=i,2)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{i}_{3}C^{i}_{0}$	${}^{i}_{3}C^{i}_{1}$	${}^{i}_{3}C^{i}_{2}$	${}^{i}_{3}C^{i}_{3}$	$\frac{{}^{i}_{3}E^{i}}{b^3 f^{(4)}(\xi)}$	
0.70	-0.622471211	-0.071428571	0.823529412	-0.129629630	0.0583	0.70
.71	- .607863751	- .091549296	.830409357	- .130996310	.0592	.71
.72	- .593745250	- .111111111	.837209302	- .132352941	.0600	.72
.73	- .580094016	- .130136986	.843930636	- .133699634	.0608	.73
.74	- .566889568	- .148648649	.850574713	- .135036496	.0617	.74
.75	- .554112554	- .166666667	.857142857	- .136363636	.0625	.75
.76	- .541744678	- .184210526	.863636364	- .137681159	.0633	.76
.77	- .529768626	- .201298701	.870056497	- .138989170	.0642	.77
.78	- .518168007	- .217948718	.876404494	- .140287770	.0650	.78
.79	- .506927288	- .234177215	.882681564	- .141577061	.0658	.79
.80	- .496031746	- .250000000	.888888889	- .142857143	.0667	.80
.81	- .485467412	- .265432099	.895027624	- .144128114	.0675	.81
.82	- .475221025	- .280487805	.901098901	- .145390071	.0683	.82
.83	- .465279993	- .295180723	.907103825	- .146643110	.0692	.83
.84	- .455632345	- .309523810	.913043478	- .147887324	.0700	.84
.85	- .446266700	- .323529412	.918918919	- .149122807	.0708	.85
.86	- .437172230	- .337209302	.924731183	- .150349650	.0717	.86
.87	- .428338627	- .350574713	.930481283	- .151567944	.0725	.87
.88	- .419756071	- .363636364	.936170213	- .152777778	.0733	.88
.89	- .411415209	- .376404494	.941798942	- .153979239	.0742	.89
.90	- .403307118	- .388888889	.947368421	- .155172414	.0750	.90
.91	- .395423292	- .401098901	.952879581	- .156357388	.0758	.91
.92	- .387755608	- .413043478	.958333333	- .157534247	.0767	.92
.93	- .380296315	- .424731183	.963730570	- .158703072	.0775	.93
.94	- .373038007	- .436170213	.969072165	- .159863946	.0783	.94
.95	- .365973604	- .447368421	.974358974	- .161016949	.0792	.95
.96	- .359096341	- .458333333	.979591837	- .162162162	.0800	.96
.97	- .352399745	- .469072165	.984771574	- .163299663	.0808	.97
.98	- .345877623	- .479591837	.989898990	- .164429530	.0817	.98
.99	- .339524045	- .489898990	.994974874	- .165551839	.0825	.99
1.00	- .333333333	- .500000000	1.00000000	- .166666667	.0833	1.00
1.01	- .327300048	- .509900990	1.00497512	- .167774086	.0842	1.01
1.02	- .321418975	- .519607843	1.00990099	- .168874172	.0850	1.02
1.03	- .315685115	- .529126214	1.01477833	- .169966997	.0858	1.03
1.04	- .310093673	- .538461538	1.01960784	- .171052632	.0867	1.04
1.05	- .304640049	- .547619048	1.02439024	- .172131148	.0875	1.05
1.06	- .299319826	- .556603774	1.02912621	- .173202614	.0883	1.06
1.07	- .294128763	- .565420561	1.03381643	- .174267101	.0892	1.07
1.08	- .289062789	- .574074074	1.03846154	- .175324675	.0900	1.08
1.09	- .284117989	- .582568807	1.04306220	- .176375405	.0908	1.09
1.10	- .279290602	- .590909091	1.04761905	- .177419355	.0917	1.10
1.11	- .274577011	- .599099099	1.05213270	- .178456592	.0925	1.11
1.12	- .269973737	- .607142857	1.05660377	- .179487179	.0933	1.12
1.13	- .265477434	- .615044248	1.06103286	- .180511182	.0942	1.13
1.14	- .261084881	- .622807018	1.06542056	- .181528662	.0950	1.14
1.15	- .256792977	- .630434783	1.06976744	- .182539683	.0958	1.15
1.16	- .252598736	- .637931034	1.07407407	- .183544304	.0967	1.16
1.17	- .248499282	- .645299145	1.07834101	- .184542587	.0975	1.17
1.18	- .244491843	- .652542373	1.08256881	- .185534591	.0983	1.18
1.19	- .240573749	- .659663866	1.08675799	- .186520376	.0992	1.19
1.20	- .236742424	- .666666667	1.09090909	- .187500000	.1000	1.20
1.21	- .232995385	- .673553719	1.09502262	- .188473520	.1008	1.21
1.22	- .229330236	- .680327869	1.09909910	- .189440994	.1017	1.22
1.23	- .225744667	- .686991870	1.10313901	- .190402477	.1025	1.23
1.24	- .222236445	- .693548387	1.10714286	- .191358025	.1033	1.24
1.25	- .218803419	- .700000000	1.11111111	- .192307692	.1042	1.25
1.26	- .215443508	- .706349206	1.11504425	- .193251534	.1050	1.26
1.27	- .212154704	- .712598425	1.11894273	- .194189602	.1058	1.27
1.28	- .208935066	- .718750000	1.12280702	- .195121951	.1067	1.28
1.29	- .205782721	- .724806202	1.12663755	- .196048632	.1075	1.29
	$-{}^{1}_{3}C^{2}_{3}$	$-{}^{1}_{3}C^{2}_{2}$	$-{}^{1}_{3}C^{2}_{1}$	$-{}^{1}_{3}C^{2}_{0}$	$-\frac{{}^{1}_{3}E^{2}}{b^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

6(b)

FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{h^i} \sum_{j=0}^{3} {}^{i}_{3}C^{i}_{j}\, y_j + {}^{i}_{3}R^{i} \quad (i=2,1)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{1}_{3}C^{2}_{0}$	${}^{1}_{3}C^{2}_{1}$	${}^{1}_{3}C^{2}_{2}$	${}^{1}_{3}C^{2}_{3}$	$\frac{{}^{1}_{3}E^{2}}{h^2 f^{(4)}(\xi)}$	
0.10	4.32900433	-5.50000000	0.909090909	0.261904762	-0.0458	0.10
.11	3.88152047	-5.04545455	.900900901	.263033175	- .0463	.11
.12	3.50965858	-4.66666667	.892857143	.264150943	- .0467	.12
.13	3.19593988	-4.34615385	.884955752	.265258216	- .0471	.13
.14	2.92788045	-4.07142857	.877192982	.266355140	- .0475	.14
.15	2.69632626	-3.83333333	.869565217	.267441860	- .0479	.15
.16	2.49441252	-3.62500000	.862068966	.268518519	- .0483	.16
.17	2.31689036	-3.44117647	.854700855	.269585253	- .0488	.17
.18	2.15967795	-3.27777778	.847457627	.270642202	- .0492	.18
.19	2.01955332	-3.13157895	.840336134	.271689498	- .0496	.19
.20	1.89393939	-3.00000000	.833333333	.272727273	- .0500	.20
.21	1.78075044	-2.88095238	.826446281	.273755656	- .0504	.21
.22	1.67828036	-2.77272727	.819672131	.274774775	- .0508	.22
.23	1.58512016	-2.67391304	.813008130	.275784753	- .0513	.23
.24	1.50009601	-2.58333333	.806451613	.276785714	- .0517	.24
.25	1.42222222	-2.50000000	.800000000	.277777778	- .0521	.25
.26	1.35066506	-2.42307692	.793650794	.278761062	- .0525	.26
.27	1.28471460	-2.35185185	.787401575	.279735683	- .0529	.27
.28	1.22376253	-2.28571429	.781250000	.280701754	- .0533	.28
.29	1.16728475	-2.22413793	.775193798	.281659389	- .0538	.29
.30	1.11482720	-2.16666667	.769230769	.282608696	- .0542	.30
.31	1.06599467	-2.11290323	.763358779	.283549784	- .0546	.31
.32	1.02044149	-2.06250000	.757575758	.284482759	- .0550	.32
.33	.977864090	-2.01515152	.751879699	.285407725	- .0554	.33
.34	.937994790	-1.97058824	.746268657	.286324786	- .0558	.34
.35	.900596645	-1.92857143	.740740741	.287234043	- .0563	.35
.36	.865459180	-1.88888889	.735294118	.288135593	- .0567	.36
.37	.832394810	-1.85135135	.729927007	.289029536	- .0571	.37
.38	.801255825	-1.81578947	.724637681	.289915966	- .0575	.38
.39	.771831845	-1.78205128	.719424460	.290794979	- .0579	.39
.40	.74404762	-1.75000000	.714285714	.291666667	- .0583	.40
.41	.717761215	-1.71951220	.709219858	.292531120	- .0588	.41
.42	.692862410	-1.69047619	.704225352	.293388430	- .0592	.42
.43	.669251315	-1.66279070	.699300699	.294238683	- .0596	.43
.44	.646837225	-1.63636364	.694444444	.295081967	- .0600	.44
.45	.625537571	-1.61111111	.689655172	.295918367	- .0604	.45
.46	.605277047	-1.58695652	.684931507	.296747967	- .0608	.46
.47	.585986828	-1.56382979	.680272109	.297570850	- .0613	.47
.48	.567603894	-1.54166667	.675675676	.298387097	- .0617	.48
.49	.550070437	-1.52040816	.671140940	.299196787	- .0621	.49
.50	.533333333	-1.50000000	.666666667	.300000000	- .0625	.50
.51	.517343688	-1.48039216	.662251656	.300796813	- .0629	.51
.52	.502056423	-1.46153846	.657894737	.301587302	- .0633	.52
.53	.487429914	-1.44339623	.653594771	.302371542	- .0638	.53
.54	.473425670	-1.42592593	.649350649	.303149606	- .0642	.54
.55	.460008050	-1.40909091	.645161290	.303921569	- .0646	.55
.56	.447144002	-1.39285714	.641025641	.304687500	- .0650	.56
.57	.434802836	-1.37719298	.636942675	.305447471	- .0654	.57
.58	.422956023	-1.36206897	.632911392	.306201550	- .0658	.58
.59	.411577003	-1.34745763	.628930818	.306949807	- .0663	.59
.60	.400641026	-1.33333333	.625000000	.307692308	- .0667	.60
.61	.390125000	-1.31967213	.621118012	.308429119	- .0671	.61
.62	.380007357	-1.30645161	.617283951	.309160305	- .0675	.62
.63	.370267930	-1.29365079	.613496933	.309885932	- .0679	.63
.64	.360887842	-1.28125000	.609756098	.310606061	- .0683	.64
.65	.351849408	-1.26923077	.606060606	.311320755	- .0688	.65
.66	.343136044	-1.25757576	.602409639	.312030075	- .0692	.66
.67	.334732179	-1.24626866	.598802395	.312734082	- .0696	.67
.68	.326623187	-1.23529412	.595238095	.313432836	- .0700	.68
.69	.318795311	-1.22463768	.591715976	.314126394	- .0704	.69
	$-{}^{1}_{3}C^{1}_{3}$	$-{}^{1}_{3}C^{1}_{2}$	$-{}^{1}_{3}C^{1}_{1}$	$-{}^{1}_{3}C^{1}_{0}$	$-\frac{{}^{1}_{3}E^{1}}{h^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

7(a)

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x-x_i=\frac{1}{b}} \sum_{j=0}^{3} {}^{i}_{3}C^{i}_{j}\, y_j + {}^{i}_{3}R^{i} \qquad (1\text{-}2.1)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{1}_{3}C^{2}_{0}$	${}^{1}_{3}C^{2}_{1}$	${}^{1}_{3}C^{2}_{2}$	${}^{1}_{3}C^{2}_{3}$	$\frac{{}^{1}_{3}E^{2}}{b^3 f^{(4)}(\xi)}$	
0.70	0.311235605	-1.21428571	0.588235294	0.314814815	-0.0708	0.70
.71	.303931875	-1.20422535	.584795322	.315498155	- .0713	.71
.72	.296872625	-1.19444444	.581395349	.316176471	- .0717	.72
.73	.290047008	-1.18493151	.578034682	.316849817	- .0721	.73
.74	.283444784	-1.17567568	.574712644	.317518248	- .0725	.74
.75	.277056277	-1.16666667	.571428571	.318181818	- .0729	.75
.76	.270872339	-1.15789474	.568181818	.318840580	- .0733	.76
.77	.264884313	-1.14935065	.564971751	.319494585	- .0738	.77
.78	.259084003	-1.14102564	.561797753	.320143885	- .0742	.78
.79	.253463644	-1.13291139	.558659218	.320788530	- .0746	.79
.80	.248015873	-1.12500000	.555555556	.321428571	- .0750	.80
.81	.242733706	-1.11728395	.552486188	.322064057	- .0754	.81
.82	.237610513	-1.10975610	.549450549	.322695035	- .0758	.82
.83	.232639996	-1.10240964	.546448087	.323321555	- .0763	.83
.84	.227816172	-1.09523810	.543478261	.323943662	- .0767	.84
.85	.223133350	-1.08823529	.540540541	.324561404	- .0771	.85
.86	.218586115	-1.08139535	.537634409	.325174825	- .0775	.86
.87	.214169313	-1.07471264	.534759358	.325783972	- .0779	.87
.88	.209878036	-1.06818182	.531914894	.326388889	- .0783	.88
.89	.205707604	-1.06179775	.529100529	.326989619	- .0788	.89
.90	.201653559	-1.05555556	.526315789	.327586207	- .0792	.90
.91	.197711646	-1.04945055	.523560209	.328178694	- .0796	.91
.92	.193877804	-1.04347826	.520833333	.328767123	- .0800	.92
.93	.190148158	-1.03763441	.518134715	.329351536	- .0804	.93
.94	.186519003	-1.03191489	.515463918	.329931973	- .0808	.94
.95	.182986802	-1.02631579	.512820513	.330508475	- .0813	.95
.96	.179548171	-1.02083333	.510204082	.331081081	- .0817	.96
.97	.176199873	-1.01546392	.507614213	.331649832	- .0821	.97
.98	.172938811	-1.01020408	.505050505	.332214765	- .0825	.98
.99	.169762023	-1.00505051	.502512563	.332775920	- .0829	.99
1.00	.166666667	-1.00000000	.500000000	.333333333	- .0833	1.00
1.01	.163650024	- .995049505	.497512438	.333887043	- .0838	1.01
1.02	.160709487	- .990196078	.495049505	.334437086	- .0842	1.02
1.03	.157842557	- .985436893	.492610837	.334983498	- .0846	1.03
1.04	.155046837	- .980769231	.490196078	.335526316	- .0850	1.04
1.05	.152320024	- .976190476	.487804878	.336065574	- .0854	1.05
1.06	.149659913	- .971698113	.485436893	.336601307	- .0858	1.06
1.07	.147064382	- .967289720	.483091787	.337133550	- .0863	1.07
1.08	.144531395	- .962962963	.480769231	.337662338	- .0867	1.08
1.09	.142058995	- .958715596	.478468900	.338187702	- .0871	1.09
1.10	.139645301	- .954545455	.476190476	.338709677	- .0875	1.10
1.11	.137288505	- .950450450	.473933649	.339228296	- .0879	1.11
1.12	.134986868	- .946428571	.471698113	.339743590	- .0883	1.12
1.13	.132738717	- .942477876	.469483568	.340255591	- .0888	1.13
1.14	.130542441	- .938596491	.467289720	.340764331	- .0892	1.14
1.15	.128396489	- .934782609	.465116279	.341269841	- .0896	1.15
1.16	.126299368	- .931034483	.462962963	.341772152	- .0900	1.16
1.17	.124249641	- .927350427	.460829493	.342271293	- .0904	1.17
1.18	.122245922	- .923728814	.458715596	.342767296	- .0908	1.18
1.19	.120286875	- .920168067	.456621005	.343260188	- .0913	1.19
1.20	.118371212	- .916666667	.454545455	.343750000	- .0917	1.20
1.21	.116497693	- .913223140	.452488688	.344236760	- .0921	1.21
1.22	.114665118	- .909836066	.450450450	.344720497	- .0925	1.22
1.23	.112872334	- .906504065	.448430493	.345201238	- .0929	1.23
1.24	.111118223	- .903225806	.446428571	.345679012	- .0933	1.24
1.25	.109401710	- .900000000	.444444444	.346153846	- .0938	1.25
1.26	.107721754	- .896825397	.442477876	.346625767	- .0942	1.26
1.27	.106077352	- .893700787	.440528634	.347094801	- .0946	1.27
1.28	.104467533	- .890625000	.438596491	.347560976	- .0950	1.28
1.29	.102891361	- .887596899	.436681223	.348024316	- .0954	1.29
	$-{}^{1}_{3}C^{1}_{3}$	$-{}^{1}_{3}C^{1}_{2}$	$-{}^{1}_{3}C^{1}_{1}$	$-{}^{1}_{3}C^{1}_{0}$	$-\frac{{}^{1}_{3}E^{1}}{b^3 f^{(4)}(\xi)}$	$\rho=\frac{c}{b}\frac{c}{a}$

7(b)

FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}_3^iC_j^i \, y_j + {}_3^iR^i \qquad (i=3,0)$$

$r=\frac{a}{b}\frac{a}{c}$	${}_3^1C_0^3$	${}_3^1C_1^3$	${}_3^1C_2^3$	${}_3^1C_3^3$	$\frac{{}_3^1E^3}{b^3 f^{(4)}(\xi)}$	
0.10	-8.65800866	10.50000000	-3.81818182	1.97619048	0.1750	0.10
.11	-7.76304094	9.59090909	-3.80180180	1.97393365	.1758	.11
.12	-7.01931716	8.83333333	-3.78571429	1.97169811	.1767	.12
.13	-6.39187976	8.19230769	-3.76991150	1.96948357	.1775	.13
.14	-5.85576090	7.64285714	-3.75438596	1.96728972	.1783	.14
.15	-5.39265251	7.16666667	-3.73913043	1.96511628	.1792	.15
.16	-4.98882503	6.75000000	-3.72413793	1.96296296	.1800	.16
.17	-4.63378072	6.38235294	-3.70940171	1.96082949	.1808	.17
.18	-4.31935590	6.05555556	-3.69491525	1.95871560	.1817	.18
.19	-4.03910663	5.76315789	-3.68067227	1.95662100	.1825	.19
.20	-3.78787879	5.50000000	-3.66666667	1.95454545	.1833	.20
.21	-3.56150089	5.26190476	-3.65289256	1.95248869	.1842	.21
.22	-3.35656073	5.04545454	-3.63934426	1.95045045	.1850	.22
.23	-3.17024032	4.84782609	-3.62601626	1.94843049	.1858	.23
.24	-3.00019201	4.66666667	-3.61290323	1.94642857	.1867	.24
.25	-2.84444444	4.50000000	-3.60000000	1.94444444	.1875	.25
.26	-2.70133013	4.34615385	-3.58730159	1.94247788	.1883	.26
.27	-2.56942919	4.20370370	-3.57480314	1.94052863	.1892	.27
.28	-2.44752506	4.07142857	-3.56250000	1.93859649	.1900	.28
.29	-2.33456949	3.94827586	-3.55038760	1.93668122	.1908	.29
.30	-2.22965440	3.83333333	-3.53846154	1.93478261	.1917	.30
.31	-2.13198933	3.72580645	-3.52671756	1.93290043	.1925	.31
.32	-2.04088297	3.62500000	-3.51515152	1.93103448	.1933	.32
.33	-1.95572818	3.53030303	-3.50375940	1.92918455	.1942	.33
.34	-1.87598958	3.44117647	-3.49253732	1.92735043	.1950	.34
.35	-1.80119329	3.35714286	-3.48148148	1.92553191	.1958	.35
.36	-1.73091836	3.27777778	-3.47058824	1.92372881	.1967	.36
.37	-1.66478962	3.20270270	-3.45985402	1.92194093	.1975	.37
.38	-1.60247165	3.13157895	-3.44927536	1.92016807	.1983	.38
.39	-1.54366369	3.06410256	-3.43884892	1.91841004	.1992	.39
.40	-1.48809524	3.00000000	-3.42857142	1.91666667	.2000	.40
.41	-1.43552243	2.93902439	-3.41843972	1.91493776	.2008	.41
.42	-1.38572482	2.88095238	-3.40845070	1.91322314	.2017	.42
.43	-1.33850263	2.82558140	-3.39860140	1.91152263	.2025	.43
.44	-1.29367445	2.77272727	-3.38888889	1.90983607	.2033	.44
.45	-1.25107514	2.72222222	-3.37931034	1.90816327	.2042	.45
.46	-1.21055409	2.67391304	-3.36986301	1.90650407	.2050	.46
.47	-1.17197366	2.62765957	-3.36054422	1.90485830	.2058	.47
.48	-1.13520779	2.58333333	-3.35135135	1.90322581	.2067	.48
.49	-1.10014087	2.54081633	-3.34228188	1.90160643	.2075	.49
.50	-1.06666667	2.50000000	-3.33333333	1.90000000	.2083	.50
.51	-1.03468738	2.46078431	-3.32450331	1.89840637	.2092	.51
.52	-1.00411285	2.42307692	-3.31578947	1.89682540	.2100	.52
.53	- .974859827	2.38679245	-3.30718954	1.89525692	.2108	.53
.54	- .946851341	2.35185185	-3.29870130	1.89370079	.2117	.54
.55	- .920016100	2.31818182	-3.29032258	1.89215686	.2125	.55
.56	- .894288004	2.28571429	-3.28205128	1.89062500	.2133	.56
.57	- .869605673	2.25438596	-3.27388835	1.88910506	.2142	.57
.58	- .845912045	2.22413793	-3.26582278	1.88759690	.2150	.58
.59	- .823154005	2.19491525	-3.25786164	1.88610039	.2158	.59
.60	- .801282051	2.16666667	-3.25000000	1.88461538	.2167	.60
.61	- .780250000	2.13934426	-3.24223602	1.88314176	.2175	.61
.62	- .760014714	2.11290323	-3.23456790	1.88167939	.2183	.62
.63	- .740535859	2.08730159	-3.22699387	1.88022814	.2192	.63
.64	- .721775684	2.06250000	-3.21951219	1.87878788	.2200	.64
.65	- .703698817	2.03846154	-3.21212121	1.87735849	.2208	.65
.66	- .686272088	2.01515152	-3.20481928	1.87593985	.2217	.66
.67	- .669464358	1.99253731	-3.19760479	1.87453184	.2225	.67
.68	- .653246373	1.97058824	-3.19047619	1.87313433	.2233	.68
.69	- .637590622	1.94927536	-3.18343195	1.87174721	.2242	.69
	$-{}_3^1C_3^0$	$-{}_3^1C_2^0$	$-{}_3^1C_1^0$	$-{}_3^1C_0^0$	$-\frac{{}_3^1E^0}{b^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

8(a)

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{3} {}_3^iC_j^i \, y_j + {}_3^iR^i \quad (i=3,0)$$

$r=\frac{a}{b}\frac{a}{c}$	${}_3^iC_0^3$	${}_3^iC_1^3$	${}_3^iC_2^3$	${}_3^iC_3^3$	$\frac{{}_3^iE^3}{b^3 f^{(4)}(\xi)}$	
0.70	-0.622471211	1.92857143	-3.17647059	1.87037037	0.2250	0.70
.71	- .607863751	1.90845070	-3.16959064	1.86900369	.2258	.71
.72	- .593745250	1.88888889	-3.16279070	1.86764706	.2267	.72
.73	- .580094016	1.86986301	-3.15606936	1.86630037	.2275	.73
.74	- .566889568	1.85135135	-3.14942529	1.86496356	.2283	.74
.75	- .554112554	1.83333333	-3.14285714	1.86363636	.2292	.75
.76	- .541744678	1.81578947	-3.13636364	1.86231884	.2300	.76
.77	- .529768626	1.79870130	-3.12994350	1.86101083	.2308	.77
.78	- .518168007	1.78205128	-3.12359550	1.85971223	.2317	.78
.79	- .506927288	1.76582278	-3.11731844	1.85842294	.2325	.79
.80	- .496031746	1.75000000	-3.11111111	1.85714286	.2333	.80
.81	- .485467412	1.73456790	-3.10497238	1.85587189	.2342	.81
.82	- .475221025	1.71951220	-3.09890110	1.85460993	.2350	.82
.83	- .465279993	1.70481928	-3.09289617	1.85335689	.2358	.83
.84	- .455632345	1.69047619	-3.08695652	1.85211268	.2367	.84
.85	- .446266700	1.67647059	-3.08108108	1.85087719	.2375	.85
.86	- .437178230	1.66279070	-3.07526882	1.84965035	.2383	.86
.87	- .428338627	1.64942529	-3.06951872	1.84843206	.2392	.87
.88	- .419756071	1.63636364	-3.06382979	1.84722222	.2400	.88
.89	- .411415209	1.62359551	-3.05820106	1.84602076	.2408	.89
.90	- .403307118	1.61111111	-3.05263158	1.84482759	.2417	.90
.91	- .395423292	1.59890110	-3.04712042	1.84364261	.2425	.91
.92	- .387755608	1.58695652	-3.04166667	1.84246575	.2433	.92
.93	- .380296315	1.57526882	-3.03626943	1.84129693	.2442	.93
.94	- .373038007	1.56382979	-3.03092783	1.84013605	.2450	.94
.95	- .365973604	1.55263158	-3.02564103	1.83898305	.2458	.95
.96	- .359096341	1.54166667	-3.02040816	1.83783784	.2467	.96
.97	- .352399745	1.53092784	-3.01522843	1.83670034	.2475	.97
.98	- .345877623	1.52040816	-3.01010101	1.83557047	.2483	.98
.99	- .339524045	1.51010101	-3.00502513	1.83444816	.2492	.99
1.00	- .333333333	1.50000000	-3.00000000	1.83333333	.2500	1.00
1.01	- .327300048	1.49009901	-2.99502488	1.83222591	.2508	1.01
1.02	- .321418975	1.48039216	-2.99009901	1.83112583	.2517	1.02
1.03	- .315685115	1.47087379	-2.98522167	1.83003300	.2525	1.03
1.04	- .310093673	1.46153846	-2.98039216	1.82894737	.2533	1.04
1.05	- .304640049	1.45238095	-2.97560976	1.82786885	.2542	1.05
1.06	- .299319826	1.44339623	-2.97087379	1.82679739	.2550	1.06
1.07	- .294128763	1.43457944	-2.96618357	1.82573290	.2558	1.07
1.08	- .289062789	1.42592593	-2.96153846	1.82467532	.2567	1.08
1.09	- .284117989	1.41743119	-2.95693780	1.82362460	.2575	1.09
1.10	- .279290602	1.40909091	-2.95238095	1.82258065	.2583	1.10
1.11	- .274577011	1.40090090	-2.94786730	1.82154341	.2592	1.11
1.12	- .269973737	1.39285714	-2.94339623	1.82051282	.2600	1.12
1.13	- .265477434	1.38495575	-2.93896714	1.81948882	.2608	1.13
1.14	- .261084881	1.37719298	-2.93457944	1.81847134	.2617	1.14
1.15	- .256792977	1.36956522	-2.93023256	1.81746032	.2625	1.15
1.16	- .252598736	1.36206897	-2.92592593	1.81645570	.2633	1.16
1.17	- .248499282	1.35470085	-2.92165899	1.81545741	.2642	1.17
1.18	- .244491843	1.34745763	-2.91743119	1.81446541	.2650	1.18
1.19	- .240573749	1.34033613	-2.91324201	1.81347962	.2658	1.19
1.20	- .236742424	1.33333333	-2.90909091	1.81250000	.2667	1.20
1.21	- .232995385	1.32644628	-2.90497738	1.81152648	.2675	1.21
1.22	- .229330236	1.31967213	-2.90090090	1.81055901	.2683	1.22
1.23	- .225744667	1.31300813	-2.89686099	1.80959752	.2692	1.23
1.24	- .222236445	1.30645161	-2.89285714	1.80864198	.2700	1.24
1.25	- .218803419	1.30000000	-2.88888889	1.80769231	.2708	1.25
1.26	- .215443508	1.29365079	-2.88495575	1.80674847	.2717	1.26
1.27	- .212154704	1.28740157	-2.88105727	1.80581040	.2725	1.27
1.28	- .208935066	1.28125000	-2.87719298	1.80487805	.2733	1.28
1.29	- .205782721	1.27519380	-2.87336245	1.80395137	.2742	1.29
	$-{}_3^iC_3^0$	$-{}_3^iC_2^0$	$-{}_3^iC_1^0$	$-{}_3^iC_0^0$	$-\frac{{}_3^iE^0}{b^3 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

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$$(D^2y)_{x=x_i} = \frac{1}{b^2} \sum_{j=0}^{3} {}^2_3C^i_j \, y_j + {}^2_3R^i \qquad (i = 0,3)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^2_3C^0_0$	${}^2_3C^0_1$	${}^2_3C^0_2$	${}^2_3C^0_3$	$\frac{{}^2_3E^0}{b^2f^{(4)}(\xi)}$	
0.10	28.5714286	-32.0000000	4.00000000	-0.571428571	0.2192	0.10
.11	25.8509263	-29.2727273	4.00000000	- .578199052	.2247	.11
.12	23.5849057	-27.0000000	4.00000000	- .584905660	.2303	.12
.13	21.6684724	-25.0769231	4.00000000	- .591549296	.2359	.13
.14	20.0267023	-23.4285714	4.00000000	- .598130841	.2416	.14
.15	18.6046512	-22.0000000	4.00000000	- .604651163	.2473	.15
.16	17.3611111	-20.7500000	4.00000000	- .611111111	.2531	.16
.17	16.2645703	-19.6470588	4.00000000	- .617511521	.2589	.17
.18	15.2905199	-18.6666667	4.00000000	- .623853211	.2648	.18
.19	14.4196107	-17.7894737	4.00000000	- .630136986	.2707	.19
.20	13.6363636	-17.0000000	4.00000000	- .636363636	.2767	.20
.21	12.9282482	-16.2857143	4.00000000	- .642533937	.2827	.21
.22	12.2850123	-15.6363636	4.00000000	- .648648649	.2888	.22
.23	11.6981868	-15.0434783	4.00000000	- .654708520	.2949	.23
.24	11.1607143	-14.5000000	4.00000000	- .660714286	.3011	.24
.25	10.6666667	-14.0000000	4.00000000	- .666666667	.3073	.25
.26	10.2110279	-13.5384615	4.00000000	- .672566372	.3136	.26
.27	9.78952521	-13.1111111	4.00000000	- .678414097	.3199	.27
.28	9.39849624	-12.7142857	4.00000000	- .684210526	.3263	.28
.29	9.03478392	-12.3448276	4.00000000	- .689956332	.3327	.29
.30	8.69565217	-12.0000000	4.00000000	- .695652174	.3392	.30
.31	8.37871806	-11.6774194	4.00000000	- .701298701	.3457	.31
.32	8.08189655	-11.3750000	4.00000000	- .706896552	.3523	.32
.33	7.80335544	-11.0909091	4.00000000	- .712446352	.3589	.33
.34	7.54147813	-10.8235294	4.00000000	- .717948718	.3656	.34
.35	7.29483283	-10.5714286	4.00000000	- .723404255	.3723	.35
.36	7.06214689	-10.3333333	4.00000000	- .728813559	.3791	.36
.37	6.84228532	-10.1081081	4.00000000	- .734177215	.3859	.37
.38	6.63423264	- 9.89473684	4.00000000	- .739495798	.3928	.38
.39	6.43707757	- 9.69230769	4.00000000	- .744769874	.3997	.39
.40	6.25000000	- 9.50000000	4.00000000	- .750000000	.4067	.40
.41	6.07225989	- 9.31707317	4.00000000	- .755186722	.4137	.41
.42	5.90318772	- 9.14285714	4.00000000	- .760330579	.4208	.42
.43	5.74217628	- 8.97674419	4.00000000	- .765432099	.4279	.43
.44	5.58867362	- 8.81818182	4.00000000	- .770491803	.4351	.44
.45	5.44217687	- 8.66666667	4.00000000	- .775510204	.4423	.45
.46	5.30222694	- 8.52173913	4.00000000	- .780487805	.4496	.46
.47	5.16840382	- 8.38297872	4.00000000	- .785425101	.4569	.47
.48	5.04032258	- 8.25000000	4.00000000	- .790322581	.4643	.48
.49	4.91762970	- 8.12244898	4.00000000	- .795180723	.4717	.49
.50	4.80000000	- 8.00000000	4.00000000	- .800000000	.4792	.50
.51	4.68713382	- 7.88235294	4.00000000	- .804780877	.4867	.51
.52	4.57875458	- 7.76923077	4.00000000	- .809523809	.4943	.52
.53	4.47460661	- 7.66037736	4.00000000	- .814229249	.5019	.53
.54	4.37445319	- 7.55555556	4.00000000	- .818897638	.5096	.54
.55	4.27807487	- 7.45454545	4.00000000	- .823529412	.5173	.55
.56	4.18526786	- 7.35714286	4.00000000	- .828125000	.5251	.56
.57	4.09584272	- 7.26315789	4.00000000	- .832684825	.5329	.57
.58	4.00962310	- 7.17241379	4.00000000	- .837209302	.5408	.58
.59	3.92644460	- 7.08474576	4.00000000	- .841698842	.5487	.59
.60	3.84615385	- 7.00000000	4.00000000	- .846153846	.5567	.60
.61	3.76860750	- 6.91803279	4.00000000	- .850574713	.5647	.61
.62	3.69367151	- 6.83870968	4.00000000	- .854961832	.5728	.62
.63	3.62122035	- 6.76190476	4.00000000	- .859315589	.5809	.63
.64	3.55113636	- 6.68750000	4.00000000	- .863636364	.5891	.64
.65	3.48330914	- 6.61538462	4.00000000	- .867924528	.5971	.65
.66	3.41763500	- 6.54545455	4.00000000	- .872180451	.6056	.66
.67	3.35401643	- 6.47761194	4.00000000	- .876404494	.6139	.67
.68	3.29236172	- 6.41176471	4.00000000	- .880597015	.6223	.68
.69	3.23258445	- 6.34782609	4.00000000	- .884758364	.6307	.69
	${}^2_3C^3_3$	${}^2_3C^3_2$	${}^2_3C^3_1$	${}^2_3C^3_0$	$\frac{{}^2_3E^3}{b^2f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

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$$(D^2y)_{x=x_i} = \frac{1}{b^2} \sum_{j=0}^{3} {}^{2}_{3}C^{i}_{j}\, y_j + {}^{2}_{3}R^{i} \qquad (i=0,3)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{2}_{3}C^{0}_{0}$	${}^{2}_{3}C^{0}_{1}$	${}^{2}_{3}C^{0}_{2}$	${}^{2}_{3}C^{0}_{3}$	$\frac{{}^{2}_{3}E^{0}}{b^2 f^{(4)}(\xi)}$	
0.70	3.17460317	- 6.28571429	4.00000000	-0.888888889	0.6392	0.70
.71	3.11834104	- 6.22535211	4.00000000	- .892988930	.6477	.71
.72	3.06372549	- 6.16666667	4.00000000	- .897058824	.6563	.72
.73	3.01068794	- 6.10958904	4.00000000	- .901098901	.6649	.73
.74	2.95916354	- 6.05405405	4.00000000	- .905109489	.6736	.74
.75	2.90909091	- 6.00000000	4.00000000	- .909090909	.6823	.75
.76	2.86041190	- 5.94736842	4.00000000	- .913043478	.6911	.76
.77	2.81307141	- 5.89610390	4.00000000	- .916967509	.6999	.77
.78	2.76701716	- 5.84615385	4.00000000	- .920863309	.7088	.78
.79	2.72219954	- 5.79746835	4.00000000	- .924731183	.7177	.79
.80	2.67857143	- 5.75000000	4.00000000	- .928571429	.7267	.80
.81	2.63608804	- 5.70370370	4.00000000	- .932384342	.7357	.81
.82	2.59470680	- 5.65853659	4.00000000	- .936170213	.7448	.82
.83	2.55438716	- 5.61445783	4.00000000	- .939929328	.7539	.83
.84	2.51509054	- 5.57142857	4.00000000	- .943661972	.7631	.84
.85	2.47678019	- 5.52941176	4.00000000	- .947368421	.7723	.85
.86	2.43942104	- 5.48837209	4.00000000	- .951048951	.7816	.86
.87	2.40297970	- 5.44827586	4.00000000	- .954703833	.7909	.87
.88	2.36742424	- 5.40909091	4.00000000	- .958333333	.8003	.88
.89	2.33272423	- 5.37078652	4.00000000	- .961937716	.8097	.89
.90	2.29885057	- 5.33333333	4.00000000	- .965517241	.8192	.90
.91	2.26577546	- 5.29670330	4.00000000	- .969072165	.8287	.91
.92	2.23347230	- 5.26086957	4.00000000	- .972602740	.8383	.92
.93	2.20191567	- 5.22580645	4.00000000	- .976109215	.8479	.93
.94	2.17108120	- 5.19148936	4.00000000	- .979591837	.8576	.94
.95	2.14094558	- 5.15789474	4.00000000	- .983050847	.8673	.95
.96	2.11145649	- 5.12500000	4.00000000	- .986486486	.8771	.96
.97	2.08268250	- 5.09278351	4.00000000	- .989898990	.8869	.97
.98	2.05451308	- 5.06122449	4.00000000	- .993288591	.8968	.98
.99	2.02695855	- 5.03030303	4.00000000	- .996655518	.9067	.99
1.00	2.00000000	- 5.00000000	4.00000000	-1.00000000	.9167	1.00
1.01	1.97361929	- 4.97029703	4.00000000	-1.00332226	.9267	1.01
1.02	1.94779899	- 4.94117647	4.00000000	-1.00662252	.9363	1.02
1.03	1.92252235	- 4.91262136	4.00000000	-1.00990099	.9469	1.03
1.04	1.89777328	- 4.88461538	4.00000000	-1.01315790	.9571	1.04
1.05	1.87353630	- 4.85714286	4.00000000	-1.01639344	.9673	1.05
1.06	1.84979652	- 4.83018868	4.00000000	-1.01960784	.9776	1.06
1.07	1.82653962	- 4.80373832	4.00000000	-1.02280130	.9879	1.07
1.08	1.80375180	- 4.77777778	4.00000000	-1.02597403	.9983	1.08
1.09	1.78141979	- 4.75229358	4.00000000	-1.02912621	1.0087	1.09
1.10	1.75953079	- 4.72727273	4.00000000	-1.03225807	1.0192	1.10
1.11	1.73807248	- 4.70270270	4.00000000	-1.03536978	1.0297	1.11
1.12	1.71703297	- 4.67857143	4.00000000	-1.03846154	1.0403	1.12
1.13	1.69640080	- 4.65486726	4.00000000	-1.04153355	1.0509	1.13
1.14	1.67616493	- 4.63157895	4.00000000	-1.04458599	1.0616	1.14
1.15	1.65631470	- 4.60869565	4.00000000	-1.04761905	1.0723	1.15
1.16	1.63683981	- 4.58620690	4.00000000	-1.05063291	1.0831	1.16
1.17	1.61773032	- 4.56410256	4.00000000	-1.05362776	1.0939	1.17
1.18	1.59897665	- 4.54237288	4.00000000	-1.05660377	1.1048	1.18
1.19	1.58056953	- 4.52100840	4.00000000	-1.05956113	1.1157	1.19
1.20	1.56250000	- 4.50000000	4.00000000	-1.06250000	1.1267	1.20
1.21	1.54475940	- 4.47933884	4.00000000	-1.06542056	1.1377	1.21
1.22	1.52733937	- 4.45901639	4.00000000	-1.06832298	1.1488	1.22
1.23	1.51023182	- 4.43902439	4.00000000	-1.07120743	1.1599	1.23
1.24	1.49342891	- 4.41935484	4.00000000	-1.07407407	1.1711	1.24
1.25	1.47692308	- 4.40000000	4.00000000	-1.07692308	1.1823	1.25
1.26	1.46070698	- 4.38095238	4.00000000	-1.07975460	1.1936	1.26
1.27	1.44477353	- 4.36220472	4.00000000	-1.08256881	1.2049	1.27
1.28	1.42911585	- 4.34375000	4.00000000	-1.08536585	1.2163	1.28
1.29	1.41372729	- 4.32558140	4.00000000	-1.08814590	1.2277	1.29
	${}^{2}_{3}C^{3}_{3}$	${}^{2}_{3}C^{3}_{2}$	${}^{2}_{3}C^{3}_{1}$	${}^{2}_{3}C^{3}_{0}$	$\frac{{}^{2}_{3}E^{3}}{b^2 f^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

