REPORT 1033

COMPARISON BETWEEN THEORY AND EXPERIMENT FOR WINGS AT SUPERSONIC SPEEDS [1]

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SUMMARY

In this paper, a critical comparison is made between experimental and theoretical results for the aerodynamic characteristics of wings at supersonic flight speeds. As a preliminary, a brief, nonmathematical review is given of the basic assumptions and general findings of supersonic wing theory in two and three dimensions. Published data from two-dimensional pressure-distribution tests are then used to illustrate the effects of fluid viscosity and to assess the accuracy of linear theory as compared with the more exact theories which are available in the two-dimensional case. Finally, an account is presented of an NACA study of the over-all force characteristics of three-dimensional wings at supersonic speed. In this study, the lift, pitching moment, and drag characteristics of several families of wings of varying plan form and section were measured in the wind tunnel and compared with values predicted by the three-dimensional linear theory. The regions of agreement and disagreement between experiment and theory are noted and discussed.

INTRODUCTION

The aerodynamics of wings at supersonic flight speeds is currently the subject of much research and discussion. As a result of many recent investigations, based on the earlier work of Prandtl, Ackeret, Busemann, and von Kármán, the theory of the subject is well advanced, both as applied to airfoil sections in two-dimensional flow and to complete, three-dimensional wings. Experimental knowledge is, by contrast, considerably less extensive, particularly with regard to the three-dimensional case. There are, however, sufficient experimental data in hand to permit a reasonably systematic comparison between theory and experiment. It is the purpose of this paper to present such a comparison insofar as the current availability of experimental results will allow.

THEORETICAL CONSIDERATIONS

To provide background for those who are unacquainted with the fundamentals of supersonic wing theory, it may be useful to review briefly the assumptions and findings of work in this field. (For a more complete discussion of the theory and a bibliography of pertinent references, the reader is referred to the Tenth Wright Brothers Lecture by Theodore von Kármán, reference 1.)

In the solution of problems in supersonic wing theory, the following assumptions are usually made concerning the flow field which surrounds the wing:

(a) The fluid medium is continuous and homogeneous.

(b) The fluid has the thermodynamic characteristics of a perfect gas with constant specific heats.

(c) Viscosity and thermal conductivity are vanishingly small.

(d) External forces (such as gravity) are negligible.

For flight at ordinary altitudes and air temperatures, the most drastic of these assumptions is that of vanishingly small viscosity and thermal conductivity. This assumption allows the effects of fluid friction and heat transfer to be disregarded except as they are necessary to explain the existence of shock waves and vortices within the flow field. The assumption thus retains the essential features of supersonic flow as it is known to occur away from the immediate vicinity of the wing surface. It results, however, in the omission of the friction drag and of any changes in pressure distribution caused by growth or separation of the boundary layer.

On the basis of the foregoing assumptions, it is possible to obtain explicit relations for the sudden changes in flow which occur across a shock wave as well as a differential equation for the gradual changes which take place in the regions between such waves. When expressed with the geometrical coordinates as the independent variables, the differential equation governing the flow in the region between shock waves is nonlinear. It is therefore difficult to apply rigorously to most problems of practical interest.

Fortunately, in the special case of an airfoil section in a two-dimensional supersonic stream, results can be obtained with a high degree of mathematical rigor despite the nonlinearity of the governing differential equation. For reasons of mathematical practicality, it has been usual to restrict the solutions to instances in which the local velocity in the flow field is everywhere supersonic. This limits the solutions to airfoils with a sharp leading edge and to angles of attack and free-stream Mach numbers such that the shock wave from the leading edge is attached to the airfoil and the flow on

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the downstream side of the wave is supersonic. (It has also been customary to neglect the rotation of the fluid particles which will exist aft of the leading-edge wave in those cases in which the wave is curved, although this approximation is not essential.) Within these restrictions, section characteristics can be calculated to a high degree of precision for sections of even appreciable thickness. The method of computation reduces in practice to a stepwise application of the known relations for the compression through a shock wave and for the expansion around a convex corner. The procedure has therefore been termed the "shock-expansion" method (see, for example, reference 2). For rapid calculations, more restricted methods, such as Ackeret's linear theory (references 3 and 4) and Busemann's second-order theory (references 5, 6, and 7), can be obtained by means of series approximation to the complete equations for the shock wave and the expansion.

In the more practical case of a complete, three-dimensional wing, the general mathematical problem is forbiddingly complex, and it is necessary to simplify the nonlinear differential equation at the outset in order to obtain a solution. To accomplish this, it is assumed that the local velocity at all points in the flow field differs only slightly in magnitude and direction from the velocity of the undisturbed stream. This implies, in effect, that the thickness, camber, and angle of attack of the wing are small. With this approximation, the complete, nonlinear differential equation reduces, through the omission of terms of higher than the first order in the flow disturbances, to a linear equation which can be solved by established mathematical methods. On the basis of this equation, an extensive body of theory has been formulated covering a wide range of practical wings. For the present it will suffice to mention certain general concepts and results of this theory. Examples of specific calculations will be presented in the course of the later discussion.

A fundamental result of the linear theory, well known by now, is the concept of the Mach cone. According to this concept, the effect of a given disturbance in a uniform supersonic stream is felt only within the interior of a circular cone with vertex located at the point of the disturbance and axis extending downstream parallel to the original flow. The geometry of the cone is determined by the requirement that the component of free-stream velocity normal to the surface of the cone is equal to the speed of sound in the undisturbed stream. It follows that the semivertex angle of the cone is a function of the free-stream Mach number only. These considerations apply not only to the effects of an isolated disturbance but to the region of influence of each disturbance in a distributed system as well.

The concept of the Mach cone has immediate implications with regard to the aerodynamic problems of three-dimensional wings. This is illustrated in figure 1, which shows certain features of the flow over three flat lifting surfaces of representative plan form. In the case of the rectangular plan form A, for example, it follows from the concept of the Mach cone that, to a first approximation, the effects of the finite span are confined to the regions of the wing lying within the cone from the leading edge of each tip. The flow over

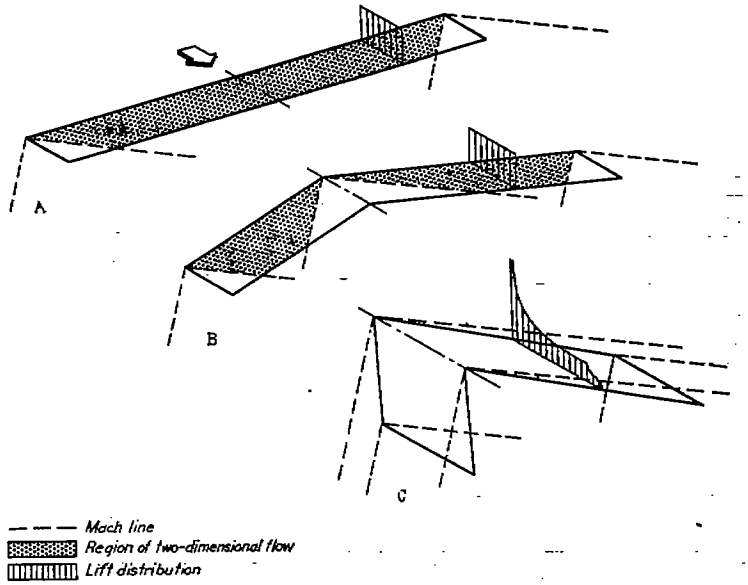


FIGURE 1.—Flat lifting surfaces in supersonic flow (linear theory).

the remainder of the wing (shown shaded) is identical with the two-dimensional flow over a wing of infinite span. On the moderately swept plan form B, the flow over the shaded regions is, by the same reasoning, unaffected by the presence of either the tips or root of the wing. Within these regions the flow can be treated as essentially two-dimensional by evaluating the velocity and the deflection angle in the direction normal to the leading edge. On the highly swept plan form C, all of the wing is within the fields of influence of the root and tips, and no regions of purely two-dimensional flow are to be expected.

