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TECHNICAL NOTE 2164

AXIAL-MOMENTUM THEORY FOR PROPELLERS IN COMPRESSIBLE FLOW

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AXIAL-MOMENTUM THEORY FOR PROPELLERS IN COMPRESSIBLE FLOW[1]

By Arthur W. Vogeley

SUMMARY

The axial-momentum theory for compressible flow has been developed as a first step in the formulation of a rational propeller theory for compressible flow. The simple theory, although neglecting such important factors as rotation and profile drag, predicts flow conditions through a propeller that are significantly different from the conditions predicted by the incompressible-flow theory. These differences are greatest at high Mach numbers and high power loadings, but, because the magnitudes of the effects of these differences cannot yet be evaluated, the possibility of encountering important effects under less extreme conditions should not be overlooked.

INTRODUCTION

Conventional propeller theory is based on the assumption of an incompressible fluid. Thus far this theory, with the use of appropriate airfoil characteristics, has been adequate even to high-subsonic Mach numbers. Several minor difficulties have been noted, however, which, although not destroying the present practical usefulness of the theory, indicate a basic inadequacy that may become important as propeller speeds and disk loadings continue to increase.

The basic problem consists in the determination of the flow conditions resulting from the addition of energy to a compressible fluid. In studies of this problem that have been made with compressors and internal-flow systems (see, for example, references 1 and 2), either the entrance and exit flow conditions or the stream boundary conditions were known. These conditions are unknown in the case of the propeller so that these related analyses cannot be applied directly. (Compressor research may become more useful in the future after the general flow pattern has been established and when the details of the flow through the disk are being studied.) Although several attempts have been made to develop a propeller theory for compressible flow (references 3 and 4,

[1]Corrected version issued to supersede Aug. 1950 version.

for example), they have, in general, been limited by the use of first-order approximations, which are of decreasing applicability as the Mach number approaches unity.

The purpose of this paper is to develop the simple axial-momentum theory for a compressible fluid in order to provide a starting point for the development of a satisfactory propeller theory. Use is made of the familiar compressible-flow relationships as given in many standard references, an example is calculated, and a discussion of the significant features is presented. Finally, an attempt is made to predict some of the phenomena which might be expected in an actual propeller.

SYMBOLS

a	speed of sound in air
A	slipstream cross-sectional area
H	total pressure
m	mass flow (ρAV)
M	Mach number
p	static pressure
P	total power
T	thrust
V	slipstream velocity
γ	ratio of specific heats (assumed equal to 1.4)
ρ	air density

Subscripts:

0	far ahead of propeller disk
1	immediately ahead of disk
2	immediately behind disk
3	far behind disk (final wake)

DEVELOPMENT OF THEORY

Assumptions

The assumption is made that the propeller is an actuator disk of zero thickness so that the slipstream area is continuous through the disk. Energy is added to the slipstream instantaneously and evenly over the disk area. Rotational losses are neglected.

In setting up the analysis the flow pattern given in figure 1 was assumed. The locations of the stations 0 to 3 were chosen as follows:

Station	Location
0	Far ahead of propeller disk (free stream)
1	Immediately ahead of disk
2	Immediately behind disk
3	In the final wake where the flow has expanded back to free-stream static pressure (i.e., $p_3 = p_0$)

Fundamental Relations

From the law of conservation of mass,

$$\rho_0 A_0 V_0 = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \rho_3 A_3 V_3 \tag{1}$$

and since, by assumption, $A_1 = A_2$,

$$\rho_1 V_1 = \rho_2 V_2 \tag{2}$$

Thrust is equal to the sum of the change in momentum plus any pressure force. When thrust is measured between stations 0 and 3, where $p_3 = p_0$, no pressure force exists and

$$T = \rho_0 A_0 V_0 \left(V_3 - V_0\right) \tag{3}$$

but across the propeller disk a pressure difference exists and the expression for thrust becomes

$$T = \rho_1 A_1 V_1 \left(V_2 - V_1\right) + A_1 \left(p_2 - p_1\right) \tag{4}$$