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$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{j=0}^{3} {}^{2}_{3}C^{i}_{j}\, y_j + {}^{2}_{3}R^{i} \qquad (i = 1,2)$$

$r=\frac{a}{b}=\frac{a}{c}$	${}^{2}_{3}C^{1}_{0}$	${}^{2}_{3}C^{1}_{1}$	${}^{2}_{3}C^{1}_{2}$	${}^{2}_{3}C^{1}_{3}$	$\frac{{}^{2}_{3}E^{1}}{b^2 f^{(4)}(\xi)}$	
0.10	25.9740260	-29.0000000	3.45454545	-0.428571429	0.1417	0.10
.11	23.2891228	-26.2727273	3.40540541	- .421800948	.1392	.11
.12	21.0579515	-24.0000000	3.35714286	- .415094340	.1367	.12
.13	19.1756393	-22.0769231	3.30973451	- .408450704	.1342	.13
.14	17.5672827	-20.4285714	3.26315789	- .401869159	.1317	.14
.15	16.1779575	-19.0000000	3.21739130	- .395348837	.1292	.15
.16	14.9664751	-17.7500000	3.17241379	- .388888889	.1267	.16
.17	13.9013422	-16.6470588	3.12820513	- .382488479	.1242	.17
.18	12.9580677	-15.6666667	3.08474576	- .376146789	.1217	.18
.19	12.1173199	-14.7894737	3.04201681	- .369863014	.1192	.19
.20	11.3636364	-14.0000000	3.00000000	- .363636364	.1167	.20
.21	10.6845027	-13.2857143	2.95867769	- .357466063	.1142	.21
.22	10.0696822	-12.6363636	2.91803279	- .351351351	.1117	.22
.23	9.51072096	-12.0434783	2.87804878	- .345291480	.1092	.23
.24	9.00057603	-11.5000000	2.83870968	- .339285714	.1067	.24
.25	8.53333332	-11.0000000	2.80000000	- .333333333	.1042	.25
.26	8.10399039	-10.5384615	2.76190476	- .327433628	.1017	.26
.27	7.70828757	-10.1111111	2.72440945	- .321585903	.0992	.27
.28	7.34257518	- 9.71428571	2.68750000	- .315789474	.0967	.28
.29	7.00370847	- 9.34482759	2.65116279	- .310043668	.0942	.29
.30	6.68896320	- 9.00000000	2.61538462	- .304347826	.0917	.30
.31	6.39596799	- 8.67741935	2.58015267	- .298701299	.0892	.31
.32	6.12264891	- 8.37500000	2.54545455	- .293103448	.0867	.32
.33	5.86718454	- 8.09090909	2.51127820	- .287553648	.0842	.33
.34	5.62796874	- 7.82352941	2.47761194	- .282051282	.0817	.34
.35	5.40357987	- 7.57142857	2.44444444	- .276595745	.0792	.35
.36	5.19275508	- 7.33333333	2.41176470	- .271186441	.0767	.36
.37	4.99436886	- 7.10810811	2.37956204	- .265822785	.0742	.37
.38	4.80741495	- 6.89473684	2.34782609	- .260504202	.0717	.38
.39	4.63099107	- 6.69230769	2.31654676	- .255230126	.0692	.39
.40	4.46428572	- 6.50000000	2.28571429	- .250000000	.0667	.40
.41	4.30656729	- 6.31707317	2.25531915	- .244813278	.0642	.41
.42	4.15717446	- 6.14285714	2.22535211	- .239669421	.0617	.42
.43	4.01550789	- 5.97674419	2.19580420	- .234567901	.0592	.43
.44	3.88102336	- 5.81818182	2.16666667	- .229508197	.0567	.44
.45	3.75322543	- 5.66666667	2.13793103	- .224489796	.0542	.45
.46	3.63166228	- 5.52173913	2.10958904	- .219512195	.0517	.46
.47	3.51592097	- 5.38297872	2.08163265	- .214574899	.0492	.47
.48	3.40562337	- 5.25000000	2.05405405	- .209677419	.0467	.48
.49	3.30042262	- 5.12244898	2.02684564	- .204819277	.0442	.49
.50	3.20000000	- 5.00000000	2.00000000	- .200000000	.0417	.50
.51	3.10406213	- 4.88235294	1.97350993	- .195219124	.0392	.51
.52	3.01233854	- 4.76923077	1.94736842	- .190476190	.0367	.52
.53	2.92457948	- 4.66037736	1.92156863	- .185770751	.0342	.53
.54	2.84055402	- 4.55555556	1.89610390	- .181102362	.0317	.54
.55	2.76004830	- 4.45454545	1.87096774	- .176470588	.0292	.55
.56	2.68286401	- 4.35714286	1.84615385	- .171875000	.0267	.56
.57	2.60881702	- 4.26315789	1.82165605	- .167315175	.0242	.57
.58	2.53773614	- 4.17241379	1.79746835	- .162790698	.0217	.58
.59	2.46946202	- 4.08474576	1.77358491	- .158301158	.0192	.59
.60	2.40384615	- 4.00000000	1.75000000	- .153846154	.0167	.60
.61	2.34075000	- 3.91803279	1.72670807	- .149425287	.0142	.61
.62	2.28004414	- 3.83870968	1.70370370	- .145038168	.0117	.62
.63	2.22160758	- 3.76190476	1.68098160	- .140684411	.0092	.63
.64	2.16532704	- 3.68750000	1.65853659	- .136363636	.0067	.64
.65	2.11109646	- 3.61538462	1.63636364	- .132075472	.0042	.65
.66	2.05881627	- 3.54545455	1.61445783	- .127819549	.0017	.66
.67	2.00839308	- 3.47761194	1.59281437	- .123595506	- .0008	.67
.68	1.95973911	- 3.41176471	1.57142857	- .119402985	- .0033	.68
.69	1.91277186	- 3.34782609	1.55029586	- .115241636	- .0058	.69
	${}^{2}_{3}C^{2}_{3}$	${}^{2}_{3}C^{2}_{2}$	${}^{2}_{3}C^{2}_{1}$	${}^{2}_{3}C^{2}_{0}$	$\frac{{}^{2}_{3}E^{2}}{b^2 f^{(4)}(\xi)}$	$p=\frac{c}{b}=\frac{c}{a}$

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{j=0}^{3} {}^{2}_{3}C^{i}_{j}\, y_j + {}^{2}_{3}R^{i} \qquad (i = 1,2)$$

$r=\frac{a\,a}{b\,c}$	${}^{2}_{3}C^{1}_{0}$	${}^{2}_{3}C^{1}_{1}$	${}^{2}_{3}C^{1}_{2}$	${}^{2}_{3}C^{1}_{3}$	$\frac{{}^{2}_{3}E^{1}}{b^2 f^{(4)}(\xi)}$	
0.70	1.86741363	- 3.28571429	1.52941176	-0.111111111	-0.0083	0.70
.71	1.82359125	- 3.22535211	1.50877193	- .107011070	- .0108	.71
.72	1.78123575	- 3.16666667	1.48837209	- .102941176	- .0133	.72
.73	1.74028206	- 3.10958904	1.46820809	- .098901099	- .0158	.73
.74	1.70066871	- 3.05405405	1.44827586	- .094890511	- .0183	.74
.75	1.66233765	- 3.00000000	1.42857143	- .090909091	- .0208	.75
.76	1.62523404	- 2.94736842	1.40909091	- .086956522	- .0233	.76
.77	1.58930589	- 2.89610390	1.38983051	- .083032491	- .0258	.77
.78	1.55450403	- 2.84615385	1.37078652	- .079136691	- .0283	.78
.79	1.52078187	- 2.79746835	1.35195531	- .075268817	- .0308	.79
.80	1.48809524	- 2.75000000	1.33333333	- .0714285714	- .0333	.80
.81	1.45640225	- 2.70370370	1.31491713	- .0676156584	- .0358	.81
.82	1.42566308	- 2.65853659	1.29670330	- .0638297872	- .0383	.82
.83	1.39583998	- 2.61445783	1.27868852	- .0600706714	- .0408	.83
.84	1.36689703	- 2.57142857	1.26086957	- .0563380282	- .0433	.84
.85	1.33880010	- 2.52941176	1.24324324	- .0526315790	- .0458	.85
.86	1.31151669	- 2.48837209	1.22580645	- .0489510489	- .0483	.86
.87	1.28501588	- 2.44827586	1.20855615	- .0452961673	- .0510	.87
.88	1.25926821	- 2.40909091	1.19148936	- .0416666667	- .0533	.88
.89	1.23424563	- 2.37078652	1.17460317	- .0380622837	- .0558	.89
.90	1.20992136	- 2.33333333	1.15789474	- .0344827586	- .0583	.90
.91	1.18626988	- 2.29670330	1.14136126	- .0309278350	- .0608	.91
.92	1.16326683	- 2.26086957	1.12500000	- .0273972603	- .0633	.92
.93	1.14088895	- 2.22580645	1.10880829	- .0238907850	- .0658	.93
.94	1.11911402	- 2.19148936	1.09278351	- .0204081633	- .0683	.94
.95	1.09792081	- 2.15789474	1.07692308	- .0169491525	- .0708	.95
.96	1.07728902	- 2.12500000	1.06122449	- .0135135135	- .0733	.96
.97	1.05719924	- 2.09278351	1.04568528	- .0101010101	- .0758	.97
.98	1.03763287	- 2.06122449	1.03030303	- .00671140940	- .0783	.98
.99	1.01857214	- 2.03030303	1.01507538	- .00334448161	- .0808	.99
1.00	1.00000000	- 2.00000000	1.00000000	0	- .0833	1.00
1.01	.981900144	- 1.97029703	.985074627	.00332225914	- .0858	1.01
1.02	.964256925	- 1.94117647	.970297030	.00662251656	- .0883	1.02
1.03	.947055345	- 1.91262136	.955665025	.00990099010	- .0908	1.03
1.04	.930281019	- 1.88461538	.941176471	.0131578947	- .0933	1.04
1.05	.913920147	- 1.85714286	.926829268	.0163934426	- .0958	1.05
1.06	.897959478	- 1.83018868	.912621359	.0196078431	- .0983	1.06
1.07	.882386289	- 1.80373832	.898550725	.0228013029	- .1008	1.07
1.08	.867188367	- 1.77777778	.884615385	.0259740260	- .1033	1.08
1.09	.852363967	- 1.75229358	.870813397	.0291262136	- .1058	1.09
1.10	.837871806	- 1.72727273	.857142857	.0322580645	- .1083	1.10
1.11	.823731033	- 1.70270270	.843601896	.0353697749	- .1108	1.11
1.12	.809921211	- 1.67857143	.830188679	.0384615385	- .1133	1.12
1.13	.796432302	- 1.65486726	.816901408	.0415335463	- .1158	1.13
1.14	.783254643	- 1.63157895	.803738318	.0445859873	- .1183	1.14
1.15	.770378931	- 1.60869565	.790697674	.0476190476	- .1208	1.15
1.16	.757796208	- 1.58620690	.777777778	.0506329114	- .1233	1.16
1.17	.745497846	- 1.56410256	.764976959	.0536277603	- .1258	1.17
1.18	.733475529	- 1.54237288	.752293578	.0566037736	- .1283	1.18
1.19	.721721247	- 1.52100840	.739726027	.0595611285	- .1308	1.19
1.20	.710227272	- 1.50000000	.727272727	.0625000000	- .1333	1.20
1.21	.698986155	- 1.47933884	.714932127	.0654205607	- .1358	1.21
1.22	.687990708	- 1.45901639	.702702703	.0683229814	- .1383	1.22
1.23	.677234001	- 1.43902439	.690582960	.0712074303	- .1408	1.23
1.24	.666709335	- 1.41935484	.678571429	.0740740741	- .1433	1.24
1.25	.656410257	- 1.40000000	.666666667	.0769230769	- .1458	1.25
1.26	.646330524	- 1.38095238	.654867257	.0797546012	- .1483	1.26
1.27	.636464112	- 1.36220472	.643171806	.0825688073	- .1508	1.27
1.28	.626805198	- 1.34375000	.631578947	.0853658537	- .1533	1.28
1.29	.617348163	- 1.32558140	.620087336	.0881458967	- .1558	1.29
	${}^{2}_{3}C^{2}_{3}$	${}^{2}_{3}C^{2}_{2}$	${}^{2}_{3}C^{2}_{1}$	${}^{2}_{3}C^{2}_{0}$	$\frac{{}^{2}_{3}E^{2}}{b^2 f^{(4)}(\xi)}$	$p=\frac{r\,c}{h\,a}$

10(b)

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

For any value of $r=\frac{a}{b}=\frac{a}{c}$ 1411

$$(D^2y)_{x=x_2}=\frac{y_1-2y_2+y_3}{b^2}+\tfrac{2}{3}R^2$$

where

$$\tfrac{2}{3}R^2=-\frac{b^2}{12}f^{(4)}(\xi)+\ldots$$

For any value of $p=\frac{c}{b}=\frac{c}{a}$

$$(D^2y)_{x=x_1}=\frac{y_0-2y_1+y_2}{b^2}+\tfrac{2}{3}R'$$

where

$$\tfrac{2}{3}R'=-\frac{b^2}{12}f^{(4)}(\xi)+\ldots$$

FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2} \sum_{j=0}^{3} {}^{2}_{3}C^{i}_{j}\, y_j + {}^{2}_{3}R^{i} \qquad (i=3,0)$$

$\rho=\frac{a}{b}\frac{a}{c}$	${}^{2}_{3}C^{3}_{0}$	${}^{2}_{3}C^{3}_{1}$	${}^{2}_{3}C^{3}_{2}$	${}^{2}_{3}C^{3}_{3}$	$\frac{{}^{2}_{3}R^{3}}{b^2 f^{(4)}(\xi)}$	
0.10	-25.9740260	31.0000000	-7.45454545	2.42857143	0.6917	0.10
.11	-23.2891228	28.2727273	-7.40540541	2.42180095	.6942	.11
.12	-21.0579515	26.0000000	-7.35714286	2.41509434	.6967	.12
.13	-19.1756393	24.0769231	-7.30973451	2.40845070	.6992	.13
.14	-17.5672827	22.4285714	-7.26315789	2.40186916	.7017	.14
.15	-16.1779575	21.0000000	-7.21739130	2.39534884	.7042	.15
.16	-14.9664751	19.7500000	-7.17241379	2.38888889	.7067	.16
.17	-13.9013422	18.6470588	-7.12820513	2.38248848	.7092	.17
.18	-12.9580677	17.6666667	-7.08474576	2.37614679	.7117	.18
.19	-12.1173199	16.7894737	-7.04201681	2.36986301	.7142	.19
.20	-11.3636364	16.0000000	-7.00000000	2.36363636	.7167	.20
.21	-10.6845027	15.2857143	-6.95867769	2.35746606	.7192	.21
.22	-10.0696822	14.6363636	-6.91803279	2.35135135	.7217	.22
.23	- 9.51072096	14.0434783	-6.87804878	2.34529148	.7242	.23
.24	- 9.00057603	13.5000000	-6.83870968	2.33928571	.7267	.24
.25	- 8.53333332	13.0000000	-6.80000000	2.33333333	.7292	.25
.26	- 8.10399039	12.5384615	-6.76190476	2.32743363	.7317	.26
.27	- 7.70828757	12.1111111	-6.72440945	2.32158590	.7342	.27
.28	- 7.34257518	11.7142857	-6.68750000	2.31578947	.7367	.28
.29	- 7.00370847	11.3448276	-6.65116279	2.31004367	.7392	.29
.30	- 6.68896320	11.0000000	-6.61538462	2.30434783	.7417	.30
.31	- 6.39596799	10.6774194	-6.58015267	2.29870130	.7442	.31
.32	- 6.12264891	10.3750000	-6.54545455	2.29310345	.7467	.32
.33	- 5.86718454	10.0909091	-6.51127820	2.28755365	.7492	.33
.34	- 5.62796874	9.82352941	-6.47761194	2.28205128	.7517	.34
.35	- 5.40357987	9.57142857	-6.44444444	2.27659574	.7542	.35
.36	- 5.19275508	9.33333333	-6.41176470	2.27118644	.7567	.36
.37	- 4.99436886	9.10810811	-6.37956204	2.26582278	.7592	.37
.38	- 4.80741495	8.89473684	-6.34782609	2.26050420	.7617	.38
.39	- 4.63099107	8.69230769	-6.31654676	2.25523013	.7642	.39
.40	- 4.46428572	8.50000000	-6.28571429	2.25000000	.7667	.40
.41	- 4.30656729	8.31707317	-6.25531915	2.24481328	.7692	.41
.42	- 4.15717446	8.14285714	-6.22535211	2.23966942	.7717	.42
.43	- 4.01550789	7.97674419	-6.19580420	2.23456790	.7742	.43
.44	- 3.88102336	7.81818182	-6.16666667	2.22950820	.7767	.44
.45	- 3.75322543	7.66666667	-6.13793103	2.22448980	.7792	.45
.46	- 3.63166228	7.52173913	-6.10958904	2.21951220	.7817	.46
.47	- 3.51592097	7.38297872	-6.08163265	2.21457490	.7842	.47
.48	- 3.40562337	7.25000000	-6.05405405	2.20967742	.7867	.48
.49	- 3.30042262	7.12244898	-6.02684564	2.20481928	.7892	.49
.50	- 3.20000000	7.00000000	-6.00000000	2.20000000	.7917	.50
.51	- 3.10406213	6.88235294	-5.97350993	2.19521912	.7942	.51
.52	- 3.01233854	6.76923077	-5.94736842	2.19047619	.7967	.52
.53	- 2.92457948	6.66037736	-5.92156863	2.18577075	.7992	.53
.54	- 2.84055402	6.55555556	-5.89610390	2.18110236	.8017	.54
.55	- 2.76004830	6.45454545	-5.87096774	2.17647059	.8042	.55
.56	- 2.68286401	6.35714286	-5.84615385	2.17187500	.8067	.56
.57	- 2.60881702	6.26315789	-5.82165605	2.16731518	.8092	.57
.58	- 2.53773614	6.17241379	-5.79746835	2.16279070	.8117	.58
.59	- 2.46946202	6.08474576	-5.77358491	2.15830116	.8142	.59
.60	- 2.40384615	6.00000000	-5.75000000	2.15384615	.8167	.60
.61	- 2.34075000	5.91803279	-5.72670807	2.14942529	.8192	.61
.62	- 2.28004414	5.83870968	-5.70370370	2.14503817	.8217	.62
.63	- 2.22160758	5.76190476	-5.68098160	2.14068441	.8242	.63
.64	- 2.16532704	5.68750000	-5.65853689	2.13636364	.8267	.64
.65	- 2.11109646	5.61538462	-5.63636364	2.13207547	.8292	.65
.66	- 2.05881627	5.54545455	-5.61445783	2.12781955	.8317	.66
.67	- 2.00839308	5.47761194	-5.59281437	2.12359551	.8342	.67
.68	- 1.95973911	5.41176471	-5.57142857	2.11940299	.8367	.68
.69	- 1.91277186	5.34782609	-5.55029586	2.11524164	.8392	.69
	${}^{2}_{3}C^{0}_{3}$	${}^{2}_{3}C^{0}_{2}$	${}^{2}_{3}C^{0}_{1}$	${}^{2}_{3}C^{0}_{0}$	$\frac{{}^{2}_{3}R^{0}}{b^2 f^{(4)}(\xi)}$	$\rho=\frac{c}{b}\frac{c}{a}$

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FOUR-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2} \sum_{j=0}^{3} {}^{2}_{3}C^{i}_{j}\, y_j + {}^{2}_{3}R^{i} \qquad (i=3,0)$$

$r=\frac{a}{b}\frac{a}{c}$	${}^{2}_{3}C^{3}_{0}$	${}^{2}_{3}C^{3}_{1}$	${}^{2}_{3}C^{3}_{2}$	${}^{2}_{3}C^{3}_{3}$	$\frac{{}^{2}_{3}E^{3}}{b^2 f^{(4)}(\xi)}$	
0.70	- 1.86741363	5.28571429	-5.52941176	2.11111111	0.8417	0.70
.71	- 1.88359125	5.22535211	-5.50877193	2.10701107	.8442	.71
.72	- 1.78123575	5.16666667	-5.48837209	2.10294118	.8467	.72
.73	- 1.74028206	5.10958904	-5.46820809	2.09890110	.8492	.73
.74	- 1.70066871	5.05405405	-5.44827586	2.09489051	.8517	.74
.75	- 1.66233765	5.00000000	-5.42857143	2.09090909	.8542	.75
.76	- 1.62523404	4.94736842	-5.40909091	2.08695652	.8567	.76
.77	- 1.58930589	4.89610390	-5.38983051	2.08303249	.8592	.77
.78	- 1.55450403	4.84615385	-5.37078652	2.07913669	.8617	.78
.79	- 1.52078187	4.79746835	-5.35195531	2.07526882	.8642	.79
.80	- 1.48809524	4.75000000	-5.33333333	2.07142857	.8667	.80
.81	- 1.45640223	4.70370370	-5.31491713	2.06761566	.8692	.81
.82	- 1.42566308	4.65853659	-5.29670330	2.06382979	.8717	.82
.83	- 1.39583998	4.61445783	-5.27868852	2.06007067	.8742	.83
.84	- 1.36689703	4.57142857	-5.26086957	2.05633803	.8767	.84
.85	- 1.33880010	4.52941176	-5.24324324	2.05263158	.8792	.85
.86	- 1.31151669	4.48837209	-5.22580645	2.04895105	.8817	.86
.87	- 1.28501588	4.44827586	-5.20855615	2.04529617	.8842	.87
.88	- 1.25926821	4.40909091	-5.19148936	2.04166667	.8867	.88
.89	- 1.23424563	4.37078652	-5.17460317	2.03806228	.8892	.89
.90	- 1.20992136	4.33333333	-5.15789474	2.03448276	.8917	.90
.91	- 1.18626988	4.29670330	-5.14136126	2.03092784	.8942	.91
.92	- 1.16326683	4.26086957	-5.12500000	2.02739726	.8967	.92
.93	- 1.14088895	4.22580645	-5.10880829	2.02389078	.8992	.93
.94	- 1.11911402	4.19148936	-5.09278351	2.02040816	.9017	.94
.95	- 1.09792081	4.15789474	-5.07692308	2.01694915	.9042	.95
.96	- 1.07728902	4.12500000	-5.06122449	2.01351351	.9067	.96
.97	- 1.05719924	4.09278351	-5.04568528	2.01010101	.9092	.97
.98	- 1.03763287	4.06122449	-5.03030303	2.00671141	.9117	.98
.99	- 1.01857214	4.03030303	-5.01507538	2.00334448	.9142	.99
1.00	- 1.00000000	4.00000000	-5.00000000	2.00000000	.9167	1.00
1.01	- .981900144	3.97029703	-4.98507463	1.99667774	.9192	1.01
1.02	- .964256925	3.94117647	-4.97029703	1.99337748	.9217	1.02
1.03	- .947055345	3.91262136	-4.95566502	1.99009901	.9242	1.03
1.04	- .930281019	3.88461538	-4.94117647	1.98684211	.9267	1.04
1.05	- .913920147	3.85714286	-4.92682927	1.98360656	.9292	1.05
1.06	- .897959478	3.83018868	-4.91262136	1.98039216	.9317	1.06
1.07	- .882386289	3.80373832	-4.89855072	1.97719870	.9342	1.07
1.08	- .867188367	3.77777778	-4.88461538	1.97402597	.9367	1.08
1.09	- .852353967	3.75229358	-4.87081340	1.97087379	.9392	1.09
1.10	- .837871806	3.72727273	-4.85714286	1.96774194	.9417	1.10
1.11	- .823731033	3.70270270	-4.84360190	1.96463023	.9442	1.11
1.12	- .809921211	3.67857143	-4.83018868	1.96153846	.9467	1.12
1.13	- .796432302	3.65486726	-4.81690141	1.95846645	.9492	1.13
1.14	- .783254643	3.63157895	-4.80373832	1.95541401	.9517	1.14
1.15	- .770378931	3.60869565	-4.79069767	1.95238095	.9542	1.15
1.16	- .757796208	3.58620690	-4.77777778	1.94936709	.9567	1.16
1.17	- .745497846	3.56410256	-4.76497696	1.94637224	.9592	1.17
1.18	- .733475529	3.54237288	-4.75229358	1.94339623	.9617	1.18
1.19	- .721721247	3.52100840	-4.73972603	1.94043887	.9642	1.19
1.20	- .710227272	3.50000000	-4.72727273	1.93750000	.9667	1.20
1.21	- .698986155	3.47933884	-4.71493213	1.93457944	.9692	1.21
1.22	- .687990708	3.45901639	-4.70270270	1.93167702	.9717	1.22
1.23	- .677234001	3.43902439	-4.69058296	1.92879257	.9742	1.23
1.24	- .666709335	3.41935484	-4.67857143	1.92592593	.9767	1.24
1.25	- .656410257	3.40000000	-4.66666667	1.92307692	.9792	1.25
1.26	- .646330524	3.38095238	-4.65486726	1.92024540	.9817	1.26
1.27	- .636464112	3.36220472	-4.64317181	1.91743119	.9842	1.27
1.28	- .626805198	3.34375000	-4.63157895	1.91463415	.9867	1.28
1.29	- .617348163	3.32558140	-4.62008734	1.91185410	.9892	1.29
	${}^{2}_{3}C^{0}_{3}$	${}^{2}_{3}C^{0}_{2}$	${}^{2}_{3}C^{0}_{1}$	${}^{2}_{3}C^{0}_{0}$	$\frac{{}^{2}_{3}E^{0}}{b^2 f^{(4)}(\xi)}$	$p=\frac{b}{a}\frac{c}{c}$

12(b)

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$$(D^3y)_{x=x_i} \frac{1}{b^3} \sum_{j=0}^{3} {}_3^3C_j^i \, y_j + {}_3^3R^i \qquad (i=0,1,2,3)$$

$r=\frac{a}{b}=\frac{a}{c}$	${}_3^3C_0^i$	${}_3^3C_1^i$	${}_3^3C_2^i$	${}_3^3C_3^i$	${}_3^3E^0 / bf^{(4)}(\xi)$	${}_3^3E^1 / bf^{(4)}(\xi)$	${}_3^3E^2 / bf^{(4)}(\xi)$	${}_3^3E^3 / bf^{(4)}(\xi)$	
0.10	-25.9740260	30.0000000	-5.45454545	1.42857143	-0.8250	-0.7250	0.2750	1.2750	0.10
.11	-23.2891228	27.2727273	-5.40540541	1.42180095	- .8325	- .7225	.2775	1.2775	.11
.12	-21.0579515	25.0000000	-5.35714286	1.41509434	- .8400	- .7200	.2800	1.2800	.12
.13	-19.1756393	23.0769231	-5.30973451	1.40845070	- .8475	- .7175	.2825	1.2825	.13
.14	-17.5672827	21.4285714	-5.26315789	1.40186916	- .8550	- .7150	.2850	1.2850	.14
.15	-16.1779575	20.0000000	-5.21739130	1.39534884	- .8625	- .7125	.2875	1.2875	.15
.16	-14.9664751	18.7500000	-5.17241379	1.38888889	- .8700	- .7100	.2900	1.2900	.16
.17	-13.9013422	17.6470588	-5.12820513	1.38248848	- .8775	- .7075	.2925	1.2925	.17
.18	-12.9580677	16.6666667	-5.08474576	1.37614679	- .8850	- .7050	.2950	1.2950	.18
.19	-12.1173199	15.7894737	-5.04201681	1.36986301	- .8925	- .7025	.2975	1.2975	.19
.20	-11.3636364	15.0000000	-5.00000000	1.36363636	- .9000	- .7000	.3000	1.3000	.20
.21	-10.6845027	14.2857143	-4.95867769	1.35746606	- .9075	- .6975	.3025	1.3025	.21
.22	-10.0696822	13.6363636	-4.91803279	1.35135135	- .9150	- .6950	.3050	1.3050	.22
.23	- 9.51072096	13.0434783	-4.87804878	1.34529148	- .9225	- .6925	.3075	1.3075	.23
.24	- 9.00057603	12.5000000	-4.83870968	1.33928571	- .9300	- .6900	.3100	1.3100	.24
.25	- 8.53333332	12.0000000	-4.80000000	1.33333333	- .9375	- .6875	.3125	1.3125	.25
.26	- 8.10399039	11.5384615	-4.76190476	1.32743363	- .9450	- .6850	.3150	1.3150	.26
.27	- 7.70828757	11.1111111	-4.72440945	1.32158590	- .9525	- .6825	.3175	1.3175	.27
.28	- 7.34257518	10.7142857	-4.68750000	1.31578947	- .9600	- .6800	.3200	1.3200	.28
.29	- 7.00370847	10.3448276	-4.65116279	1.31004367	- .9675	- .6775	.3225	1.3225	.29
.30	- 6.68896320	10.0000000	-4.61538462	1.30434783	- .9750	- .6750	.3250	1.3250	.30
.31	- 6.39596799	9.67741935	-4.58015267	1.29870130	- .9825	- .6725	.3275	1.3275	.31
.32	- 6.12264891	9.37500000	-4.54545455	1.29310345	- .9900	- .6700	.3300	1.3300	.32
.33	- 5.86718454	9.09090909	-4.51127820	1.28755365	- .9975	- .6675	.3325	1.3325	.33
.34	- 5.62796874	8.82352941	-4.47761194	1.28205128	-1.0050	- .6650	.3350	1.3350	.34
.35	- 5.40357987	8.57142857	-4.44444444	1.27659574	-1.0125	- .6625	.3375	1.3375	.35
.36	- 5.19275508	8.33333333	-4.41176471	1.27118644	-1.0200	- .6600	.3400	1.3400	.36
.37	- 4.99436886	8.10810811	-4.37956204	1.26582278	-1.0275	- .6575	.3425	1.3425	.37
.38	- 4.80741495	7.89473684	-4.34782609	1.26050420	-1.0350	- .6550	.3450	1.3450	.38
.39	- 4.63099107	7.69230769	-4.31654676	1.25523013	-1.0425	- .6525	.3475	1.3475	.39
.40	- 4.46428572	7.50000000	-4.28571429	1.25000000	-1.0500	- .6500	.3500	1.3500	.40
.41	- 4.30656729	7.31707317	-4.25531915	1.24481328	-1.0575	- .6475	.3525	1.3525	.41
.42	- 4.15717446	7.14285714	-4.22535211	1.23966942	-1.0650	- .6450	.3550	1.3550	.42
.43	- 4.01550789	6.97674419	-4.19580420	1.23456790	-1.0725	- .6425	.3575	1.3575	.43
.44	- 3.88102336	6.81818182	-4.16666667	1.22950820	-1.0800	- .6400	.3600	1.3600	.44
.45	- 3.75322543	6.66666667	-4.13793103	1.22448980	-1.0875	- .6375	.3625	1.3625	.45
.46	- 3.63166228	6.52173913	-4.10958904	1.21951220	-1.0950	- .6350	.3650	1.3650	.46
.47	- 3.51592097	6.38297872	-4.08163265	1.21457490	-1.1025	- .6325	.3675	1.3675	.47
.48	- 3.40562337	6.25000000	-4.05405405	1.20967742	-1.1100	- .6300	.3700	1.3700	.48
.49	- 3.30042262	6.12244898	-4.02684564	1.20481928	-1.1175	- .6275	.3725	1.3725	.49
.50	- 3.20000000	6.00000000	-4.00000000	1.20000000	-1.1250	- .6250	.3750	1.3750	.50
.51	- 3.10406213	5.88235294	-3.97350993	1.19521912	-1.1325	- .6225	.3775	1.3775	.51
.52	- 3.01233854	5.76923077	-3.94736842	1.19047619	-1.1400	- .6200	.3800	1.3800	.52
.53	- 2.92457948	5.66037736	-3.92156863	1.18577075	-1.1475	- .6175	.3825	1.3825	.53
.54	- 2.84055402	5.55555556	-3.89610390	1.18110236	-1.1550	- .6150	.3850	1.3850	.54
.55	- 2.76004830	5.45454545	-3.87096774	1.17647059	-1.1625	- .6125	.3875	1.3875	.55
.56	- 2.68286401	5.35714286	-3.84615385	1.17187500	-1.1700	- .6100	.3900	1.3900	.56
.57	- 2.60881702	5.26315789	-3.82165605	1.16731518	-1.1775	- .6075	.3925	1.3925	.57
.58	- 2.53773614	5.17241379	-3.79746835	1.16279070	-1.1850	- .6050	.3950	1.3950	.58
.59	- 2.46946202	5.08474576	-3.77358491	1.15830116	-1.1925	- .6025	.3975	1.3975	.59
.60	- 2.40384615	5.00000000	-3.75000000	1.15384615	-1.2000	- .6000	.4000	1.4000	.60
.61	- 2.34075000	4.91803279	-3.72670807	1.14942529	-1.2075	- .5975	.4025	1.4025	.61
.62	- 2.28004414	4.83870968	-3.70370370	1.14503817	-1.2150	- .5950	.4050	1.4050	.62
.63	- 2.22160758	4.76190476	-3.68098160	1.14068441	-1.2225	- .5925	.4075	1.4075	.63
.64	- 2.16532704	4.68750000	-3.65853659	1.13636364	-1.2300	- .5900	.4100	1.4100	.64
.65	- 2.11109646	4.61538462	-3.63636364	1.13207547	-1.2375	- .5875	.4125	1.4125	.65
.66	- 2.05881627	4.54545455	-3.61445783	1.12781955	-1.2450	- .5850	.4150	1.4150	.66
.67	- 2.00839308	4.47761194	-3.59281437	1.12359551	-1.2525	- .5825	.4175	1.4175	.67
.68	- 1.95973911	4.41176471	-3.57142857	1.11940299	-1.2600	- .5800	.4200	1.4200	.68
.69	- 1.91277186	4.34782609	-3.55029586	1.11524164	-1.2675	- .5775	.4225	1.4225	.69
	$-{}_3^3C_3^i$	$-{}_3^3C_2^i$	$-{}_3^3C_1^i$	$-{}_3^3C_0^i$	${}_3^3E^3 / bf^{(4)}(\xi)$	${}_3^3E^2 / bf^{(4)}(\xi)$	${}_3^3E^1 / bf^{(4)}(\xi)$	${}_3^3E^0 / bf^{(4)}(\xi)$	$p=\frac{c}{b}=\frac{c}{a}$

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$$(D^3y)_{x=x_i}=\frac{1}{b^3}\sum_{j=0}^{3}{}_3C_j^i\,y_j+{}_3R^i \qquad (i=0,1,2,3)$$