Carrying these considerations a step farther, we may also examine the effect which the relationship between the plan form and the Mach cones has upon the chordwise lift distribution for the three wings. On both wings A and B, where the leading edge lies ahead of the Mach cones from the corners of the plan form, the Mach number of the component of free-stream velocity normal to the leading edge is greater than one. For reasons just examined, the lift distribution at the spanwise stations for which it is shown will be the same as the distribution over a flat lifting surface in a two-dimensional supersonic stream. Characteristic features of this distribution are that the intensity of lift at the leading edge is finite and has zero gradient in the chordwise direction. On plan form C, where the leading edge is swept behind the Mach cone, the Mach number of the flow component normal to the leading edge is less than one. It develops from the theory that in this case the lift distribution near the edge resembles the theoretical distribution predicted by linear theory for a flat lifting surface in a purely subsonic flow—that is, the lift intensity tends to an infinite value at the leading edge and drops off rapidly along the chord toward the trailing edge.

The foregoing differences in lift distribution provide one example of a general principle, the significance of which was first noted by R. T. Jones (reference 8). This principle, which arises throughout the study of wings by the linear theory, can be stated as follows: When the component of free-stream velocity normal to a wing element (i. e., lead-

ing edge, ridge line, or trailing edge) is greater than the speed of sound, the theoretical flow in the vicinity of the element has the essential character of the two-dimensional supersonic flow about an element of the same geometric type; similarly, when the velocity component normal to the element is less than the speed of sound, the theoretical local flow resembles that which prevails in the two-dimensional subsonic case. Because of the utility of this general result, it has become customary to describe the wing elements themselves as either "supersonic" or "subsonic." To determine which category an element occupies, it is obviously sufficient, as in figure 1, to note whether it is swept ahead of or behind the Mach cone. It is apparent that a wing element may change from one classification to the other as its orientation relative to the Mach cone is changed. This can be brought about by variation in either the free-stream Mach number or the geometry of the wing.

As a result of the inherent differences in the flow about supersonic and subsonic elements, theoretical calculations for three-dimensional wings indicate marked and interesting changes in the flight characteristics with changes in Mach number or wing geometry. By studying these effects, wing shapes can be found which afford optimum aerodynamic characteristics for a given flight condition. The results of such studies, indeed, provide a valuable guidance to the aircraft designer. In anticipation of the experimental results to be presented later, however, a word of caution is in order here. As exemplified in figure 1, the differences in theoretical pressure distribution between a supersonic and subsonic element may be characterized by large differences in chordwise pressure gradient. These differences may, in a real, viscous medium, give rise to corresponding differences in boundary-layer flow and hence to aerodynamic effects which are beyond the scope of the inviscid theory. As a result, the true variation of the wing characteristics with change in Mach number or wing geometry may be considerably different from that predicted by the theory. The later experimental results with regard to the drag of triangular wings supply an excellent example of such an effect.

In anticipation of the experimental data, it should also be pointed out that the concepts and results of the linear theory, based as they are upon the assumption of small disturbances, constitute only a first-order approximation to the truth even for the supposedly inviscid gas. When disturbances of appreciable magnitude are considered, the previous concept of a Mach cone traversing the entire flow field is no longer tenable. On the contrary, a given disturbance in a supersonic stream is then confined, not to the interior of a cone, but to the interior of some more complex surface whose shape and position depend upon the magnitude of the disturbance as well as upon other conditions in the general flow field. It follows that the regions of influence of a wing tip or wing root are not strictly as shown in figure 1, and the previous distinction between a supersonic and subsonic element cannot be applied without qualification. The ideas of the linear theory with regard to pressure propagation, therefore, should not be taken literally nor should deductions based upon them be accepted without reservation.

It is apparent from these brief theoretical considerations that calculations by the linear theory may be expected to fall short of the truth for two primary reasons. These are

(a) the omission from the theory of all viscous phenomena, and

(b) the theoretical assumption that the flow disturbances are small.

The importance of these approximations cannot be assessed at present from purely theoretical knowledge. Some insight is provided, however, by the available experimental results.

PRESSURE-DISTRIBUTION MEASUREMENTS IN TWO DIMENSIONS

It is desirable to begin the comparison between theory and experiment by examining some typical pressure-distribution results for an airfoil section in a two-dimensional supersonic stream. Because of the availability in the two-dimensional case of theories of greater accuracy than the linear theory, it is possible here to distinguish between the effects of viscosity and the effects of the terms neglected through the assumption of small disturbances.

A typical two-dimensional pressure distribution is given in figure 2, which shows the calculated and measured results for a 10-percent-thick, symmetrical, biconvex section at a Mach number of 2.13 and an angle of attack of 10°. The local pressure coefficient is plotted as a function of the chordwise position on the airfoil, positive values being plotted below the horizontal axis and negative values above. The theoretical pressure distributions given by the linear and shock-

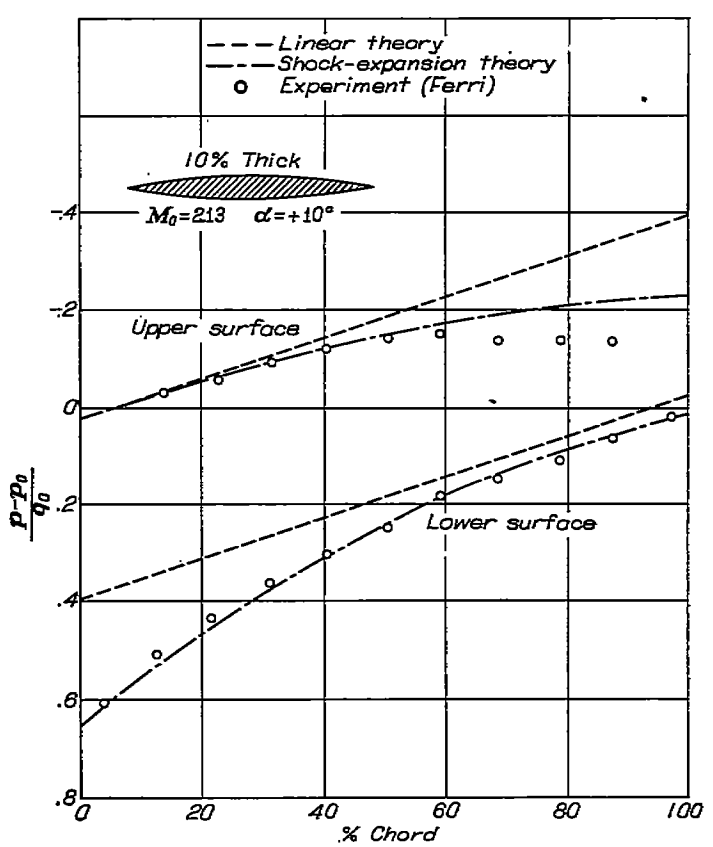


FIGURE 2.—Pressure distribution for symmetrical biconvex section.

expansion theories are shown by curves as noted. The individual circles indicate experimental points obtained from the results of Ferri (reference 9).