Bernoulli's form of the energy equation states that

$$H = p\left(1 + \frac{\gamma - 1}{2\gamma} \frac{\rho V^2}{p}\right)^{\frac{\gamma}{\gamma-1}} \tag{5}$$

and, because no energy change occurs in the stream except across the propeller disk,

$$\left.\begin{aligned} H_0 &= H_1 \\ H_2 &= H_3 \end{aligned}\right\} \tag{6}$$

The value of H_0 is, of course, determined by the free-stream conditions. On the other hand, H_3 depends on the energy added to the air and may be evaluated from considerations of the general energy equation, where

$$\frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0} + \frac{V_0^2}{2} + \frac{P}{m} = \frac{\gamma}{\gamma - 1} \frac{p_3}{\rho_3} + \frac{V_3^2}{2}$$

This equation may be rearranged to read

$$P = \frac{1}{2} m\left(V_3^2 - V_0^2\right) + \frac{\gamma}{\gamma - 1} m p_0 \left(\frac{1}{\rho_3} - \frac{1}{\rho_0}\right)$$

where the term

$$\frac{1}{2} m\left(V_3^2 - V_0^2\right) \tag{7}$$

may be recognized from the incompressible momentum theory as a measure of the power used in producing thrust and induced losses, and the term

$$\frac{\gamma}{\gamma - 1} mp_0\left(\frac{1}{\rho_3} - \frac{1}{\rho_0}\right) \tag{8}$$

is an additional term arising from the occurrence of additional losses such as profile drag. This additional term is a measure of the increase in entropy in the final wake. In the general case when losses occur to make ρ_3 differ from ρ_0 the flow process is nonisentropic. When ρ_3 and ρ_0 are assumed equal, the process becomes isentropic.

A restriction is imposed in the analysis whenever the Mach number ahead of the propeller reaches 1.0. Presumably, because signals are transmitted at the speed of sound, the Mach number ahead of the disk can never exceed 1.0 if the free-stream Mach number is less than 1.0. Of course, if the free-stream Mach number is equal to or greater than 1.0, the Mach number in the slipstream ahead of the disk is everywhere equal to the free-stream value.

The speed of sound at any point, used to calculate the Mach number, is obtained from the equation

$$a^2 = \frac{\gamma p}{\rho} \tag{9}$$

Finally, the equation

$$\frac{p}{\rho^\gamma} = \text{Constant} \tag{10}$$

is used where isentropic conditions exist.

METHODS OF OBTAINING SOLUTIONS

In general, the flow conditions are desired for certain values of altitude, forward speed, power, and propeller-disk area. A first approximation of A_0, only a few percent greater than the propeller area, may be used to calculate the mass flow $\rho_0 A_0 V_0$, and conditions in the final wake may then be determined from expressions (7) and (8). Conditions at stations 1 and 2 may be found from conditions at stations 0 and 3 by means of compressible-flow relationships. From equation (4) the thrust may be calculated. If this value does not agree with the value given by equation (3), further calculations with different assumed values of A_0 are necessary.

The mechanics of obtaining a solution may be varied considerably. The method presented is laborious but yields results to any degree of accuracy desired.

RESULTS

Because of the difficulty of presenting in general terms the results of the development of the momentum theory for compressible flow, the solution of a typical problem is presented here.

Solutions were obtained for an ideal propeller $(\rho_3 = \rho_0)$ for which the flow process is isentropic. A flight Mach number of 0.7 and an altitude of 40,000 feet were assumed. Power loading was varied from 0 to approximately 77.5 horsepower per square foot of disk area, the value at which sonic velocity into the disk was reached. (After sonic velocity is reached the process becomes nonisentropic. Solutions in this range are not presented because the physical significance of the results is not clear at this time.) Results of the calculations are presented in figure 2 with corresponding results of calculations for an incompressible fluid obtained by using the familiar axial-momentum equations.