$r=\frac{a}{b}\frac{a'}{c}$	${}_3C_0^i$	${}_3C_1^i$	${}_3C_2^i$	${}_3C_3^i$	$\frac{{}_3E^0}{bf^{(4)}(\xi)}$	$\frac{{}_3E^1}{bf^{(4)}(\xi)}$	$\frac{{}_3E^2}{bf^{(4)}(\xi)}$	$\frac{{}_3E^3}{bf^{(4)}(\xi)}$	
0.70	- 1.86741363	4.28571429	-3.52941176	1.11111111	-1.2750	-0.5750	0.4250	1.4250	0.70
.71	- 1.82359125	4.22535211	-3.50877193	1.10701107	-1.2825	- .5725	.4275	1.4275	.71
.72	- 1.78123575	4.16666667	-3.48837209	1.10294118	-1.2900	- .5700	.4300	1.4300	.72
.73	- 1.74028206	4.10958904	-3.46820809	1.09890110	-1.2975	- .5675	.4325	1.4325	.73
.74	- 1.70066871	4.05405405	-3.44827586	1.09489051	-1.3050	- .5650	.4350	1.4350	.74
.75	- 1.66233765	4.00000000	-3.42857143	1.09090909	-1.3125	- .5625	.4375	1.4375	.75
.76	- 1.62523404	3.94736842	-3.40909091	1.08695652	-1.3200	- .5600	.4400	1.4400	.76
.77	- 1.58930589	3.89610390	-3.38983051	1.08303249	-1.3275	- .5575	.4425	1.4425	.77
.78	- 1.55450403	3.84615385	-3.37078652	1.07913669	-1.3350	- .5550	.4450	1.4450	.78
.79	- 1.52078187	3.79746835	-3.35195531	1.07526882	-1.3425	- .5525	.4475	1.4475	.79
.80	- 1.48809824	3.75000000	-3.33333333	1.07142857	-1.3500	- .5500	.4500	1.4500	.80
.81	- 1.45640223	3.70370370	-3.31491713	1.06761566	-1.3575	- .5475	.4525	1.4525	.81
.82	- 1.42566308	3.65853659	-3.29670330	1.06382979	-1.3650	- .5450	.4550	1.4550	.82
.83	- 1.39583998	3.61445783	-3.27868852	1.06007067	-1.3725	- .5425	.4575	1.4575	.83
.84	- 1.36689703	3.57142857	-3.26086957	1.05633803	-1.3800	- .5400	.4600	1.4600	.84
.85	- 1.33880010	3.52941176	-3.24324324	1.05263158	-1.3875	- .5375	.4625	1.4625	.85
.86	- 1.31151669	3.48837209	-3.22580645	1.04895105	-1.3950	- .5350	.4650	1.4650	.86
.87	- 1.28501588	3.44827586	-3.20855615	1.04529617	-1.4025	- .5325	.4675	1.4675	.87
.88	- 1.25926821	3.40909091	-3.19148936	1.04166667	-1.4100	- .5300	.4700	1.4700	.88
.89	- 1.23424563	3.37078652	-3.17460317	1.03806228	-1.4175	- .5275	.4725	1.4725	.89
.90	- 1.20992136	3.33333333	-3.15789474	1.03448276	-1.4250	- .5250	.4750	1.4750	.90
.91	- 1.18626988	3.29670330	-3.14136126	1.03092784	-1.4325	- .5225	.4775	1.4775	.91
.92	- 1.16326683	3.26086957	-3.12500000	1.02739726	-1.4400	- .5200	.4800	1.4800	.92
.93	- 1.14088895	3.22580645	-3.10880829	1.02389078	-1.4475	- .5175	.4825	1.4825	.93
.94	- 1.11911402	3.19148936	-3.09278351	1.02040816	-1.4550	- .5150	.4850	1.4850	.94
.95	- 1.09792081	3.15789474	-3.07692308	1.01694915	-1.4625	- .5125	.4875	1.4875	.95
.96	- 1.07728902	3.12500000	-3.06122449	1.01351351	-1.4700	- .5100	.4900	1.4900	.96
.97	- 1.05719924	3.09278351	-3.04568528	1.01010101	-1.4775	- .5075	.4925	1.4925	.97
.98	- 1.03763287	3.06122449	-3.03030303	1.00671141	-1.4850	- .5050	.4950	1.4950	.98
.99	- 1.01857214	3.03030303	-3.01507538	1.00334448	-1.4925	- .5025	.4975	1.4975	.99
1.00	- 1.00000000	3.00000000	-3.00000000	1.00000000	-1.5000	- .5000	.5000	1.5000	1.00
1.01	- .981900144	2.97029703	-2.98507463	.996677741	-1.5075	- .4975	.5025	1.5025	1.01
1.02	- .964256925	2.94117647	-2.97029703	.993377483	-1.5150	- .4950	.5050	1.5050	1.02
1.03	- .947055345	2.91262136	-2.95566502	.990099010	-1.5225	- .4925	.5075	1.5075	1.03
1.04	- .930281019	2.88461538	-2.94117647	.986842105	-1.5300	- .4900	.5100	1.5100	1.04
1.05	- .913920147	2.85714286	-2.92682927	.983606557	-1.5375	- .4875	.5125	1.5125	1.05
1.06	- .897959478	2.83018868	-2.91262136	.980392157	-1.5450	- .4850	.5150	1.5150	1.06
1.07	- .882386289	2.80373832	-2.89855072	.977198697	-1.5525	- .4825	.5175	1.5175	1.07
1.08	- .867188367	2.77777778	-2.88461538	.974025974	-1.5600	- .4800	.5200	1.5200	1.08
1.09	- .852353967	2.75229358	-2.87081340	.970873786	-1.5675	- .4775	.5225	1.5225	1.09
1.10	- .837871806	2.72727273	-2.85714286	.967741936	-1.5750	- .4750	.5250	1.5250	1.10
1.11	- .823731033	2.70270270	-2.84360190	.964630225	-1.5825	- .4725	.5275	1.5275	1.11
1.12	- .809921211	2.67857143	-2.83018868	.961538462	-1.5900	- .4700	.5300	1.5300	1.12
1.13	- .796432302	2.65486726	-2.81690141	.958466454	-1.5975	- .4675	.5325	1.5325	1.13
1.14	- .783254643	2.63157895	-2.80373832	.95541401	-1.6050	- .4650	.5350	1.5350	1.14
1.15	- .770378931	2.60869565	-2.79069767	.95238095	-1.6125	- .4625	.5375	1.5375	1.15
1.16	- .757796208	2.58620690	-2.77777778	.94936709	-1.6200	- .4600	.5400	1.5400	1.16
1.17	- .745497846	2.56410256	-2.76497696	.94637224	-1.6275	- .4575	.5425	1.5425	1.17
1.18	- .733475529	2.54237288	-2.75229358	.94339623	-1.6350	- .4550	.5450	1.5450	1.18
1.19	- .721721247	2.52100840	-2.73972603	.94043887	-1.6425	- .4525	.5475	1.5475	1.19
1.20	- .710227272	2.50000000	-2.72727273	.93750000	-1.6500	- .4500	.5500	1.5500	1.20
1.21	- .698986155	2.47933884	-2.71493213	.93457944	-1.6575	- .4475	.5525	1.5525	1.21
1.22	- .687990708	2.45901639	-2.70270270	.83167702	-1.6650	- .4450	.5550	1.5550	1.22
1.23	- .677234001	2.43902439	-2.69058296	.92879257	-1.6725	- .4425	.5575	1.5575	1.23
1.24	- .666709335	2.41935484	-2.67857143	.92592593	-1.6800	- .4400	.5600	1.5600	1.24
1.25	- .656410257	2.40000000	-2.66666667	.92307692	-1.6875	- .4375	.5625	1.5625	1.25
1.26	- .646330524	2.38095238	-2.65486726	.92024540	-1.6950	- .4350	.5650	1.5650	1.26
1.27	- .636464112	2.36220472	-2.64317181	.91743119	-1.7025	- .4325	.5675	1.5675	1.27
1.28	- .626805198	2.34375000	-2.63157895	.91463415	-1.7100	- .4300	.5700	1.5700	1.28
1.29	- .617348163	2.32558140	-2.62008734	.91185410	-1.7175	- .4275	.5725	1.5725	1.29
	$-{}_3C_3^i$	$-{}_3C_2^i$	$-{}_3C_1^i$	$-{}_3C_0^i$	$-\frac{{}_3E^3}{bf^{(4)}(\xi)}$	$-\frac{{}_3E^2}{bf^{(4)}(\xi)}$	$-\frac{{}_3E^1}{bf^{(4)}(\xi)}$	$-\frac{{}_3E^0}{bf^{(4)}(\xi)}$	$p=\frac{c}{b}\frac{c}{a}$

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$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{4} {}^{i}_{4}C^{j}_{j}\, y_j + {}^{i}_{4}R^{i} \qquad (i=0,4)$$

$r = \frac{a\,a\,a}{b\,c\,d}$	${}^{i}_{4}C^{0}_{0}$	${}^{i}_{4}C^{0}_{1}$	${}^{i}_{4}C^{0}_{2}$	${}^{i}_{4}C^{0}_{3}$	${}^{i}_{4}C^{0}_{4}$	$\frac{{}^{i}_{4}E^{0}}{b^4 f^{(5)}(\xi)}$	
0.10	-11.7078620	11.9350000	-0.295909092	0.0811904762	-0.0124193548	0.0060	0.10
.11	-10.7872870	11.0362591	- .325149099	.0899836493	- .0138065916	.0067	.11
.12	-10.0184014	10.2890667	- .354342857	.0988981132	- .0152205128	.0074	.12
.13	- 9.36623583	9.65845769	- .383494248	.107933568	- .0166611821	.0082	.13
.14	- 8.80581118	9.11945714	- .412607018	.117089720	- .0181286624	.0089	.14
.15	- 8.31880848	8.65375000	- .441684783	.126366279	- .0196230159	.0097	.15
.16	- 7.89148762	8.24760000	- .470731034	.135762963	- .0211443038	.0106	.16
.17	- 7.51334070	7.89050294	- .499749145	.145279493	- .0226925868	.0114	.17
.18	- 7.17619419	7.57428889	- .528742373	.154915596	- .0242679245	.0123	.18
.19	- 6.87359466	7.29250789	- .557713266	.164671005	- .0258703762	.0132	.19
.20	- 6.60037879	7.04000000	- .586666667	.174545455	- .0275000000	.0141	.20
.21	- 6.35236621	6.81259810	- .615603719	.184538688	- .0291568536	.0150	.21
.22	- 6.12613613	6.60685455	- .644527869	.194650450	- .0308409938	.0160	.22
.23	- 5.91886223	6.41997609	- .673441870	.204880493	- .0325524768	.0170	.23
.24	- 5.72818882	6.24960000	- .702348387	.215228571	- .0342913580	.0180	.24
.25	- 5.55213675	6.09375000	- .731250000	.225694444	- .0360576923	.0190	.25
.26	- 5.38903098	5.95075385	- .760149206	.236277876	- .0378515337	.0201	.26
.27	- 5.23744431	5.81918704	- .789048425	.246978634	- .0396729358	.0212	.27
.28	- 5.09615311	5.69782857	- .817950000	.257796491	- .0415219512	.0223	.28
.29	- 4.96410225	5.58582586	- .846856202	.268731223	- .0433986322	.0235	.29
.30	- 4.84037701	5.48166667	- .875769231	.279782609	- .0453030303	.0247	.30
.31	- 4.72418046	5.38515645	- .904691221	.290950433	- .0472351964	.0259	.31
.32	- 4.61481506	5.29540000	- .933624242	.302234483	- .0491951807	.0271	.32
.33	- 4.51166758	5.21178636	- .962570301	.313634549	- .0511830330	.0284	.33
.34	- 4.41419675	5.13377647	- .991531343	.325150427	- .0531988024	.0297	.34
.35	- 4.32192297	5.06089286	-1.02050926	.336781915	- .0552425373	.0310	.35
.36	- 4.23441975	4.99271111	-1.04950588	.348528814	- .0573142857	.0324	.36
.37	- 4.15130654	4.92885270	-1.07852299	.360390928	- .0594140950	.0337	.37
.38	- 4.07224268	4.86897895	-1.10756232	.372368067	- .0615420118	.0352	.38
.39	- 3.99692232	4.81278590	-1.13662554	.384460042	- .0636980826	.0366	.39
.40	- 3.92507003	4.76000000	-1.16571429	.396666667	- .0658823529	.0381	.40
.41	- 3.85643714	4.71037439	-1.19483014	.408987759	- .0680948680	.0396	.41
.42	- 3.79079853	4.66368572	-1.22397465	.421423140	- .0703356725	.0411	.42
.43	- 3.72794992	4.61973140	-1.25314930	.433972634	- .0726048105	.0427	.43
.44	- 3.66770546	4.57832727	-1.28235556	.446636066	- .0749023256	.0442	.44
.45	- 3.60989573	4.53930556	-1.31159483	.459413265	- .0772282609	.0460	.45
.46	- 3.55436596	4.50251304	-1.34086849	.472304065	- .0795826590	.0476	.46
.47	- 3.50097442	4.46780957	-1.37017789	.485308300	- .0819655620	.0494	.47
.48	- 3.44959114	4.43506667	-1.39952432	.498425806	- .0843770115	.0511	.48
.49	- 3.40009664	4.40416633	-1.42890906	.511656426	- .0868170487	.0529	.49
.50	- 3.35238095	4.37500000	-1.45833333	.525000000	- .0892857143	.0547	.50
.51	- 3.30634263	4.34746765	-1.48779834	.538456375	- .0917830484	.0565	.51
.52	- 3.26188796	4.32147692	-1.51730526	.552025397	- .0943090909	.0584	.52
.53	- 3.21893026	4.29694245	-1.54685523	.565706917	- .0968638810	.0604	.53
.54	- 3.17738916	4.27378519	-1.57644935	.579500787	- .0994474570	.0623	.54
.55	- 3.13719011	4.25193182	-1.60608871	.593406863	- .102059859	.0643	.55
.56	- 3.09826380	4.23131429	-1.63577436	.607425000	- .104701124	.0664	.56
.57	- 3.06054574	4.21186930	-1.66550732	.621555058	- .107371289	.0684	.57
.58	- 3.02397583	4.19353793	-1.69528861	.635796899	- .110070391	.0705	.58
.59	- 2.98849799	4.17626525	-1.72511918	.650150386	- .112798468	.0727	.59
.60	- 2.95405983	4.16000000	-1.75500000	.664615385	- .115555556	.0749	.60
.61	- 2.92061235	4.14469426	-1.78493199	.679191762	- .118341690	.0771	.61
.62	- 2.88810966	4.13030323	-1.81491605	.693879389	- .121156906	.0794	.62
.63	- 2.85650875	4.11678492	-1.84495307	.708678137	- .124001240	.0817	.63
.64	- 2.82576925	4.10410000	-1.87504390	.723587878	- .126874725	.0841	.64
.65	- 2.79585324	4.09221154	-1.90518939	.738608490	- .129777397	.0864	.65
.66	- 2.76672505	4.08108485	-1.93539036	.753739850	- .132709290	.0889	.66
.67	- 2.73835111	4.07068731	-1.96564760	.768981835	- .135670436	.0914	.67
.68	- 2.71069980	4.06098824	-1.99596190	.784334328	- .138660870	.0939	.68
.69	- 2.68374126	4.05195870	-2.02633402	.799797212	- .141680623	.0965	.69
	$-{}^{i}_{4}C^{4}_{4}$	$-{}^{i}_{4}C^{4}_{3}$	$-{}^{i}_{4}C^{4}_{2}$	$-{}^{i}_{4}C^{4}_{1}$	$-{}^{i}_{4}C^{4}_{0}$	$\frac{{}^{i}_{4}E^{4}}{b^4 f^{(5)}(\xi)}$	$r = \frac{d\,d\,d}{c\,b\,a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b^i} \sum_{j=0}^{4} {}^{i}_{4}C^{i}_{j}\, y_j + {}^{i}_{4}R^{i} \qquad (i=0,4)$$

$i=\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^{i}_{4}C^{0}_{0}$	${}^{i}_{4}C^{0}_{1}$	${}^{i}_{4}C^{0}_{2}$	${}^{i}_{4}C^{0}_{3}$	${}^{i}_{4}C^{0}_{4}$	$\frac{{}^{i}_{4}E^{0}}{b^4 f^{(5)}(\xi)}$	
0.70	- 2.65744736	4.04357143	-2.05676471	0.815370370	-0.144729730	0.0991	0.70
.71	- 2.63179149	4.03580070	-2.08725468	.831053690	- .147808221	.1017	.71
.72	- 2.60674850	4.02862222	-2.11780465	.846847059	- .150916129	.1044	.72
.73	- 2.58229457	4.02201301	-2.14841532	.862750366	- .154053485	.1072	.73
.74	- 2.55840717	4.01595135	-2.17908736	.878763504	- .157220321	.1100	.74
.75	- 2.53506493	4.01041667	-2.20982143	.894886364	- .160416667	.1128	.75
.76	- 2.51224758	4.00538947	-2.24061818	.911118841	- .163642553	.1157	.76
.77	- 2.48993587	4.00085130	-2.27147825	.927460830	- .166898011	.1186	.77
.78	- 2.46811153	3.99678462	-2.30240225	.943912230	- .170183069	.1216	.78
.79	- 2.44675718	3.99317278	-2.33339078	.960472939	- .173497757	.1246	.79
.80	- 2.42585631	3.99000000	-2.36444444	.977142857	- .176842105	.1277	.80
.81	- 2.40539316	3.98725123	-2.39556381	.993921886	- .180216142	.1308	.81
.82	- 2.38535278	3.98491220	-2.42674945	1.01080993	- .183619895	.1340	.82
.83	- 2.36572086	3.98296927	-2.45800191	1.02780689	- .187053394	.1372	.83
.84	- 2.34648379	3.98140953	-2.48932174	1.04491268	- .190516667	.1405	.84
.85	- 2.32762858	3.98022058	-2.52070946	1.06212719	- .194009740	.1438	.85
.86	- 2.30914281	3.97939071	-2.55216559	1.07945035	- .197532642	.1472	.86
.87	- 2.29101463	3.97890862	-2.58369064	1.09688206	- .201085401	.1506	.87
.88	- 2.27323271	3.97876364	-2.61528511	1.11442222	- .204668041	.1541	.88
.89	- 2.25578620	3.97894550	-2.64694947	1.13207076	- .208280591	.1576	.89
.90	- 2.23866474	3.97944444	-2.67868421	1.14982759	- .211923077	.1612	.90
.91	- 2.22185839	3.98025110	-2.71048979	1.16769261	- .215595524	.1648	.91
.92	- 2.20535765	3.98135653	-2.74236667	1.18566575	- .219297959	.1685	.92
.93	- 2.18915339	3.98275215	-2.77431528	1.20374693	- .223030407	.1722	.93
.94	- 2.17323686	3.98442979	-2.80633608	1.22193605	- .226792893	.1760	.94
.95	- 2.15759970	3.98638157	-2.83842949	1.24023305	- .230585443	.1799	.95
.96	- 2.14223384	3.98860001	-2.87059592	1.25863784	- .234408081	.1838	.96
.97	- 2.12713155	3.99107784	-2.90283579	1.27715034	- .238260831	.1878	.97
.98	- 2.11228542	3.99380816	-2.93514949	1.29577047	- .242143719	.1918	.98
.99	- 2.09768830	3.99678434	-2.96753744	1.31449816	- .246056767	.1959	.99
1.00	- 2.08333333	4.00000000	-3.00000000	1.33333333	- .250000000	.2000	1.00
1.01	- 2.06921392	4.00344901	-3.03253756	1.35227591	- .253973441	.2042	1.01
1.02	- 2.05532371	4.00712549	-3.06515050	1.37132583	- .257977114	.2085	1.02
1.03	- 2.04165658	4.01102379	-3.09783916	1.39048300	- .262011042	.2128	1.03
1.04	- 2.02820666	4.01513846	-3.13060392	1.40974737	- .266075248	.2171	1.04
1.05	- 2.01496826	4.01946429	-3.16344512	1.42911885	- .270169753	.2216	1.05
1.06	- 2.00193592	4.02399623	-3.19636311	1.44859739	- .274294581	.2261	1.06
1.07	- 1.98910437	4.02872944	-3.22935821	1.46818290	- .278449754	.2306	1.07
1.08	- 1.97646852	4.03365926	-3.26243077	1.48787532	- .282635294	.2352	1.08
1.09	- 1.96402346	4.03878119	-3.29558110	1.50767460	- .286851222	.2399	1.09
1.10	- 1.95176447	4.04409091	-3.32880952	1.52758065	- .291097561	.2447	1.10
1.11	- 1.93968696	4.04958423	-3.36211635	1.54759341	- .295374331	.2495	1.11
1.12	- 1.92778652	4.05525714	-3.39550189	1.56771282	- .299681553	.2543	1.12
1.13	- 1.91605889	4.06110575	-3.42896643	1.58793882	- .304019249	.2593	1.13
1.14	- 1.90449993	4.06712632	-3.46251028	1.60827134	- .308387440	.2643	1.14
1.15	- 1.89310567	4.07331521	-3.49613372	1.62871032	- .312786145	.2694	1.15
1.16	- 1.88187224	4.07966897	-3.52983704	1.64925570	- .317215385	.2745	1.16
1.17	- 1.87079591	4.08618419	-3.56362051	1.66990741	- .321675180	.2797	1.17
1.18	- 1.85987308	4.09285763	-3.59748440	1.69066541	- .326165550	.2849	1.18
1.19	- 1.84910025	4.09968613	-3.63142900	1.71152962	- .330686516	.2903	1.19
1.20	- 1.83847403	4.10666667	-3.66545455	1.73250000	- .335238095	.2957	1.20
1.21	- 1.82799114	4.11379628	-3.69956131	1.75357648	- .339820309	.3012	1.21
1.22	- 1.81764841	4.12107214	-3.73374955	1.77475901	- .344433175	.3067	1.22
1.23	- 1.80744277	4.12849146	-3.76801951	1.79604752	- .349076714	.3123	1.23
1.24	- 1.79737122	4.13605162	-3.80237143	1.81744198	- .353750943	.3180	1.24
1.25	- 1.78743087	4.14374999	-3.83680556	1.83894231	- .358455882	.3237	1.25
1.26	- 1.77761892	4.15158413	-3.87132212	1.86054847	- .363191549	.3296	1.26
1.27	- 1.76793264	4.15955157	-3.90592137	1.88226040	- .367957963	.3355	1.27
1.28	- 1.75836940	4.16765001	-3.94060351	1.90407805	- .372755140	.3414	1.28
1.29	- 1.74892662	4.17587713	-3.97536878	1.92600137	- .377583100	.3475	1.29
	$-{}^{i}_{4}C^{4}_{4}$	$-{}^{i}_{4}C^{4}_{3}$	$-{}^{i}_{4}C^{4}_{2}$	$-{}^{i}_{4}C^{4}_{1}$	$-{}^{i}_{4}C^{4}_{0}$	$\frac{{}^{i}_{4}E^{4}}{b^4 f^{(5)}(\xi)}$	$p=\frac{d}{c}\frac{d}{b}\frac{d}{a}$

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$$(D^i y)_{x=x_1} = \frac{1}{b} \sum_{j=0}^{4} {}^{i}_{4}C^{i}_{j}\, y_j + {}^{i}_{4}R^{i} \qquad (i=1,3)$$

$\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^{i}_{4}C^{i}_{0}$	${}^{i}_{4}C^{i}_{1}$	${}^{i}_{4}C^{i}_{2}$	${}^{i}_{4}C^{i}_{3}$	${}^{i}_{4}C^{i}_{4}$	$\frac{{}^{i}_{4}E^{i}}{b^4 f^{(5)}(\xi)}$	
0.10	-8.37871806	8.16666667	.272727273	-.0714285714	.0107526882	-.0050	0.10
.11	-7.48846393	7.25757576	.297297297	-.0781990521	.0117899250	-.0055	.11
.12	-6.74934342	6.50000000	.321428571	-.0849056604	.0128205128	-.0060	.12
.13	-6.12640232	5.85897436	.345132743	-.0915492958	.0138445154	-.0065	.13
.14	-5.59467602	5.30952381	.368421053	-.0981308411	.0148619958	-.0070	.14
.15	-5.13585953	4.83333333	.391304348	-.104651163	.0158730159	-.0075	.15
.16	-4.73622630	4.41666667	.413793103	-.111111111	.0168776371	-.0080	.16
.17	-4.38528144	4.04901961	.435897436	-.117511521	.0178759201	-.0085	.17
.18	-4.07486405	3.72222222	.457627119	-.123853211	.0188679245	-.0090	.18
.19	-3.79853288	3.42982456	.478991597	-.130136986	.0198537095	-.0095	.19
.20	-3.55113636	3.16666667	.500000000	-.136363636	.0208333333	-.0100	.20
.21	-3.32850550	2.92857143	.520661157	-.142533937	.0218068536	-.0105	.21
.22	-3.12723050	2.71212121	.540983607	-.148648649	.0227743271	-.0110	.22
.23	-2.94449585	2.51449275	.560975610	-.154708520	.0237358101	-.0115	.23
.24	-2.77795557	2.33333333	.580645161	-.160714286	.0246913580	-.0120	.24
.25	-2.62564103	2.16666667	.600000000	-.166666667	.0256410256	-.0125	.25
.26	-2.48588663	2.01282051	.619047619	-.172566372	.0265848671	-.0130	.26
.27	-2.35727448	1.87037037	.637795276	-.178414097	.0275229358	-.0135	.27
.28	-2.23859000	1.73809524	.656250000	-.184210526	.0284552846	-.0140	.28
.29	-2.12878677	1.61494253	.674418605	-.189956332	.0293819656	-.0145	.29
.30	-2.02695855	1.50000000	.692307692	-.195652174	.0303030303	-.0150	.30
.31	-1.93231661	1.39247312	.709923664	-.201298701	.0312185297	-.0155	.31
.32	-1.84417136	1.29166667	.727272727	-.206896552	.0321285141	-.0160	.32
.33	-1.76191728	1.19696970	.744360902	-.212446352	.0330330330	-.0165	.33
.34	-1.68502058	1.10784314	.761194030	-.217948718	.0339321357	-.0170	.34
.35	-1.61300892	1.02380952	.777777778	-.223404255	.0348258706	-.0175	.35
.36	-1.54546282	.944444445	.794117647	-.228813559	.0357142857	-.0180	.36
.37	-1.48200856	.869369370	.810218978	-.234177215	.0365974283	-.0185	.37
.38	-1.42231212	.798245614	.826086957	-.239495798	.0374753452	-.0190	.38
.39	-1.36607406	.730769231	.841726619	-.244769874	.0383480826	-.0195	.39
.40	-1.31302521	.666666667	.857142857	-.250000000	.0392156863	-.0200	.40
.41	-1.26292296	.605691057	.872340426	-.255186722	.0400782014	-.0205	.41
.42	-1.21554809	.547619048	.887323944	-.260330579	.0409356725	-.0210	.42
.43	-1.17070201	.492248062	.902097902	-.265432099	.0417881438	-.0215	.43
.44	-1.12820446	.439393939	.916666667	-.270491803	.0426356589	-.0220	.44
.45	-1.08789143	.388888889	.931034483	-.275510204	.0434782609	-.0225	.45
.46	-1.04961338	.340579710	.945205479	-.280487805	.0443159923	-.0230	.46
.47	-1.01323371	.294326241	.959183673	-.285425101	.0451488953	-.0235	.47
.48	- .978627404	.250000000	.972972973	-.290322581	.0459770115	-.0240	.48
.49	- .945679834	.207482993	.986577181	-.295180923	.0468003820	-.0245	.49
.50	- .914285714	.166666667	1.00000000	-.300000000	.0476190476	-.0250	.50
.51	- .884348185	.127450980	1.01324503	-.304780876	.0484330484	-.0255	.51
.52	- .855777994	.0897435897	1.02631579	-.309523810	.0492424242	-.0260	.52
.53	- .828492771	.0534591195	1.03921569	-.314229249	.0500472143	-.0265	.53
.54	- .802416390	.0185185185	1.05194805	-.318897638	.0508474576	-.0270	.54
.55	- .777478394	- .0151515152	1.06451613	-.323529412	.0516431925	-.0275	.55
.56	- .753613486	- .0476190476	1.07692308	-.328125000	.0524344569	-.0280	.56
.57	- .730761070	- .0889473684	1.08917197	-.332684825	.0532212885	-.0285	.57
.58	- .708864842	- .109195402	1.10126582	-.337209302	.0540037244	-.0290	.58
.59	- .687872428	- .138418079	1.11320755	-.341698842	.0547818013	-.0295	.59
.60	- .667735043	- .166666667	1.12500000	-.346153846	.0555555556	-.0300	.60
.61	- .648407202	- .193989071	1.13664596	-.350574713	.0563250231	-.0305	.61
.62	- .629846448	- .220430108	1.14814815	-.354961832	.0570902394	-.0310	.62
.63	- .612013107	- .246031746	1.15950920	-.359315589	.0578512396	-.0315	.63
.64	- .594870069	- .270833333	1.17073171	-.363636364	.0586080586	-.0320	.64
.65	- .578382589	- .294871794	1.18181818	-.367924528	.0593607305	-.0325	.65
.66	- .562518105	- .318181818	1.19277108	-.372180451	.0601092896	-.0330	.66
.67	- .547246069	- .340796019	1.20359281	-.376404494	.0608537693	-.0335	.67
.68	- .532537804	- .362745097	1.21428571	-.380597015	.0615942028	-.0340	.68
.69	- .518366359	- .384057971	1.22485207	-.384758364	.0623306233	-.0345	.69
	$-{}^{i}_{4}C^{3}_{4}$	$-{}^{i}_{4}C^{3}_{3}$	$-{}^{i}_{4}C^{3}_{2}$	$-{}^{i}_{4}C^{3}_{1}$	$-{}^{i}_{4}C^{3}_{0}$	$\frac{{}^{i}_{4}E^{3}}{b^4 f^{(5)}(\xi)}$	$\frac{d}{c}\frac{d}{b}\frac{d}{a}$

15(a)

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b^i} \sum_{j=0}^{4} {}^i_4C^j_j \, y_j + {}^i_4R^i \qquad (i=1,3)$$

$\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^i_4C^i_0$	${}^i_4C^i_1$	${}^i_4C^i_2$	${}^i_4C^i_3$	${}^i_4C^i_4$	$\frac{{}^i_4E^i}{b^4 f^{(5)}(\xi)}$	
0.70	-.504706387	-0.404761904	1.23529412	-.388888889	.0830630630	-.0350	0.70
.71	-.491534030	-.424882629	1.24561404	-.392988930	.0837915543	-.0355	.71
.72	-.478826815	-.444444444	1.25581395	-.397058824	.0845161290	-.0360	.72
.73	-.466563552	-.463470319	1.26589595	-.401098901	.0852368185	-.0365	.73
.74	-.454724252	-.481981982	1.27586207	-.405109489	.0859536541	-.0370	.74
.75	-.443290043	-.500000000	1.28571429	-.409090909	.0866666667	-.0375	.75
.76	-.432243094	-.517543859	1.29545455	-.413043478	.0873758865	-.0380	.76
.77	-.421566546	-.534632034	1.30508475	-.416967509	.0880813439	-.0385	.77
.78	-.411244450	-.551282051	1.31460674	-.420863309	.0887830687	-.0390	.78
.79	-.401261706	-.567510548	1.32402235	-.424731183	.0894810905	-.0395	.79
.80	-.391604010	-.583333333	1.33333333	-.428571429	.0901754386	-.0400	.80
.81	-.382257805	-.598765432	1.34254144	-.432384342	.0908661417	-.0405	.81
.82	-.373210228	-.613821138	1.35164835	-.436170213	.0915532286	-.0410	.82
.83	-.364449081	-.628514056	1.36065574	-.439929329	.0922367276	-.0415	.83
.84	-.355962768	-.642857143	1.36956522	-.443661972	.0929166667	-.0420	.84
.85	-.347740287	-.656862745	1.37837838	-.447368421	.0935930736	-.0425	.85
.86	-.339771162	-.670542636	1.38709677	-.451048951	.0942659758	-.0430	.86
.87	-.332045447	-.683908046	1.39572192	-.454703833	.0949354005	-.0435	.87
.88	-.324553663	-.696969697	1.40425532	-.458333333	.0956013746	-.0440	.88
.89	-.317286793	-.709737828	1.41269841	-.461937716	.0962639246	-.0445	.89
.90	-.310236245	-.722222222	1.42105263	-.465517241	.0969230769	-.0450	.90
.91	-.303393830	-.734432234	1.42931937	-.469072165	.0975788876	-.0455	.91
.92	-.296751741	-.746376812	1.43750000	-.472602740	.0982312925	-.0460	.92
.93	-.290302531	-.758064516	1.44559585	-.476109215	.0988804071	-.0465	.93
.94	-.284039091	-.769503546	1.45360825	-.479591837	.0995262267	-.0470	.94
.95	-.277954637	-.780701754	1.46153846	-.483050847	.0801687764	-.0475	.95
.96	-.272042682	-.791666667	1.46938776	-.486486486	.0808080808	-.0480	.96
.97	-.266297037	-.802405498	1.47715736	-.489898990	.0814441646	-.0485	.97
.98	-.260711776	-.812925170	1.48484848	-.493288591	.0820770519	-.0490	.98
.99	-.255281237	-.823232323	1.49246231	-.496655518	.0827067669	-.0495	.99
1.00	-.250000000	-.833333333	1.50000000	-.500000000	.0833333333	-.0500	1.00
1.01	-.244862879	-.843234323	1.50746269	-.503322259	.0839567747	-.0505	1.01
1.02	-.239864907	-.852941176	1.51485149	-.506622517	.0845771144	-.0510	1.02
1.03	-.235001326	-.862459547	1.52216749	-.509900990	.0851943755	-.0515	1.03
1.04	-.230267579	-.871794872	1.52941176	-.513157895	.0858085809	-.0520	1.04
1.05	-.225659295	-.880952381	1.53658537	-.516393443	.0864197531	-.0525	1.05
1.06	-.221172285	-.889937107	1.54368932	-.519607843	.0870279146	-.0530	1.06
1.07	-.216802528	-.898753894	1.55072464	-.522801303	.0876330876	-.0535	1.07
1.08	-.212546168	-.907407407	1.55769231	-.525974026	.0882352941	-.0540	1.08
1.09	-.208399503	-.915902141	1.56459330	-.529126214	.0888345558	-.0545	1.09
1.10	-.204358977	-.924242424	1.57142857	-.532258065	.0894308943	-.0550	1.10
1.11	-.200421176	-.932432432	1.57819905	-.535369775	.0900243309	-.0555	1.11
1.12	-.196582818	-.940476190	1.58490566	-.538461538	.0906148867	-.0560	1.12
1.13	-.192840751	-.948377581	1.59154930	-.541533546	.0912025827	-.0565	1.13
1.14	-.189191942	-.956140351	1.59813084	-.544585987	.0917874396	-.0570	1.14
1.15	-.185633477	-.963768116	1.60465116	-.547619048	.0923694779	-.0575	1.15
1.16	-.182162550	-.971264368	1.61111111	-.550632911	.0929487179	-.0580	1.16
1.17	-.178776462	-.978632479	1.61751152	-.553627760	.0935251799	-.0585	1.17
1.18	-.175472615	-.985875706	1.62385321	-.556603774	.0940988836	-.0590	1.18
1.19	-.172248508	-.992997199	1.63013699	-.559561129	.0946698488	-.0595	1.19
1.20	-.169101732	-1.00000000	1.63636364	-.562500000	.0952380952	-.0600	1.20
1.21	-.166029966	-1.00688705	1.64253394	-.565420561	.0958036421	-.0605	1.21
1.22	-.163030974	-1.01366120	1.64864865	-.568322981	.0963665087	-.0610	1.22
1.23	-.160102601	-1.02032520	1.65470852	-.571207430	.0969267139	-.0615	1.23
1.24	-.157242768	-1.02688172	1.66071429	-.574074074	.0974842767	-.0620	1.24
1.25	-.154449472	-1.03333333	1.66666667	-.576923077	.0980392157	-.0625	1.25
1.26	-.151720780	-1.03968254	1.67256637	-.579754601	.0985915493	-.0630	1.26
1.27	-.149054827	-1.04593176	1.67841410	-.582568807	.0991412959	-.0635	1.27
1.28	-.146449813	-1.05208333	1.68421053	-.585365854	.0996884735	-.0640	1.28
1.29	-.143904001	-1.05813953	1.68995633	-.588145897	.100233100	-.0645	1.29
	$-{}^i_4C^3_4$	$-{}^i_4C^3_3$	$-{}^i_4C^3_2$	$-{}^i_4C^3_1$	$-{}^i_4C^3_0$	$\frac{{}^i_4E^3}{b^4 f^{(5)}(\xi)}$	$\frac{d}{c}\frac{d}{b}\frac{d}{a}$

15(b)

1411

1411

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^l y)_{x=x_i} = \frac{1}{b}\sum_{j=0}^{4} {}^{l}_{4}C^{l}_{j}\, y_j + {}^{l}_{4}R^{l} \qquad (l = 2)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{l}_{4}C^{2}_{0}$	${}^{l}_{4}C^{2}_{1}$	${}^{l}_{4}C^{2}_{2}$	${}^{l}_{4}C^{2}_{3}$	${}^{l}_{4}C^{2}_{4}$	$\frac{{}^{l}_{4}E^{2}}{b^4 f^{(5)}(\xi)}$	
0.10	2.79290602	-3.66666667	0.409090909	0.523809524	-0.0591397849	0.0183	0.10
.11	2.49615464	-3.36363636	.400900901	.526066351	- .0594855305	.0185	.11
.12	2.24978114	-3.11111111	.392857143	.528301887	- .0598290598	.0187	.12
.13	2.04213411	-2.89743590	.384955752	.530516432	- .0601703940	.0188	.13
.14	1.86489201	-2.71428571	.377192982	.532710280	- .0605095541	.0190	.14
.15	1.71195318	-2.55555556	.369565217	.534883721	- .0608465608	.0192	.15
.16	1.57874210	-2.41666667	.362068966	.537037037	- .0611814346	.0193	.16
.17	1.46176048	-2.29411765	.354700855	.539170507	- .0615141956	.0195	.17
.18	1.35828802	-2.18518519	.347457627	.541284404	- .0618448637	.0197	.18
.19	1.26617763	-2.08771930	.340336134	.543378995	- .0621734587	.0198	.19
.20	1.18371212	-2.00000000	.333333333	.545454545	- .0625000000	.0200	.20
.21	1.10950183	-1.92063492	.326446281	.547511312	- .0628245087	.0202	.21
.22	1.04241017	-1.84848485	.319672131	.549549550	- .0631469979	.0203	.22
.23	.981498550	-1.78260870	.313008130	.551569507	- .0634674923	.0205	.23
.24	.925985190	-1.72222222	.306451613	.553571429	- .0637860082	.0207	.24
.25	.875213677	-1.66666667	.300000000	.555555556	- .0641025641	.0208	.25
.26	.828628877	-1.61538462	.293650794	.557522124	- .0644171779	.0210	.26
.27	.785758160	-1.56790123	.287401575	.559471366	- .0647298675	.0212	.27
.28	.746196667	-1.52380952	.281250000	.561403509	- .0650406504	.0213	.28
.29	.709595590	-1.48275862	.275193798	.563318777	- .0653495441	.0215	.29
.30	.675652850	-1.44444444	.269230769	.565217391	- .0656565657	.0217	.30
.31	.644105537	-1.40860215	.263358779	.567099567	- .0659617321	.0218	.31
.32	.614723787	-1.37500000	.257575757	.568965517	- .0662650602	.0220	.32
.33	.587305760	-1.34343434	.251879699	.570815451	- .0665665666	.0222	.33
.34	.561673527	-1.31372549	.246268657	.572649573	- .0668662675	.0223	.34
.35	.537669640	-1.28571429	.240740740	.574468085	- .0671641791	.0225	.35
.36	.515154273	-1.25925926	.235294118	.576271186	- .0674603175	.0227	.36
.37	.494002853	-1.23423423	.229927007	.578059072	- .0677546983	.0228	.37
.38	.474104040	-1.21052632	.224637681	.579831933	- .0680473373	.0230	.38
.39	.455358020	-1.18803419	.219422460	.581589958	- .0683382498	.0232	.39
.40	.437675070	-1.16666667	.214285714	.583333333	- .0686274510	.0233	.40
.41	.420974320	-1.14634146	.209219858	.585062241	- .0689149560	.0235	.41
.42	.405182697	-1.12698413	.204225352	.586776860	- .0692007797	.0237	.42
.43	.390234003	-1.10852713	.199300699	.588477366	- .0694849368	.0238	.43
.44	.376068153	-1.09090909	.194444444	.590163934	- .0697674419	.0240	.44
.45	.362630476	-1.07407407	.189655172	.591836735	- .0700483092	.0242	.45
.46	.349871126	-1.05797101	.184931507	.593495935	- .0703275530	.0243	.46
.47	.337744570	-1.04255319	.180272109	.595141700	- .0706051873	.0245	.47
.48	.326209135	-1.02777778	.175675676	.596774194	- .0708812261	.0247	.48
.49	.315226611	-1.01360544	.171140940	.598393574	- .0711556829	.0248	.49
.50	.304761905	-1.00000000	.166666667	.600000000	- .0714285714	.0250	.50
.51	.294782728	- .986928105	.162251656	.601593625	- .0716999050	.0252	.51
.52	.285259331	- .974358974	.157894737	.603174603	- .0719696970	.0253	.52
.53	.276164257	- .962264151	.153594771	.604743083	- .0722379603	.0255	.53
.54	.267472130	- .950617284	.149350649	.606299213	- .0725047081	.0257	.54
.55	.259159465	- .939393939	.145161290	.607843137	- .0727699531	.0258	.55
.56	.251204495	- .928571429	.141025641	.609375000	- .0730337079	.0260	.56
.57	.243587023	- .918128655	.136942675	.610894942	- .0732959851	.0262	.57
.58	.236288281	- .908045977	.132911392	.612403101	- .0735567970	.0263	.58
.59	.229290809	- .898305085	.128930818	.613899614	- .0738161560	.0265	.59
.60	.222578348	- .888888889	.125000000	.615384615	- .0740740741	.0267	.60
.61	.216135734	- .879781421	.121118012	.616858238	- .0743305633	.0268	.61
.62	.209948816	- .870967742	.117283951	.618320611	- .0745856354	.0270	.62
.63	.204004370	- .862433862	.113496933	.619771863	- .0748393021	.0272	.63
.64	.198290023	- .854166667	.109756098	.621212121	- .0750915750	.0273	.64
.65	.192794196	- .846153846	.106060606	.622641509	- .0753424657	.0275	.65
.66	.187506035	- .838383838	.102409639	.624060150	- .0755919854	.0277	.66
.67	.182415356	- .830845771	.098802395	.625468165	- .0758401453	.0278	.67
.68	.177512601	- .823529412	.095238095	.626865672	- .0760869565	.0280	.68
.69	.172788786	- .816425121	.091715976	.628252788	- .0763324299	.0282	.69
	$-{}^{l}_{4}C^{2}_{4}$	$-{}^{l}_{4}C^{2}_{3}$	$-{}^{l}_{4}C^{2}_{2}$	$-{}^{l}_{4}C^{2}_{1}$	$-{}^{l}_{4}C^{2}_{0}$	$\frac{{}^{l}_{4}E^{2}}{b^4 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