The data of figure 2 show that considerable accuracy is gained by going from the linear to the shock-expansion theory. Over most of the airfoil section, the linear theory predicts the correct sense for the pressure gradient, but the quantitative agreement between the curve given by this theory and the experimental points is poor compared with the excellent check given by the shock-expansion method. Over the rear 40 percent of the upper surface, neither of the theories agrees with the trend exhibited by experiment.

The discrepancy between the theoretical pressure distributions calculated by the linear and shock-expansion theories is of importance primarily for its effect upon the chordwise distribution of lift. Examination of figure 2 reveals that the total lift of the section, as approximated by the area between the curves for the upper and lower surfaces, is given almost identically by the two theories. This illustrates the fact that in the two-dimensional case the higher-order terms neglected in the linear theory have little effect upon the over-all lift of the section. They do, however, serve to concentrate the lift farther forward on the chord than the linear theory would predict. This effect is essentially a consequence of the airfoil thickness and diminishes as the thickness is reduced.

The failure of even the shock-expansion theory to predict the pressure variation over the rear part of the upper surface is due to shock-wave, boundary-layer interaction (reference 9). In the idealized, inviscid fluid, the two-dimensional flow over a lifting airfoil at supersonic speeds is characterized by an oblique compression wave originating on the upper surface at the trailing edge. In the real, viscous fluid, this flow pattern is modified by an interaction between the oblique wave and the viscous boundary layer on the airfoil surface. The boundary layer separates from the upper surface some distance forward of the trailing edge, with the formation of a weak compression wave at the separation point and a consequent increase in pressure between this point and the trailing edge. There is, as a result, a noticeable loss of lift over the rear of the airfoil.

The foregoing results, of course, imply certain deviations of the true aerodynamic coefficients from the curves predicted by the linear theory. For the reasons outlined, the higher-order pressure effects neglected in the linear theory have little influence upon the lift-curve slope, although they do result in a relatively forward shift of the center of pressure (or aerodynamic center). The interaction between the trailing shock wave and the viscous boundary layer acts both to decrease the lift-curve slope slightly and to displace the center of pressure still farther forward. Viscous friction, the effects of which are not visible in the pressure distribution, tends to increase the true drag relative to the calculated value, though this tendency is opposed here by the unpredicted increase in pressure near the trailing edge as the result of the shock-wave, boundary-layer interaction. All of these effects are apparent in the available force-test data for airfoils in two-dimensional flow (references 9 and 10). As will be seen, they are also observed in the results for three-dimensional wings, at least for those cases in which the wing elements are predominantly supersonic.

FORCE TESTS IN THREE DIMENSIONS

The discussion to this point has been confined to theoretical considerations and to a comparison between theoretical and experimental results for a typical airfoil section in two-dimensional flow. The remainder of the paper will be concerned with a more general comparison between theory and experiment for complete, three-dimensional wings.

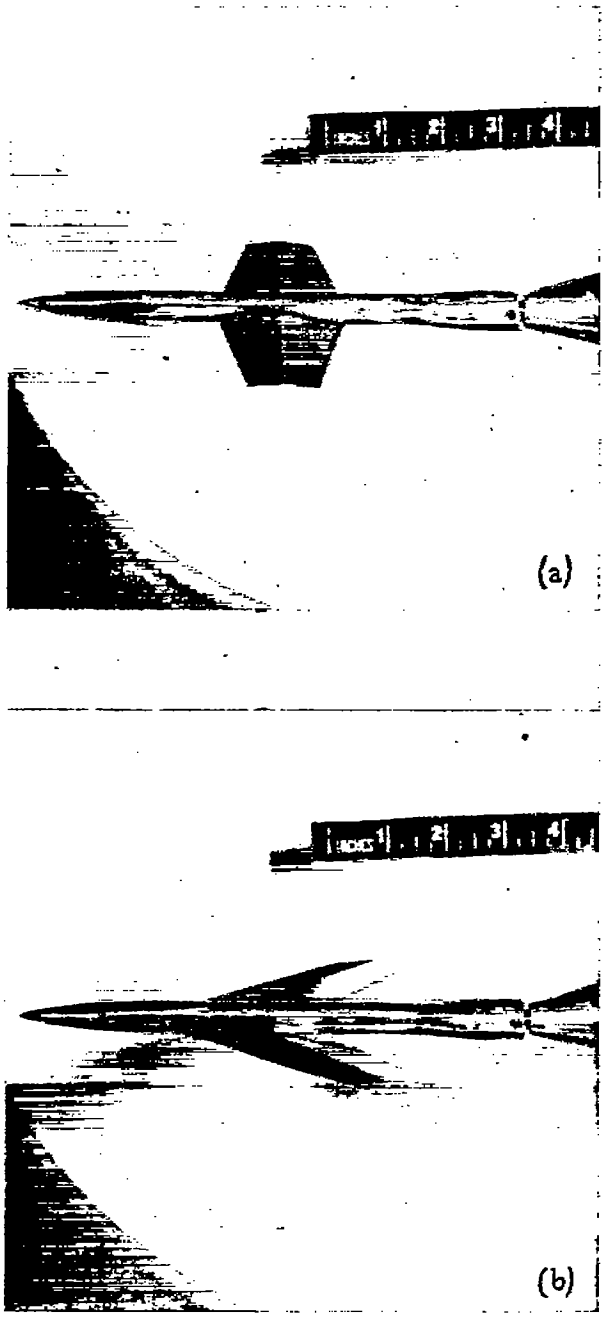
The results upon which this comparison is based were obtained in 1946 as part of an investigation of wing characteristics conducted at the Ames Aeronautical Laboratory of the NACA. The portion of the general investigation to be discussed here was concerned with force tests at supersonic speeds of approximately 30 wing models chosen to cover a wide range of geometric variables and to include examples with both supersonic and subsonic wing elements. The experimental work was performed in the Ames 1- by 3-foot supersonic wind tunnel No. 1, which is a continuous-flow, closed-return tunnel of approximately 10,000 horsepower.[2]

The wing models were supported in the wind tunnel on a slender body of revolution mounted directly ahead of a three-component, strain-gage balance as shown in figure 3. For the majority of the models, the airfoil section taken in the streamwise direction was a 5-percent-thick isosceles triangle, that is, a triangle with maximum thickness of 5 percent located at midchord. This cambered section was chosen primarily for ease of construction. The models were made of hardened, ground tool steel with the leading and trailing edges maintained sharp to less than a 0.001-inch radius, except for certain tests in which the leading edge was purposely rounded. The support body, which was the same for all models, was kept as small as possible consistent with the requirement that it could be used with a wide range of plan forms.

Because of the presence of the support body, the experimental results to be presented apply, strictly speaking, to wing-body combinations rather than to the wings alone. The theoretical curves are, on the other hand, for simple, isolated wings. A detailed examination of the interference problem indicates that, for the particular body used here, the effects of the body are small insofar as the lift and pitching moment are concerned. The influence on minimum drag may, however, be considerable. The measured values of the minimum drag coefficient must therefore be regarded as of primarily qualitative significance in comparison with theory.

Because of limitations of time and space, it is obviously impossible in a paper of this kind to discuss more than a small portion of the results obtained in the investigation. The data presented will therefore be chosen primarily for their value in illustrating certain general ideas or typical conclusions. This approach will result in the omission of

[2] As with most experimental investigations, many people contributed to the final results of the study. Particular credit is due, however, to Jack N. Nielsen, Milton D. Van Dyke, and Frederick H. Matteson, who participated in the analysis of the results, to Robert T. Madden, Richard Scherrer, and John A. Blackburn, who conducted the wind-tunnel tests, and to Albert G. Oswald, who was in charge of the wind-tunnel instrumentation.



(a) Unswept wing.
(b) Swept wing.

FIGURE 3.—Typical wing models mounted on support body in Ames 1- by 3-foot supersonic wind tunnel.

many interesting items dear to the heart of the experimentalist, but it is hoped that an adequate over-all picture of the significant results will emerge. In all of the figures presented, the aerodynamic coefficients will be referred to the plan form area of the wing, including that portion of the plan form enclosed by the support body. All of the results are for a free-stream Mach number of 1.53 and a test Reynolds number of 0.75 million based upon the mean geometric chord of the wing. Unless stated otherwise, it may be assumed that the results were obtained using models with the cambered, isosceles-triangle section previously described.

In the discussion of the results, it is convenient to consider first the lift and pitching moment, since these characteristics depend primarily upon the distribution of normal pressure over the surface of the wing. The consideration of drag, which depends upon the frictional forces as well, will be deferred until later.

LIFT AND PITCHING MOMENT

According to the linear theory, the lift and pitching-moment curves for any given wing are each a straight line. At a given Mach number, the slope of the line depends solely upon the plan form of the wing and is independent of the camber and thickness. The intercept—that is, the angle of zero lift or the moment at zero lift—is a function of both the camber and the plan form, but is independent of the wing thickness. Only the slope of the curves will be discussed here, since this is the characteristic of greatest practical importance.

Lift-curve slope.—The nature of the agreement between theory and experiment with regard to the lift-curve slope for unswept wings is illustrated in figure 4. Here $dC_L/d\alpha$ is plotted as a function of aspect ratio for a series of four unswept wings having a common taper ratio of 0.5. The wing corresponding to each test point is indicated by a small sketch, which shows also the trace of the Mach cones from the forwardmost point of the wing. On this and later figures, the variation predicted by the linear theory is shown over as wide a range as is practicable on the basis of existing computational methods.

The agreement between theory and experiment in figure 4 is seen to be excellent over the entire range of aspect ratios. The exact coincidence for aspect ratios from 2 to 6 is, in fact, too good to be absolutely true. It appears likely that the secondary effects of viscosity and support-body interference, which must certainly be present in some degree, are completely compensating for these wings. The decrease in lift-curve slope observed both experimentally and theoretically at the low aspect ratios is caused by a loss of lift within the Mach cones which originate at the leading edge of the wing tips. As the aspect ratio is reduced, a greater and greater percentage of the plan form is included within these Mach

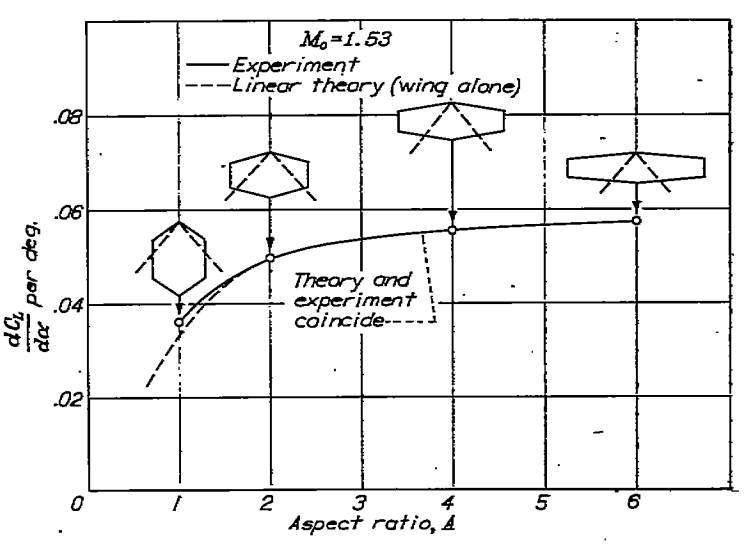


FIGURE 4.—Effect of aspect ratio on lift-curve slope.

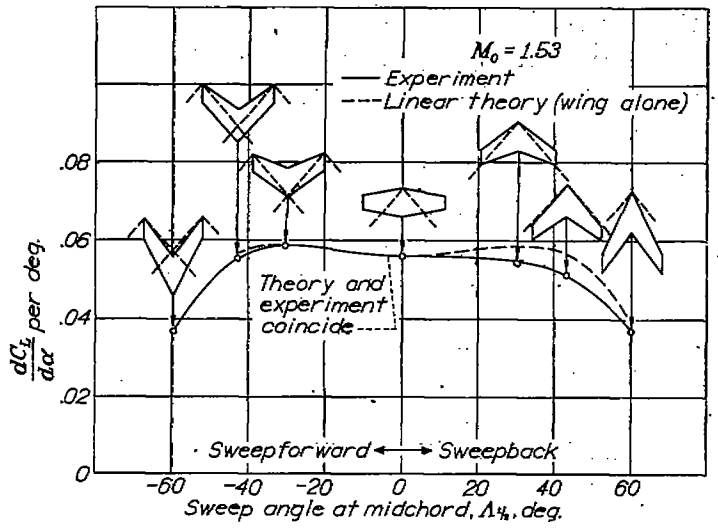


FIGURE 5.—Effect of sweep on lift-curve slope.

cones, with a resulting decrease in the lifting effectiveness of the wing.

The effect of wing sweep on the slope of the lift curve is illustrated in figure 5. Here $dC_L/d\alpha$ is shown as a function of the sweep angle at the midchord line for a series of seven wings also of taper ratio 0.5. The unswept wing of this series is identical with the aspect ratio 4 wing of the previous figure. In the design of the swept wings, the aspect ratio was made to decrease as the cosine of the angle of sweep, since wings of constant aspect ratio did not appear structurally feasible. The sweep angles were chosen to provide representative plan forms with both supersonic and subsonic leading and trailing edges. The wing of 43° sweepback was designed to have its leading edge coincident with the Mach cone, which has a sweep angle of 49.2° at the test Mach number of 1.53. Since the sweep angle of these wings is specified at the midchord line, a given swept-forward wing can be obtained from the corresponding swept-back wing by a simple reversal of the direction of motion.