16(a)

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b}\sum_{j=0}^{4} {}^{i}_{4}C^{i}_{j}\, y_j + {}^{i}_{4}R^{i} \qquad (i = 2)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{i}_{4}C^{2}_{0}$	${}^{i}_{4}C^{2}_{1}$	${}^{i}_{4}C^{2}_{2}$	${}^{i}_{4}C^{2}_{3}$	${}^{i}_{4}C^{2}_{4}$	${}^{i}_{4}E^{2}/b^4 f^{(5)}(\xi)$	
0.70	0.168235462	-0.809523810	0.0882352941	0.629629630	-0.0765765765	0.0283	0.70
.71	.163844677	- .802816901	.0847953216	.630996310	- .0768194070	.0285	.71
.72	.159608938	- .796296296	.0813953488	.632352941	- .0770609318	.0287	.72
.73	.155521184	- .789954338	.0780346820	.633699634	- .0773011617	.0288	.73
.74	.151574751	- .783783784	.0747126436	.635036496	- .0775401069	.0290	.74
.75	.147765348	- .777777778	.0714285714	.636363636	- .0777777778	.0292	.75
.76	.144081031	- .771929825	.0681818182	.637681159	- .0780141843	.0293	.76
.77	.140522182	- .766233766	.0649717514	.638989170	- .0782493368	.0295	.77
.78	.137081463	- .760683761	.0617977528	.640287770	- .0784832451	.0297	.78
.79	.133753902	- .755274262	.0586592178	.641577061	- .0787159190	.0298	.79
.80	.130534670	- .750000000	.0555555556	.642857143	- .0789473684	.0300	.80
.81	.127419268	- .744855967	.0524861878	.644128114	- .0791776028	.0302	.81
.82	.124403409	- .739837398	.0494505494	.645390071	- .0794066318	.0303	.82
.83	.121483027	- .734939759	.0464480874	.646643110	- .0796344648	.0305	.83
.84	.118654256	- .730158730	.0434782609	.647887324	- .0798611111	.0307	.84
.85	.115913429	- .725490196	.0405405405	.649122807	- .0800865801	.0308	.85
.86	.113257054	- .720930232	.0376344086	.650349650	- .0803108808	.0310	.86
.87	.110681816	- .716475096	.0347593583	.651567944	- .0805340224	.0312	.87
.88	.108184554	- .712121212	.0319148936	.652777778	- .0807560137	.0313	.88
.89	.105762264	- .707865168	.0291005291	.653979239	- .0809768638	.0315	.89
.90	.103412082	- .703703704	.0263157895	.655172414	- .0811965812	.0317	.90
.91	.101131277	- .699633699	.0235602094	.656357388	- .0814151748	.0318	.91
.92	.0989172470	- .695652174	.0208333333	.657534247	- .0816326531	.0320	.92
.93	.0967675103	- .691756272	.0181347150	.658703072	- .0818490246	.0322	.93
.94	.0946796970	- .687943262	.0154639175	.659863946	- .0820642978	.0323	.94
.95	.0926515457	- .684210526	.0128205128	.661016949	- .0822784810	.0325	.95
.96	.0906808940	- .680555555	.0102040816	.662162162	- .0824915825	.0327	.96
.97	.088765679	- .676975945	.00761421320	.663299663	- .0827036104	.0328	.97
.98	.0869039253	- .673469388	.00505050505	.664429530	- .0829145729	.0330	.98
.99	.0850937457	- .670033670	.00251256281	.665551839	- .0831244779	.0332	.99
1.00	.0833333333	- .666666667	0	.666666667	- .0833333333	.0333	1.00
1.01	.0816209597	- .663366337	- .00248756219	.667774086	- .0835411471	.0335	1.01
1.02	.0799549690	- .660130719	- .00495049505	.668874172	- .0837479270	.0337	1.02
1.03	.0783337753	- .656957929	- .00738916256	.669966997	- .0839536807	.0338	1.03
1.04	.0767558597	- .653846154	- .00980392157	.671052632	- .0841584158	.0340	1.04
1.05	.0752197650	- .650793651	- .0121951220	.672131148	- .0843621399	.0342	1.05
1.06	.0737240950	- .647798742	- .0145631068	.673202614	- .0845648604	.0343	1.06
1.07	.0722675093	- .644859813	- .0169082126	.674267101	- .0847665848	.0345	1.07
1.08	.0708487227	- .641975309	- .0192307692	.675324675	- .0849673203	.0347	1.08
1.09	.0694665010	- .639143731	- .0215311005	.676375405	- .0851670742	.0348	1.09
1.10	.0681196590	- .636363636	- .0238095238	.677419355	- .0853658537	.0350	1.10
1.11	.0668070587	- .633633634	- .0260663507	.678456592	- .0855636659	.0352	1.11
1.12	.0655276060	- .630952381	- .0283018868	.679487179	- .0857605178	.0353	1.12
1.13	.0642802503	- .628318584	- .0305164319	.680511182	- .0859564165	.0355	1.13
1.14	.0630639807	- .625730994	- .0327102804	.681528662	- .0861513688	.0357	1.14
1.15	.0618778257	- .623188406	- .0348837209	.682539683	- .0863453815	.0358	1.15
1.16	.0607208500	- .620689655	- .0370370370	.683544304	- .0865384615	.0360	1.16
1.17	.0595921540	- .618233618	- .0391705069	.684542587	- .0867306155	.0362	1.17
1.18	.0584908717	- .615819209	- .0412844037	.685534591	- .0869218501	.0363	1.18
1.19	.0574161693	- .613445378	- .0433789954	.686520376	- .0871121718	.0365	1.19
1.20	.0563672440	- .611111111	- .0454545455	.687500000	- .0873015873	.0367	1.20
1.21	.0553433220	- .608815427	- .0475113122	.688473520	- .0874901029	.0368	1.21
1.22	.0543436580	- .606557377	- .0495495495	.689440994	- .0876777251	.0370	1.22
1.23	.0533675337	- .604336043	- .0515695067	.690402477	- .0878644602	.0372	1.23
1.24	.0524142559	- .602150538	- .0535714286	.691358025	- .0880505145	.0373	1.24
1.25	.0514831573	- .600000000	- .0555555556	.692307692	- .0882352941	.0375	1.25
1.26	.0505735933	- .597883598	- .0575221239	.693251534	- .0884194053	.0377	1.26
1.27	.0496849423	- .595800525	- .0594713656	.694189603	- .0886026542	.0378	1.27
1.28	.0488166042	- .593750000	- .0614035088	.695121951	- .0887850467	.0380	1.28
1.29	.0479680003	- .591731266	- .0633187773	.696048632	- .0889665890	.0382	1.29
	$-{}^{i}_{4}C^{2}_{4}$	$-{}^{i}_{4}C^{2}_{3}$	$-{}^{i}_{4}C^{2}_{2}$	$-{}^{i}_{4}C^{2}_{1}$	$-{}^{i}_{4}C^{2}_{0}$	${}^{i}_{4}E^{2}/b^4 f^{(5)}(\xi)$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^l y)_{x=x_i} = \frac{1}{b}\sum_{j=0}^{4} {}^{l}_{4}C^{l}_{j}\, y_j + {}^{l}_{4}R^{l} \qquad (l = 3, 1)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{1}_{4}C^{3}_{0}$	${}^{1}_{4}C^{3}_{1}$	${}^{1}_{4}C^{3}_{2}$	${}^{1}_{4}C^{3}_{3}$	${}^{1}_{4}C^{3}_{4}$	$\frac{{}^{1}_{4}E^{3}}{b^4 f^{(5)}(\xi)}$	
0.10	-2.79290602	3.50000000	-1.90909091	0.976190476	0.225806452	-0.0350	0.10
.11	-2.49615464	3.19696970	-1.90090090	.973933649	.226152197	- .0352	.11
.12	-2.24978114	2.94444444	-1.89285714	.971698113	.226495726	- .0353	.12
.13	-2.04213411	2.73076923	-1.88495575	.969483568	.226837061	- .0355	.13
.14	-1.86489201	2.54761905	-1.87719298	.967289720	.227176221	- .0357	.14
.15	-1.71195318	2.38888889	-1.86956522	.965116279	.227513228	- .0358	.15
.16	-1.57874210	2.25000000	-1.86206897	.962962963	.227848101	- .0360	.16
.17	-1.46176048	2.12745098	-1.85470085	.960829493	.228180862	- .0362	.17
.18	-1.35828802	2.01851852	-1.84745763	.958715596	.228511530	- .0363	.18
.19	-1.26617763	1.92105263	-1.84033613	.956621005	.228840125	- .0365	.19
.20	-1.18371212	1.83333333	-1.83333333	.954545455	.229166667	- .0367	.20
.21	-1.10950183	1.75396825	-1.82644628	.952488688	.229491173	- .0368	.21
.22	-1.04241017	1.68181818	-1.81967213	.950450450	.229813665	- .0370	.22
.23	- .981498550	1.61594203	-1.81300813	.948430493	.230134159	- .0372	.23
.24	- .925985190	1.55555556	-1.80645161	.946428571	.230452675	- .0373	.24
.25	- .875213677	1.50000000	-1.80000000	.944444444	.230769231	- .0375	.25
.26	- .828628877	1.44871795	-1.79365079	.942477876	.231083845	- .0377	.26
.27	- .785758160	1.40123457	-1.78740157	.940528634	.231396534	- .0378	.27
.28	- .746196667	1.35714286	-1.78125000	.938596491	.231707317	- .0380	.28
.29	- .709595590	1.31609195	-1.77519380	.936681223	.232016211	- .0382	.29
.30	- .675652850	1.27777778	-1.76923077	.934782609	.232323232	- .0383	.30
.31	- .644105537	1.24193548	-1.76335878	.932900433	.232628399	- .0385	.31
.32	- .614723787	1.20833333	-1.75757576	.931034483	.232931727	- .0387	.32
.33	- .587305760	1.17676768	-1.75187970	.929184549	.233233233	- .0388	.33
.34	- .561673527	1.14705882	-1.74626866	.927350427	.233532934	- .0390	.34
.35	- .537669640	1.11904762	-1.74074074	.925531915	.233830846	- .0392	.35
.36	- .515154273	1.09259259	-1.73529412	.923728814	.234126984	- .0393	.36
.37	- .494002853	1.06756757	-1.72992701	.921940928	.234421365	- .0395	.37
.38	- .474104040	1.04385965	-1.72463768	.920168067	.234714004	- .0397	.38
.39	- .455358020	1.02136752	-1.71942446	.918410042	.235004916	- .0398	.39
.40	- .437675070	1.00000000	-1.71428571	.916666667	.235294118	- .0400	.40
.41	- .420974320	.979674797	-1.70921986	.914937759	.235581623	- .0402	.41
.42	- .405182697	.960317461	-1.70422535	.913223140	.235867446	- .0403	.42
.43	- .390234003	.941860465	-1.69930070	.911522634	.236151603	- .0405	.43
.44	- .376068153	.924242424	-1.69444444	.909836066	.236434109	- .0407	.44
.45	- .362630476	.907407407	-1.68965517	.908163265	.236714976	- .0408	.45
.46	- .349871126	.891304348	-1.68493151	.906504065	.236994220	- .0410	.46
.47	- .337744570	.875886525	-1.68027211	.904858300	.237271854	- .0412	.47
.48	- .326209135	.861111111	-1.67567568	.903225806	.237547893	- .0413	.48
.49	- .315226611	.846938776	-1.67114094	.901606426	.237822350	- .0415	.49
.50	- .304761905	.833333333	-1.66666667	.900000000	.238095238	- .0417	.50
.51	- .294782728	.820261438	-1.66225166	.898406375	.238366572	- .0418	.51
.52	- .285259331	.807692308	-1.65789474	.896825397	.238636364	- .0420	.52
.53	- .276164257	.795597484	-1.65359477	.895256917	.238904627	- .0422	.53
.54	- .267472130	.783950617	-1.64935065	.893700787	.239171375	- .0423	.54
.55	- .259159465	.772727273	-1.64516129	.892156863	.239436620	- .0425	.55
.56	- .251204495	.761904762	-1.64102564	.890625000	.239700375	- .0427	.56
.57	- .243587023	.751461988	-1.63694268	.889105058	.239962652	- .0428	.57
.58	- .236288281	.741379310	-1.63291139	.887596899	.240223464	- .0430	.58
.59	- .229290809	.731638418	-1.62893082	.886100386	.240482823	- .0432	.59
.60	- .222578348	.722222222	-1.62500000	.884615385	.240740741	- .0433	.60
.61	- .216135734	.713114754	-1.62111801	.883141762	.240997230	- .0435	.61
.62	- .209948816	.704301075	-1.61728395	.881679389	.241252302	- .0437	.62
.63	- .204004370	.695767196	-1.61349693	.880228137	.241505969	- .0438	.63
.64	- .198290023	.687500000	-1.60975610	.878787879	.241758242	- .0440	.64
.65	- .192794196	.679487179	-1.60606061	.877358491	.242009132	- .0442	.65
.66	- .187506035	.671717172	-1.60240964	.875939850	.242258652	- .0443	.66
.67	- .182415356	.664179104	-1.59880240	.874531835	.242506812	- .0445	.67
.68	- .177512601	.656862745	-1.59523810	.873134328	.242753623	- .0447	.68
.69	- .172788786	.649758454	-1.59171598	.871747212	.242999097	- .0448	.69
	$-{}^{1}_{4}C^{1}_{4}$	$-{}^{1}_{4}C^{1}_{3}$	$-{}^{1}_{4}C^{1}_{2}$	$-{}^{1}_{4}C^{1}_{1}$	$-{}^{1}_{4}C^{1}_{0}$	$\frac{{}^{1}_{4}E^{1}}{b^4 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b}\sum_{j=0}^{4} {}^{i}_{4}C^{i}_{j}\, y_j + {}^{i}_{4}R^{i} \qquad (i = 3, 1)$$

$r = \frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^{i}_{4}C^{3}_{0}$	${}^{i}_{4}C^{3}_{1}$	${}^{i}_{4}C^{3}_{2}$	${}^{i}_{4}C^{3}_{3}$	${}^{i}_{4}C^{3}_{4}$	$\frac{{}^{i}_{4}E^{3}}{b^4 f^{(5)}(\xi)}$	
0.70	-0.168235462	0.642857143	-1.58823529	0.870370370	0.243243243	-0.0450	0.70
.71	- .163844677	.636150235	-1.58479532	.869003690	.243486074	- .0452	.71
.72	- .159608938	.629629630	-1.58139535	.867647059	.243727599	- .0453	.72
.73	- .155521184	.623287671	-1.57803468	.866300366	.243967828	- .0455	.73
.74	- .151574751	.617117117	-1.57471264	.864963504	.244206774	- .0457	.74
.75	- .147763348	.611111111	-1.57142857	.863636364	.244444444	- .0458	.75
.76	- .144081031	.605263158	-1.56818182	.862318841	.244680851	- .0460	.76
.77	- .140522182	.599567100	-1.56497175	.861010830	.244916004	- .0462	.77
.78	- .137081483	.594017094	-1.56179775	.859712230	.245149912	- .0463	.78
.79	- .133753902	.588607595	-1.55865922	.858422939	.245382586	- .0465	.79
.80	- .130534670	.583333333	-1.55555556	.857142857	.245614035	- .0467	.80
.81	- .127419268	.578189300	-1.55248619	.855871886	.245844269	- .0468	.81
.82	- .124403409	.573170732	-1.54945055	.854609929	.246073298	- .0470	.82
.83	- .121483027	.568273092	-1.54644809	.853356890	.246301131	- .0472	.83
.84	- .118654256	.563492063	-1.54347826	.852112676	.246527778	- .0473	.84
.85	- .115913429	.558823529	-1.54054054	.850877193	.246753247	- .0475	.85
.86	- .113257054	.554263566	-1.53763441	.849650350	.246977547	- .0477	.86
.87	- .110681816	.549808429	-1.53475936	.848432056	.247200689	- .0478	.87
.88	- .108184554	.545454545	-1.53191489	.847222222	.247422680	- .0480	.88
.89	- .105762264	.541198502	-1.52910053	.846020761	.247643530	- .0482	.89
.90	- .103412082	.537037037	-1.52631579	.844827586	.247863248	- .0483	.90
.91	- .101131277	.532967033	-1.52356021	.843642612	.248081841	- .0485	.91
.92	- .0989172470	.528985507	-1.52083333	.842465753	.248299320	- .0487	.92
.93	- .0967675103	.525089606	-1.51813472	.841296928	.248515691	- .0488	.93
.94	- .0946796970	.521276596	-1.51546392	.840136054	.248730964	- .0490	.94
.95	- .0926515457	.517543860	-1.51282051	.838983051	.248945148	- .0492	.95
.96	- .0906808940	.513888889	-1.51020408	.837837838	.249158249	- .0493	.96
.97	- .0887656790	.510309278	-1.50761421	.836700337	.249370277	- .0495	.97
.98	- .0869039253	.506802721	-1.50505051	.835570470	.249581240	- .0497	.98
.99	- .0850937457	.503367003	-1.50251256	.834448161	.249791145	- .0498	.99
1.00	- .0833333333	.500000000	-1.50000000	.833333333	.250000000	- .0500	1.00
1.01	- .0816209597	.496699670	-1.49751244	.832225914	.250207814	- .0502	1.01
1.02	- .0799549690	.493464052	-1.49504950	.831125828	.250414594	- .0503	1.02
1.03	- .0783337753	.490291262	-1.49261084	.830033003	.250620347	- .0505	1.03
1.04	- .0767558597	.487179487	-1.49019608	.828947368	.250825083	- .0507	1.04
1.05	- .0752197650	.484126984	-1.48780488	.827868852	.251028807	- .0508	1.05
1.06	- .0737240950	.481132075	-1.48543689	.826797386	.251231527	- .0510	1.06
1.07	- .0722675093	.478193146	-1.48309179	.825732899	.251433251	- .0512	1.07
1.08	- .0708487227	.475308642	-1.48076923	.824675325	.251633987	- .0513	1.08
1.09	- .0694665010	.472477064	-1.47846890	.823624595	.251833741	- .0515	1.09
1.10	- .0681196590	.469696970	-1.47619048	.822580645	.252032520	- .0517	1.10
1.11	- .0668070587	.466966967	-1.47393365	.821543408	.252230333	- .0518	1.11
1.12	- .0655276060	.464285714	-1.47169811	.820512821	.252427184	- .0520	1.12
1.13	- .0642802503	.461651917	-1.46948357	.819488818	.252623083	- .0522	1.13
1.14	- .0630639807	.459064327	-1.46728972	.818471338	.252818035	- .0523	1.14
1.15	- .0618778257	.456521739	-1.46511628	.817460317	.253012048	- .0525	1.15
1.16	- .0607208500	.454022989	-1.46296296	.816455696	.253205128	- .0527	1.16
1.17	- .0595921540	.451566952	-1.46082949	.815457413	.253397282	- .0528	1.17
1.18	- .0584908717	.449152542	-1.45871560	.814465409	.253588517	- .0530	1.18
1.19	- .0574161693	.446778711	-1.45662100	.813479624	.253778838	- .0532	1.19
1.20	- .0563672440	.444444444	-1.45454545	.812500000	.253968254	- .0533	1.20
1.21	- .0553433220	.442148760	-1.45248869	.811526480	.254156770	- .0535	1.21
1.22	- .0543436580	.439890710	-1.45045045	.810559006	.254344392	- .0537	1.22
1.23	- .0533675337	.437669377	-1.44843049	.809597523	.254531127	- .0538	1.23
1.24	- .0524142559	.435483871	-1.44642857	.808641975	.254716981	- .0540	1.24
1.25	- .0514831573	.433333333	-1.44444444	.807692308	.254901961	- .0542	1.25
1.26	- .0505735933	.431216931	-1.44247788	.806748466	.255086072	- .0543	1.26
1.27	- .0496849423	.429133858	-1.44052863	.805810398	.255269321	- .0545	1.27
1.28	- .0488166042	.427083333	-1.43859649	.804878049	.255451713	- .0547	1.28
1.29	- .0479680003	.425064599	-1.43668122	.803951368	.255633256	- .0548	1.29
	$-{}^{i}_{4}C^{i}_{4}$	$-{}^{i}_{4}C^{i}_{3}$	$-{}^{i}_{4}C^{i}_{2}$	$-{}^{i}_{4}C^{i}_{1}$	$-{}^{i}_{4}C^{i}_{0}$	$\frac{{}^{i}_{4}E^{i}}{b^4 f^{(5)}(\xi)}$	$p = \frac{d}{c}\frac{d}{b}\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^l y)_{x=x_l} = \frac{1}{b}\sum_{j=0}^{4} {}^{l}_{4}C^{l}_{j}\, y_j + {}^{l}_{4}R^{l} \qquad (l = 4,0)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{l}_{4}C^{4}_{c}$	${}^{l}_{4}C^{4}_{1}$	${}^{l}_{4}C^{4}_{2}$	${}^{l}_{4}C^{4}_{3}$	${}^{l}_{4}C^{4}_{4}$	$\frac{{}^{l}_{4}E^{4}}{b^4 f^{(5)}(\xi)}$	
0.10	8.37871806	-10.3333333	4.22727273	-4.42857143	2.15591398	0.1550	0.10
.11	7.48846393	- 9.42424242	4.20270270	-4.42180095	2.15487674	.1555	.11
.12	6.74934342	- 8.66666667	4.17857143	-4.41509434	2.15384615	.1560	.12
.13	6.12640232	- 8.02564103	4.15486726	-4.40845070	2.15282215	.1565	.13
.14	5.59467602	- 7.47619048	4.13157895	-4.40186916	2.15180467	.1570	.14
.15	5.13585953	- 7.00000000	4.10869565	-4.39534884	2.15079365	.1575	.15
.16	4.73622630	- 6.58333333	4.08620690	-4.38888889	2.14978903	.1580	.16
.17	4.38528144	- 6.21568627	4.06410256	-4.38248848	2.14879075	.1585	.17
.18	4.07486405	- 5.88888889	4.04237288	-4.37614679	2.14779874	.1590	.18
.19	3.79853288	- 5.59649123	4.02100840	-4.36986301	2.14681296	.1595	.19
.20	3.55113636	- 5.33333333	4.00000000	-4.36363636	2.14583333	.1600	.20
.21	3.32850550	- 5.09523810	3.97933884	-4.35746606	2.14485981	.1605	.21
.22	3.12723050	- 4.87878788	3.95901639	-4.35135135	2.14389234	.1610	.22
.23	2.94449565	- 4.68115942	3.93902439	-4.34529148	2.14293086	.1615	.23
.24	2.77795557	- 4.50000000	3.91935484	-4.33928571	2.14197531	.1620	.24
.25	2.62564103	- 4.33333333	3.90000000	-4.33333333	2.14102564	.1625	.25
.26	2.48588663	- 4.17948718	3.88095238	-4.32743363	2.14008180	.1630	.26
.27	2.35727448	- 4.03703704	3.86220472	-4.32158590	2.13914373	.1635	.27
.28	2.23859000	- 3.90476190	3.84375000	-4.31578947	2.13821138	.1640	.28
.29	2.12878677	- 3.78160920	3.82558140	-4.31004367	2.13728470	.1645	.29
.30	2.02695855	- 3.66666667	3.80769231	-4.30434783	2.13636364	.1650	.30
.31	1.93231661	- 3.55913978	3.79007634	-4.29870130	2.13544814	.1655	.31
.32	1.84417136	- 3.45833333	3.77272727	-4.29310345	2.13453815	.1660	.32
.33	1.76191728	- 3.36363636	3.75563910	-4.28755365	2.13363363	.1665	.33
.34	1.68502058	- 3.27450980	3.73880597	-4.28205128	2.13273453	.1670	.34
.35	1.61300892	- 3.19047619	3.72222222	-4.27659574	2.13184080	.1675	.35
.36	1.54546282	- 3.11111111	3.70588235	-4.27118644	2.13095238	.1680	.36
.37	1.48200856	- 2.03603604	3.68978102	-4.26582278	2.13006924	.1685	.37
.38	1.42231212	- 2.96491228	3.67391304	-4.26050420	2.12919132	.1690	.38
.39	1.36607406	- 2.89743590	3.65827338	-4.25523013	2.12831858	.1695	.39
.40	1.31302521	- 2.83333333	3.64285714	-4.25000000	2.12745098	.1700	.40
.41	1.26292296	- 2.77235772	3.62765957	-4.24481328	2.12658847	.1705	.41
.42	1.21554809	- 2.71428571	3.61267606	-4.23966942	2.12573099	.1710	.42
.43	1.17070201	- 2.65891473	3.59790210	-4.23456790	2.12487852	.1715	.43
.44	1.12820446	- 2.60606061	3.58333333	-4.22950820	2.12403101	.1720	.44
.45	1.08789143	- 2.55555556	3.56896552	-4.22448980	2.12318841	.1725	.45
.46	1.04961338	- 2.50724638	3.55479452	-4.21951220	2.12235067	.1730	.46
.47	1.01323371	- 2.46099291	3.54081633	-4.21457490	2.12151777	.1735	.47
.48	.978627404	- 2.41666667	3.52702703	-4.20967742	2.12068966	.1740	.48
.49	.945679834	- 2.37414966	3.51342282	-4.20481928	2.11986628	.1745	.49
.50	.914285714	- 2.33333333	3.50000000	-4.20000000	2.11904762	.1750	.50
.51	.884348185	- 2.29411765	3.48675497	-4.19521912	2.11823362	.1755	.51
.52	.855777994	- 2.25641026	3.47368421	-4.19047619	2.11742424	.1760	.52
.53	.828492771	- 2.22012579	3.46078431	-4.18577075	2.11661945	.1765	.53
.54	.802416390	- 2.18518519	3.44805195	-4.18110236	2.11581921	.1770	.54
.55	.777478394	- 2.15151515	3.43548387	-4.17647059	2.11502347	.1775	.55
.56	.753613486	- 2.11904762	3.42307692	-4.17187500	2.11423221	.1780	.56
.57	.730761070	- 2.08771930	3.41082803	-4.16731518	2.11344538	.1785	.57
.58	.708864842	- 2.05747126	3.39873418	-4.16279070	2.11266294	.1790	.58
.59	.687872428	- 2.02824859	3.38679245	-4.15830116	2.11188487	.1795	.59
.60	.667735043	- 2.00000000	3.37500000	-4.15384615	2.11111111	.1800	.60
.61	.648407202	- 1.97267760	3.36335404	-4.14942529	2.11034164	.1805	.61
.62	.629846448	- 1.94623656	3.35185185	-4.14503817	2.10957643	.1810	.62
.63	.612013107	- 1.92063492	3.34049080	-4.14068441	2.10881543	.1815	.63
.64	.594870069	- 1.89583333	3.32926829	-4.13636364	2.10805861	.1820	.64
.65	.578382589	- 1.87179487	3.31818182	-4.13207547	2.10730594	.1825	.65
.66	.562518105	- 1.84846485	3.30722892	-4.12781955	2.10655738	.1830	.66
.67	.547246069	- 1.82587065	3.29640719	-4.12359551	2.10581290	.1835	.67
.68	.532537804	- 1.80392157	3.28571429	-4.11940299	2.10507246	.1840	.68
.69	.518366359	- 1.78260870	3.27514793	-4.11524164	2.10433604	.1845	.69
.	$-{}^{l}_{4}C^{0}_{4}$	$-{}^{l}_{4}C^{0}_{3}$	$-{}^{l}_{4}C^{0}_{2}$	$-{}^{l}_{4}C^{0}_{1}$	$-{}^{l}_{4}C^{0}_{0}$	$\frac{{}^{l}_{4}E^{0}}{b^4 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

18(a)

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^i y)_{x=x_i} = \frac{1}{b} \sum_{j=0}^{4} {}^{i}_{4}C^{i}_{j}\ y_j + {}^{i}_{4}R^{i} \quad (i = 4,0)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{i}_{4}C^{4}_{0}$	${}^{i}_{4}C^{4}_{1}$	${}^{i}_{4}C^{4}_{2}$	${}^{i}_{4}C^{4}_{3}$	${}^{i}_{4}C^{4}_{4}$	$\frac{{}^{i}_{4}E^{4}}{b^4 f^{(5)}(\xi)}$	
0.70	0.504706387	-1.76190476	3.26470588	-4.11111111	2.10360360	.1850	.70
.71	.491534030	-1.74178404	3.25438596	-4.10701107	2.10287511	.1855	.71
.72	.478826815	-1.72222222	3.24418605	-4.10294118	2.10215054	.1860	.72
.73	.466563552	-1.70319635	3.23410405	-4.09890110	2.10142985	.1865	.73
.74	.454724252	-1.68468468	3.22413793	-4.09489051	2.10071301	.1870	.74
.75	.443290043	-1.66666667	3.21428571	-4.09090909	2.10000000	.1875	.75
.76	.432243094	-1.64912281	3.20454545	-4.08695652	2.09929078	.1880	.76
.77	.421566546	-1.63203463	3.19491525	-4.08303249	2.09858532	.1885	.77
.78	.411244450	-1.61538462	3.18539326	-4.07913669	2.09788360	.1890	.78
.79	.401261706	-1.59915612	3.17597765	-4.07526882	2.09718558	.1895	.79
.80	.391604010	-1.58333333	3.16666667	-4.07142857	2.09649123	.1900	.80
.81	.382257805	-1.56790123	3.15745856	-4.06761566	2.09580052	.1905	.81
.82	.373210228	-1.55284553	3.14835165	-4.06382979	2.09511344	.1910	.82
.83	.364449081	-1.53815261	3.13934426	-4.06007067	2.09442994	.1915	.83
.84	.355962768	-1.52380952	3.13043478	-4.05633803	2.09375000	.1920	.84
.85	.347740287	-1.50980392	3.12162162	-4.05263158	2.09307359	.1925	.85
.86	.339771162	-1.49612403	3.11290322	-4.04895105	2.09240069	.1930	.86
.87	.332045447	-1.48275862	3.10427807	-4.04529617	2.09173127	.1935	.87
.88	.324553663	-1.46969697	3.09574468	-4.04166667	2.09106529	.1940	.88
.89	.317286793	-1.45692884	3.08730159	-4.03806228	2.09040274	.1945	.89
.90	.310236245	-1.44444444	3.07894737	-4.03448276	2.08974359	.1950	.90
.91	.303393830	-1.43223443	3.07068063	-4.03092784	2.08908781	.1955	.91
.92	.296751741	-1.42028986	3.06250000	-4.02739726	2.08843537	.1960	.92
.93	.290302531	-1.40860215	3.05440414	-4.02389078	2.08778626	.1965	.93
.94	.284039091	-1.39716312	3.04639175	-4.02040816	2.08714044	.1970	.94
.95	.277954637	-1.38596491	3.03846154	-4.01694915	2.08649789	.1975	.95
.96	.272042682	-1.37500000	3.03061224	-4.01351351	2.08585859	.1980	.96
.97	.266297037	-1.36426117	3.02284264	-4.01010101	2.08522250	.1985	.97
.98	.260711776	-1.35374150	3.01515152	-4.00671141	2.08458961	.1990	.98
.99	.255281237	-1.34343434	3.00753769	-4.00334448	2.08395990	.1995	.99
1.00	.250000000	-1.33333333	3.00000000	-4.00000000	2.08333333	.2000	1.00
1.01	.244862879	-1.32343234	2.99253731	-3.99667774	2.08270989	.2005	1.01
1.02	.239864907	-1.31372549	2.98514851	-3.99337748	2.08208955	.2010	1.02
1.03	.235001326	-1.30420712	2.97783251	-3.99009901	2.08147229	.2015	1.03
1.04	.230267579	-1.29487179	2.97058824	-3.98684211	2.08085809	.2020	1.04
1.05	.225659295	-1.28571429	2.96341463	-3.98360656	2.08024691	.2025	1.05
1.06	.221172285	-1.27672956	2.95631068	-3.98039216	2.07963875	.2030	1.06
1.07	.216802528	-1.26791277	2.94927536	-3.97719870	2.07903358	.2035	1.07
1.08	.212546168	-1.25925926	2.94230769	-3.97402597	2.07843137	.2040	1.08
1.09	.208399503	-1.25076453	2.93540670	-3.97087379	2.07783211	.2045	1.09
1.10	.204358977	-1.24242424	2.92857143	-3.96774194	2.07723577	.2050	1.10
1.11	.200421176	-1.23423423	2.92180095	-3.96463023	2.07664234	.2055	1.11
1.12	.196582818	-1.22619048	2.91509434	-3.96153846	2.07605178	.2060	1.12
1.13	.192840751	-1.21828909	2.90845070	-3.95846645	2.07546408	.2065	1.13
1.14	.189191942	-1.21052632	2.90186916	-3.95541401	2.07487923	.2070	1.14
1.15	.185633477	-1.20289855	2.89534884	-3.95238095	2.07429719	.2075	1.15
1.16	.182162550	-1.19540230	2.88888889	-3.94936709	2.07371795	.2080	1.16
1.17	.178776462	-1.18803419	2.88248848	-3.94637224	2.07314149	.2085	1.17
1.18	.175472615	-1.18079096	2.87614679	-3.94339623	2.07256778	.2090	1.18
1.19	.172248508	-1.17366947	2.86986301	-3.94043887	2.07199682	.2095	1.19
1.20	.169101732	-1.16666667	2.86363636	-3.93750000	2.07142857	.2100	1.20
1.21	.166029966	-1.15977961	2.85746606	-3.93457944	2.07086302	.2105	1.21
1.22	.163030974	-1.15300546	2.85135135	-3.93167702	2.07030016	.2110	1.22
1.23	.160102601	-1.14634146	2.84529148	-3.92879257	2.06973995	.2115	1.23
1.24	.157242768	-1.13978495	2.83928571	-3.92592593	2.06918239	.2120	1.24
1.25	.154449472	-1.13333333	2.83333333	-3.92307692	2.06862745	.2125	1.25
1.26	.151720780	-1.12698413	2.82743363	-3.92024540	2.06807512	.2130	1.26
1.27	.149054827	-1.12073491	2.82158590	-3.91743119	2.06752537	.2135	1.27
1.28	.146449813	-1.11458333	2.81578947	-3.91463415	2.06697819	.2140	1.28
1.29	.143904001	-1.10852713	2.81004367	-3.91185410	2.06643357	.2145	1.29
	$-{}^{i}_{4}C^{0}_{4}$	$-{}^{i}_{4}C^{0}_{3}$	$-{}^{i}_{4}C^{0}_{2}$	$-{}^{i}_{4}C^{0}_{1}$	$-{}^{i}_{4}C^{0}_{0}$	$\frac{{}^{i}_{4}E^{0}}{b^4 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_l} = \frac{1}{b^2}\sum_{j=0}^{4} {}^{2}_{4}C^{l}_{j}\, y_j + {}^{2}_{4}R^{l} \qquad (l = 0,4)$$