The agreement between theory and experiment in figure 5 is almost exact over the range of sweep angles from 0° to 43° sweepforward, the forwardmost limit of the theoretical results. For all of the swept-back wings, the experimental slopes fall consistently below the theoretical values by from 8 to 10 percent. In both the swept-back and swept-forward direction, the experimental results exhibit a marked reduction in $dC_L/d\alpha$ as the edges of the plan form are swept increasingly farther behind the Mach cone. This trend is predicted by the theoretical curve in the swept-back case and would undoubtedly be confirmed for the swept-forward wings if complete theoretical results were available.[3] It is interesting to note, incidentally, that the 43° swept-back wing, which has its leading edge coincident with the Mach cone, shows no departure from the general trend of the experimental results.

[3] For the range of sweep angles from 43° to 60° sweepback, the shape of the theoretical curve is somewhat approximate. Strictly speaking, small discontinuities in the slope of the curve would be expected at approximately 43° and 55° where the leading edge and trailing edge of the plan form coincide, respectively, with the Mach cone. No attempt was made to determine these discontinuities, the theoretical curve being faired smoothly through the available calculated points.

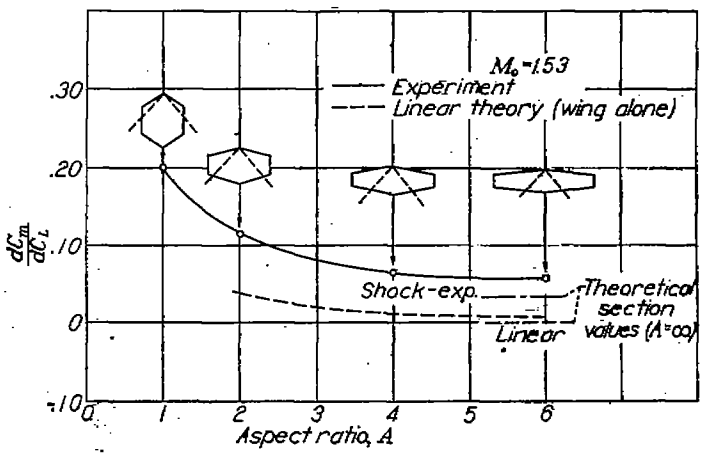


FIGURE 6.—Effect of aspect ratio on moment-curve slope.

For the range of sweep angles between ±43°, the theoretical curve of figure 5 is exactly symmetrical about the vertical axis. This means that, within this range, the theoretical lift-curve slope of a plan form of the present series is unchanged by a reversal of the direction of motion. Similar result has been obtained by several authors for other, more general classes of wings (see, for example, references 11 and 12), though the limits of generality have not, to the writer's knowledge, been completely established.[4] The observed departure of the experimental results from the theoretical symmetry may be due to differences in aeroelastic deformation between corresponding swept-forward and swept-back wings or to asymmetry in the effects of other secondary factors such as viscosity and support body interference.

To summarize, we may say that the agreement between experiment and linear theory with regard to the lift-curve slope of three-dimensional wings is satisfactory for most practical purposes. In view of the situation previously observed in the two-dimensional case, however, it cannot be assumed that agreement in the integrated lift implies complete agreement in the details of the lift distribution.

Moment-curve slope.—Further indication that the details of the flow over the wings are, as in the two-dimensional case, somewhat different from the predictions of the linear theory is given by the pitching-moment data. Figure 6 shows the moment-curve slope as a function of aspect ratio for the series of unswept wings previously discussed. The moment coefficient is here taken about the centroid of plan-form area, with the mean aerodynamic chord as the reference length. The moment-curve slope is thus an approximate measure of the displacement of the aerodynamic center of the wing forward of the centroid of area, expressed as a fraction of the mean aerodynamic chord.

It can be seen from figure 6 that the linear theory predicts a progressively forward displacement of the aerodynamic center as the aspect ratio is reduced. As in the case of the

[4] Since the present paper was written, the theoretical result observed here has been established with complete generality with regard to plan form by Clinton E. Brown of The Langley Aeronautical Laboratory of the NACA. (See Brown, Clinton E.: The Reversibility Theorem for Thin Airfoils in Subsonic and Supersonic Flow. NACA TN 1044, 1940.) According to Brown's proof, which is based upon previous work by Max M. Munk, the theoretical lift-curve slope of a given wing is, to the first order, invariant with respect to a reversal of the direction of motion, irrespective of the Mach number or shape of the plan form.

lift-curve slope, this variation is due to the loss of lift which occurs over the rear portion of the wing within the Mach cones from the tips. The trend of the experimental values is in agreement with the theoretical curve, but the forward displacement is uniformly greater than the theory predicts. The reason for this discrepancy becomes apparent if we imagine the wing series of figure 6 to be extended to indefinitely high aspect ratios. In the limit of infinite aspect ratio, the flow over the wing would be purely two-dimensional, and the theoretical characteristics would be simply those of the wing section. For the present isosceles-triangle section, the values of dC_m/dC_L given by the linear and shock-expansion theories are as indicated by the two horizontal lines to the right. The theoretical curve for the finite-span wings, of course, approaches the linear section value as an asymptote. If only nonviscous effects were important in the experiments, the measured curve would be expected to approach the section value predicted by the shock-expansion method. The fact that it seems to approach an asymptote above this latter value is consistent with the occurrence of shock-wave, boundary-layer interaction near the supersonic trailing edge as previously observed in the two-dimensional results (fig. 2). We may thus infer that the discrepancy between experiment and linear theory over the entire range of aspect ratios is due to a combination of both higher-order pressure effects and fluid viscosity.

The effect of sweep on the moment-curve slope is shown in figure 7 for the same series of wings used before. It is apparent that here experiment and theory agree neither quantitatively nor qualitatively. For the unswept wing, the observed discrepancy can be accounted for as explained in connection with figure 6. The disagreement in the variation with angle of sweep is, however, difficult to reconcile on the basis of present knowledge. In general, the effects of boundary-layer separation may be expected to have a major influence on the moment characteristics of swept wings, particularly in those cases in which the wing elements are predominately subsonic. The possible importance of the higher-order pressure effects should not be overlooked, however. It can be shown from quite general considerations that the calculation by the linear theory of the aerodynamic-center position for any given wing is subject to a possible error of the same order of magnitude as the percent thickness of the airfoil section. For this reason, the development of a reasonably general, second-order wing theory may prove essential to a complete understanding of the pitching-moment problem.

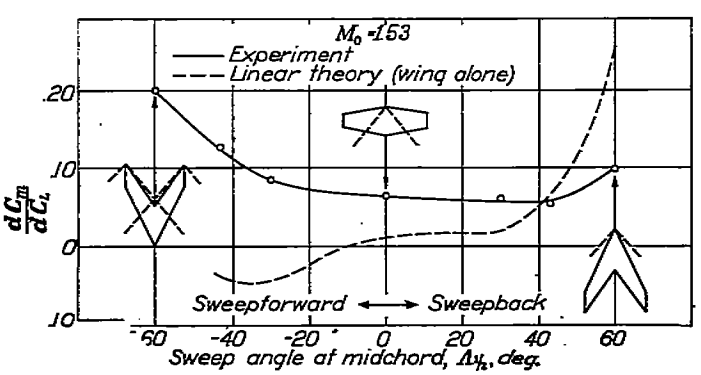


FIGURE 7.—Effect of sweep on moment-curve slope.

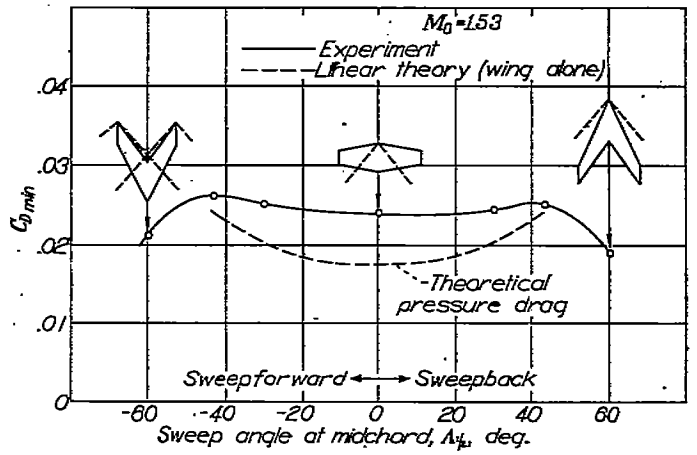


FIGURE 8.—Effect of sweep on minimum drag.

DRAG

The calculation of wing drag by the linear theory leads to a parabolic curve of drag versus lift. The value of the minimum drag coefficient depends, for a given Mach number, upon the thickness, camber, and plan form of the wing, while the lift coefficient at which the minimum occurs is a function of the camber and plan form. The rise in drag as the lift coefficient departs from that for minimum drag depends, according to the linear theory, upon the geometry of the plan form only.

Minimum drag.—A typical illustration of the effect of change in plan form on the minimum drag is given in figure 8, which shows the variation in minimum drag coefficient for the previous series of swept wings. The theoretical curve shown is for the pressure drag only—that is, no attempt has been made to estimate the skin friction. Because of the mathematical complications introduced by camber when the edges of the wing are subsonic, it was not practicable here to extend the theoretical curve beyond 43° in either direction. Within these limits, the theoretical drag increases with increasing sweep. Extension of the curve to higher angles of sweep would be expected to show a marked decrease in the calculated drag, similar to the well-known results for uncambered wings swept behind the Mach cone.

The experimental curve of figure 8 follows the general trend indicated by theory. As the sweep increases from zero in either direction, the measured drag first rises to a maximum in the vicinity of the Mach cone and then decreases markedly with further increase in sweep. The large decrease in drag obtained by sweeping the wing behind the Mach cone has been observed by numerous investigators and need not be enlarged upon here. What is more interesting in the present results is the failure of the experimental values to rise as rapidly as does the theoretical curve in the lower range of sweep angles. For the wings of 0° and ±30° sweep, the displacement of the experimental points above the theoretical curve is consistent with a reasonable allowance for skin friction and support-body interference. For the wings of ±43° sweep, however, the experimental values are almost

coincident with the theoretical. This result suggests that the linear theory may be overly pessimistic regarding wing drag when the Mach number normal to the wing elements is near unity. Support for this conjecture is found in the work of Hilton and Pruden (reference 10), who report a similar situation in two-dimensional tests of an airfoil section at moderately supersonic speeds. It is likely that in both instances the results are due to transonic effects which are beyond the scope of the linear theory.

The symmetry of the curves of figure 8 is also worthy of note. It has been shown by several authors (see, for example, references 1 and 12) that, to the order of accuracy of the linear theory, the minimum pressure drag of a wing of any plan form is unchanged by a reversal of the direction of motion, provided the wing section is without camber. For cambered wings, the corresponding drag theorem is probably less general with regard to plan form, though, as in the case of the lift-curve slope, the limits of generality have not been defined. For the present wings, reversibility is readily proven over the range of sweep angles between ±43°. As a result, the theoretical curve of figure 8 is, like the corresponding curve for $dC_L/d\alpha$ in figure 5, exactly symmetrical over this interval. In spite of the theoretical result, however, the almost perfect symmetry of the experimental curve of figure 8 comes as somewhat of a surprise. It might be expected that secondary differences between corresponding swept-forward and swept-back wings would cause an asymmetry here akin to that observed in the experimental values of lift-curve slope.

The most interesting results with regard to drag, however, are concerned with the effects of thickness distribution on the minimum drag of triangular wings. At about the time the present study was beginning, theoretical results by Puckett appeared (reference 13) which indicated that the minimum pressure drag of an uncambered triangular wing with a subsonic leading edge could be held to a relatively low value by proper location of the position of maximum thickness. To check these results, two triangular wings of aspect ratio 2 were included in the present study. Both wings had an uncambered double-wedge section with a thickness ratio of 5 percent. In one case the maximum thickness was located at midchord, in the other at a position 20 percent of the chord aft of the leading edge.

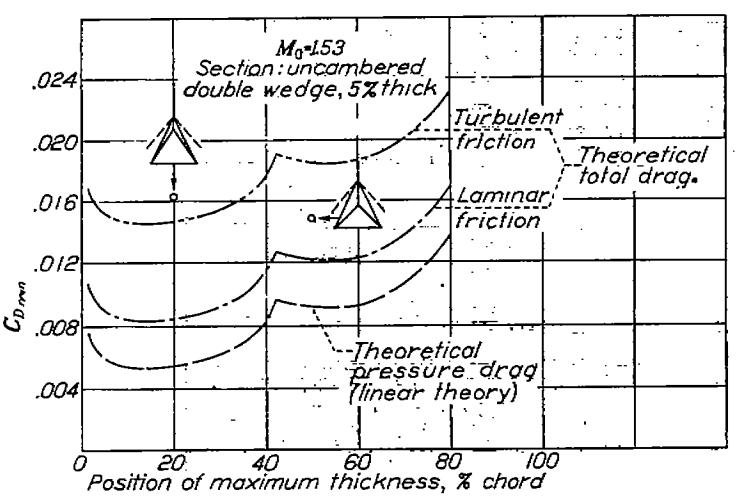
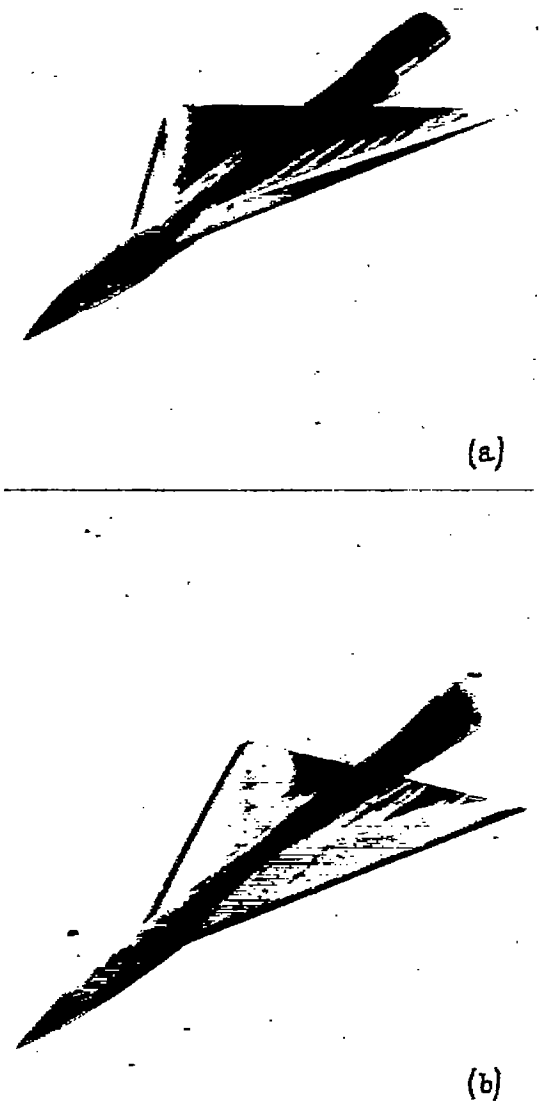


FIGURE 9.—Effect of position of maximum thickness on minimum drag of triangular wings.