$r = \frac{a}{b} = \frac{a}{c} = \frac{a}{d}$	${}^{2}_{4}C^{0}_{0}$	${}^{2}_{4}C^{0}_{1}$	${}^{2}_{4}C^{0}_{2}$	${}^{2}_{4}C^{0}_{3}$	${}^{2}_{4}C^{0}_{4}$	$\frac{{}^{2}_{4}E^{0}}{b^3 f^{(5)}(\xi)}$	
0.10	35.9167714	-40.7666667	6.39090909	-1.82380952	0.282795699	-0.1397	0.10
.11	32.5813081	-37.4433333	6.42909910	-1.85606635	.288992497	- .1441	.11
.12	29.8015009	-34.6755556	6.46714286	-1.88830189	.295213675	- .1484	.12
.13	27.4491414	-32.3351282	6.50504425	-1.92051643	.301458999	- .1529	.13
.14	25.4326512	-30.3304762	6.54280702	-1.95271028	.307728238	- .1574	.14
.15	23.6848722	-28.5944444	6.58043478	-1.98488372	.314021164	- .1620	.15
.16	22.1554351	-27.0766667	6.61793103	-2.01703704	.320337553	- .1666	.16
.17	20.8058216	-25.7386275	6.65529915	-2.04917051	.326677182	- .1713	.17
.18	19.6060726	-24.5503704	6.69254237	-2.08128440	.333039832	- .1761	.18
.19	18.5325355	-23.4882456	6.72966387	-2.11337900	.339425287	- .1810	.19
.20	17.5662879	-22.5333333	6.76666667	-2.14545455	.345833333	- .1859	.20
.21	16.6920113	-21.6703175	6.80355372	-2.17751131	.352263759	- .1908	.21
.22	15.8971720	-20.8866667	6.84032787	-2.20954955	.358716356	- .1959	.22
.23	15.1714157	-20.1720290	6.87699187	-2.24156951	.365190918	- .2010	.23
.24	14.5061136	-19.5177778	6.91354839	-2.27357143	.371687243	- .2062	.24
.25	13.8940171	-18.9166667	6.95000000	-2.30555556	.378205128	- .2115	.25
.26	13.3289926	-18.3625641	6.98634921	-2.33752212	.384744376	- .2168	.26
.27	12.8058151	-17.8502469	7.02259843	-2.36947137	.391304791	- .2222	.27
.28	12.3200054	-17.3752381	7.05875000	-2.40140351	.397886179	- .2277	.28
.29	11.8677024	-16.9336782	7.09480620	-2.43331878	.404488349	- .2332	.29
.30	11.4455593	-16.5222222	7.13076923	-2.46521739	.411111111	- .2388	.30
.31	11.0506611	-16.1379570	7.16664122	-2.49709957	.417754280	- .2445	.31
.32	10.6804569	-15.7783333	7.20242424	-2.52896552	.424417671	- .2502	.32
.33	10.3327052	-15.4411111	7.23812030	-2.56081545	.431101101	- .2561	.33
.34	10.0054276	-15.1243137	7.27373134	-2.59264951	.437804391	- .2620	.34
.35	9.69687194	-14.8261905	7.30925926	-2.62446809	.444527363	- .2679	.35
.36	9.40548065	-14.5451852	7.34470588	-2.65627119	.451269841	- .2740	.36
.37	9.12986434	-14.2799099	7.38007299	-2.68805907	.458031652	- .2801	.37
.38	8.86877980	-14.0291228	7.41536232	-2.71983193	.464812623	- .2863	.38
.39	8.62111123	-13.7917094	7.45057554	-2.75158996	.471612586	- .2926	.39
.40	8.38585434	-15.5666667	7.48571429	-2.78333333	.478431373	- .2989	.40
.41	8.16210271	-13.3530894	7.52078014	-2.81506224	.485268817	- .3054	.41
.42	7.94903619	-13.1501587	7.55577465	-2.84677686	.492124756	- .3119	.42
.43	7.74591082	-12.9571318	7.59069930	-2.87847737	.498999028	- .3184	.43
.44	7.55205024	-12.7733333	7.62555556	-2.91016393	.505891473	- .3251	.44
.45	7.36683812	-12.5981481	7.66034483	-2.94183673	.512801932	- .3318	.45
.46	7.18971168	-12.4310145	7.69506849	-2.97349593	.519730250	- .3386	.46
.47	7.02015598	-12.2714184	7.72972789	-3.00514170	.526676273	- .3455	.47
.48	6.85769891	-12.1188889	7.76432432	-3.03677419	.533639847	- .3525	.48
.49	6.70190689	-11.9729932	7.79885906	-3.06839357	.540620821	- .3595	.49
.50	6.55238095	-11.8333333	7.83333333	-3.10000000	.547619048	- .3667	.50
.51	6.40875339	-11.6995425	7.86774834	-3.13159363	.554634378	- .3739	.51
.52	6.27068472	-11.5712821	7.90210526	-3.16317460	.561666667	- .3812	.52
.53	6.13786108	-11.4482390	7.93640523	-3.19474308	.568715770	- .3885	.53
.54	6.00999177	-11.3301235	7.97064935	-3.22629921	.575781544	- .3960	.54
.55	5.88680724	-11.2166667	8.00483871	-3.25784314	.582863850	- .4035	.55
.56	5.76805714	-11.1076190	8.03897436	-3.28937500	.589962547	- .4111	.56
.57	5.65350866	-11.0027485	8.07305732	-3.32089494	.597077498	- .4188	.57
.58	5.54294501	-10.9018391	8.10708861	-3.35240310	.604208566	- .4266	.58
.59	5.43616408	-10.8046893	8.14106918	-3.38389961	.611355617	- .4345	.59
.60	5.33297721	-10.7111111	8.17500000	-3.41538462	.618518519	- .4424	.60
.61	5.23320807	-10.6209290	8.20888199	-3.44685824	.625697138	- .4504	.61
.62	5.13669171	-10.5339785	8.24271605	-3.47832061	.632891344	- .4585	.62
.63	5.04327361	-10.4501058	8.27650307	-3.50977186	.640101010	- .4667	.63
.64	4.95280888	-10.3691666	8.31024390	-3.54121212	.647326007	- .4750	.64
.65	4.86516155	-10.2910256	8.34393939	-3.57264151	.654566210	- .4834	.65
.66	4.78020385	-10.2155556	8.37759036	-3.60406015	.661821494	- .4918	.66
.67	4.69781564	-10.1426368	8.41119760	-3.63546816	.669091735	- .5004	.67
.68	4.61788382	-10.0721569	8.44476190	-3.66686567	.676376812	- .5090	.68
.69	4.54030182	-10.0040096	8.47828402	-3.69825279	.683676603	- .5177	.69
	${}^{2}_{4}C^{4}_{4}$	${}^{2}_{4}C^{4}_{3}$	${}^{2}_{4}C^{4}_{2}$	${}^{2}_{4}C^{4}_{1}$	${}^{2}_{4}C^{4}_{0}$	$\frac{{}^{2}_{4}E^{4}}{b^3 f^{(5)}(\xi)}$	$p = \frac{d}{c} = \frac{d}{b} = \frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{j=0}^{4} {}^{2}_{4}C^{i}_{j}\, y_j + {}^{2}_{4}R^{i} \qquad (i = 0,4)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{2}_{4}C^{0}_{0}$	${}^{2}_{4}C^{0}_{1}$	${}^{2}_{4}C^{0}_{2}$	${}^{2}_{4}C^{0}_{3}$	${}^{2}_{4}C^{0}_{4}$	$\frac{{}^{2}_{4}E^{0}}{b^3 f^{(5)}(\xi)}$	
0.70	4.46496917	-9.93809525	8.51176471	-3.72962963	0.690990991	-0.5265	0.70
.71	4.39179102	-9.87431925	8.54520468	-3.76099631	.698319856	- .5354	.71
.72	4.32067780	-9.81259260	8.57860465	-3.79235294	.705663082	- .5444	.72
.73	4.25154481	-9.75283105	8.61196532	-3.82369963	.713020554	- .5535	.73
.74	4.18431194	-9.69495495	8.64528736	-3.85503650	.720392157	- .5626	.74
.75	4.11890332	-9.63888890	8.67857143	-3.88636364	.727777778	- .5719	.75
.76	4.05524708	-9.58456140	8.71181818	-3.91768116	.735177305	- .5812	.76
.77	3.99327506	-9.53190475	8.74502825	-3.94898917	.742590628	- .5906	.77
.78	3.93292259	-9.48085470	8.77820225	-3.98028777	.750017637	- .6002	.78
.79	3.87412827	-9.43135020	8.81134078	-4.01157706	.757458223	- .6098	.79
.80	3.81683375	-9.38333333	8.84444444	-4.04285714	.764912281	- .6195	.80
.81	3.76098357	-9.33674897	8.87751381	-4.07412811	.772379703	- .6293	.81
.82	3.70652494	-9.29154471	8.91054945	-4.10539007	.779860384	- .6391	.82
.83	3.65340766	-9.24767068	8.94355191	-4.13664311	.787354221	- .6491	.83
.84	3.60158383	-9.20507936	8.97652174	-4.16788732	.794861111	- .6592	.84
.85	3.55100790	-9.16372549	9.00945946	-4.19912281	.802380952	- .6694	.85
.86	3.50163630	-9.12356589	9.04236559	-4.23034965	.809913644	- .6796	.86
.87	3.45342760	-9.08455939	9.07524064	-4.26156794	.817459087	- .6900	.87
.88	3.40634215	-9.04666667	9.10808511	-4.29277778	.825017182	- .7004	.88
.89	3.36034212	-9.00985019	9.14089947	-4.32397924	.832587832	- .7110	.89
.90	3.31539134	-8.97407407	9.17368421	-4.35517241	.840170940	- .7216	.90
.91	3.27145522	-8.93930403	9.20643979	-4.38635739	.847766411	- .7323	.91
.92	3.22850067	-8.90550725	9.23916667	-4.41753425	.855374150	- .7432	.92
.93	3.18649606	-8.87265233	9.27186528	-4.44870307	.862994063	- .7541	.93
.94	3.14541102	-8.84070922	9.30453608	-4.47986395	.870626058	- .7651	.94
.95	3.10521655	-8.80964912	9.33717949	-4.51101695	.878270042	- .7762	.95
.96	3.06588476	-8.77944444	9.36979592	-4.54216216	.885925926	- .7875	.96
.97	3.02738899	-8.75006873	9.40238579	-4.57329966	.893593619	- .7988	.97
.98	2.98970360	-8.72149660	9.43494949	-4.60442953	.901273032	- .8102	.98
.99	2.95280403	-8.69370370	9.46748744	-4.63555184	.908964077	- .8217	.99
1.00	2.91666667	-8.66666667	9.50000000	-4.66666667	.916666667	- .8333	1.00
1.01	2.88126885	-8.64036304	9.53248756	-4.69777409	.924380715	- .8450	1.01
1.02	2.84658878	-8.61477124	9.56495050	-4.72887417	.932106136	- .8569	1.02
1.03	2.81260554	-8.58987055	9.59738916	-4.75996700	.939842845	- .8688	1.03
1.04	2.77929898	-8.56564103	9.62980392	-4.79105263	.947590759	- .8808	1.04
1.05	2.74664972	-8.54206349	9.66219512	-4.82213115	.955349794	- .8929	1.05
1.06	2.71463914	-8.51911950	9.69456311	-4.85320261	.963119869	- .9051	1.06
1.07	2.68324926	-8.49679128	9.72690821	-4.88426710	.970900901	- .9175	1.07
1.08	2.65246282	-8.47506173	9.75923077	-4.91532468	.978692810	- .9299	1.08
1.09	2.62226316	-8.45391437	9.79153110	-4.94637540	.986495518	- .9424	1.09
1.10	2.59263422	-8.43333333	9.82380952	-4.97741935	.994308943	- .9551	1.10
1.11	2.56356054	-8.41330330	9.85606635	-5.00845659	1.00213301	- .9678	1.11
1.12	2.53502718	-8.39380952	9.88830189	-5.03948718	1.00996764	- .9807	1.12
1.13	2.50701976	-8.37483776	9.92051643	-5.07051118	1.01781275	- .9936	1.13
1.14	2.47952437	-8.35637427	9.95271028	-5.10152866	1.02566828	-1.0067	1.14
1.15	2.45252763	-8.33840580	9.98488372	-5.13253968	1.03353414	-1.0199	1.15
1.16	2.42601655	-8.32091954	10.0170370	-5.16354430	1.04141026	-1.0331	1.16
1.17	2.39997865	-8.30390313	10.0491705	-5.19454259	1.04929656	-1.0465	1.17
1.18	2.37440184	-8.28734463	10.0812844	-5.22553459	1.05719298	-1.0599	1.18
1.19	2.34927443	-8.27123249	10.1133790	-5.25652038	1.06509944	-1.0735	1.19
1.20	2.32458514	-8.25555556	10.1454545	-5.28750000	1.07301587	-1.0872	1.20
1.21	2.30032304	-8.24030303	10.1775113	-5.31847352	1.08094220	-1.1010	1.21
1.22	2.27647757	-8.22546449	10.2095495	-5.34944099	1.08887836	-1.1149	1.22
1.23	2.25303851	-8.21102981	10.2415695	-5.38040248	1.09682427	-1.1289	1.23
1.24	2.22999597	-8.19698925	10.2735714	-5.41135802	1.10477987	-1.1431	1.24
1.25	2.20734037	-8.18333333	10.3055556	-5.44230769	1.11274510	-1.1573	1.25
1.26	2.18506244	-8.17005291	10.3375221	-5.47325153	1.12071987	-1.1717	1.26
1.27	2.16315321	-8.15713911	10.3694714	-5.50418960	1.12870414	-1.1861	1.27
1.28	2.14160395	-8.14458333	10.4014035	-5.53512195	1.13669782	-1.2007	1.28
1.29	2.12040626	-8.13237726	10.4333188	-5.56604863	1.14470085	-1.2154	1.29
	${}^{2}_{4}C^{4}_{4}$	${}^{2}_{4}C^{4}_{3}$	${}^{2}_{4}C^{4}_{2}$	${}^{2}_{4}C^{4}_{1}$	${}^{2}_{4}C^{4}_{0}$	$\frac{{}^{2}_{4}E^{4}}{b^3 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i}=\frac{1}{b^2}\sum_{j=0}^{4}{}_4^2C_j^i\,y_j+{}_4^2R^i \qquad (i = 1,3)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}_4^2C_0^1$	${}_4^2C_1^1$	${}_4^2C_2^1$	${}_4^2C_3^1$	${}_4^2C_4^1$	$\frac{{}_4^2E^1}{b^3f^{(5)}(\xi)}$	
0.10	30.7219662	-34.6666667	5.00000000	-1.23809254	0.182795699	-0.0817	0.10
.11	27.4577011	-31.3333333	4.90990991	-1.21327014	.178992497	- .0798	.11
.12	24.7475926	-28.5555556	4.82142857	-1.18867925	.175213675	- .0780	.12
.13	22.4634752	-26.2051282	4.73451327	-1.16431925	.171458999	- .0762	.13
.14	20.5138121	-24.1904762	4.64912281	-1.14018692	.167728238	- .0743	.14
.15	18.8314850	-22.4444444	4.56521739	-1.11627907	.164021164	- .0725	.15
.16	17.3661631	-20.9166667	4.48275862	-1.09259259	.160337553	- .0707	.16
.17	16.0793653	-19.5686275	4.40170940	-1.06912442	.156677182	- .0688	.17
.18	14.9411682	-18.3703704	4.32203390	-1.04587156	.153039832	- .0670	.18
.19	13.9279539	-17.2982456	4.24369748	-1.02283105	.149425287	- .0652	.19
.20	13.0208333	-16.3333333	4.16666667	-1.00000000	.145833333	- .0633	.20
.21	12.2045202	-15.4603175	4.09090909	- .977375565	.142263759	- .0615	.21
.22	11.4665118	-14.6666667	4.01639344	- .954954954	.138716356	- .0597	.22
.23	10.7964841	-13.9420290	3.94308943	- .932735426	.135190918	- .0578	.23
.24	10.1858371	-13.2777778	3.87096774	- .910714285	.131687243	- .0560	.24
.25	9.62735043	-12.6666667	3.80000000	- .888888889	.128205128	- .0542	.25
.26	9.11491763	-12.1025641	3.73015873	- .867256637	.124744376	- .0523	.26
.27	8.64333978	-11.5802469	3.66141732	- .845814978	.121304791	- .0505	.27
.28	8.20816332	-11.0952381	3.59375000	- .824561404	.117886179	- .0487	.28
.29	7.80555148	-10.6436782	3.52713178	- .803493450	.114488349	- .0468	.29
.30	7.43218135	-10.2222222	3.46153846	- .782608696	.111111111	- .0450	.30
.31	7.08516091	- 9.82795699	3.39694656	- .761904762	.107754280	- .0432	.31
.32	6.76196164	- 9.45833333	3.33333333	- .741379310	.104417671	- .0413	.32
.33	6.46036336	- 9.11111111	3.27067669	- .721030043	.101101101	- .0395	.33
.34	6.17840881	- 8.78431373	3.20895522	- .700854701	.0978043912	- .0377	.34
.35	5.91436603	- 8.47619048	3.14814815	- .680851064	.0945283632	- .0358	.35
.36	5.66669700	- 8.18518519	3.08823529	- .661016949	.0912698413	- .0340	.36
.37	5.43403139	- 7.90990991	3.02919708	- .641350211	.0880316518	- .0322	.37
.38	5.21514443	- 7.64912281	2.97101449	- .621848739	.0848126233	- .0303	.38
.39	5.00893821	- 7.40170940	2.91366906	- .602510460	.0816125860	- .0285	.39
.40	4.81442577	- 7.16666667	2.85714286	- .583333333	.0784313725	- .0267	.40
.41	4.63071753	- 6.94308943	2.80141844	- .564315353	.0752688172	- .0248	.41
.42	4.45700965	- 6.73015873	2.74647887	- .545454545	.0721247563	- .0230	.42
.43	4.29257403	- 6.52713178	2.69230769	- .526748971	.0689990282	- .0212	.43
.44	4.13674969	- 6.33333333	2.63888889	- .508196121	.0658914729	- .0193	.44
.45	3.98893524	- 6.14814815	2.58620690	- .489795918	.0628019324	- .0175	.45
.46	3.84858238	- 5.97101449	2.53424658	- .471544715	.0597302505	- .0157	.46
.47	3.71519027	- 5.80141844	2.48299320	- .453441296	.0566762728	- .0138	.47
.48	3.58830048	- 5.63888889	2.43243243	- .435483872	.0536398467	- .0120	.48
.49	3.46749272	- 5.48299320	2.38255034	- .417670683	.0506208214	- .0102	.49
.50	3.35238095	- 5.33333333	2.33333333	- .400000000	.0476190476	- .0083	.50
.51	3.24261001	- 5.18954248	2.28476821	- .382470120	.0446343780	- .0065	.51
.52	3.13785265	- 5.05128205	2.23684211	- .365079365	.0416666667	- .0047	.52
.53	3.03780683	- 4.91823899	2.18954248	- .347826087	.0387157696	- .0028	.53
.54	2.94219343	- 4.79012346	2.14285714	- .330708661	.0357815443	- .0010	.54
.55	2.85075411	- 4.66666667	2.09677419	- .313725490	.0328638498	.0008	.55
.56	2.76324945	- 4.54761905	2.05128205	- .296875000	.0299625468	.0027	.56
.57	2.67945726	- 4.43274854	2.00636943	- .280155642	.0270774977	.0045	.57
.58	2.59917109	- 4.32183908	1.96202532	- .263565891	.0242085661	.0063	.58
.59	2.52219890	- 4.21468927	1.91823899	- .247104247	.0213556175	.0082	.59
.60	2.44836182	- 4.11111111	1.87500000	- .230769231	.0185185185	.0100	.60
.61	2.37749307	- 4.01092896	1.83229814	- .214559388	.0156971376	.0118	.61
.62	2.30943698	- 3.91397849	1.79012346	- .198473282	.0128913444	.0137	.62
.63	2.24404806	- 3.82010582	1.74846626	- .182509506	.0101010101	.0155	.63
.64	2.18119025	- 3.72916667	1.70731707	- .166666667	.00732600733	.0173	.64
.65	2.12073616	- 3.64102564	1.66666667	- .150943396	.00456621005	.0192	.65
.66	2.06256638	- 3.55555556	1.62650602	- .135338346	.00182149362	.0210	.66
.67	2.00656892	- 3.47263682	1.58682635	- .119850187	- .000908265213	.0228	.67
.68	1.95263862	- 3.39215686	1.54761905	- .104477612	- .00362318841	.0247	.68
.69	1.90067665	- 3.31400966	1.50887574	- .089219331	- .00632339657	.0265	.69
	${}_4^2C_4^3$	${}_4^2C_3^3$	${}_4^2C_2^3$	${}_4^2C_1^3$	${}_4^2C_0^3$	$-\frac{{}_4^2E^3}{b^3f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

20(a)

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{0}^{4} {}^{2}_{4}C^{i}_{j}\, y_i + {}^{2}_{4}R^{i} \quad (i = 1,3)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{2}_{4}C^{1}_{0}$	${}^{2}_{4}C^{1}_{1}$	${}^{2}_{4}C^{1}_{2}$	${}^{2}_{4}C^{1}_{3}$	${}^{2}_{4}C^{1}_{4}$	$\frac{{}^{2}_{4}E^{1}}{b^3 f^{(5)}(\xi)}$	
0.70	1.85059009	− 3.23809524	1.47058824	−0.074074074	−0.00900900901	0.0283	0.70
.71	1.80229144	− 3.16431925	1.43274854	− .059040590	− .0116801438	.0302	.71
.72	1.75569832	− 3.09259259	1.39534884	− .044117647	− .0143369188	.0320	.72
.73	1.71073302	− 3.02283105	1.35838150	− .029304029	− .0169794459	.0338	.73
.74	1.66732226	− 2.95495495	1.32183908	− .014598540	− .0196078431	.0357	.74
.75	1.62539683	− 2.88888889	1.28571429	0	− .0222222222	.0375	.75
.76	1.58489134	− 2.82456140	1.25000000	.014492754	− .0248226950	.0393	.76
.77	1.54574400	− 2.76190476	1.21468927	.028880866	− .0274093722	.0412	.77
.78	1.50789832	− 2.70085470	1.17977528	.043165468	− .0299823633	.0430	.78
.79	1.47129292	− 2.64135021	1.14525140	.057347670	− .0325417766	.0448	.79
.80	1.43588137	− 2.58333333	1.11111111	.071428571	− .0350877193	.0467	.80
.81	1.40161195	− 2.52674897	1.07734807	.085409253	− .0376202975	.0485	.81
.82	1.36843750	− 2.47154472	1.04395604	.099290780	− .0401396161	.0503	.82
.83	1.33631330	− 2.41767068	1.01092896	.113074205	− .0426457789	.0522	.83
.84	1.30519682	− 2.36507936	.978260869	.126760563	− .0451388889	.0540	.84
.85	1.27504772	− 2.31372549	.945945946	.140350877	− .0476190476	.0558	.85
.86	1.24582760	− 2.26356589	.913978494	.153846154	− .0500863558	.0577	.86
.87	1.21749997	− 2.21455939	.882352941	.167247387	− .0525409130	.0595	.87
.88	1.19003010	− 2.16666667	.851063829	.180555556	− .0549828179	.0613	.88
.89	1.16338491	− 2.11985019	.820105820	.193771626	− .0574121680	.0632	.89
.90	1.13753290	− 2.07407407	.789473684	.206896552	− .0598290598	.0650	.90
.91	1.11244404	− 2.02930403	.759162303	.219931271	− .0622335891	.0668	.91
.92	1.08808972	− 1.98550725	.729166667	.232876712	− .0646258503	.0687	.92
.93	1.06444261	− 1.94265233	.699481865	.245733788	− .0670059372	.0705	.93
.94	1.04147667	− 1.90070922	.670103092	.258503401	− .0693739425	.0723	.94
.95	1.01916700	− 1.85964912	.641025641	.271186441	− .0717299578	.0742	.95
.96	.997489835	− 1.81944444	.612244897	.283783784	− .0740740741	.0760	.96
.97	.976422468	− 1.78006873	.583756345	.296296296	− .0764063812	.0778	.97
.98	.955943179	− 1.74149660	.555555556	.308724832	− .0787269682	.0791	.98
.99	.936031202	− 1.70370370	.527638191	.321070234	− .0810359231	.0815	.99
1.00	.916666667	− 1.66666667	.500000000	.333333333	− .0833333333	.0833	1.00
1.01	.897830555	− 1.63036304	.472636816	.345514950	− .0856192851	.0852	1.01
1.02	.879504657	− 1.59477124	.445544554	.357615894	− .0878938640	.0870	1.02
1.03	.861671529	− 1.55987055	.418719212	.369636964	− .0901571547	.0888	1.03
1.04	.844314456	− 1.52564103	.392156863	.381578947	− .0924092409	.0907	1.04
1.05	.827417416	− 1.49206349	.365853659	.393442623	− .0946502058	.0925	1.05
1.06	.810965045	− 1.45911950	.339805825	.405228758	− .0968801314	.0943	1.06
1.07	.794942604	− 1.42679128	.314009662	.416938111	− .0990990991	.0962	1.07
1.08	.779335951	− 1.39506173	.288461538	.428571429	− .101307190	.0980	1.08
1.09	.764131511	− 1.36391437	.263157895	.440129450	− .103504482	.0998	1.09
1.10	.749316249	− 1.33333333	.238095238	.451612903	− .105691057	.1017	1.10
1.11	.734877644	− 1.30330330	.213270142	.463022508	− .107866991	.1035	1.11
1.12	.720803667	− 1.27380952	.188679245	.474358974	− .110032362	.1053	1.12
1.13	.707082754	− 1.24483776	.164319249	.485623003	− .112187248	.1072	1.13
1.14	.693703789	− 1.21637427	.140186916	.496815287	− .114331723	.1090	1.14
1.15	.680656084	− 1.18840580	.116279070	.507936508	− .116465863	.1108	1.15
1.16	.667929349	− 1.16091954	.0925925926	.518987342	− .118589744	.1127	1.16
1.17	.655513693	− 1.13390313	.0691244240	.529968454	− .120703437	.1145	1.17
1.18	.643399587	− 1.10734463	.0458715596	.540880503	− .122807018	.1163	1.18
1.19	.631577862	− 1.08123249	.0228310502	.551724138	− .124900557	.1182	1.19
1.20	.620039683	− 1.05555556	0.0000000000	.562500000	− .126984127	.1200	1.20
1.21	.608776541	− 1.03030303	− .0226244344	.573208723	− .129057799	.1218	1.21
1.22	.597780237	− 1.00546448	− .0450450450	.583850932	− .131121643	.1237	1.22
1.23	.587042869	− .981029810	− .0672645740	.594427245	− .133175729	.1255	1.23
1.24	.576556815	− .956989247	− .0892857143	.604938272	− .135220126	.1273	1.24
1.25	.566314732	− .933333333	− .111111111	.615384615	− .137254902	.1292	1.25
1.26	.556309526	− .910052911	− .132743363	.625766871	− .139280125	.1310	1.26
1.27	.546534365	− .887139108	− .154185022	.636085627	− .141295863	.1328	1.27
1.28	.536982647	− .864583333	− .175438596	.646341463	− .143302181	.1347	1.28
1.29	.527648002	− .842377261	− .196506550	.656534954	− .145299145	.1365	1.29
	${}^{2}_{4}C^{3}_{4}$	${}^{2}_{4}C^{3}_{3}$	${}^{2}_{4}C^{3}_{2}$	${}^{2}_{4}C^{3}_{1}$	${}^{2}_{4}C^{3}_{0}$	$\frac{{}^{2}_{4}E^{3}}{b^3 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_2}=\frac{1}{b^2}\sum_{j=0}^{4}{}^{2}_{4}C^2_j\,y_j+{}^{2}_{4}R^2$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{2}_{4}C^2_0$	${}^{2}_{4}C^2_1$	${}^{2}_{4}C^2_2$	${}^{2}_{4}C^2_3$	${}^{2}_{4}C^2_4$	$\frac{{}^{2}_{4}E^2}{b^3f^{(5)}(\xi)}$	
0.10	-2.79290602	4.33333333	-2.90909091	1.47619048	-0.107526882	0.0150	0.10
.11	-2.49615464	4.03030303	-2.90090090	1.47393365	- .107181136	.0148	.11
.12	-2.24978114	3.77777778	-2.89285714	1.47169811	- .106837607	.0147	.12
.13	-2.04213411	3.56410256	-2.88495575	1.46948357	- .106496273	.0145	.13
.14	-1.86489201	3.38095238	-2.87719298	1.46728972	- .106157113	.0143	.14
.15	-1.71195318	3.22222222	-2.86956522	1.46511628	- .105820106	.0142	.15
.16	-1.57874210	3.08333333	-2.86206897	1.46296296	- .105485232	.0140	.16
.17	-1.46176048	2.96078431	-2.85470085	1.46082949	- .105152471	.0138	.17
.18	-1.35828802	2.85185185	-2.84745763	1.45871560	- .104821803	.0137	.18
.19	-1.26617763	2.75438596	-2.84033613	1.45662100	- .104493208	.0135	.19
.20	-1.18371212	2.66666667	-2.83333333	1.45454545	- .104166667	.0133	.20
.21	-1.10950183	2.58730159	-2.82644628	1.45248869	- .103842160	.0132	.21
.22	-1.04241017	2.51515152	-2.81967213	1.45045045	- .103519669	.0130	.22
.23	- .981498550	2.44927536	-2.81300813	1.44843049	- .103199174	.0128	.23
.24	- .925985190	2.38888889	-2.80645161	1.44642857	- .102880658	.0127	.24
.25	- .875213677	2.33333333	-2.80000000	1.44444444	- .102564103	.0125	.25
.26	- .828628877	2.28205128	-2.79365079	1.44247788	- .102249489	.0123	.26
.27	- .785758160	2.23456790	-2.78740157	1.44052863	- .101936799	.0122	.27
.28	- .746196667	2.19047619	-2.78125000	1.43859649	- .101626016	.0120	.28
.29	- .709595590	2.14942529	-2.77519380	1.43668122	- .101317123	.0118	.29
.30	- .675652850	2.11111111	-2.76923077	1.43478261	- .101010101	.0117	.30
.31	- .644105537	2.07526882	-2.76335878	1.43290043	- .100704935	.0115	.31
.32	- .614723787	2.04166667	-2.75757576	1.43103448	- .100401601	.0113	.32
.33	- .587305760	2.01010101	-2.75187970	1.42918455	- .100100100	.0112	.33
.34	- .561673527	1.98039216	-2.74626866	1.42735043	- .0998003992	.0111	.34
.35	- .537669640	1.95238095	-2.74074074	1.42553192	- .0995024876	.0108	.35
.36	- .515154273	1.92592593	-2.73529412	1.42372881	- .0992063492	.0107	.36
.37	- .494002853	1.90090090	-2.72992701	1.42194093	- .0989119683	.0105	.37
.38	- .474104040	1.87719298	-2.72463768	1.42016807	- .0986193294	.0103	.38
.39	- .455358020	1.85470085	-2.71942446	1.41841004	- .0983284169	.0102	.39
.40	- .437675070	1.83333333	-2.71428571	1.41666667	- .0980392157	.0100	.40
.41	- .420974320	1.81300813	-2.70921986	1.41493776	- .0977517107	.0098	.41
.42	- .405182697	1.79365079	-2.70422535	1.41322314	- .0974658869	.0097	.42
.43	- .390236003	1.77519380	-2.69930070	1.41152263	- .0971817298	.0095	.43
.44	- .376068153	1.75757576	-2.69444444	1.40983607	- .0968992248	.0093	.44
.45	- .362630476	1.74074074	-2.68965517	1.40816327	- .0966183575	.0092	.45
.46	- .349871126	1.72463768	-2.68493151	1.40650407	- .0963391137	.0090	.46
.47	- .337744570	1.70921986	-2.68027211	1.40485830	- .0960614793	.0088	.47
.48	- .326209135	1.69444444	-2.67567568	1.40322581	- .0957854406	.0087	.48
.49	- .315226611	1.68027211	-2.67114094	1.40160643	- .0955109838	.0085	.49
.50	- .304761905	1.66666667	-2.66666667	1.40000000	- .0952380952	.0083	.50
.51	- .294782728	1.65359477	-2.66225166	1.39840637	- .0949667616	.0082	.51
.52	- .285259331	1.64102564	-2.65789474	1.39682540	- .0946969897	.0080	.52
.53	- .276164257	1.62893082	-2.65359477	1.39525692	- .0944287063	.0078	.53
.54	- .267472130	1.61728395	-2.64935065	1.39370079	- .0941619586	.0077	.54
.55	- .259159465	1.60606061	-2.64516129	1.39215686	- .0938967136	.0075	.55
.56	- .251204495	1.59523810	-2.64102564	1.39062500	- .0936329588	.0073	.56
.57	- .243587023	1.58479532	-2.63694268	1.38910506	- .0933706816	.0072	.57
.58	- .236288281	1.57471264	-2.63291139	1.38759690	- .0931098696	.0070	.58
.59	- .229290809	1.56497175	-2.62893082	1.38610039	- .0928505107	.0068	.59
.60	- .222578348	1.55555556	-2.62500000	1.38461538	- .0925925926	.0067	.60
.61	- .216135734	1.54644809	-2.62111801	1.38314176	- .0923361034	.0065	.61
.62	- .209948816	1.53763441	-2.61728395	1.38167939	- .0920810313	.0063	.62
.63	- .204004370	1.52910053	-2.61349693	1.38022814	- .0918273645	.0062	.63
.64	- .198290023	1.52083333	-2.60975610	1.37878788	- .0915750915	.0060	.64
.65	- .192794196	1.51282051	-2.60606061	1.37735849	- .0913242009	.0058	.65
.66	- .187506035	1.50505051	-2.60240964	1.37593985	- .0910746812	.0057	.66
.67	- .182415356	1.49751244	-2.59880240	1.37453184	- .0908265213	.0056	.67
.68	- .177512601	1.49019608	-2.59523810	1.37313433	- .0905797101	.0053	.68
.69	- .172788786	1.48309179	-2.59171598	1.37174721	- .0903342366	.0052	.69
	${}^{2}_{4}C^2_4$	${}^{2}_{4}C^2_3$	${}^{2}_{4}C^2_2$	${}^{2}_{4}C^2_1$	${}^{2}_{4}C^2_0$	$\frac{{}^{2}_{4}E^2}{b^3f^{(5)}(\xi)}$	$\rho=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_2} = \frac{1}{b^2}\sum_{j=0}^{4} {}^2_4C^2_j\, y_j + {}^2_4R^2$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^2_4C^2_0$	${}^2_4C^2_1$	${}^2_4C^2_2$	${}^2_4C^2_3$	${}^2_4C^2_4$	$\frac{{}^2_4E^2}{b^3f^{(5)}(\xi)}$	
0.70	−0.168235462	1.47619048	−2.58823529	1.37037037	−0.0900900901	0.0050	0.70
.71	− .163844677	1.46948357	−2.58479532	1.36900369	− .0898472896	.0048	.71
.72	− .159608938	1.46296296	−2.58139535	1.36764706	− .0896057347	.0047	.72
.73	− .155521184	1.45662100	−2.57803468	1.36630037	− .0893655049	.0045	.73
.74	− .151574751	1.45045045	−2.57471264	1.36496350	− .0891265597	.0043	.74
.75	− .147763348	1.44444444	−2.57142857	1.36363636	− .0888888889	.0042	.75
.76	− .144081031	1.43859648	−2.56818182	1.36231884	− .0886524822	.0040	.76
.77	− .140522182	1.43290043	−2.56497175	1.36101083	− .0884173297	.0038	.77
.78	− .137081483	1.42735043	−2.56179775	1.35971223	− .0881834215	.0037	.78
.79	− .133753902	1.42194093	−2.55865922	1.35842294	− .0879507475	.0035	.79
.80	− .130534670	1.41666667	−2.55555556	1.35714286	− .0877192982	.0033	.80
.81	− .127419268	1.41152263	−2.55248619	1.35587189	− .0874890639	.0032	.81
.82	− .124403409	1.40650406	−2.54945055	1.35460993	− .0872600349	.0030	.82
.83	− .121483027	1.40160642	−2.54644809	1.35335689	− .0870322019	.0028	.83
.84	− .118654256	1.39682540	−2.54347826	1.35211268	− .0868055556	.0027	.84
.85	− .115913429	1.39215686	−2.54054054	1.35087719	− .0865800866	.0025	.85
.86	− .113257054	1.38759690	−2.53763441	1.34965035	− .0863557858	.0023	.86
.87	− .110681816	1.38314176	−2.53475936	1.34843206	− .0861326443	.0022	.87
.88	− .108184554	1.37878788	−2.53191489	1.34722222	− .0859106529	.0020	.88
.89	− .105762264	1.37453184	−2.52910053	1.34602076	− .0856898029	.0018	.89
.90	− .103412082	1.37037037	−2.52631579	1.34482759	− .0854700855	.0017	.90
.91	− .101131277	1.36630037	−2.52356021	1.34364261	− .0852514919	.0015	.91
.92	− .0989172470	1.36231884	−2.52083333	1.34246575	− .0850340136	.0013	.92
.93	− .0967675103	1.35842294	−2.51813472	1.34129693	− .0848176421	.0012	.93
.94	− .0946796970	1.35460993	−2.51546392	1.34013605	− .0846023689	.0010	.94
.95	− .0926515457	1.35087719	−2.51282051	1.33898305	− .0843881857	.0008	.95
.96	− .0906808940	1.34722222	−2.51020408	1.33783784	− .0841750842	.0007	.96
.97	− .0887656790	1.34364261	−2.50761421	1.33670034	− .0839630563	.0005	.97
.98	− .0869039253	1.34013605	−2.50505051	1.33557047	− .0837520938	.0003	.98
.99	− .0850937457	1.33670034	−2.50251256	1.33444816	− .0835421888	.0002	.99
1.00	− .0833333333	1.33333333	−2.50000000	1.33333333	− .0833333333	[a]0	1.00
1.01	− .0816209597	1.33003300	−2.49751244	1.33222591	− .0831255195	−0.0002	1.01
1.02	− .0799549690	1.32679739	−2.49504950	1.33112583	− .0829187396	− .0003	1.02
1.03	− .0783337753	1.32362460	−2.49261084	1.33003300	− .0827129859	− .0005	1.03
1.04	− .0767558597	1.32051282	−2.49019608	1.32894737	− .0825082508	− .0007	1.04
1.05	− .0752197650	1.31746032	−2.48780488	1.32786885	− .0823045267	− .0008	1.05
1.06	− .0737240950	1.31446541	−2.48543689	1.32679739	− .0821018062	− .0010	1.06
1.07	− .0722675093	1.31152648	−2.48309179	1.32573290	− .0819000819	− .0012	1.07
1.08	− .0708487227	1.30864198	−2.48076923	1.32467532	− .0816993464	− .0013	1.08
1.09	− .0694665010	1.30581040	−2.47846890	1.32362460	− .0814995925	− .0015	1.09
1.10	− .0681196590	1.30303030	−2.47619048	1.32258065	− .0813008130	− .0017	1.10
1.11	− .0668070587	1.30030030	−2.47393365	1.32154341	− .0811030008	− .0018	1.11
1.12	− .0655276060	1.29761905	−2.47169811	1.32051282	− .0809061489	− .0020	1.12
1.13	− .0642802503	1.29498525	−2.46948557	1.31948882	− .0807102502	− .0021	1.13
1.14	− .0630639807	1.29239766	−2.46728972	1.31847134	− .0805152979	− .0023	1.14
1.15	− .0618778257	1.28985507	−2.46511628	1.31746032	− .0803212851	− .0025	1.15
1.16	− .0607208500	1.28735632	−2.46296296	1.31645570	− .0801282051	− .0027	1.16
1.17	− .0595921540	1.28490028	−2.46082949	1.31545741	− .0799360512	− .0028	1.17
1.18	− .0584908717	1.28248588	−2.45871560	1.31446541	− .0797448166	− .0030	1.18
1.19	− .0574161693	1.28011204	−2.45662100	1.31347962	− .0795549484	− .0032	1.19
1.20	− .0563672440	1.27777778	−2.45454545	1.31250000	− .0793650794	− .0033	1.20
1.21	− .0553433220	1.27548209	−2.45248869	1.31152648	− .0791765637	− .0035	1.21
1.22	− .0543436580	1.27322404	−2.45045045	1.31055901	− .0789889415	− .0037	1.22
1.23	− .0533675337	1.27100271	−2.44843049	1.30959752	− .0788022065	− .0038	1.23
1.24	− .0524142559	1.26881720	−2.44642857	1.30864198	− .0786163522	− .0040	1.24
1.25	− .0514831573	1.26666667	−2.44444444	1.30769231	− .0784313725	− .0042	1.25
1.26	− .0505735933	1.26455026	−2.44247788	1.30674847	− .0782472613	− .0043	1.26
1.27	− .0496849423	1.26246719	−2.44052863	1.30581040	− .0780640125	− .0045	1.27
1.28	− .0488166042	1.26041667	−2.43859649	1.30487805	− .0778816199	− .0047	1.28
1.29	− .0479680003	1.25839793	−2.43668122	1.30395137	− .0777000777	− .0048	1.29
	${}^2_4C^2_4$	${}^2_4C^2_3$	${}^2_4C^2_2$	${}^2_4C^2_1$	${}^2_4C^2_0$	$\frac{{}^2_4E^2}{b^3f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

[a]The next order remainder term is $0.0111\ b^4\left[f^{(6)}(x)\right]_{x=x_2}$

21(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{j=0}^{4} {}^2_4C^i_j\, y_j + {}^2_4R^2 \qquad (i = 3, 1)$$

$r = \frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^2_4C^3_0$	${}^2_4C^3_1$	${}^2_4C^3_2$	${}^2_4C^3_3$	${}^2_4C^3_4$	$\frac{{}^2_4E^3}{b^3f^{(5)}(\xi)}$	
0.10	-2.79290602	3.33333333	0.0909090901	-1.52380952	0.892473118	-0.0683	0.10
.11	-2.49615464	3.03030303	.0990990991	-1.52606635	.892818864	- .0685	.11
.12	-2.24978114	2.77777778	.107142857	-1.52830189	.893162393	- .0687	.12
.13	-2.04213411	2.56410256	.115044247	-1.53051643	.893503727	- .0688	.13
.14	-1.86489201	2.38095238	.122807018	-1.53271028	.893842887	- .0690	.14
.15	-1.71195318	2.22222222	.130434783	-1.53488372	.894179894	- .0692	.15
.16	-1.57874210	2.08333333	.137931034	-1.53703703	.894514768	- .0693	.16
.17	-1.46176048	1.96078431	.145299145	-1.53917051	.894847529	- .0695	.17
.18	-1.35828802	1.85185185	.152542373	-1.54128440	.895178197	- .0697	.18
.19	-1.26617763	1.75438596	.159663866	-1.54337900	.895506792	- .0698	.19
.20	-1.18371212	1.66666667	.166666667	-1.54545455	.895833333	- .0700	.20
.21	-1.10950183	1.58730159	.173553719	-1.54751131	.896157840	- .0702	.21
.22	-1.04241017	1.51515152	.180327869	-1.54954955	.896480331	- .0703	.22
.23	- .981498550	1.44927536	.186991870	-1.55156951	.896800826	- .0705	.23
.24	- .925985190	1.38888889	.193548387	-1.55357143	.897119342	- .0707	.24
.25	- .875213677	1.33333333	.200000000	-1.55555556	.897435897	- .0708	.25
.26	- .828628877	1.28205128	.206349206	-1.55752212	.897750511	- .0710	.26
.27	- .785758160	1.23456790	.212598425	-1.55947137	.898063201	- .0712	.27
.28	- .746196667	1.19047619	.218750000	-1.56140351	.898373984	- .0713	.28
.29	- .709595590	1.14942529	.224806202	-1.56331878	.898682877	- .0715	.29
.30	- .675652850	1.11111111	.230769231	-1.56521739	.898989899	- .0717	.30
.31	- .644105537	1.07526882	.236641221	-1.56709957	.899295065	- .0718	.31
.32	- .614723787	1.04166667	.242424242	-1.56896552	.899598394	- .0720	.32
.33	- .587305760	1.01010101	.248120301	-1.57081545	.899899900	- .0722	.33
.34	- .561673527	.980392157	.253731343	-1.57264957	.900199601	- .0723	.34
.35	- .537669640	.952380952	.259259259	-1.57446809	.900497512	- .0725	.35
.36	- .515154273	.925925926	.264705882	-1.57627199	.900793651	- .0727	.36
.37	- .494002853	.900900901	.270072993	-1.57805907	.901088032	- .0728	.37
.38	- .474104040	.877192982	.275362319	-1.57983193	.901380671	- .0730	.38
.39	- .455358020	.854700855	.280575540	-1.58158996	.901671583	- .0732	.39
.40	- .437675070	.833333333	.285714286	-1.58333333	.901960784	- .0733	.40
.41	- .420974320	.813008130	.290780142	-1.58506224	.902248289	- .0735	.41
.42	- .405182697	.793650794	.295774648	-1.58677686	.902534113	- .0737	.42
.43	- .390236003	.775193798	.300699301	-1.58847737	.902818270	- .0738	.43
.44	- .376068153	.757575758	.305555556	-1.59016393	.903100775	- .0740	.44
.45	- .362630476	.740740741	.310344828	-1.59183673	.903381643	- .0742	.45
.46	- .349871126	.724637681	.315068493	-1.59349593	.903660886	- .0743	.46
.47	- .337744570	.709219858	.319727891	-1.59514170	.903938521	- .0745	.47
.48	- .326209135	.694444444	.324324324	-1.59677419	.904214559	- .0747	.48
.49	- .315226611	.680272109	.328859060	-1.59839357	.904489016	- .0748	.49
.50	- .304761905	.666666667	.333333333	-1.60000000	.904761905	- .0750	.50
.51	- .294782728	.653594771	.337748344	-1.60159363	.905033238	- .0752	.51
.52	- .285259331	.641025641	.342105263	-1.60317460	.905303030	- .0753	.52
.53	- .276164257	.628930818	.346405229	-1.60474308	.905571294	- .0755	.53
.54	- .267472130	.617283951	.350649351	-1.60629921	.905838041	- .0757	.54
.55	- .259159465	.606060606	.354838710	-1.60784314	.906103286	- .0758	.55
.56	- .251204495	.595238095	.358974359	-1.60937500	.906367041	- .0760	.56
.57	- .243587023	.584795322	.363057325	-1.61089494	.906629318	- .0762	.57
.58	- .236288281	.574712644	.367088608	-1.61240310	.906890130	- .0763	.58
.59	- .229290809	.564971751	.371069182	-1.61389961	.907149489	- .0765	.59
.60	- .222578348	.555555556	.375000000	-1.61538462	.907407407	- .0767	.60
.61	- .216135734	.546448087	.378881988	-1.61685824	.907663897	- .0768	.61
.62	- .209948816	.537634409	.382716049	-1.61832061	.907918969	- .0770	.62
.63	- .204004370	.529100529	.386503067	-1.61977186	.908172635	- .0772	.63
.64	- .198290023	.520833333	.390243903	-1.62121212	.908424908	- .0773	.64
.65	- .192794196	.512820513	.393939394	-1.62264151	.908675799	- .0775	.65
.66	- .187506035	.505050505	.397590360	-1.62406015	.908925319	- .0777	.66
.67	- .182415356	.497512438	.401197603	-1.62546817	.909173479	- .0779	.67
.68	- .177512601	.490196078	.404761903	-1.62686567	.909420290	- .0780	.68
.69	- .172788786	.483091787	.408284023	-1.62825279	.909665763	- .0782	.69
	${}^2_4C^1_4$	${}^2_4C^1_3$	${}^2_4C^1_2$	${}^2_4C^1_1$	${}^2_4C^1_0$	$\frac{{}^2_4E^1}{b^3f^{(5)}(\xi)}$	$p = \frac{d}{c}\frac{d}{b}\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_i} = \frac{1}{b^2}\sum_{j=0}^{4} {}^{2}_{4}C^{i}_{j}\, y_j + {}^{2}_{4}R^{2} \qquad (i = 3, 1)$$

$\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^{2}_{4}C^{3}_{0}$	${}^{2}_{4}C^{3}_{1}$	${}^{2}_{4}C^{3}_{2}$	${}^{2}_{4}C^{3}_{3}$	${}^{2}_{4}C^{3}_{4}$	$\frac{{}^{2}_{4}E^{3}}{b^3 f^{(5)}(\xi)}$	
0.70	-0.168235462	0.476190476	0.411764707	-1.62962963	0.909909910	-0.0783	0.70
.71	- .163844677	.469483568	.415204680	-1.63099631	.910152740	- .0785	.71
.72	- .159608938	.462962963	.418604650	-1.63235294	.910394265	- .0787	.72
.73	- .155521184	.456621005	.421965317	-1.63369963	.910634495	- .0788	.73
.74	- .151574751	.450450450	.425287357	-1.63503650	.910873440	- .0790	.74
.75	- .147763348	.444444444	.428571430	-1.63636364	.911111111	- .0792	.75
.76	- .144081031	.438596491	.431818183	-1.63768116	.911347518	- .0793	.76
.77	- .140522182	.432900433	.435028250	-1.63898917	.911582670	- .0795	.77
.78	- .137081483	.427350427	.438202247	-1.64028777	.911816578	- .0797	.78
.79	- .133753902	.421940928	.441340783	-1.64157706	.912049252	- .0798	.79
.80	- .130534670	.416666667	.444444443	-1.64285714	.912280702	- .0800	.80
.81	- .127419268	.411522633	.447513813	-1.64412811	.912510936	- .0802	.81
.82	- .124403409	.406504065	.450549450	-1.64539007	.912739965	- .0803	.82
.83	- .121483027	.401606425	.453551913	-1.64664311	.912967798	- .0805	.83
.84	- .118654256	.396825396	.456521740	-1.64788732	.913194444	- .0807	.84
.85	- .115913429	.392156862	.459459460	-1.64912281	.913419913	- .0808	.85
.86	- .113257054	.387596899	.462365590	-1.65034965	.913644214	- .0810	.86
.87	- .110681816	.383141762	.465240640	-1.65156794	.913867356	- .0812	.87
.88	- .108184554	.378787879	.468085107	-1.65277778	.914089347	- .0813	.88
.89	- .105762264	.374531836	.470899470	-1.65397924	.914310197	- .0815	.89
.90	- .103412082	.370370370	.473684210	-1.65517241	.914529914	- .0817	.90
.91	- .101131277	.366300366	.476439790	-1.65635739	.914748508	- .0818	.91
.92	- .0989172470	.362318840	.479166667	-1.65753425	.914965986	- .0820	.92
.93	- .0967675103	.358422939	.481865283	-1.65870307	.915182358	- .0822	.93
.94	- .0946796970	.354609929	.484536083	-1.65986395	.915397631	- .0823	.94
.95	- .0926515457	.350877192	.487179487	-1.66101695	.915611814	- .0825	.95
.96	- .0906808940	.347222222	.489795920	-1.66216216	.915824916	- .0827	.96
.97	- .0887656790	.343642612	.492385787	-1.66329966	.916036944	- .0828	.97
.98	- .0869039253	.340136054	.494949495	-1.66442953	.916247906	- .0830	.98
.99	- .0850937457	.336700337	.497487437	-1.66555184	.916457811	- .0832	.99
1.00	- .0833333333	.333333333	.500000000	-1.66666667	.916666667	- .0833	1.00
1.01	- .0816209597	.330033003	.502487562	-1.66777409	.916874480	- .0835	1.01
1.02	- .0799549690	.326797386	.504950495	-1.66887417	.917081260	- .0837	1.02
1.03	- .0783337753	.323624595	.507389163	-1.66996700	.917287014	- .0838	1.03
1.04	- .0767558597	.320512821	.509803922	-1.67105263	.917491749	- .0840	1.04
1.05	- .0752197650	.317460317	.512195122	-1.67213115	.917695473	- .0842	1.05
1.06	- .0737240950	.314465409	.514563107	-1.67320261	.917898194	- .0843	1.06
1.07	- .0722675093	.311526480	.516908213	-1.67426710	.918099918	- .0845	1.07
1.08	- .0708487227	.308641975	.519230769	-1.67532468	.918300654	- .0847	1.08
1.09	- .0694665010	.305810398	.521531100	-1.67637540	.918500407	- .0848	1.09
1.10	- .0681196590	.303030303	.523809524	-1.67741936	.918699187	- .0850	1.10
1.11	- .0668070587	.300300300	.526066351	-1.67845659	.918896999	- .0852	1.11
1.12	- .0655276060	.297619048	.528301887	-1.67948718	.919093851	- .0853	1.12
1.13	- .0642802503	.294985251	.530516432	-1.68051118	.919289750	- .0855	1.13
1.14	- .0630639807	.292397661	.532710280	-1.68152866	.919484702	- .0857	1.14
1.15	- .0618778257	.289855072	.534883721	-1.68253968	.919678715	- .0858	1.15
1.16	- .0607208500	.287356322	.537037037	-1.68354430	.919871795	- .0860	1.16
1.17	- .0595921540	.284900285	.539170507	-1.68454259	.920063949	- .0862	1.17
1.18	- .0584908717	.282485876	.541284404	-1.68553459	.920255183	- .0864	1.18
1.19	- .0574161693	.280112045	.543378995	-1.68652038	.920445505	- .0865	1.19
1.20	- .0563672440	.277777778	.545454545	-1.68750000	.920634921	- .0853	1.20
1.21	- .0553433220	.275482094	.547511312	-1.68847352	.920823436	- .0868	1.21
1.22	- .0543436580	.273224044	.549549550	-1.68944099	.921011058	- .0870	1.22
1.23	- .0533675337	.271002710	.551569507	-1.69040248	.921197794	- .0872	1.23
1.24	- .0524142559	.268817204	.553571429	-1.69135802	.921383648	- .0873	1.24
1.25	- .0514831573	.266666667	.555555556	-1.69230769	.921568627	- .0875	1.25
1.26	- .0505735933	.264550265	.557522124	-1.69325153	.921752739	- .0877	1.26
1.27	- .0496849423	.262467192	.559471366	-1.69418960	.921935988	- .0878	1.27
1.28	- .0488166042	.260416667	.561403509	-1.69512195	.922118380	- .0880	1.28
1.29	- .0479680003	.258397933	.563318777	-1.69604863	.922299922	- .0882	1.29
	${}^{2}_{4}C^{1}_{4}$	${}^{2}_{4}C^{1}_{3}$	${}^{2}_{4}C^{1}_{2}$	${}^{2}_{4}C^{1}_{1}$	${}^{2}_{4}C^{1}_{0}$	$\frac{{}^{2}_{4}E^{1}}{b^3 f^{(5)}(\xi)}$	$p=\frac{d}{c}\frac{d}{b}\frac{d}{a}$

22(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_2} = \frac{1}{b^2} \sum_{j=0}^{4} {}^{2}_{4}C^{2}_{j}\, y_j + {}^{2}_{4}R^{2} \qquad (i=4,0)$$

$r = \frac{a}{b} = \frac{a}{c} = \frac{a}{d}$	${}^{2}_{4}C^{4}_{0}$	${}^{2}_{4}C^{4}_{1}$	${}^{2}_{4}C^{4}_{2}$	${}^{2}_{4}C^{4}_{3}$	${}^{2}_{4}C^{4}_{4}$	$\frac{{}^{2}_{4}E^{4}}{b^3 f^{(5)}(\xi)}$	
0.10	30.7219662	-37.6666667	14.0000000	-10.2380952	3.18279570	0.6683	0.10
.11	27.4577011	-34.3333333	13.9099099	-10.2132701	3.17899250	.6701	.11
.12	24.7475926	-31.5555556	13.8214286	-10.1886792	3.17521368	.6720	.12
.13	22.4634752	-29.2051282	13.7345133	-10.1643192	3.17145900	.6738	.13
.14	20.5138121	-27.1904762	13.6491228	-10.1401869	3.16772824	.6756	.14
.15	18.8314850	-25.4444445	13.5652174	-10.1162791	3.16402116	.6776	.15
.16	17.3661631	-23.9166667	13.4827586	-10.0925926	3.16033755	.6793	.16
.17	16.0793653	-22.5686275	13.4017094	-10.0691244	3.15667718	.6811	.17
.18	14.9411682	-21.3703704	13.3220339	-10.0458716	3.15303983	.6831	.18
.19	13.9279539	-20.2982456	13.2436975	-10.0228311	3.14942529	.6848	.19
.20	13.0208333	-19.3333333	13.1666667	-10.0000000	3.14583333	.6866	.20
.21	12.2045202	-18.4603175	13.0909091	- 9.97737557	3.14226376	.6885	.21
.22	11.4665118	-17.6666667	13.0163934	- 9.95495495	3.13871636	.6903	.22
.23	10.7964841	-16.9420290	12.9430894	- 9.93273543	3.13519092	.6923	.23
.24	10.1858371	-16.2777778	12.8709677	- 9.91071429	3.13168724	.6940	.24
.25	9.62735043	-15.6666667	12.8000000	- 9.88888889	3.12820513	.6958	.25
.26	9.11491763	-15.1025641	12.7301587	- 9.86725664	3.12474438	.6977	.26
.27	8.64333978	-14.5802469	12.6614173	- 9.84581498	3.12130479	.6995	.27
.28	8.20816332	-14.0952381	12.5937500	- 9.82456140	3.11788618	.7014	.28
.29	7.80555148	-13.6436782	12.5271318	- 9.80349345	3.11448835	.7031	.29
.30	7.43218135	-13.2222222	12.4615385	- 9.78260870	3.11111111	.7051	.30
.31	7.08516091	-12.8279570	12.3969466	- 9.76190476	3.10775428	.7068	.31
.32	6.76196164	-12.4583333	12.3333333	- 9.74137931	3.10441767	.7088	.32
.33	6.46036336	-12.1111111	12.2706767	- 9.72103004	3.10110110	.7105	.33
.34	6.17840881	-11.7843137	12.2089552	- 9.70085470	3.09780439	.7124	.34
.35	5.91436603	-11.4761905	12.1481482	- 9.68085106	3.09452736	.7142	.35
.36	5.66669700	-11.1851852	12.0882353	- 9.66101695	3.09126984	.7161	.36
.37	5.43403139	-10.9099099	12.0291971	- 9.64135021	3.08803165	.7178	.37
.38	5.21514443	-10.6491228	11.9710145	- 9.62184874	3.08481262	.7197	.38
.39	5.00893821	-10.4017094	11.9136691	- 9.60251046	3.08161259	.7215	.39
.40	4.81442577	-10.1666667	11.8571429	- 9.58333333	3.07843137	.7234	.40
.41	4.63071753	- 9.94308943	11.8014184	- 9.56431535	3.07526882	.7251	.41
.42	4.45700965	- 9.73015873	11.7464789	- 9.54545455	3.07212476	.7270	.42
.43	4.29257403	- 9.52713178	11.6923077	- 9.52674897	3.06899903	.7289	.43
.44	4.13674969	- 9.33333333	11.6388889	- 9.50819672	3.06589147	.7307	.44
.45	3.98893524	- 9.14814815	11.5862069	- 9.48979592	3.06280193	.7326	.45
.46	3.84858238	- 8.97101449	11.5342466	- 9.47154472	3.05973025	.7343	.46
.47	3.71519027	- 8.80141844	11.4829932	- 9.45344130	3.05667627	.7362	.47
.48	3.58830048	- 8.63888889	11.4324324	- 9.43548387	3.05363985	.7379	.48
.49	3.46749272	- 8.48299320	11.3825503	- 9.41767068	3.05062082	.7398	.49
.50	3.35238095	- 8.33333333	11.3333333	- 9.40000000	3.04761905	.7417	.50
.51	3.24261001	- 8.18954248	11.2847682	- 9.38247012	3.04463438	.7434	.51
.52	3.13785265	- 8.05128205	11.2368421	- 9.36507936	3.04166667	.7453	.52
.53	3.03780683	- 7.91823899	11.1895425	- 9.34782609	3.03871577	.7472	.53
.54	2.94219343	- 7.79012346	11.1428571	- 9.33070866	3.03578154	.7489	.54
.55	2.85075411	- 7.66666667	11.0967742	- 9.31372549	3.03286385	.7508	.55
.56	2.76324945	- 7.54761905	11.0512821	- 9.29687500	3.02996255	.7527	.56
.57	2.67945726	- 7.43274854	11.0063694	- 9.28015564	3.02707750	.7544	.57
.58	2.59917109	- 7.32183908	10.9620253	- 9.26356589	3.02420857	.7563	.58
.59	2.52219890	- 7.21468927	10.9182390	- 9.24710425	3.02135562	.7582	.59
.60	2.44836182	- 7.11111111	10.8750000	- 9.23076923	3.01851852	.7601	.60
.61	2.37749307	- 7.01092896	10.8322981	- 9.21455939	3.01569714	.7618	.61
.62	2.30943698	- 6.91397849	10.7901235	- 9.19847328	3.01289134	.7637	.62
.63	2.24404806	- 6.82010582	10.7484663	- 9.18250951	3.01010101	.7656	.63
.64	2.18119025	- 6.72916667	10.7073171	- 9.16666667	3.00732601	.7672	.64
.65	2.12073616	- 6.64102564	10.6666667	- 9.15094340	3.00456621	.7691	.65
.66	2.06256638	- 6.55555556	10.6265060	- 9.13533835	3.00182149	.7710	.66
.67	2.00656892	- 6.47263682	10.5868264	- 9.11985019	2.99909174	.7729	.67
.68	1.95263862	- 6.39215686	10.5476191	- 9.10447761	2.99637681	.7746	.68
.69	1.90067665	- 6.31400966	10.5088757	- 9.08921933	2.99367660	.7764	.69
	${}^{2}_{4}C^{0}_{4}$	${}^{2}_{4}C^{0}_{3}$	${}^{2}_{4}C^{0}_{2}$	${}^{2}_{4}C^{0}_{1}$	${}^{2}_{4}C^{0}_{0}$	$\frac{{}^{2}_{4}E^{0}}{b^3 f^{(5)}(\xi)}$	$p = \frac{d}{c} = \frac{d}{b} = \frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^2y)_{x=x_2} = \frac{1}{b^2} \sum_{j=0}^{4} {}^{2}_{4}C^{2}_{j}\, y_j + {}^{2}_{4}R^{2} \qquad (i=4,0)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{2}_{4}C^{4}_{0}$	${}^{2}_{4}C^{4}_{1}$	${}^{2}_{4}C^{4}_{2}$	${}^{2}_{4}C^{4}_{3}$	${}^{2}_{4}C^{4}_{4}$	$\frac{{}^{2}_{4}E^{4}}{b^3 f^{(5)}(\xi)}$	
0.70	1.85059009	− 6.23809524	10.4705882	− 9.07407407	2.99099099	0.7783	0.70
.71	1.80229144	− 6.16431925	10.4327485	− 9.05904059	2.98831986	.7800	.71
.72	1.75569832	− 6.09259259	10.3953488	− 9.04411765	2.98566308	.7821	.72
.73	1.71073302	− 6.02283105	10.3583815	− 9.02930403	2.98302055	.7837	.73
.74	1.66732226	− 5.95495495	10.3218391	− 9.01459854	2.98039216	.7856	.74
.75	1.62539683	− 5.88888889	10.2857143	− 9.00000000	2.97777778	.7875	.75
.76	1.58489134	− 5.82456140	10.2500000	− 8.98550725	2.97517731	.7896	.76
.77	1.54574400	− 5.76190476	10.2146893	− 8.97111913	2.97259063	.7912	.77
.78	1.50789632	− 5.70085470	10.1797753	− 8.95683453	2.97001764	.7929	.78
.79	1.47129292	− 5.64135021	10.1452514	− 8.94265233	2.96745822	.7948	.79
.80	1.43588137	− 5.58333333	10.1111111	− 8.92857143	2.96491228	.7966	.80
.81	1.40161195	− 5.52674897	10.0773481	− 8.91459075	2.96237970	.7985	.81
.82	1.36843750	− 5.47154472	10.0439560	− 8.90070922	2.95986038	.8004	.82
.83	1.33631130	− 5.41767068	10.0109290	− 8.88692580	2.95735422	.8022	.83
.84	1.30519682	− 5.36507936	9.97826087	− 8.87323944	2.95486111	.8041	.84
.85	1.27504772	− 5.31372549	9.94594594	− 8.85964912	2.95238095	.8057	.85
.86	1.24582760	− 5.26356589	9.91397849	− 8.84615385	2.94991364	.8076	.86
.87	1.21749997	− 5.21455939	9.88235294	− 8.83275261	2.94745909	.8094	.87
.88	1.19003010	− 5.16666667	9.85106383	− 8.81944444	2.94501718	.8113	.88
.89	1.16338491	− 5.11985019	9.82010582	− 8.80622837	2.94258783	.8132	.89
.90	1.13753290	− 5.07407407	9.78947368	− 8.79310345	2.94017094	.8150	.90
.91	1.11244404	− 5.02930403	9.75916230	− 8.78006873	2.93776641	.8169	.91
.92	1.08808972	− 4.98550725	9.72916667	− 8.76712329	2.93537415	.8187	.92
.93	1.06444261	− 4.94265233	9.69948186	− 8.75426621	2.93299406	.8206	.93
.94	1.04147667	− 4.90070922	9.67010309	− 8.74149660	2.93062606	.8224	.94
.95	1.01916700	− 4.85964912	9.64102564	− 8.72881356	2.92827004	.8240	.95
.96	.997489835	− 4.81944444	9.61224490	− 8.71621622	2.92592593	.8259	.96
.97	.976422468	− 4.78006873	9.58375635	− 8.70370370	2.92359362	.8277	.97
.98	.955943179	− 4.74149660	9.55555556	− 8.69127517	2.92127303	.8296	.98
.99	.936031202	− 4.70370370	9.52763819	− 8.67892977	2.91896408	.8314	.99
1.00	.916666667	− 4.66666667	9.50000000	− 8.66666667	2.91666667	.8333	1.00
1.01	.897830555	− 4.63036304	9.47263682	− 8.65448505	2.91438071	.8351	1.01
1.02	.879504657	− 4.59477124	9.44554455	− 8.64238411	2.91210614	.8370	1.02
1.03	.861671529	− 4.55987055	9.41871921	− 8.63036304	2.90984285	.8388	1.03
1.04	.844314456	− 4.52564103	9.39215686	− 8.61842105	2.90759076	.8406	1.04
1.05	.827417416	− 4.49206349	9.36585366	− 8.60655738	2.90534979	.8425	1.05
1.06	.810965045	− 4.45911950	9.33980583	− 8.59477124	2.90311987	.8443	1.06
1.07	.794942604	− 4.42679128	9.31400966	− 8.58306189	2.90090090	.8462	1.07
1.08	.779335951	− 4.39506173	9.28846154	− 8.57142857	2.89869281	.8480	1.08
1.09	.764131511	− 4.36391437	9.26315789	− 8.55987055	2.89649552	.8498	1.09
1.10	.749316249	− 4.33333333	9.23809524	− 8.54838710	2.89430894	.8517	1.10
1.11	.734877644	− 4.30330330	9.21327014	− 8.53697749	2.89213301	.8535	1.11
1.12	.720803667	− 4.27380952	9.18867925	− 8.52564103	2.88996764	.8553	1.12
1.13	.707082754	− 4.24483776	9.16431925	− 8.51437700	2.88781275	.8571	1.13
1.14	.693703789	− 4.21637427	9.14018692	− 8.50318471	2.88566828	.8590	1.14
1.15	.680656084	− 4.18840580	9.11627907	− 8.49206349	2.88353414	.8608	1.15
1.16	.667929349	− 4.16091954	9.09259259	− 8.48101266	2.88141026	.8627	1.16
1.17	.655513693	− 4.13390313	9.06912442	− 8.47003155	2.87929656	.8645	1.17
1.18	.643399587	− 4.10734463	9.04587156	− 8.45911950	2.87719298	.8663	1.18
1.19	.631577862	− 4.08123249	9.02283105	− 8.44827586	2.87509944	.8682	1.19
1.20	.620039683	− 4.05555556	9.00000000	− 8.43750000	2.87301587	.8700	1.20
1.21	.608776541	− 4.03030303	8.97737557	− 8.42679128	2.87094220	.8718	1.21
1.22	.597780237	− 4.00546448	8.95495495	− 8.41614907	2.86887836	.8737	1.22
1.23	.587042869	− 3.98102981	8.93273543	− 8.40557276	2.86682427	.8755	1.23
1.24	.576556815	− 3.95698925	8.91071429	− 8.39506173	2.86477987	.8773	1.24
1.25	.566314732	− 3.93333333	8.88888889	− 8.38461538	2.86274500	.8792	1.25
1.26	.556309526	− 3.91005291	8.86725664	− 8.37423313	2.86071987	.8810	1.26
1.27	.546534365	− 3.88713911	8.84581498	− 8.36391437	2.85870414	.8828	1.27
1.28	.536982647	− 3.86458333	8.82456140	− 8.35365854	2.85669782	.8847	1.28
1.29	.527648002	− 3.84237726	8.80349345	− 8.34346505	2.85470085	.8865	1.29
	${}^{2}_{4}C^{0}_{4}$	${}^{2}_{4}C^{0}_{3}$	${}^{2}_{4}C^{0}_{2}$	${}^{2}_{4}C^{0}_{1}$	${}^{2}_{4}C^{0}_{0}$	$\frac{{}^{2}_{4}E^{0}}{b^3 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3} \sum_{j=0}^{4} {}^3_4C^i_j \, y_j + {}^3_4R^i \qquad (i=0,4)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^3_4C^0_0$	${}^3_4C^0_1$	${}^3_4C^0_2$	${}^3_4C^0_3$	${}^3_4C^0_4$	$\frac{{}^3_4E^0}{b^2f^{(5)}(\xi)}$	
0.10	-53.6237956	63.0000000	-14.4545455	6.14285714	-1.06451613	0.6430	0.10
.11	-48.2257077	57.5454545	-14.4054054	6.15639810	-1.07073955	.6526	.11
.12	-43.7357454	53.0000000	-14.3571429	6.16981132	-1.07692308	.6623	.12
.13	-39.9441431	49.1538462	-14.3097345	6.18309859	-1.08306709	.6721	.13
.14	-36.7010747	45.8571429	-14.2631579	6.19626168	-1.08917197	.6819	.14
.15	-33.8966729	43.0000000	-14.2173913	6.20930233	-1.09523810	.6918	.15
.16	-31.4485426	40.5000000	-14.1724138	6.22222222	-1.10126582	.7017	.16
.17	-29.2936800	38.2941176	-14.1282051	6.23502304	-1.10725552	.7117	.17
.18	-27.3830864	36.3333333	-14.0847457	6.24770642	-1.11320755	.7217	.18
.19	-25.6780823	34.5789474	-14.0420168	6.26027397	-1.11912226	.7318	.19
.20	-24.1477273	33.0000000	-14.0000000	6.27272727	-1.12500000	.7420	.20
.21	-22.7669776	31.5714286	-13.9586777	6.28506787	-1.13084112	.7522	.21
.22	-21.5153458	30.2727273	-13.9180328	6.29729730	-1.13664596	.7625	.22
.23	-20.3759099	29.0869565	-13.8780488	6.30941704	-1.14241486	.7729	.23
.24	-19.3345707	28.0000000	-13.8387097	6.32142857	-1.14814815	.7833	.24
.25	-18.3794872	27.0000000	-13.8000000	6.33333333	-1.15384615	.7938	.25
.26	-17.5006419	26.0769231	-13.7619048	6.34513274	-1.15950920	.8043	.26
.27	-16.6895034	25.2222222	-13.7244094	6.35682819	-1.16513761	.8149	.27
.28	-15.9387608	24.4285714	-13.6875000	6.36842105	-1.17073171	.8255	.28
.29	-15.2421133	23.6896552	-13.6511628	6.37991266	-1.17629179	.8362	.29
.30	-14.5941016	23.0000000	-13.6153846	6.39130435	-1.18181818	.8470	.30
.31	-13.9899723	22.3548387	-13.5801527	6.40259740	-1.18731118	.8578	.31
.32	-13.4255675	21.7500000	-13.5454545	6.41379310	-1.19277108	.8687	.32
.33	-12.8972345	21.1818182	-13.5112782	6.42489270	-1.19819820	.8797	.33
.34	-12.4017515	20.6470588	-13.4776119	6.43589744	-1.20359281	.8907	.34
.35	-11.9362660	20.1428571	-13.4444444	6.44680851	-1.20895522	.9018	.35
.36	-11.4982434	19.6666667	-13.4117647	6.45762712	-1.21428571	.9129	.36
.37	-11.0854240	19.2162162	-13.3795620	6.46835443	-1.21958457	.9241	.37
.38	-10.6957871	18.7894737	-13.3478261	6.47899160	-1.22485207	.9353	.38
.39	-10.3275199	18.3846154	-13.3165468	6.48953975	-1.23008850	.9466	.39
.40	- 9.97899160	18.0000000	-13.2857143	6.50000000	-1.23529412	.9580	.40
.41	- 9.64873143	17.6341463	-13.2553192	6.51037344	-1.24046921	.9694	.41
.42	- 9.33540929	17.2857143	-13.2253521	6.52066116	-1.24561404	.9809	.42
.43	- 9.03781951	16.9534884	-13.1958042	6.53086420	-1.25072886	.9925	.43
.44	- 8.75486662	16.6363636	-13.1666667	6.54098361	-1.25581395	1.004	.44
.45	- 8.48555314	16.3333333	-13.1379310	6.55102041	-1.26086957	1.016	.45
.46	- 8.22896888	16.0434783	-13.1095890	6.56097561	-1.26589595	1.027	.46
.47	- 7.98428162	15.7659574	-13.0816327	6.57085020	-1.27089337	1.039	.47
.48	- 7.75072904	15.5000000	-13.0540541	6.58064516	-1.27586207	1.051	.48
.49	- 7.52761148	15.2448980	-13.0268456	6.59036145	-1.28080229	1.063	.49
.50	- 7.31428571	15.0000000	-13.0000000	6.60000000	-1.28571429	1.075	.50
.51	- 7.11015941	14.7647058	-12.9735099	6.60956175	-1.29059829	1.087	.51
.52	- 6.91468619	14.5384615	-12.9473684	6.61904762	-1.29545455	1.099	.52
.53	- 6.72736130	14.3207547	-12.9215686	6.62845850	-1.30028329	1.111	.53
.54	- 6.54771774	14.1111111	-12.8961039	6.63779528	-1.30508475	1.123	.54
.55	- 6.37532284	13.9090909	-12.8709677	6.64705882	-1.30985915	1.136	.55
.56	- 6.20977513	13.7142857	-12.8461538	6.65625000	-1.31460674	1.148	.56
.57	- 6.05070166	13.5263158	-12.8216561	6.66536965	-1.31932773	1.160	.57
.58	- 5.89775549	13.3448276	-12.7974684	6.67441860	-1.32402235	1.173	.58
.59	- 5.75061350	13.1694915	-12.7735849	6.68339768	-1.32869081	1.185	.59
.60	- 5.60897436	13.0000000	-12.7500000	6.69230769	-1.33333333	1.198	.60
.61	- 5.47255679	12.8360656	-12.7267081	6.70114943	-1.33795014	1.211	.61
.62	- 5.34109788	12.6774194	-12.7037037	6.70992366	-1.34254144	1.223	.62
.63	- 5.21435167	12.5238095	-12.6809816	6.71863118	-1.34710744	1.236	.63
.64	- 5.09208779	12.3750000	-12.6585366	6.72727273	-1.35164835	1.249	.64
.65	- 4.97409027	12.2307692	-12.6363636	6.73584906	-1.35616438	1.258	.65
.66	- 4.86015642	12.0909091	-12.6144578	6.74436090	-1.36066574	1.275	.66
.67	- 4.75009588	11.9552239	-12.5928144	6.75280899	-1.36512262	1.288	.67
.68	- 4.64372965	11.8235294	-12.5714286	6.76119403	-1.36956522	1.301	.68
.69	- 4.54088930	11.6956522	-12.5502959	6.76951673	-1.37398374	1.314	.69
	$-{}^3_4C^4_4$	$-{}^3_4C^4_3$	$-{}^3_4C^4_2$	$-{}^3_4C^4_1$	$-{}^3_4C^4_0$	$\frac{{}^3_4E^4}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3} \sum_{j=0}^{4} {}^3_4C^i_j \, y_j + {}^3_4R^i \qquad (i=0,4)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^3_4C^0_0$	${}^3_4C^0_1$	${}^3_4C^0_2$	${}^3_4C^0_3$	${}^3_4C^0_4$	$\frac{{}^3_4E^0}{b^2f^{(5)}(\xi)}$	
0.70	- 4.44141621	11.5714286	-12.5294118	6.77777778	-1.37837838	1.327	0.70
.71	- 4.34516083	11.4507042	-12.5087719	6.78597786	-1.38274933	1.340	.71
.72	- 4.25198211	11.3333333	-12.4883721	6.79411765	-1.38709677	1.354	.72
.73	- 4.16174688	11.2191781	-12.4682081	6.80219780	-1.39142091	1.367	.73
.74	- 4.07432930	11.1081081	-12.4482759	6.81021898	-1.39572192	1.380	.74
.75	- 3.98961039	11.0000000	-12.4285714	6.81818182	-1.40000000	1.394	.75
.76	- 3.90747757	10.8947368	-12.4090909	6.82608696	-1.40425532	1.407	.76
.77	- 3.82782424	10.7922078	-12.3898305	6.83393502	-1.40848806	1.421	.77
.78	- 3.75054938	10.6923077	-12.3707865	6.84172662	-1.41269841	1.435	.78
.79	- 3.67555722	10.5949367	-12.3519553	6.84946237	-1.41688654	1.448	.79
.80	- 3.60275689	10.5000000	-12.3333333	6.85714286	-1.42105263	1.462	.80
.81	- 3.53206212	10.4074074	-12.3149171	6.86476868	-1.42519685	1.476	.81
.82	- 3.46339092	10.3170732	-12.2967033	6.87234043	-1.42931937	1.490	.82
.83	- 3.39666543	10.2289157	-12.2786885	6.87985866	-1.43342037	1.504	.83
.84	- 3.33181151	10.1428571	-12.2608696	6.88732394	-1.43750000	1.518	.84
.85	- 3.26875870	10.0588235	-12.2432432	6.89473684	-1.44155844	1.532	.85
.86	- 3.20743977	9.97674419	-12.2258065	6.90209790	-1.44559586	1.546	.86
.87	- 3.14779084	9.89655172	-12.2085562	6.90940767	-1.44961240	1.560	.87
.88	- 3.08975087	9.81818182	-12.1914894	6.91666667	-1.45360825	1.574	.88
.89	- 3.03326174	9.74157303	-12.1746032	6.92387543	-1.45758355	1.589	.89
.90	- 2.97826795	9.66666667	-12.1578947	6.93103448	-1.46153846	1.603	.90
.91	- 2.92471652	9.59340659	-12.1413613	6.93814433	-1.46547315	1.617	.91
.92	- 2.87255685	9.52173913	-12.1250000	6.94520548	-1.46938776	1.632	.92
.93	- 2.82174060	9.45161290	-12.1088083	6.95221843	-1.47328244	1.646	.93
.94	- 2.77222153	9.38297872	-12.0927835	6.95918367	-1.47715736	1.661	.94
.95	- 2.72395544	9.31578947	-12.0769231	6.96610169	-1.48101266	1.676	.95
.96	- 2.67689999	9.25000000	-12.0612245	6.97297297	-1.48484849	1.690	.96
.97	- 2.63101472	9.18556701	-12.0456853	6.97979798	-1.48866499	1.705	.97
.98	- 2.58626082	9.12244898	-12.0303030	6.98657718	-1.49246231	1.720	.98
.99	- 2.54260112	9.06060606	-12.0150754	6.99331104	-1.49624060	1.735	.99
1.00	- 2.50000000	9.00000000	-12.0000000	7.00000000	-1.50000000	1.750	1.00
1.01	- 2.45842330	8.94059406	-11.9850746	7.00664452	-1.50374065	1.765	1.01
1.02	- 2.41783826	8.88235294	-11.9702970	7.01324503	-1.50746269	1.780	1.02
1.03	- 2.37821342	8.82524272	-11.9556650	7.01980198	-1.51116625	1.795	1.03
1.04	- 2.33951860	8.76923077	-11.9411765	7.02631579	-1.51485149	1.810	1.04
1.05	- 2.30172481	8.71428571	-11.9268293	7.03278689	-1.51851852	1.826	1.05
1.06	- 2.26480420	8.66037736	-11.9126214	7.03921569	-1.52216749	1.841	1.06
1.07	- 2.22872999	8.60747664	-11.8985507	7.04560261	-1.52579853	1.856	1.07
1.08	- 2.19347646	8.55555556	-11.8846154	7.05194805	-1.52941176	1.872	1.08
1.09	- 2.15901885	8.50458716	-11.8708134	7.05825243	-1.53300733	1.887	1.09
1.10	- 2.12533336	8.45454545	-11.8571429	7.06451613	-1.53658537	1.903	1.10
1.11	- 2.09239707	8.40540541	-11.8436019	7.07073955	-1.54014599	1.919	1.11
1.12	- 2.06018793	8.35714286	-11.8301887	7.07692308	-1.54368932	1.934	1.12
1.13	- 2.02868470	8.30973451	-11.8169014	7.08306709	-1.54721550	1.950	1.13
1.14	- 1.99786691	8.26315789	-11.8037383	7.08917197	-1.55072464	1.966	1.14
1.15	- 1.96771486	8.21739130	-11.7906977	7.09523810	-1.55421687	1.982	1.15
1.16	- 1.93820953	8.17241379	-11.7777778	7.10126582	-1.55769231	1.998	1.16
1.17	- 1.90933261	8.12820513	-11.7649770	7.10725552	-1.56115108	2.014	1.17
1.18	- 1.88106643	8.08474576	-11.7522936	7.11320755	-1.56459330	2.030	1.18
1.19	- 1.85339394	8.04201681	-11.7397260	7.11912226	-1.56801909	2.046	1.19
1.20	- 1.82629870	8.00000000	-11.7272727	7.12500000	-1.57142857	2.062	1.20
1.21	- 1.79976483	7.95867769	-11.7149321	7.13084112	-1.57482185	2.078	1.21
1.22	- 1.77377699	7.91803279	-11.7027027	7.13664596	-1.57819905	2.095	1.22
1.23	- 1.74832040	7.87804878	-11.6905830	7.14241486	-1.58156028	2.111	1.23
1.24	- 1.72338073	7.83870968	-11.6785714	7.14814815	-1.58490566	2.127	1.24
1.25	- 1.69894420	7.80000000	-11.6666667	7.15384615	-1.58823529	2.144	1.25
1.26	- 1.67499741	7.76190476	-11.6548673	7.15950920	-1.59154930	2.160	1.26
1.27	- 1.65152748	7.72440945	-11.6431718	7.16513762	-1.59484778	2.177	1.27
1.28	- 1.62852192	7.68750000	-11.6315789	7.17073171	-1.59813084	2.194	1.28
1.29	- 1.60596865	7.65116279	-11.6200873	7.17629179	-1.60139860	2.210	1.29
	$-{}^3_4C^4_4$	$-{}^3_4C^4_3$	$-{}^3_4C^4_2$	$-{}^3_4C^4_1$	$-{}^3_4C^4_0$	$\frac{{}^3_4E^4}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