The findings for these wings are summarized in figure 9, which shows the theoretical and experimental values of the minimum drag coefficient plotted as a function of the position of maximum thickness. The curve of theoretical pressure drag, which is representative of Puckett's results, is divided into two parts by a sharp break in slope, located in this instance at 42 percent of the chord. For points to the right of this break, the ridge line defined by the position of maximum thickness is supersonic, and the flow around the ridge resembles the supersonic flow around a convex corner. Under these conditions, there is little pressure recovery over the rear of the wing, and the drag is relatively high. For points to the left of the break, the ridge line is subsonic, and the local flow is of the characteristically subsonic type. Under these conditions, the pressure recovery over the rear of the wing is considerable, and the drag is correspondingly reduced. For the wings under consideration, the net result of moving the maximum thickness forward from the 50-percent to the 20-percent station is to reduce the computed pressure-drag coefficient from 0.0092 to 0.0054. Unfortunately, the measured values of the minimum drag, indicated by the two small circles, do not follow the theoretical trend. The apparent effect of the forward displacement is, in fact, to increase the drag slightly.

When this result was first noted, the experimental data were suspected of being in error. Repeated tests, however, gave identical results. It was next thought that support-body interference might be to blame. Estimates indicated, however, that such interference could hardly account for the large difference in the increments by which the measured total drag exceeded the computed pressure drag for the two wings. Consideration of the friction drag finally supplied the key to a possible explanation. To examine this possibility, curves of theoretical total drag were computed on the basis of the skin-friction coefficients corresponding to completely laminar and completely turbulent flow in the boundary layer. When this was done, it was found, as is apparent in the figure, that the experimental point for the wing with maximum thickness at 50 percent fell midway between the two resulting curves, while that for the wing with maximum thickness at 20 percent was slightly above the curve for completely turbulent flow. This suggested that the failure of the experimental points to follow the trend of the theoretical pressure drag might be due to a difference in the extent of laminar boundary-layer flow on the two wings.

To check this hypothesis, the liquid film method developed by Gray of the R.A.E. for the indication of transition at subsonic speeds (reference 14) was adapted for use in a supersonic stream. This method depends upon the fact that the rate of evaporation of a film of liquid on the surface of a model is, on the average, greater where the boundary layer is turbulent than where it is laminar. In applying this principle at the Ames Laboratory, the model is first coated with



(a) Maximum thickness at 20-percent chord.
(b) Maximum thickness at 50-percent chord.

FIGURE 10.—Results of liquid-film tests on triangular wings at zero lift. Section: uncambered double wedge, 5-percent thick, $M_0=1.53$.

flat black lacquer and then, immediately prior to installation in the tunnel, with a liquid mixture containing glycerin. A run is then made at the desired test condition for a sufficient time to allow the liquid to evaporate completely in the turbulent region but remain moist over most of the laminar area. Upon removal from the tunnel, the model is dusted with talcum powder, which adheres to the laminar but not to the turbulent area, thus increasing the contrast for photographic purposes and providing a clear indication of the extent of the two types of boundary-layer flow.

The results of liquid film tests of the two triangular wings at zero lift are shown in figure 10. For the wing with maximum thickness at midchord, the region of turbulent flow, which appears as the dark region on the model, constitutes only about half of the surface area aft of the ridge line. For the wing with maximum thickness displaced forward, the turbulent region occupies almost all of the considerably larger area which is aft of the ridge line on this wing.[5] These results were repeated many times during the numerous tests necessary to perfect the liquid-film technique. Examination of calculated pressure distributions for the two wings shows in each case excellent correlation between the experimentally determined region of turbulent flow and the calculated region of adverse pressure gradient. Because of the effects of support-body interference, it is not possible to make a decisive comparison between the measured values of total drag and theoretical values calculated on the basis of the observed areas of laminar and turbulent flow. The evidence of the liquid-film tests, however, leaves little doubt as to the primary reason why forward displacement of the maximum thickness fails to produce the reduction in minimum drag predicted by the inviscid, linear theory.

The foregoing result has important implications with regard to the degree of drag reduction possible at supersonic speeds through the use of sweepback. The relatively high pressure drag of an unswept wing at speeds above the speed of sound is a direct result of an absence of pressure recovery over the rear of the wing. The high pressure drag is thus associated with a chordwise pressure gradient which is, for the most part, favorable to the boundary-layer flow. The reduction of pressure drag by means of sweepback depends, on the other hand, upon the presence of an appreciable pressure recovery, or in other words, upon the existence of a region of adverse gradient. If the region of such gradient occupies the major portion of the wing, then, as was seen in the case of the triangular wing with thickness forward, the detrimental effects upon the skin friction may more than offset the gains in pressure drag. This suggests that it may be desirable here, as in the case of the subsonic, low-drag airfoil, to look for wing shapes which have their pressure recovery confined to a relatively small part of the wing area. Wings of this type may, in fact, prove more practical at supersonic than at subsonic speeds, since there is indication (reference 15) that the boundary-layer phenomena at the higher speeds may be more conducive to long runs of laminar flow.

Drag rise and lift drag ratio.—The final question to be discussed is that of the variation in drag with change in lift. As previously mentioned, the theoretical curve of drag versus lift is, for any given wing, parabolic in shape. The rise in drag as the lift coefficient departs from that for minimum drag depends, for a given Mach number, on the wing plan form only and is independent of the camber and thickness. The shape of the theoretical parabola for a given wing is thus identical with that for a flat lifting surface of the same plan form as the wing in question.

In the case of a plan form with a supersonic leading edge, the determination of the rise of the theoretical parabola is relatively simple. In this case, which is exemplified by plan forms A and B of figure 1, the local pressure on the flat lifting surface is everywhere finite. The variation in drag with change in lift can thus be found by simple integration of

[5] The white streaks extending back into the otherwise dark turbulent area are streamers of excess liquid blown back from the laminar region. These streamers may at times be used as a valuable indication of the direction of flow within the boundary layer, particularly on highly swept wings.

the pressures acting on the top and bottom of the surface. For all of the wings of the present study having a supersonic leading edge, the shape of the drag curve given by the theoretical calculation shows good agreement with experiment.