24(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3}\sum_{j=0}^{4} {}^3_4C^i_j\, y_j + {}^3_4R^i \qquad (i=1,3)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^3_4C^1_0$	${}^3_4C^1_1$	${}^3_4C^1_2$	${}^3_4C^1_3$	${}^3_4C^1_4$	$\frac{{}^3_4E^1}{b^2f^{(5)}(\xi)}$	
0.10	-50.2723084	59.0000000	-13.3636364	5.57142857	-0.935483871	0.5200	0.10
.11	-44.9307836	53.5454545	-13.2162162	5.53080569	- .929260450	.5170	.11
.12	-40.4980605	49.0000000	-13.0714286	5.49056604	- .923076923	.5140	.12
.13	-36.7584139	45.1538462	-12.9292035	5.45070423	- .916932907	.5110	.13
.14	-33.5680561	41.8571429	-12.7894737	5.41121495	- .910828025	.5080	.14
.15	-30.8151572	39.0000000	-12.6521739	5.37209302	- .904761905	.5050	.15
.16	-28.4173578	36.5000000	-12.5172414	5.33333333	- .898734177	.5020	.16
.17	-26.3116886	34.2941176	-12.3846154	5.29493088	- .892744479	.4990	.17
.18	-24.4491843	32.3333333	-12.2542373	5.25688073	- .886792453	.4960	.18
.19	-22.7911973	30.5789474	-12.1260504	5.21917808	- .880877743	.4930	.19
.20	-21.3068182	29.0000000	-12.0000000	5.18181818	- .875000000	.4900	.20
.21	-19.9710330	27.5714286	-11.8760331	5.14479638	- .869158879	.4870	.21
.22	-18.7633830	26.2727273	-11.7540984	5.10810811	- .863354037	.4840	.22
.23	-17.6669739	25.0869565	-11.6341463	5.07174888	- .857585139	.4810	.23
.24	-16.6677334	24.0000000	-11.5161290	5.03571429	- .851851852	.4780	.24
.25	-15.7538462	23.0000000	-11.4000000	5.00000000	- .846153846	.4750	.25
.26	-14.9153198	22.0769231	-11.2857143	4.96460177	- .840490798	.4720	.26
.27	-14.1436469	21.2222222	-11.1732284	4.92951542	- .834862385	.4690	.27
.28	-13.4315400	20.4285714	-11.0625000	4.89473684	- .829268293	.4660	.28
.29	-12.7727206	19.6896552	-10.9534884	4.86026201	- .823708207	.4630	.29
.30	-12.1617513	19.0000000	-10.8461539	4.82608696	- .818181818	.4600	.30
.31	-11.5938997	18.3548387	-10.7404580	4.79220779	- .812688822	.4570	.31
.32	-11.0650282	17.7500000	-10.6363636	4.75862069	- .807228916	.4540	.32
.33	-10.5715037	17.1818182	-10.5338346	4.72532189	- .801801802	.4510	.33
.34	-10.1101235	16.6470588	-10.4328358	4.69230769	- .796407186	.4480	.34
.35	- 9.67805352	16.1428571	-10.3333333	4.65957447	- .791044776	.4450	.35
.36	- 9.27277692	15.6666667	-10.2352941	4.62711864	- .785714286	.4420	.36
.37	- 8.89205136	15.2162162	-10.1386861	4.59493671	- .780415430	.4390	.37
.38	- 8.53387272	14.7894737	-10.0434783	4.56302521	- .775147929	.4360	.38
.39	- 8.19644436	14.3846154	- 9.94964028	4.53138075	- .769911504	.4330	.39
.40	- 7.87815126	14.0000000	- 9.85714284	4.50000000	- .764705882	.4300	.40
.41	- 7.57753776	13.6341463	- 9.76595745	4.46887967	- .759530792	.4270	.41
.42	- 7.29328854	13.2857143	- 9.67605633	4.43801653	- .754385965	.4240	.42
.43	- 7.02421206	12.9534884	- 9.58741260	4.40740741	- .749271137	.4210	.43
.44	- 6.76922676	12.6363636	- 9.50000000	4.37704918	- .744186047	.4180	.44
.45	- 6.52734857	12.3333333	- 9.41379310	4.34693878	- .739130435	.4150	.45
.46	- 6.29768026	12.0434783	- 9.32876712	4.31707317	- .734104046	.4120	.46
.47	- 6.07940225	11.7659574	- 9.24489796	4.28744939	- .729106628	.4090	.47
.48	- 5.87176442	11.5000000	- 9.16216216	4.25806452	- .724137931	.4060	.48
.49	- 5.67407900	11.2448980	- 9.08053691	4.22891566	- .719197708	.4030	.49
.50	- 5.48571429	11.0000000	- 9.00000000	4.20000000	- .714285714	.4000	.50
.51	- 5.30608911	10.7647058	- 8.92052980	4.17131474	- .709401709	.3970	.51
.52	- 5.13466797	10.5384615	- 8.84210526	4.14285714	- .704545455	.3940	.52
.53	- 4.97095663	10.3207547	- 8.76470588	4.11462451	- .699716714	.3910	.53
.54	- 4.81449834	10.1111111	- 8.68831169	4.08661417	- .694915254	.3880	.54
.55	- 4.66487037	9.90909091	- 8.61290323	4.05882353	- .690140845	.3850	.55
.56	- 4.52168092	9.71428571	- 8.53846154	4.03125000	- .685393258	.3820	.56
.57	- 4.38456642	9.52631579	- 8.46496815	4.00389105	- .680672269	.3790	.57
.58	- 4.25318905	9.34482759	- 8.39240506	3.97674419	- .675977654	.3760	.58
.59	- 4.12723457	9.16949153	- 8.32075472	3.94980695	- .671309192	.3730	.59
.60	- 4.00641026	9.00000000	- 8.25000000	3.92307692	- .666666667	.3700	.60
.61	- 3.89044321	8.83606557	- 8.18012422	3.89655172	- .662049861	.3670	.61
.62	- 3.77907869	8.67741935	- 8.11111111	3.87022901	- .657458564	.3640	.62
.63	- 3.67207864	8.52380952	- 8.04294480	3.84410646	- .652892562	.3610	.63
.64	- 3.56922041	8.37500000	- 7.97560977	3.81818182	- .648351648	.3580	.64
.65	- 3.47029554	8.23076923	- 7.90909092	3.79245283	- .643835616	.3550	.65
.66	- 3.37510863	8.09090909	- 7.84337349	3.76691729	- .639633262	.3520	.66
.67	- 3.28347642	7.95522388	- 7.77844311	3.74157303	- .634877384	.3490	.67
.68	- 3.19522683	7.82352941	- 7.71428571	3.71641791	- .630434783	.3460	.68
.69	- 3.11019815	7.69565217	- 7.65088758	3.69144981	- .626016260	.3430	.69
	$-{}^3_4C^3_4$	$-{}^3_4C^3_3$	$-{}^3_4C^3_2$	$-{}^3_4C^3_1$	$-{}^3_4C^3_0$	$\frac{{}^3_4E^3}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3}\sum_{j=0}^{4} {}^3_4C^i_j\, y_j + {}^3_4R^i \qquad (i=1,3)$$

$r=\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^3_4C^1_0$	${}^3_4C^1_1$	${}^3_4C^1_2$	${}^3_4C^1_3$	${}^3_4C^1_4$	$\frac{{}^3_4E^1}{b^2f^{(5)}(\xi)}$	
0.70	− 3.02823832	7.57142857	− 7.58823528	3.66666667	−0.621621622	0.3400	0.70
.71	− 2.94920418	7.45070423	− 7.52631579	3.64206642	− .617250674	.3370	.71
.72	− 2.87296089	7.33333333	− 7.46511627	3.61764706	− .612903226	.3340	.72
.73	− 2.79938131	7.21917808	− 7.40462427	3.59340659	− .608579088	.3310	.73
.74	− 2.72834551	7.10810811	− 7.34482758	3.56934307	− .604278075	.3280	.74
.75	− 2.65974026	7.00000000	− 7.28571429	3.54545455	− .600000000	.3250	.75
.76	− 2.59345856	6.89473684	− 7.22727273	3.52173913	− .595744681	.3220	.76
.77	− 2.52939928	6.79220779	− 7.16949153	3.49819495	− .591511936	.3190	.77
.78	− 2.46746670	6.69230769	− 7.11235956	3.47482014	− .587301587	.3160	.78
.79	− 2.40757023	6.59493671	− 7.05586593	3.45161290	− .583113456	.3130	.79
.80	− 2.34962406	6.50000000	− 7.00000000	3.42857143	− .578947368	.3100	.80
.81	− 2.29354683	6.40740741	− 6.94475138	3.40569395	− .574803150	.3070	.81
.82	− 2.23926137	6.31707317	− 6.89010989	3.38297872	− .570680628	.3040	.82
.83	− 2.18669449	6.22891566	− 6.83606557	3.36042403	− .566579634	.3010	.83
.84	− 2.13577661	6.14285714	− 6.78260870	3.33802817	− .562500000	.2980	.84
.85	− 2.08644172	6.05882353	− 6.72972973	3.31578947	− .558441558	.2950	.85
.86	− 2.03862697	5.97674419	− 6.67741935	3.29370629	− .554404145	.2920	.86
.87	− 1.99227268	5.89655172	− 6.62566845	3.27177700	− .550387597	.2890	.87
.88	− 1.94732198	5.81818182	− 6.57446809	3.25000000	− .546391753	.2860	.88
.89	− 1.90372076	5.74157303	− 6.52380952	3.22837370	− .542416452	.2830	.89
.90	− 1.86141747	5.66666667	− 6.47368421	3.20689655	− .538461538	.2800	.90
.91	− 1.82036298	5.59340659	− 6.42408377	3.18556701	− .534526854	.2770	.91
.92	− 1.78051045	5.52173913	− 6.37500000	3.16438356	− .530612245	.2740	.92
.93	− 1.74181519	5.45161290	− 6.32642487	3.14334471	− .526717557	.2710	.93
.94	− 1.70423455	5.38297872	− 6.27835052	3.12244898	− .522842640	.2680	.94
.95	− 1.66772782	5.31578947	− 6.23076923	3.10169492	− .518987342	.2650	.95
.96	− 1.63225609	5.25000000	− 6.18367347	3.08108108	− .515151515	.2620	.96
.97	− 1.59778222	5.18556701	− 6.13705584	3.06060606	− .511335013	.2590	.97
.98	− 1.56427066	5.12244898	− 6.09090909	3.04026846	− .507537688	.2560	.98
.99	− 1.53168742	5.06060606	− 6.04522613	3.02006689	− .503759398	.2530	.99
1.00	− 1.50000000	5.00000000	− 6.00000000	3.00000000	− .500000000	.2500	1.00
1.01	− 1.46917727	4.94059406	− 5.95522388	2.98006645	− .496259352	.2470	1.01
1.02	− 1.43918944	4.88235294	− 5.91089109	2.96026490	− .492537313	.2440	1.02
1.03	− 1.41000796	4.82524272	− 5.86699507	2.94059406	− .488833747	.2410	1.03
1.04	− 1.38160547	4.76923077	− 5.82352941	2.92105263	− .485148515	.2380	1.04
1.05	− 1.35395577	4.71428571	− 5.78048780	2.90163934	− .481481481	.2350	1.05
1.06	− 1.32703371	4.66037736	− 5.73786408	2.88235294	− .477832512	.2320	1.06
1.07	− 1.30081517	4.60747664	− 5.69565217	2.86319218	− .474201474	.2290	1.07
1.08	− 1.27527701	4.55555556	− 5.65384615	2.84415584	− .470588235	.2260	1.08
1.09	− 1.25039702	4.50458716	− 5.61244019	2.82524272	− .466992665	.2230	1.09
1.10	− 1.22615386	4.45454545	− 5.57142857	2.80645161	− .463414634	.2200	1.10
1.11	− 1.20252706	4.40540541	− 5.53080569	2.78778135	− .459854015	.2179	1.11
1.12	− 1.17949691	4.35714286	− 5.49056604	2.76923077	− .456310680	.2140	1.12
1.13	− 1.15704451	4.30973451	− 5.45070423	2.75079872	− .452784504	.2110	1.13
1.14	− 1.13515164	4.26315789	− 5.41121495	2.73248408	− .449275362	.2080	1.14
1.15	− 1.11380086	4.21739130	− 5.37209302	2.71428571	− .445783132	.2050	1.15
1.16	− 1.09297530	4.17241379	− 5.33333333	2.69620253	− .442307692	.2020	1.16
1.17	− 1.07265877	4.12820513	− 5.29493088	2.67823344	− .438848921	.1990	1.17
1.18	− 1.05283569	4.08474576	− 5.25688073	2.66037736	− .435406699	.1960	1.18
1.19	− 1.03349105	4.04201681	− 5.21917808	2.64263323	− .431980907	.1930	1.19
1.20	− 1.01461039	4.00000000	− 5.18181818	2.62500000	− .428571429	.1900	1.20
1.21	− .996179794	3.95867769	− 5.14479638	2.60747664	− .425178147	.1870	1.21
1.22	− .978185841	3.91803279	− 5.10810811	2.59006211	− .421800948	.1840	1.22
1.23	− .960615603	3.87804878	− 5.07174888	2.57275542	− .418439716	.1810	1.23
1.24	− .943456607	3.83870968	− 5.03571429	2.55555555	− .415094340	.1780	1.24
1.25	− .926696834	3.80000000	− 5.00000000	2.53846154	− .411764706	.1750	1.25
1.26	− .910324679	3.76190476	− 4.96460177	2.52147239	− .408450704	.1720	1.26
1.27	− .894328962	3.72440945	− 4.92951542	2.50458716	− .405152225	.1690	1.27
1.28	− .878898876	3.68750000	− 4.89473684	2.48780488	− .401869159	.1660	1.28
1.29	− .863424004	3.65116279	− 4.86026201	2.47112462	− .398601399	.1630	1.29
	$-{}^3_4C^3_4$	$-{}^3_4C^3_3$	$-{}^3_4C^3_2$	$-{}^3_4C^3_1$	$-{}^3_4C^3_0$	$\frac{{}^3_4E^3}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}\frac{d}{b}\frac{d}{a}$

25(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i}=\frac{1}{b^3}\sum_{j=0}^{4} {}^{3}_{4}C^{i}_{j}\, y_j + {}^{3}_{4}R^{i} \qquad (i=2)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{3}_{4}C^{2}_{0}$	${}^{3}_{4}C^{2}_{1}$	${}^{3}_{4}C^{2}_{2}$	${}^{3}_{4}C^{2}_{3}$	${}^{3}_{4}C^{2}_{4}$	$\frac{{}^{3}_{4}E^{2}}{b^2 f^{(5)}(\xi)}$	
0.10	-16.7574361	19.0000000	-2.45454545	-0.142857143	0.354838710	-0.1600	0.10
.11	-14.9769279	17.1818182	-2.40540541	- .156398104	.356913183	- .1610	.11
.12	-13.4986868	15.6666667	-2.35714286	- .169811321	.358974359	- .1620	.12
.13	-12.2528046	14.3846154	-2.30973451	- .183098592	.361022364	- .1630	.13
.14	-11.1893520	13.2857143	-2.26315789	- .196261682	.363057325	- .1640	.14
.15	-10.2717191	12.3333333	-2.21739130	- .209302326	.365079365	- .1650	.15
.16	- 9.47245266	11.5000000	-2.17241379	- .222222222	.367088608	- .1660	.16
.17	- 8.77056288	10.7647059	-2.12820513	- .235023042	.369085174	- .1670	.17
.18	- 8.14972810	10.1111111	-2.08474576	- .247706422	.371069182	- .1680	.18
.19	- 7.59706576	9.52631579	-2.04201681	- .260273972	.373040752	- .1690	.19
.20	- 7.10227272	9.00000000	-2.00000000	- .272727272	.375000000	- .1700	.20
.21	- 6.65701100	8.52380952	-1.95867769	- .285067874	.376947040	- .1710	.21
.22	- 6.25446100	8.09090909	-1.91803279	- .297297298	.378881988	- .1720	.22
.23	- 5.88899130	7.69565217	-1.87804878	- .309417040	.380804954	- .1730	.23
.24	- 5.55591114	7.33333333	-1.83870968	- .321428572	.382716049	- .1740	.24
.25	- 5.25128206	7.00000000	-1.80000000	- .333333334	.384615385	- .1750	.25
.26	- 4.97177326	6.69230769	-1.76190476	- .345132744	.386503067	- .1760	.26
.27	- 4.71454897	6.40740741	-1.72440945	- .356828194	.388379205	- .1770	.27
.28	- 4.47718000	6.14285714	-1.68750000	- .368421052	.390243902	- .1780	.28
.29	- 4.25757354	5.89655172	-1.65116279	- .379912664	.392097264	- .1790	.29
.30	- 4.05391710	5.66666667	-1.61538462	- .391304348	.393939394	- .1800	.30
.31	- 3.86463322	5.45161290	-1.58015267	- .402597402	.395770393	- .1810	.31
.32	- 3.68834272	5.25000000	-1.54545455	- .413793104	.397590361	- .1820	.32
.33	- 3.52383456	5.06060606	-1.51127820	- .424892704	.399399399	- .1830	.33
.34	- 3.37004116	4.88235294	-1.47761194	- .435897436	.401197605	- .1840	.34
.35	- 3.22601784	4.71428571	-1.44444444	- .446808510	.402985075	- .1850	.35
.36	- 3.09092564	4.55555556	-1.41176471	- .457627118	.404761905	- .1860	.36
.37	- 2.96401712	4.40540541	-1.37956204	- .468354430	.406528190	- .1870	.37
.38	- 2.84462424	4.26315789	-1.34782609	- .478991597	.408284024	- .1880	.38
.39	- 2.73214812	4.12820513	-1.31654676	- .489539748	.410029499	- .1890	.39
.40	- 2.62605042	4.00000000	-1.28571429	- .500000000	.411764706	- .1900	.40
.41	- 2.52584592	3.87804878	-1.25531915	- .510373444	.413489736	- .1910	.41
.42	- 2.43109618	3.76190476	-1.22535211	- .520661158	.415204678	- .1920	.42
.43	- 2.34140402	3.65116279	-1.19580420	- .530864198	.416909621	- .1930	.43
.44	- 2.25640892	3.54545455	-1.16666667	- .540983606	.418604651	- .1940	.44
.45	- 2.17578286	3.44444444	-1.13793103	- .551020408	.420289855	- .1950	.45
.46	- 2.09922675	3.34782609	-1.10958904	- .560975610	.421965318	- .1960	.46
.47	- 2.02646742	3.25531915	-1.08163265	- .570850202	.423631124	- .1970	.47
.48	- 1.95725481	3.16666667	-1.05405405	- .580645161	.425287356	- .1980	.48
.49	- 1.89135967	3.08163265	-1.02684564	- .590361446	.426934097	- .1990	.49
.50	- 1.82857143	3.00000000	-1.00000000	- .600000000	.428571429	- .2000	.50
.51	- 1.76869637	2.92156863	- .973509934	- .609561753	.430199430	- .2010	.51
.52	- 1.71155599	2.84615385	- .947368421	- .619047619	.431818182	- .2020	.52
.53	- 1.65698554	2.77358491	- .921568627	- .628458498	.433427762	- .2030	.53
.54	- 1.60483278	2.70370370	- .896103896	- .637795276	.435028249	- .2040	.54
.55	- 1.55495679	2.63636364	- .870967742	- .647058823	.436619718	- .2050	.55
.56	- 1.50722697	2.57142857	- .846153846	- .656250000	.438202247	- .2060	.56
.57	- 1.46152214	2.50877193	- .821656051	- .665369650	.439775910	- .2070	.57
.58	- 1.41772968	2.44827586	- .797468354	- .674418605	.441340782	- .2080	.58
.59	- 1.37574486	2.38983051	- .773584906	- .683397683	.442896936	- .2090	.59
.60	- 1.33547009	2.33333333	- .750000000	- .692307692	.444444444	- .2100	.60
.61	- 1.29681440	2.27868852	- .726708075	- .701149425	.445983380	- .2110	.61
.62	- 1.25969290	2.22580645	- .703703704	- .709923664	.447513812	- .2120	.62
.63	- 1.22402621	2.17460317	- .680981595	- .718631179	.449035813	- .2130	.63
.64	- 1.18974014	2.12500000	- .658536585	- .727272727	.450549450	- .2140	.64
.65	- 1.15676518	2.07692308	- .636363636	- .735849057	.452054793	- .2150	.65
.66	- 1.12503621	2.03030303	- .614457831	- .744360902	.453551913	- .2160	.66
.67	- 1.09449214	1.98507463	- .592814371	- .752808989	.455040873	- .2170	.67
.68	- 1.06507561	1.94117647	- .571428571	- .761194030	.456521740	- .2180	.68
.69	- 1.03673272	1.89855072	- .550295858	- .769516729	.457994580	- .2190	.69
	$-{}^{3}_{4}C^{2}_{4}$	$-{}^{3}_{4}C^{2}_{3}$	$-{}^{3}_{4}C^{2}_{2}$	$-{}^{3}_{4}C^{2}_{1}$	$-{}^{3}_{4}C^{2}_{0}$	$\frac{{}^{3}_{4}E^{2}}{b^2 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i}=\frac{1}{b^3}\sum_{j=0}^{4} {}^{3}_{4}C^{i}_{j}\, y_j + {}^{3}_{4}R^{i} \qquad (i=2)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{3}_{4}C^{2}_{0}$	${}^{3}_{4}C^{2}_{1}$	${}^{3}_{4}C^{2}_{2}$	${}^{3}_{4}C^{2}_{3}$	${}^{3}_{4}C^{2}_{4}$	$\frac{{}^{3}_{4}E^{2}}{b^2f^{(5)}(\xi)}$	
0.70	- 1.00941277	1.85714286	-0.529411765	-0.777777778	0.459459460	-0.2200	0.70
.71	- .983068060	1.81690141	- .508771930	- .785977860	.460916443	- .2210	.71
.72	- .957653630	1.77777778	- .488372093	- .794117647	.462365590	- .2220	.72
.73	- .933127104	1.73972603	- .468208092	- .802197802	.463806970	- .2230	.73
.74	- .909448504	1.70270270	- .448275862	- .810218978	.465240640	- .2240	.74
.75	- .886580086	1.66666667	- .428571429	- .818181818	.466666667	- .2250	.75
.76	- .864486188	1.63157895	- .409090909	- .826086957	.468085107	- .2260	.76
.77	- .843133092	1.59740260	- .389830508	- .833935018	.469496020	- .2270	.77
.78	- .822488900	1.56410256	- .370786517	- .841726619	.470899470	- .2280	.78
.79	- .802523410	1.53164557	- .351955307	- .849462366	.472295513	- .2290	.79
.80	- .783208020	1.50000000	- .333333333	- .857142857	.473684211	- .2300	.80
.81	- .764515610	1.46913580	- .314917127	- .864768683	.475065617	- .2310	.81
.82	- .746420456	1.43902439	- .296703297	- .872340426	.476439791	- .2320	.82
.83	- .728898162	1.40963855	- .278688525	- .879858657	.477806789	- .2330	.83
.84	- .711925536	1.38095238	- .260869565	- .887323944	.479166667	- .2340	.84
.85	- .695480574	1.35294118	- .243243243	- .894736842	.480519481	- .2350	.85
.86	- .679542324	1.32558140	- .225806452	- .902097902	.481865285	- .2360	.86
.87	- .664090894	1.29885057	- .208556150	- .909407666	.483204134	- .2370	.87
.88	- .649107326	1.27272727	- .191489362	- .916666667	.484536082	- .2380	.88
.89	- .634573587	1.24719101	- .174603175	- .923875433	.485861183	- .2390	.89
.90	- .620472490	1.22222222	- .157894737	- .931034483	.487179487	- .2400	.90
.91	- .606787660	1.19780220	- .141361257	- .938144330	.488491049	- .2410	.91
.92	- .593503482	1.17391304	- .125000000	- .945205479	.489795918	- .2420	.92
.93	- .580605063	1.15053763	- .108808290	- .952218430	.491094148	- .2430	.93
.94	- .568078182	1.12765957	- .0927835051	- .959183673	.492385787	- .2440	.94
.95	- .555909274	1.10526316	- .0769230769	- .966101695	.493670886	- .2450	.95
.96	- .544085364	1.08333333	- .0612244897	- .972972973	.494949495	- .2460	.96
.97	- .532594074	1.06185567	- .0456852792	- .979797980	.496221662	- .2470	.97
.98	- .521423552	1.04081633	- .0303030303	- .986577181	.497487437	- .2480	.98
.99	- .510562474	1.02020202	- .0150753769	- .993311037	.498746867	- .2490	.99
1.00	- .500000000	1.00000000	0	-1.00000000	.500000000	- .2500	1.00
1.01	- .489725758	.980198020	.0149253731	-1.00664452	.501246883	- .2510	1.01
1.02	- .479729814	.960784314	.0297029703	-1.01324503	.502487562	- .2520	1.02
1.03	- .470002652	.941747573	.0443349754	-1.01980198	.503722084	- .2530	1.03
1.04	- .460535158	.923076923	.0588235294	-1.02631579	.504950495	- .2540	1.04
1.05	- .451318590	.904761905	.0731707317	-1.03278689	.506172839	- .2550	1.05
1.06	- .442344570	.886792453	.0873786408	-1.03921569	.507389163	- .2560	1.06
1.07	- .433605056	.869158879	.101449275	-1.04560261	.508599509	- .2570	1.07
1.08	- .425092336	.851851852	.115384615	-1.05194805	.509803922	- .2580	1.08
1.09	- .416799006	.834862385	.129186603	-1.05825243	.511002445	- .2590	1.09
1.10	- .408717954	.818181818	.142857143	-1.06451613	.512195122	- .2600	1.10
1.11	- .400842352	.801801802	.156398104	-1.07073955	.513381995	- .2610	1.11
1.12	- .393168336	.785714286	.169811321	-1.07692308	.514563107	- .2620	1.12
1.13	- .385681502	.769911504	.183098592	-1.08306709	.515738499	- .2630	1.13
1.14	- .378383885	.754385965	.196261682	-1.08917197	.516908213	- .2640	1.14
1.15	- .371266955	.739130435	.209302326	-1.09523810	.518072289	- .2650	1.15
1.16	- .364325099	.724137931	.222222222	-1.10126582	.519230769	- .2660	1.16
1.17	- .357552924	.709401709	.235023041	-1.10725552	.520383693	- .2670	1.17
1.18	- .350945229	.694915254	.247706422	-1.11320755	.521531100	- .2680	1.18
1.19	- .344497016	.680672269	.260273973	-1.11912226	.522673031	- .2690	1.19
1.20	- .338203463	.666666667	.272727273	-1.12500000	.523809524	- .2700	1.20
1.21	- .332059932	.652892562	.285067873	-1.13084112	.524940618	- .2710	1.21
1.22	- .326061947	.639344262	.297297297	-1.13664596	.526066351	- .2720	1.22
1.23	- .320205201	.626016260	.309417040	-1.14241486	.527186761	- .2730	1.23
1.24	- .314485536	.612903226	.321428571	-1.14814815	.528301887	- .2740	1.24
1.25	- .308898945	.600000000	.333333333	-1.15384615	.529411765	- .2750	1.25
1.26	- .303441560	.587301587	.345132743	-1.15950920	.530516432	- .2760	1.26
1.27	- .298109654	.574803150	.356828194	-1.16513761	.531615925	- .2770	1.27
1.28	- .292899625	.562500000	.368421053	-1.17073171	.532710280	- .2780	1.28
1.29	- .287808001	.550387597	.379912664	-1.17629179	.533799534	- .2790	1.29
	$-{}^{3}_{4}C^{2}_{4}$	$-{}^{3}_{4}C^{2}_{3}$	$-{}^{3}_{4}C^{2}_{2}$	$-{}^{3}_{4}C^{2}_{1}$	$-{}^{3}_{4}C^{2}_{0}$	$\frac{{}^{3}_{4}E^{2}}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3} \sum_{j=0}^{4} {}^3_4C^i_j \, y_j + {}^3_4R^i \qquad (i=3,1)$$

$r=\frac{a}{b}\frac{a}{c}\frac{a}{d}$	${}^3_4C^3_0$	${}^3_4C^3_1$	${}^3_4C^3_2$	${}^3_4C^3_3$	${}^3_4C^3_4$	$\frac{{}^3_4E^3}{b^2f^{(5)}(\xi)}$	
0.10	16.7574361	-21.0000000	8.45454545	-5.85714286	1.64516129	0.1600	0.10
.11	14.9769279	-19.1818182	8.40540541	-5.84360189	1.64308682	.1610	.11
.12	13.4986868	-17.6666667	8.35714286	-5.83018868	1.64102564	.1620	.12
.13	12.2528046	-16.3846154	8.30973451	-5.81690141	1.63897764	.1630	.13
.14	11.1893520	-15.2857143	8.26315789	-5.80373832	1.63694268	.1640	.14
.15	10.2717191	-14.3333333	8.21739130	-5.79069767	1.63492063	.1650	.15
.16	9.47245266	-13.5000000	8.17241379	-5.77777778	1.63291139	.1660	.16
.17	8.77056288	-12.7647059	8.12820513	-5.76497696	1.63091483	.1670	.17
.18	8.14972810	-12.1111111	8.08474576	-5.75229358	1.62893082	.1680	.18
.19	7.59706576	-11.5263158	8.04201681	-5.73972603	1.62695925	.1690	.19
.20	7.10227272	-11.0000000	8.00000000	-5.72727273	1.62500000	.1700	.20
.21	6.65701100	-10.5238095	7.95867769	-5.71493213	1.62305296	.1710	.21
.22	6.25446100	-10.0909091	7.91803279	-5.70270270	1.62111801	.1720	.22
.23	5.88899130	- 9.69565217	7.87804878	-5.69058296	1.61919505	.1730	.23
.24	5.55591114	- 9.33333333	7.83870968	-5.67857143	1.61728395	.1740	.24
.25	5.25128206	- 9.00000000	7.80000000	-5.66666667	1.61538462	.1750	.25
.26	4.97177326	- 8.69230770	7.76190476	-5.65486726	1.61349693	.1760	.26
.27	4.71454897	- 8.40740741	7.72440945	-5.64317180	1.61162080	.1770	.27
.28	4.47718000	- 8.14285714	7.68750000	-5.63157895	1.60975610	.1780	.28
.29	4.25757354	- 7.89655172	7.65116279	-5.62008734	1.60790274	.1790	.29
.30	4.05391710	- 7.66666667	7.61538462	-5.60869565	1.60606061	.1800	.30
.31	3.86463322	- 7.45161290	7.58015267	-5.59740260	1.60422961	.1810	.31
.32	3.68834272	- 7.25000000	7.54545455	-5.58620690	1.60240964	.1820	.32
.33	3.52383456	- 7.06060606	7.51127820	-5.57510729	1.60060060	.1830	.33
.34	3.37004116	- 6.88235294	7.47761194	-5.56410256	1.59880240	.1840	.34
.35	3.22601784	- 6.71428571	7.44444444	-5.55319149	1.59701492	.1850	.35
.36	3.09092564	- 6.55555556	7.41176471	-5.54237288	1.59523810	.1860	.36
.37	2.96401712	- 6.40540541	7.37956204	-5.53164557	1.59347181	.1870	.37
.38	2.84462424	- 6.26315789	7.34782609	-5.52100840	1.59171598	.1880	.38
.39	2.73214812	- 6.12820513	7.31654676	-5.51046025	1.58997050	.1890	.39
.40	2.62605042	- 6.00000000	7.28571429	-5.50000000	1.58823529	.1900	.40
.41	2.52584592	- 5.87804878	7.25531915	-5.48962655	1.58651026	.1910	.41
.42	2.43109618	- 5.76190476	7.22535211	-5.47933884	1.58479532	.1920	.42
.43	2.34140402	- 5.65116279	7.19580420	-5.46913580	1.58309038	.1930	.43
.44	2.25640892	- 5.54545455	7.16666667	-5.45901640	1.58139535	.1940	.44
.45	2.17578286	- 5.44444444	7.13793103	-5.44897959	1.57971014	.1950	.45
.46	2.09922675	- 5.34782609	7.10958904	-5.43902439	1.57803468	.1960	.46
.47	2.02646742	- 5.25531915	7.08163265	-5.42914980	1.57636888	.1970	.47
.48	1.95725481	- 5.16666667	7.05405405	-5.41935484	1.57471264	.1980	.48
.49	1.89135967	- 5.08163265	7.02684564	-5.40963855	1.57306590	.1990	.49
.50	1.82857143	- 5.00000000	7.00000000	-5.40000000	1.57142857	.2000	.50
.51	1.76869637	- 4.92156863	6.97350993	-5.39043285	1.56980057	.2010	.51
.52	1.71155599	- 4.84615385	6.94736842	-5.38095238	1.56818182	.2020	.52
.53	1.65698554	- 4.77358491	6.92156863	-5.37154150	1.56657224	.2030	.53
.54	1.60483278	- 4.70370370	6.89610390	-5.36220472	1.56497175	.2040	.54
.55	1.55495679	- 4.63636364	6.87096774	-5.35294118	1.56338028	.2050	.55
.56	1.50722697	- 4.57142857	6.84615385	-5.34375000	1.56179775	.2060	.56
.57	1.46152214	- 4.50877193	6.82165605	-5.33463035	1.56022409	.2070	.57
.58	1.41772968	- 4.44827586	6.79746835	-5.32558140	1.55865922	.2080	.58
.59	1.37574486	- 4.38983051	6.77358491	-5.31660232	1.55710306	.2090	.59
.60	1.33547009	- 4.33333333	6.75000000	-5.30769231	1.55555556	.2100	.60
.61	1.29681440	- 4.27868852	6.72670807	-5.29885058	1.55401662	.2110	.61
.62	1.25969290	- 4.22580645	6.70370370	-5.29007634	1.55248619	.2120	.62
.63	1.22402621	- 4.17460317	6.68098160	-5.28136882	1.55096419	.2130	.63
.64	1.18974014	- 4.12500000	6.65853659	-5.27272727	1.54945055	.2140	.64
.65	1.15676518	- 4.07692308	6.63636364	-5.26415095	1.54794521	.2150	.65
.66	1.12503621	- 4.03030303	6.61445783	-5.25563910	1.54644809	.2160	.66
.67	1.09449214	- 3.98507463	6.59281437	-5.24719101	1.54495913	.2170	.67
.68	1.06507561	- 3.94117647	6.57142857	-5.23880597	1.54347826	.2180	.68
.69	1.03673272	- 3.89855072	6.55029586	-5.23048327	1.54200542	.2190	.69
	$-{}^3_4C^i_4$	$-{}^3_4C^i_3$	$-{}^3_4C^i_2$	$-{}^3_4C^i_1$	$-{}^3_4C^i_0$	$\frac{{}^3_4E^i}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}\frac{d}{b}\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3}\sum_{j=0}^{4} {}^3_4C^i_j\, y_j + {}^3_4R^i \qquad (i=3,1)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^3_4C^3_0$	${}^3_4C^3_1$	${}^3_4C^3_2$	${}^3_4C^3_3$	${}^3_4C^3_4$	$\frac{{}^3_4E^3}{b^2f^{(5)}(\xi)}$	
0.70	1.00941277	- 3.85714286	6.52941177	-5.22222222	1.54054054	0.2200	0.70
.71	.983068060	- 3.81690141	6.50877193	-5.21402214	1.53908356	.2210	.71
.72	.957653630	- 3.77777778	6.48837209	-5.20588235	1.53763441	.2220	.72
.73	.933127104	- 3.73970603	6.46820809	-5.19780220	1.53619303	.2230	.73
.74	.909448504	- 3.70270270	6.44827586	-5.18978102	1.53475936	.2240	.74
.75	.886580086	- 3.66666667	6.42857143	-5.18181818	1.53333333	.2250	.75
.76	.864486188	- 3.63157895	6.40909091	-5.17391304	1.53191489	.2260	.76
.77	.843133092	- 3.59740260	6.38983051	-5.16606498	1.53050398	.2270	.77
.78	.822488900	- 3.56410256	6.37078652	-5.15827338	1.52910053	.2280	.78
.79	.802523410	- 3.53164557	6.35195531	-5.15053763	1.52770449	.2290	.79
.80	.783208020	- 3.50000000	6.33333333	-5.14285714	1.52631579	.2300	.80
.81	.764515610	- 3.46913580	6.31491713	-5.13523132	1.52493438	.2310	.81
.82	.746420456	- 3.43902439	6.29670330	-5.12765957	1.52356021	.2320	.82
.83	.728898162	- 3.40963855	6.27868853	-5.12014134	1.52219321	.2330	.83
.84	.711925536	- 3.38095238	6.26086957	-5.11267606	1.52083333	.2340	.84
.85	.695480574	- 3.35294118	6.24324324	-5.10526316	1.51948052	.2350	.85
.86	.679542324	- 3.32558140	6.22580645	-5.09790210	1.51813472	.2360	.86
.87	.664090894	- 3.29885057	6.20855615	-5.09059234	1.51679589	.2370	.87
.88	.649107326	- 3.27272727	6.19148936	-5.08333333	1.51546392	.2380	.88
.89	.634573587	- 3.24719101	6.17460318	-5.07612457	1.51413882	.2390	.89
.90	.620472490	- 3.22222222	6.15789474	-5.06896552	1.51282051	.2400	.90
.91	.606787660	- 3.19780220	6.14136126	-5.06185567	1.51150895	.2410	.91
.92	.593503482	- 3.17391304	6.12500000	-5.05479452	1.51020408	.2420	.92
.93	.580605063	- 3.15053763	6.10880829	-5.04778157	1.50890585	.2430	.93
.94	.568078182	- 3.12765957	6.09278351	-5.04081632	1.50761421	.2440	.94
.95	.555909274	- 3.10526316	6.07692308	-5.03389831	1.50632911	.2450	.95
.96	.544085364	- 3.08333333	6.06122449	-5.02702703	1.50505051	.2460	.96
.97	.532594074	- 3.06185567	6.04568528	-5.02020202	1.50377834	.2470	.97
.98	.521423552	- 3.04081633	6.03030303	-5.01342282	1.50251256	.2480	.98
.99	.510562474	- 3.02020202	6.01507538	-5.00668896	1.50125313	.2490	.99
1.00	.500000000	- 3.00000000	6.00000000	-5.00000000	1.50000000	.2500	1.00
1.01	.489725758	- 2.98019802	5.98507463	-4.99335548	1.49875312	.2510	1.01
1.02	.479729814	- 2.96078431	5.97029703	-4.98675497	1.49751244	.2520	1.02
1.03	.470002652	- 2.94174757	5.95566502	-4.98019802	1.49627792	.2530	1.03
1.04	.460535158	- 2.92307692	5.94117647	-4.97368421	1.49504951	.2540	1.04
1.05	.451318590	- 2.90476190	5.92682927	-4.96721311	1.49382716	.2550	1.05
1.06	.442344570	- 2.88679245	5.91262136	-4.96078431	1.49261084	.2560	1.06
1.07	.433605056	- 2.86915888	5.89855072	-4.95439739	1.49140049	.2570	1.07
1.08	.425092336	- 2.85185185	5.88461538	-4.94805195	1.49019608	.2580	1.08
1.09	.416799006	- 2.83486239	5.87081340	-4.94174757	1.48899755	.2590	1.09
1.10	.408717954	- 2.81818182	5.85714286	-4.93548387	1.48780488	.2600	1.10
1.11	.400842352	- 2.80180180	5.84360190	-4.92926045	1.48661801	.2610	1.11
1.12	.393165636	- 2.78571429	5.83018868	-4.92307692	1.48543689	.2620	1.12
1.13	.385681502	- 2.76991150	5.81690141	-4.91693291	1.48426150	.2630	1.13
1.14	.378383885	- 2.75438596	5.80373832	-4.91082803	1.48309179	.2640	1.14
1.15	.371266955	- 2.73913043	5.79069767	-4.90476190	1.48192771	.2650	1.15
1.16	.364325099	- 2.72413793	5.77777778	-4.89873418	1.48076923	.2660	1.16
1.17	.357552924	- 2.70940171	5.76497696	-4.89274448	1.47961631	.2670	1.17
1.18	.350945229	- 2.69491525	5.75229358	-4.88679245	1.47846890	.2680	1.18
1.19	.344497016	- 2.68067227	5.73972603	-4.88087774	1.47732697	.2690	1.19
1.20	.338203463	- 2.66666667	5.72727273	-4.87500000	1.47619048	.2700	1.20
1.21	.332059932	- 2.65289256	5.71493213	-4.86915888	1.47505938	.2710	1.21
1.22	.326061947	- 2.63934426	5.70270270	-4.86335404	1.47393365	.2720	1.22
1.23	.320205201	- 2.62601626	5.69058296	-4.85758514	1.47281324	.2730	1.23
1.24	.314485536	- 2.61290323	5.67857143	-4.85185185	1.47169811	.2740	1.24
1.25	.308898945	- 2.60000000	5.66666667	-4.84615385	1.47058824	.2750	1.25
1.26	.303441560	- 2.58730159	5.65486726	-4.84049080	1.46948357	.2760	1.26
1.27	.298109654	- 2.57480315	5.64317181	-4.83486239	1.46838408	.2770	1.27
1.28	.292899625	- 2.56250000	5.63157895	-4.82926829	1.46728972	.2780	1.28
1.29	.287808001	- 2.55038760	5.62008734	-4.82370821	1.46620047	.2790	1.29
	$-{}^3_4C^1_4$	$-{}^3_4C^1_3$	$-{}^3_4C^1_2$	$-{}^3_4C^1_1$	$-{}^3_4C^1_0$	$\frac{{}^3_4E^1}{b^2f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

27(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i} = \frac{1}{b^3} \sum_{j=0}^{4} {}^{3}_{4}C^{i}_{j}\, y_j + {}^{3}_{4}R^{i} \qquad (i=4,0)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{3}_{4}C^{4}_{0}$	${}^{3}_{4}C^{4}_{1}$	${}^{3}_{4}C^{4}_{2}$	${}^{3}_{4}C^{4}_{3}$	${}^{3}_{4}C^{4}_{4}$	$\frac{{}^{3}_{4}E^{4}}{b^2 f^{(5)}(\xi)}$	
0.10	60.2723084	-61.0000000	19.3636364	-11.5714286	2.93548387	1.480	0.10
.11	44.9307836	-55.5454545	19.2162162	-11.5308057	2.92926045	1.483	.11
.12	40.4960605	-51.0000000	19.0714286	-11.4905660	2.92307692	1.486	.12
.13	36.7584139	-47.1538462	18.9292035	-11.4507042	2.91693291	1.489	.13
.14	33.5680561	-43.8571429	18.7894737	-11.4112150	2.91082803	1.492	.14
.15	30.8151572	-41.0000000	18.6521739	-11.3720930	2.90476190	1.495	.15
.16	28.4173578	-38.5000000	18.5172414	-11.3333333	2.89873418	1.498	.16
.17	26.3116886	-36.2941176	18.3846154	-11.2949309	2.89274448	1.501	.17
.18	24.4491843	-34.3333333	18.2542373	-11.2568807	2.88679245	1.504	.18
.19	22.7911973	-32.5789474	18.1260504	-11.2191781	2.88087774	1.507	.19
.20	21.3068182	-31.0000000	18.0000000	-11.1818182	2.87500000	1.510	.20
.21	19.9710330	-29.5714286	17.8760331	-11.1447964	2.86915888	1.513	.21
.22	18.7633830	-28.2727273	17.7540984	-11.1081081	2.86335404	1.516	.22
.23	17.6669739	-27.0869565	17.6341463	-11.0717489	2.85758574	1.519	.23
.24	16.6677334	-26.0000000	17.5161290	-11.0357143	2.85185185	1.522	.24
.25	15.7538462	-25.0000000	17.4000000	-11.0000000	2.84615385	1.525	.25
.26	14.9153198	-24.0769231	17.2857143	-10.9646018	2.84049080	1.528	.26
.27	14.1436469	-23.2222222	17.1732284	-10.9295154	2.83486239	1.531	.27
.28	13.4315400	-22.4285714	17.0625000	-10.8947368	2.82926829	1.534	.28
.29	12.7727206	-21.6896552	16.9534884	-10.8602620	2.82370821	1.537	.29
.30	12.1617613	-21.0000000	16.8461539	-10.8260870	2.81818182	1.540	.30
.31	11.5938997	-20.3548387	16.7404580	-10.7922078	2.81268882	1.543	.31
.32	11.0650282	-19.7500000	16.6363636	-10.7586207	2.80722892	1.546	.32
.33	10.5715037	-19.1818182	16.5338346	-10.7253219	2.80180180	1.549	.33
.34	10.1101235	-18.6470588	16.4328358	-10.6923077	2.79640719	1.552	.34
.35	9.67805352	-18.1428571	16.3333333	-10.6595145	2.79104478	1.555	.35
.36	9.27277692	-17.6666667	16.2352941	-10.6271186	2.78571429	1.558	.36
.37	8.89205136	-17.2162162	16.1386861	-10.5949367	2.78041543	1.561	.37
.38	8.53387272	-16.7894737	16.0434783	-10.5630252	2.77514793	1.564	.38
.39	8.19644436	-16.3846154	15.9496403	-10.5313808	2.76991150	1.567	.39
.40	7.87815126	-16.0000000	15.8571428	-10.5000000	2.76470588	1.570	.40
.41	7.57753776	-15.6341463	15.7659574	-10.4688797	2.75953079	1.573	.41
.42	7.29328854	-15.2857143	15.6760563	-10.4380165	2.75438597	1.576	.42
.43	7.02421206	-14.9534894	15.5874126	-10.4074074	2.74927114	1.579	.43
.44	6.76922676	-14.6363636	15.5000000	-10.3770492	2.74418605	1.582	.44
.45	6.52734857	-14.3333333	15.4137931	-10.3469388	2.73913043	1.585	.45
.46	6.29768026	-14.0434783	15.3287671	-10.3170732	2.73410405	1.588	.46
.47	6.07940225	-13.7659574	15.2448980	-10.2874494	2.72910663	1.591	.47
.48	5.87176442	-13.5000000	15.1621622	-10.2580645	2.72413793	1.594	.48
.49	5.67407900	-13.2448980	15.0805369	-10.2289157	2.71919771	1.597	.49
.50	5.48571429	-13.0000000	15.0000000	-10.2000000	2.71428571	1.600	.50
.51	5.30608911	-12.7647058	14.9205298	-10.1713147	2.70940171	1.603	.51
.52	5.13466797	-12.5384615	14.8421053	-10.1428571	2.70454545	1.606	.52
.53	4.97095663	-12.3207547	14.7647059	-10.1146245	2.69971671	1.609	.53
.54	4.81449834	-12.1111111	14.6883117	-10.0866142	2.69491525	1.612	.54
.55	4.66487037	-11.9090909	14.6129032	-10.0588235	2.69014085	1.615	.55
.56	4.52168092	-11.7142857	14.5384615	-10.0312500	2.68539326	1.618	.56
.57	4.38456642	-11.5263158	14.4649682	-10.0038911	2.68067227	1.621	.57
.58	4.25318905	-11.3448276	14.3924051	- 9.97674419	2.67597765	1.624	.58
.59	4.12723457	-11.1694915	14.3207547	- 9.94980695	2.67130919	1.627	.59
.60	4.00641026	-11.0000000	14.2500000	- 9.92307692	2.66666667	1.630	.60
.61	3.89044321	-10.8360656	14.1801242	- 9.89655172	2.66204986	1.633	.61
.62	3.77907869	-10.6774194	14.1111111	- 9.87022901	2.65745856	1.636	.62
.63	3.67207864	-10.5238095	14.0429448	- 9.84410646	2.65289256	1.639	.63
.64	3.56922041	-10.3750000	13.9756098	- 9.81818182	2.64835165	1.642	.64
.65	3.47029554	-10.2307692	13.9090909	- 9.79245283	2.64383562	1.645	.65
.66	3.37510863	-10.0909091	13.8433735	- 9.76691729	2.63934426	1.648	.66
.67	3.28347642	- 9.95522388	13.7784431	- 9.74157303	2.63487738	1.651	.67
.68	3.19522683	- 9.82352941	13.7142857	- 9.71641791	2.63043478	1.654	.68
.69	3.11019815	- 9.69565217	13.6508876	- 9.69144981	2.62601626	1.657	.69
	$-{}^{3}_{4}C^{0}_{4}$	$-{}^{3}_{4}C^{0}_{3}$	$-{}^{3}_{4}C^{0}_{2}$	$-{}^{3}_{4}C^{0}_{1}$	$-{}^{3}_{4}C^{0}_{0}$	$\frac{{}^{3}_{4}E^{0}}{b^2 f^{(5)}(\xi)}$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^3y)_{x=x_i}=\frac{1}{b^3}\sum_{j=0}^{4}{}^{3}_{4}C^{i}_{j}\,y_j+{}^{3}_{4}R^{i}\qquad(i=4,0)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}^{3}_{4}C^{4}_{0}$	${}^{3}_{4}C^{4}_{1}$	${}^{3}_{4}C^{4}_{2}$	${}^{3}_{4}C^{4}_{3}$	${}^{3}_{4}C^{4}_{4}$	$\frac{{}^{3}_{4}C^{4}}{b^2f^{(3)}(\xi)}$	
0.70	3.02823832	− 9.57142857	13.5882353	− 9.66666667	2.62162162	1.660	0.70
.71	2.94920418	− 9.45070423	13.5263158	− 9.64206642	2.61725067	1.663	.71
.72	2.87296089	− 9.33333333	13.4651163	− 9.61764706	2.61290323	1.666	.72
.73	2.79938131	− 9.21917808	13.4046243	− 9.59340659	2.60857909	1.669	.73
.74	2.72834551	− 9.10810811	13.3448276	− 9.56934307	2.60427808	1.672	.74
.75	2.65974026	− 9.00000000	13.2857143	− 9.54545455	2.60000000	1.675	.75
.76	2.59345856	− 8.89473684	13.2272727	− 9.52173913	2.59574468	1.678	.76
.77	2.52939928	− 8.79220779	13.1694915	− 9.49819495	2.59151194	1.681	.77
.78	2.46746670	− 8.69230769	13.1123596	− 9.47482014	2.58730159	1.684	.78
.79	2.40757023	− 8.59493671	13.0558659	− 9.45161290	2.58311346	1.687	.79
.80	2.34962406	− 8.50000000	13.00000000	− 9.42857143	2.57894737	1.690	.80
.81	2.29354683	− 8.40740741	12.9447514	− 9.40569395	2.57480315	1.693	.81
.82	2.23926137	− 8.31707317	12.8901099	− 9.38297872	2.57068068	1.696	.82
.83	2.18669449	− 8.22891566	12.8360656	− 9.36042403	2.56657963	1.699	.83
.84	2.13577661	− 8.14285714	12.7826087	− 9.33802817	2.56250000	1.702	.84
.85	2.08644172	− 8.05882353	12.7297297	− 9.31578947	2.55844156	1.705	.85
.86	2.03862697	− 7.97674419	12.6774194	− 9.29370629	2.55440415	1.708	.86
.87	1.99227268	− 7.89655172	12.6256685	− 9.27177700	2.55038760	1.711	.87
.88	1.94732198	− 7.81818182	12.5744681	− 9.25000000	2.54639175	1.714	.88
.89	1.90372076	− 7.74157303	12.5238095	− 9.22837370	2.54241645	1.717	.89
.90	1.86141747	− 7.66666667	12.4736842	− 9.20689655	2.53846154	1.720	.90
.91	1.82036298	− 7.59340659	12.4240838	− 9.18556701	2.53452685	1.723	.91
.92	1.78051045	− 7.52173913	12.3750000	− 9.16438356	2.53061225	1.726	.92
.93	1.74181519	− 7.45161290	12.3264249	− 9.14334471	2.52671756	1.729	.93
.94	1.70423455	− 7.38297872	12.2783505	− 9.12244898	2.52284265	1.732	.94
.95	1.66772782	− 7.31578947	12.2307692	− 9.10169492	2.51898734	1.735	.95
.96	1.63225609	− 7.25000000	12.1836735	− 9.08108108	2.51515152	1.738	.96
.97	1.59778222	− 7.18556701	12.1370558	− 9.06060606	2.51133501	1.741	.97
.98	1.56427066	− 7.12244898	12.0909091	− 9.04026846	2.50753769	1.744	.98
.99	1.53168742	− 7.06060606	12.0452261	− 9.02006689	2.50375940	1.747	.99
1.00	1.50000000	− 7.00000000	12.0000000	− 9.00000000	2.50000000	1.750	1.00
1.01	1.46917727	− 6.94059406	11.9552239	− 8.98006645	2.49625935	1.753	1.01
1.02	1.43918944	− 6.88235294	11.9108911	− 8.96026490	2.49253731	1.756	1.02
1.03	1.41000796	− 6.82524272	11.8669951	− 8.94059406	2.48883375	1.759	1.03
1.04	1.38160547	− 6.76923077	11.8235294	− 8.92105263	2.48514852	1.762	1.04
1.05	1.35395577	− 6.71428571	11.7804878	− 8.90163934	2.48148148	1.765	1.05
1.06	1.32703371	− 6.66037736	11.7378641	− 8.88235294	2.47783257	1.768	1.06
1.07	1.30081517	− 6.60747664	11.6956522	− 8.86319218	2.47420147	1.771	1.07
1.08	1.27527701	− 6.55555556	11.6538462	− 8.84415584	2.47058824	1.774	1.08
1.09	1.25039702	− 6.50458716	11.6124402	− 8.82524272	2.46699267	1.777	1.09
1.10	1.22615386	− 6.45454545	11.5714286	− 8.80645161	2.46341463	1.780	1.10
1.11	1.20252706	− 6.40540541	11.5308057	− 8.78778135	2.45985401	1.783	1.11
1.12	1.17949691	− 6.35714286	11.4905660	− 8.76923077	2.45631068	1.786	1.12
1.13	1.15704451	− 6.30973451	11.4507042	− 8.75079872	2.45278450	1.789	1.13
1.14	1.13515164	− 6.26315789	11.4112150	− 8.73248408	2.44927536	1.792	1.14
1.15	1.11380086	− 6.21739130	11.3720930	− 8.71428571	2.44578313	1.795	1.15
1.16	1.09297530	− 6.17241379	11.3333333	− 8.69620253	2.44230769	1.798	1.16
1.17	1.07265877	− 6.12820513	11.2949309	− 8.67823344	2.43884892	1.801	1.17
1.18	1.05283569	− 6.08474576	11.2568807	− 8.66037736	2.43540670	1.804	1.18
1.19	1.03349105	− 6.04201681	11.2191781	− 8.64263323	2.43198091	1.807	1.19
1.20	1.01461039	− 6.00000000	11.1818182	− 8.62500000	2.42857143	1.810	1.20
1.21	.996179794	− 5.95867769	11.1447964	− 8.60747664	2.42517815	1.813	1.21
1.22	.978185841	− 5.91803279	11.1081081	− 8.59006211	2.42180095	1.816	1.22
1.23	.960615603	− 5.87804878	11.0717489	− 8.57275542	2.41843972	1.819	1.23
1.24	.943456607	− 5.83870968	11.0357143	− 8.55555556	2.41509434	1.822	1.24
1.25	.926696834	− 5.80000000	11.0000000	− 8.53846154	2.41176471	1.825	1.25
1.26	.901324679	− 5.76190476	10.9646018	− 8.52147239	2.40845070	1.828	1.26
1.27	.894328962	− 5.72440945	10.9295154	− 8.50458716	2.40515222	1.831	1.27
1.28	.878698876	− 5.68750000	10.8947368	− 8.48780488	2.40186916	1.834	1.28
1.29	.863424004	− 5.65116279	10.8602620	− 8.47112462	2.39860140	1.837	1.29
	$-{}^{3}_{4}C^{0}_{4}$	$-{}^{3}_{4}C^{0}_{3}$	$-{}^{3}_{4}C^{0}_{2}$	$-{}^{3}_{4}C^{0}_{1}$	$-{}^{3}_{4}C^{0}_{0}$	$\frac{{}^{3}_{4}C^{0}}{b^2f^{(3)}(\xi)}$	$r=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^4y)_{x=x_i} = \frac{1}{b^4} \sum_{j=0}^{4} {}_4^4C_j^i \, y_i + {}_4^4R^i \qquad (i=0,1,2,3,4)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}_4^4C_0^i$	${}_4^4C_1^i$	${}_4^4C_2^i$	${}_4^4C_3^i$
0.10	33.5148722	-40.0000000	10.9090909	-5.71428571
.11	29.9538558	-36.3636364	10.8108108	-5.68720379
.12	26.9973736	-33.3333333	10.7142857	-5.66037736
.13	24.5056092	-30.7692308	10.6194690	-5.63380282
.14	22.3787040	-28.5714286	10.5263158	-5.60747664
.15	20.5434382	-26.6666667	10.4347826	-5.58139535
.16	18.9449052	-25.0000000	10.3448276	-5.55555556
.17	17.5411258	-23.5294118	10.2564103	-5.52995392
.18	16.2994562	-22.2222222	10.1694915	-5.50458716
.19	15.1941315	-21.0526316	10.0840336	-5.47945205
.20	14.2045454	-20.0000000	10.0000000	-5.45454545
.21	13.3140220	-19.0476190	9.91735537	-5.42986425
.22	12.5089220	-18.1818182	9.83606557	-5.40540541
.23	11.7779826	-17.3913043	9.75609756	-5.38116592
.24	11.1118223	-16.6666667	9.67741935	-5.35714286
.25	10.5025641	-16.0000000	9.60000000	-5.33333333
.26	9.94354652	-15.3846154	9.52380952	-5.30973451
.27	9.42909794	-14.8148148	9.44881890	-5.28634361
.28	8.95436000	-14.2857143	9.37500000	-5.26315789
.29	8.51514708	-13.7931034	9.30232558	-5.24017467
.30	8.10783420	-13.3333333	9.23076923	-5.21739130
.31	7.72926644	-12.9032258	9.16030534	-5.19480519
.32	7.37668544	-12.5000000	9.09090909	-5.17241379
.33	7.04766912	-12.1212121	9.02255639	-5.15021459
.34	6.74008232	-11.7647059	8.95522388	-5.12820513
.35	6.45203568	-11.4285714	8.88888889	-5.10638298
.36	6.18185128	-11.1111111	8.82352941	-5.08474576
.37	5.92803424	-10.8108108	8.75912409	-5.06329114
.38	5.68924848	-10.5263158	8.69565217	-5.04201681
.39	5.46429624	-10.2564103	8.63309353	-5.02092050
.40	5.25210084	-10.0000000	8.57142857	-5.00000000
.41	5.05169184	- 9.75609756	8.51063830	-4.97925311
.42	4.86219236	- 9.52380952	8.45070423	-4.95867769
.43	4.68280804	- 9.30232558	8.39160839	-4.93827160
.44	4.51281784	- 9.09090909	8.33333333	-4.91803279
.45	4.35156571	- 8.88888889	8.27586207	-4.89795918
.46	4.19845351	- 8.69565217	8.21917808	-4.87804878
.47	4.05293484	- 8.51063830	8.16326531	-4.85829960
.48	3.91450962	- 8.33333333	8.10810811	-4.83870968
.49	3.78271933	- 8.16326531	8.05369128	-4.81927711
.50	3.65714286	- 8.00000000	8.00000000	-4.80000000
.51	3.53739274	- 7.84313725	7.94701987	-4.78087649
.52	3.42311198	- 7.69230769	7.89473684	-4.76190476
.53	3.31397108	- 7.54716981	7.84313725	-4.74308300
.54	3.20966556	- 7.40740741	7.79220779	-4.72440945
.55	3.10991358	- 7.27272727	7.74193548	-4.70588235
.56	3.01445394	- 7.14285714	7.69230769	-4.68750000
.57	2.92304428	- 7.01754386	7.64331210	-4.66926070
.58	2.83545937	- 6.89655172	7.59493671	-4.65116279
.59	2.75148971	- 6.77966102	7.54716981	-4.63320463
.60	2.67094017	- 6.66666667	7.50000000	-4.61538462
.61	2.59362881	- 6.55737705	7.45341615	-4.59770115
.62	2.51938579	- 6.45161290	7.40740741	-4.58015267
.63	2.44805242	- 6.34920635	7.36196319	-4.56273764
.64	2.37948028	- 6.25000000	7.31707317	-4.54545455
.65	2.31353036	- 6.15384615	7.27272727	-4.52830189
.66	2.25007242	- 6.06060606	7.22891566	-4.51127820
.67	2.18898428	- 5.97014925	7.18562874	-4.49438202
.68	2.13015122	- 5.88235294	7.14285714	-4.47761194
.69	2.07346544	- 5.79710145	7.10059172	-4.46096654
	${}_4^4C_4^i$	${}_4^4C_3^i$	${}_4^4C_2^i$	${}_4^4C_1^i$

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^4y)_{x=x_i} = \frac{1}{b^4} \sum_{j=0}^{4} {}^4_4C^i_j \, y_j + {}^4_4R^i \qquad (i=0,1,2,3,4)$$

${}^4_4C^i_4$	${}^4_4E^0 / bf^{(5)}(\xi)$	${}^4_4E^1 / bf^{(5)}(\xi)$	${}^4_4E^2 / bf^{(5)}(\xi)$	${}^4_4E^3 / bf^{(5)}(\xi)$	${}^4_4E^4 / bf^{(5)}(\xi)$	
1.29032258	-1.280	-1.180	-0.1800	0.820	1.820	0.10
1.28617363	-1.288	-1.178	- .1780	.822	1.822	.11
1.28205128	-1.296	-1.176	- .1760	.824	1.824	.12
1.27795527	-1.304	-1.174	- .1740	.826	1.826	.13
1.27388535	-1.312	-1.172	- .1720	.828	1.828	.14
1.26984127	-1.320	-1.170	- .1700	.830	1.830	.15
1.26582278	-1.328	-1.168	- .1680	.832	1.832	.16
1.26182965	-1.336	-1.166	- .1660	.834	1.834	.17
1.25786164	-1.344	-1.164	- .1640	.836	1.836	.18
1.25391850	-1.352	-1.162	- .1620	.838	1.838	.19
1.25000000	-1.360	-1.160	- .1600	.840	1.840	.20
1.24610592	-1.368	-1.158	- .1580	.842	1.842	.21
1.24223602	-1.376	-1.156	- .1560	.844	1.844	.22
1.23839009	-1.384	-1.154	- .1540	.846	1.846	.23
1.23456790	-1.392	-1.152	- .1520	.848	1.848	.24
1.23076923	-1.400	-1.150	- .1500	.850	1.850	.25
1.22699387	-1.408	-1.148	- .1480	.852	1.852	.26
1.22324159	-1.416	-1.146	- .1460	.854	1.854	.27
1.21951220	-1.424	-1.144	- .1440	.856	1.856	.28
1.21580547	-1.432	-1.142	- .1420	.858	1.858	.29
1.21212121	-1.440	-1.140	- .1400	.860	1.860	.30
1.20845921	-1.448	-1.138	- .1380	.862	1.862	.31
1.20481928	-1.456	-1.136	- .1360	.864	1.864	.32
1.20120120	-1.464	-1.134	- .1340	.866	1.866	.33
1.19760479	-1.472	-1.132	- .1320	.868	1.868	.34
1.19402985	-1.480	-1.130	- .1300	.870	1.870	.35
1.19047619	-1.488	-1.128	- .1280	.872	1.872	.36
1.18694362	-1.496	-1.126	- .1260	.874	1.874	.37
1.18343195	-1.504	-1.124	- .1240	.876	1.876	.38
1.17994100	-1.512	-1.122	- .1220	.878	1.878	.39
1.17647059	-1.520	-1.120	- .1200	.880	1.880	.40
1.17302053	-1.528	-1.118	- .1180	.882	1.882	.41
1.16959064	-1.536	-1.116	- .1160	.884	1.884	.42
1.16618076	-1.544	-1.114	- .1140	.886	1.886	.43
1.16279070	-1.552	-1.112	- .1120	.888	1.888	.44
1.15942029	-1.560	-1.110	- .1100	.890	1.890	.45
1.15606936	-1.568	-1.108	- .1080	.892	1.892	.46
1.15273775	-1.576	-1.106	- .1060	.894	1.894	.47
1.14942529	-1.584	-1.104	- .1040	.896	1.896	.48
1.14613181	-1.592	-1.102	- .1020	.898	1.898	.49
1.14285714	-1.600	-1.100	- .1000	.900	1.900	.50
1.13960114	-1.608	-1.098	- .0980	.902	1.902	.51
1.13636364	-1.616	-1.096	- .0960	.904	1.904	.52
1.13314448	-1.624	-1.094	- .0940	.906	1.906	.53
1.12994350	-1.632	-1.092	- .0920	.908	1.908	.54
1.12676056	-1.640	-1.090	- .0900	.910	1.910	.55
1.12359551	-1.648	-1.088	- .0880	.912	1.912	.56
1.12044818	-1.656	-1.086	- .0860	.914	1.914	.57
1.11731844	-1.664	-1.084	- .0840	.916	1.916	.58
1.11420613	-1.672	-1.082	- .0820	.918	1.918	.59
1.11111111	-1.680	-1.080	- .0800	.920	1.920	.60
1.10803324	-1.688	-1.078	- .0780	.922	1.922	.61
1.10497238	-1.696	-1.076	- .0760	.924	1.924	.62
1.10192837	-1.704	-1.074	- .0740	.926	1.926	.63
1.09890110	-1.712	-1.072	- .0720	.928	1.928	.64
1.09589041	-1.720	-1.070	- .0700	.930	1.930	.65
1.09289617	-1.728	-1.068	- .0680	.932	1.932	.66
1.08991826	-1.736	-1.066	- .0660	.934	1.934	.67
1.08695652	-1.744	-1.064	- .0640	.936	1.936	.68
1.08401084	-1.752	-1.062	- .0620	.938	1.938	.69
${}^4_4C^i_0$	${}^4_4E^4 / bf^{(5)}(\xi)$	${}^4_4E^3 / bf^{(5)}(\xi)$	${}^4_4E^2 / bf^{(5)}(\xi)$	${}^4_4E^1 / bf^{(5)}(\xi)$	${}^4_4E^0 / bf^{(5)}(\xi)$	$\rho = \frac{d}{c} = \frac{d}{b} = \frac{d}{a}$

29(b)

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FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^4y)_{x=x_i}=\frac{1}{b^4}\sum_{j=0}^{4}{}_4C_j^i\,y_i+{}_4R^i \qquad (i=0,1,2,3,4)$$

$r=\frac{a}{b}=\frac{a}{c}=\frac{a}{d}$	${}_4C_0^i$	${}_4C_1^i$	${}_4C_2^i$	${}_4C_3^i$
0.70	2.01882554	− 5.71428571	7.05882353	−4.44444444
.71	1.96613612	− 5.63380282	7.01754386	−4.42804428
.72	1.91530726	− 5.55555556	6.97674419	−4.41176471
.73	1.86625421	− 5.47945205	6.93641618	−4.39560440
.74	1.81889701	− 5.40540541	6.89655172	−4.37956204
.75	1.77316017	− 5.33333333	6.85714286	−4.36363636
.76	1.72897238	− 5.26315789	6.81818182	−4.34782609
.77	1.68626618	− 5.19480519	6.77966102	−4.33212996
.78	1.64497780	− 5.12820513	6.74157303	−4.31654676
.79	1.60504682	− 5.06329114	6.70391061	−4.30107527
.80	1.56641604	− 5.00000000	6.66666667	−4.28571429
.81	1.52903122	− 4.93827160	6.62983425	−4.27046263
.82	1.49284091	− 4.87804878	6.59340659	−4.25531915
.83	1.45779632	− 4.81927711	6.55737705	−4.24028269
.84	1.42385107	− 4.76190476	6.52173913	−4.22535211
.85	1.39096115	− 4.70588235	6.48648649	−4.21052632
.86	1.35908465	− 4.65116279	6.45161290	−4.19580420
.87	1.32818179	− 4.59770115	6.41711230	−4.18118467
.88	1.29821465	− 4.54545455	6.38297872	−4.16666667
.89	1.26914717	− 4.49438202	6.34920635	−4.15224913
.90	1.24094498	− 4.44444444	6.31578947	−4.13793103
.91	1.21357532	− 4.39560440	6.28272251	−4.12371134
.92	1.18700696	− 4.34782609	6.25000000	−4.10958904
.93	1.16121012	− 4.30107527	6.21761658	−4.09556314
.94	1.13615636	− 4.25531915	6.18556701	−4.08163265
.95	1.11181855	− 4.21052632	6.15384615	−4.06779661
.96	1.08817073	− 4.16666667	6.12244898	−4.05405405
.97	1.06518815	− 4.12371134	6.09137056	−4.04040404
.98	1.04284710	− 4.08163265	6.06060606	−4.02684564
.99	1.02112495	− 4.04040404	6.03015075	−4.01337792
1.00	1.00000000	− 4.00000000	6.00000000	−4.00000000
1.01	.979451516	− 3.96039604	5.97014925	−3.98671096
1.02	.959459628	− 3.92156863	5.94059406	−3.97350993
1.03	.940005304	− 3.88349515	5.91133005	−3.96039604
1.04	.921070316	− 3.84615385	5.88235294	−3.94736842
1.05	.902637180	− 3.80952381	5.85365854	−3.93442623
1.06	.884689140	− 3.77358491	5.82524272	−3.92156863
1.07	.867210112	− 3.73831776	5.79710145	−3.90879479
1.08	.850184672	− 3.70370370	5.76923077	−3.89610390
1.09	.833598012	− 3.66972477	5.74162679	−3.88349515
1.10	.817435908	− 3.63636364	5.71428571	−3.87096774
1.11	.801684704	− 3.60360360	5.68720739	−3.85852090
1.12	.786331272	− 3.57142857	5.66037736	−3.84615385
1.13	.771363004	− 3.53982301	5.63380282	−3.83386581
1.14	.756767769	− 3.50877193	5.60747664	−3.82165605
1.15	.742533910	− 3.47826087	5.58139535	−3.80952381
1.16	.728650199	− 3.44827586	5.55555556	−3.79746835
1.17	.715105847	− 3.41880342	5.52995392	−3.78548896
1.18	.701890458	− 3.38983051	5.50458716	−3.77358491
1.19	.688994031	− 3.36134454	5.47945206	−3.76175549
1.20	.676406926	− 3.33333333	5.45454545	−3.75000000
1.21	.664119863	− 3.30578512	5.42986425	−3.73831776
1.22	.652123894	− 3.27868852	5.40540541	−3.72670807
1.23	.640410402	− 3.25203252	5.38116592	−3.71517028
1.24	.628971071	− 3.22580645	5.35714286	−3.70370370
1.25	.617797889	− 3.20000000	5.33333333	−3.69230769
1.26	.606883120	− 3.17460317	5.30973451	−3.68098160
1.27	.596219308	− 3.14960630	5.28634361	−3.66972477
1.28	.585799251	− 3.12500000	5.26315789	−3.65853659
1.29	.575616002	− 3.10077519	5.24017467	−3.64741641
	${}_4C_4^i$	${}_4C_3^i$	${}_4C_2^i$	${}_4C_1^i$

29(c)

FIVE-POINT DIFFERENTIATION COEFFICIENTS

$$(D^4y)_{x=x_i}=\frac{1}{b^4}\sum_{j=0}^{4}{}_4C_j^i\, y_j + {}_4R^i \qquad (i\ 0,1,2,3,4)$$

${}_4C_4^i$	${}_4E^0_4 / bf^{(5)}(\xi)$	${}_4E^1_4 / bf^{(5)}(\xi)$	${}_4E^2_4 / bf^{(5)}(\xi)$	${}_4E^3_4 / bf^{(5)}(\xi)$	${}_4E^4_4 / bf^{(5)}(\xi)$	
1.08108108	-1.760	-1.060	-0.0600	0.940	1.940	0.70
1.07816712	-1.768	-1.058	- .0580	.942	1.942	.71
1.07526882	-1.776	-1.056	- .0560	.944	1.944	.72
1.07238606	-1.784	-1.054	- .0540	.946	1.946	.73
1.06951872	-1.792	-1.052	- .0520	.948	1.948	.74
1.06666667	-1.800	-1.050	- .0500	.950	1.950	.75
1.06382979	-1.808	-1.048	- .0480	.952	1.952	.76
1.06100795	-1.816	-1.046	- .0460	.954	1.954	.77
1.05820106	-1.824	-1.044	- .0440	.956	1.956	.78
1.05540897	-1.832	-1.042	- .0420	.958	1.958	.79
1.05263158	-1.840	-1.040	- .0400	.960	1.960	.80
1.04986877	-1.848	-1.038	- .0380	.962	1.962	.81
1.04712042	-1.856	-1.036	- .0360	.964	1.964	.82
1.04438642	-1.864	-1.034	- .0340	.966	1.966	.83
1.04166667	-1.872	-1.032	- .0320	.968	1.968	.84
1.03896104	-1.880	-1.030	- .0330	.970	1.970	.85
1.03626943	-1.888	-1.028	- .0280	.972	1.972	.86
1.03359173	-1.896	-1.026	- .0260	.974	1.974	.87
1.03092784	-1.904	-1.024	- .0240	.976	1.976	.88
1.02827763	-1.912	-1.022	- .0220	.978	1.978	.89
1.02564103	-1.920	-1.020	- .0200	.980	1.980	.90
1.02301790	-1.928	-1.018	- .0180	.982	1.982	.91
1.02040816	-1.936	-1.016	- .0160	.984	1.984	.92
1.01781170	-1.944	-1.014	- .0140	.986	1.986	.93
1.01522843	-1.952	-1.012	- .0120	.988	1.988	.94
1.02165823	-1.960	-1.010	- .0100	.990	1.990	.95
1.01010101	-1.968	-1.008	- .0080	.992	1.992	.96
1.00755668	-1.976	-1.006	- .0060	.994	1.994	.97
1.00502513	-1.984	-1.004	- .0040	.996	1.996	.98
1.00250627	-1.992	-1.002	- .0020	.998	1.998	.99
1.00000000	-2.000	-1.000	[a]0	1.000	2.000	1.00
.997506234	-2.008	- .998	.0020	1.002	2.002	1.01
.995024876	-2.016	- .996	.0040	1.004	2.004	1.02
.992555831	-2.024	- .994	.0060	1.006	2.006	1.03
.990099010	-2.032	- .992	.0080	1.008	2.008	1.04
.987654321	-2.040	- .990	.0100	1.010	2.010	1.05
.985221675	-2.048	- .988	.0120	1.012	2.012	1.06
.982800983	-2.056	- .986	.0140	1.014	2.014	1.07
.980392157	-2.064	- .984	.0160	1.016	2.016	1.08
.977995110	-2.072	- .982	.0180	1.018	2.018	1.09
.975609756	-2.080	- .980	.0200	1.020	2.020	1.10
.973236010	-2.088	- .978	.0220	1.022	2.022	1.11
.970873786	-2.096	- .976	.0240	1.024	2.024	1.12
.968523002	-2.104	- .974	.0260	1.026	2.026	1.13
.966183575	-2.112	- .972	.0280	1.028	2.028	1.14
.963855422	-2.120	- .970	.0300	1.030	2.030	1.15
.961538462	-2.128	- .968	.0320	1.032	2.032	1.16
.959232614	-2.136	- .966	.0340	1.034	2.034	1.17
.956937799	-2.144	- .964	.0360	1.036	2.036	1.18
.954653938	-2.152	- .962	.0380	1.038	2.038	1.19
.952380952	-2.160	- .960	.0400	1.040	2.040	1.20
.950118765	-2.168	- .958	.0420	1.042	2.042	1.21
.947867299	-2.176	- .956	.0440	1.044	2.044	1.22
.945626478	-2.184	- .954	.0460	1.046	2.046	1.23
.943396226	-2.192	- .952	.0480	1.048	2.048	1.24
.941176471	-2.200	- .950	.0500	1.050	2.050	1.25
.938967136	-2.208	- .948	.0520	1.052	2.052	1.26
.936768150	-2.216	- .946	.0540	1.054	2.054	1.27
.934579439	-2.224	- .944	.0560	1.056	2.056	1.28
.932400932	-2.232	- .942	.0580	1.058	2.058	1.29
${}_4C_4^i$	$-{}_4E^4_4 / bf^{(5)}(\xi)$	$-{}_4E^3_4 / bf^{(5)}(\xi)$	$-{}_4E^2_4 / bf^{(5)}(\xi)$	$-{}_4E^1_4 / bf^{(5)}(\xi)$	$-{}_4E^0_4 / bf^{(5)}(\xi)$	$p=\frac{d}{c}=\frac{d}{b}=\frac{d}{a}$

[a]The next order remainder term is $-0.1667\, b^2 \left[f^{(6)}(x)\right]_{x=x_2}$

29(a)

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