In the case of a wing with a subsonic leading edge, the theoretical problem is more complex. In this case, exemplified by plan form C of figure 1, there is a singularity—that is, an infinite value—in the theoretical lift intensity at the leading edge of the equivalent flat surface. The effect of this singularity is to produce a finite suction force on the leading edge in the direction opposite to the free stream. This force—sometimes referred to simply as "leading-edge suction"—reduces the rise of the theoretical drag parabola below what it would be if only the pressures on the top and bottom of the wing were considered. Actually, of course, the details of the flow about the leading edge must, in any real case, be considerably different from the representations of the linear theory, since an infinite lift intensity is obviously impossible. It does not follow, however, that the theoretical forward force at the leading edge will not exist. The situation here is much the same as that encountered at the leading edge of an airfoil section in two-dimensional, incompressible flow. In this latter case, it is known, both from experiment and from the indications of more refined calculations, that the elementary theory gives an accurate prediction of the leading-edge suction within certain limits of angle-of-attack and leading-edge radius. The range of applicability of the linear theory as applied to swept wings at supersonic speeds must similarly be established by careful theoretical and experimental investigation.

The results of the present study are not, in general, conclusive with regard to the conditions necessary for the attainment of the theoretical force at the subsonic edge. The data for the triangular wings, however, do offer some possibily significant findings. These are illustrated in figure 11, which shows the effects of change in wing section upon the drag due to lift for the triangular wings previously

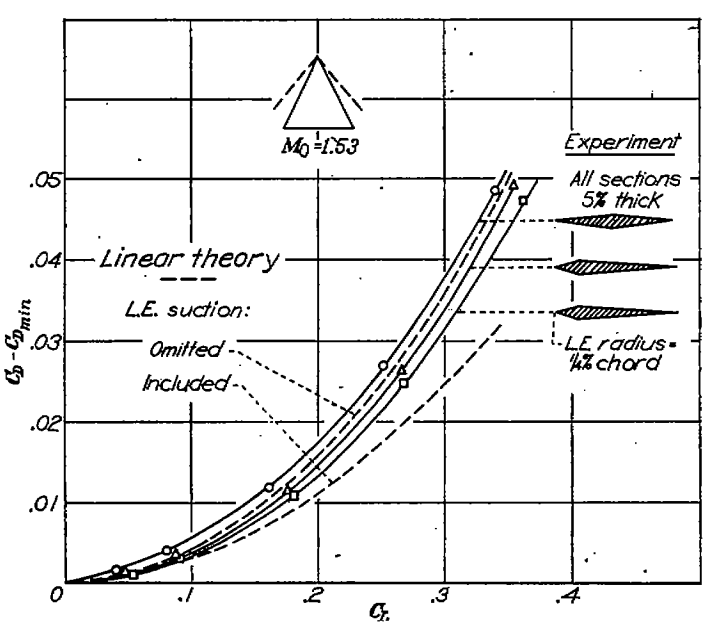


FIGURE 11.—Effect of wing section on drag rise of triangular wings.

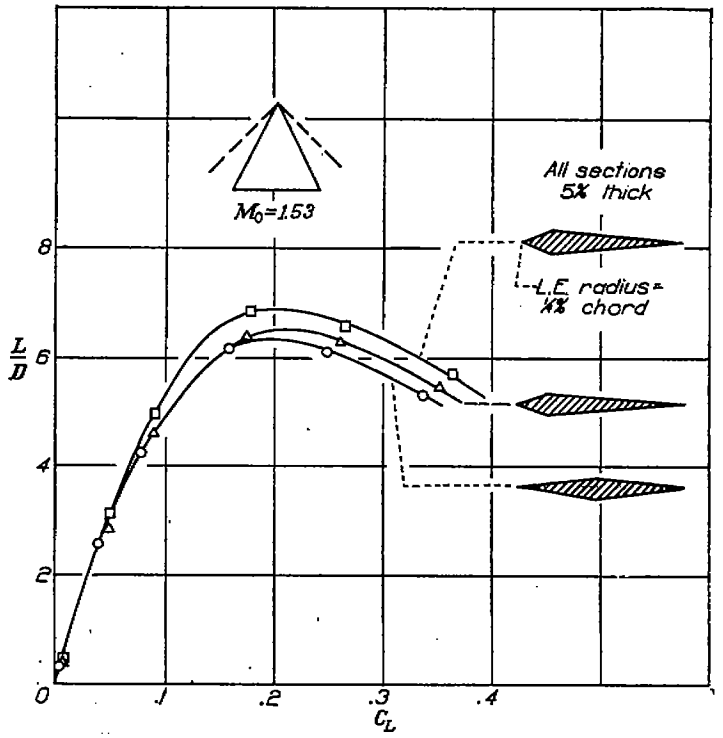


FIGURE 12.—Effect of wing section on lift-drag ratio of triangular wings.

discussed. The two theoretical curves show the calculated drag rise with the leading-edge suction both included and omitted. For the wing with maximum thickness at midchord, the experimental curve is slightly above the theoretical curve with leading-edge suction omitted. This is as might be expected for a sharp-edged wing, the slight increase above the upper theoretical curve being due possibly to an increase in friction drag with increasing lift or to support-body interference. Moving the maximum thickness forward on the wing to the 20-percent-chord position resulted in a slight reduction in drag despite the retention of a sharp leading edge. This gain may be due either to the attainment of leading-edge suction as a result of the larger leading-edge wedge angle on this wing or to a change in the variation of friction drag with lift. In an attempt to bring the drag rise of the second wing down to the values indicated by the complete theory, the edge of this wing was rounded to a radius of 0.25 percent of the chord, which is of the same order of magnitude as the radius of an NACA low-drag section of comparable thickness ratio. This rounding of the leading edge afforded some benefit, the resulting experimental values being approximately midway between the two theoretical curves. Additional rounding—to a 0.50-percent radius over the entire span and then to a still greater value over the outer half—had no further effect.

The influence of the foregoing changes on the experimental curves of lift-drag ratio is shown in figure 12. The wing with maximum thickness at midchord has a value of $(L/D)_{max}$ of about 6.3. When the maximum thickness is moved forward to the 20-percent-chord station, the decrease in drag rise apparent in figure 11 more than outweighs the slight increase in minimum drag observed in figure 9. As a result, the maximum lift-drag ratio increases slightly.

Rounding the leading edge of the second wing, while reducing the drag rise as previously noted, does not alter the minimum drag. As a consequence, the maximum lift-drag ratio is increased to approximately 6.8. These results suggest that the aerodynamic gains predicted on the basis of the theoretical leading-edge suction can be at least partially realized in practice. The determination of the optimum profile shape for this purpose may, however, involve considerable detailed research.

It is interesting for contrast with the foregoing results to point out the detrimental effects at the test Mach number of rounding the leading edge on an unswept wing. In tests of an unswept, untapered wing of aspect ratio 4, rounding the leading edge to a radius of 0.25 percent of the chord resulted in a 27-percent increase in minimum drag and a consequent reduction in maximum lift-drag ratio from 6 to about 5.5. The rise in the drag curve was unaffected by the modification.

CONCLUDING REMARKS

The foregoing results represent only a small contribution to the body of experimental and theoretical knowledge now being accumulated concerning the characteristics of wings at supersonic speeds. As is the case with most measurements of over-all forces, the data of the present study raise more questions than they answer. Detailed and patient investigations of pressure distribution and boundary-layer flow are required to develop a rational explanation for many of the observed phenomena. Several major problems have not been discussed here at all, including the important question of the adequacy of the Kutta condition to describe the real flow at a highly swept, subsonic trailing edge. There is sufficient to be done, indeed, to keep many investigators occupied for years to come.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., *May 3, 1950.*

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