Figure 2 shows that significant differences in the flow conditions through the propeller exist between compressible and incompressible flow. In incompressible flow, a discontinuity occurs only in the pressure through the disk. In compressible flow, however, discontinuities appear also in the density, velocity, and Mach number. These changes do not, of course, occur instantaneously in an actual case but take place within the propeller blading. In the first approximation, therefore, conditions noted as occurring at station 1 may be considered as existing in the vicinity of the leading edge of the blade and conditions at station 2, near the trailing edge.

Figure 2 shows that the pressure jump through the disk for compressible flow is approximately twice (at $M_0 = 0.7$) the jump for incompressible flow.

Other interesting results for this Mach number are that the velocity V_1 immediately ahead of the disk is greater than the final slipstream velocity and that V_2 is less than free-stream velocity. These results are quite different from those obtained from incompressible theory.

The variations in Mach number through the propeller disk follow, of course, the variation in velocity. It is interesting to note that the Mach number immediately behind the disk M_2 is always less than the free-stream Mach number.

For compressible flow, all the variables presented in figure 2 are seen to vary smoothly with power loading until the Mach number M_1 reaches unity. From this point on, conditions ahead of the disk would remain fixed. Conditions behind the disk would depend upon the flow process chosen.

Calculations were made of the efficiency for both compressible and incompressible flow. No charts of efficiency are presented, because within the accuracy of the calculations no difference was found.

IMPLICATIONS OF THEORY

The simple axial-momentum theory is one-dimensional, whereas the flow about an actual propeller is three-dimensional. Therefore, from the axial-momentum theory, the determination of the effects of compressibility on propeller operation is difficult. For a thorough evaluation, a more complete analysis considering such effects as rotational energy, profile drag, and finite number of blades should be made. The actuator disk may be likened to a propeller with a large number of blades operating in dual rotation (in order to remove rotation in the slipstream), and in this case simple theory should, to the first order at least, predict the general flow phenomena and indicate some of the effects of compressibility that might be experienced.

Variations in Pressure

For compressible flow the pressure variations through the propeller disk are similar to, but larger than, the variations in incompressible

flow. These larger pressure variations will tend to increase the mutual interference effects between propeller and body over the amount usually expected. The effects on body critical speeds and boundary-layer action may become significant.

The generally unfavorable pressure rise through the disk, being greater in compressible flow, would tend to produce a greater adverse effect on the blade boundary layer.

Variations in Velocity

As the air passes through the propeller disk it decreases in velocity, as shown by figure 2. This result has also been noted in reference 1 for the analogous case of the compressor. Reference 1 discusses in detail the effects of the change in velocity and shows that the effect of compressibility is to reduce the turning angle required for a given pressure rise (or thrust, in the case of a propeller). In effect, then, a propeller designed according to conventional incompressible theory may have excessive camber for compressible flow.

The variation in velocity through the propeller raises the problem of defining a velocity upon which to base section dynamic pressure, blade angle, and direction of force vectors. Ultimately, of course, all losses (whether induced or otherwise) must be resolved into forces acting on the blade sections. For the incompressible case the solution is simple since the velocity through the disk is constant, and the increase in axial velocity causes a tilt in the section force vectors, which corresponds quite logically to the induced losses. For the compressible case, however, the problem is more complicated and the determination of a representative velocity through the disk is difficult. Neither the inflow velocity (V_1) nor the mean velocity (average of V_1 and V_2) seems wholly satisfactory for this purpose.

Variations in Mach Number

At $M_0 = 0.7$, a wing or body should be favorably affected if placed immediately behind a propeller because of the lower axial Mach number. As shown by figure 2, the Mach number behind the disk is always less than free-stream Mach number. Conversely, a body ahead of the disk should be adversely affected (except for the favorable pressure gradient), since it would be operating where the Mach number is always greater than free stream and, in the case presented, is even higher than in the ultimate slipstream. The influence of the propeller is significant, however, for only a relatively short distance ahead of and behind the disk. A body would have to be placed rather close (less than 1 propeller diameter)

behind the disk to experience the reduced Mach number; otherwise it would operate in the higher Mach number of the final propeller wake. The variation in axial Mach number may make a tractor configuration preferable to a pusher installation, other things being equal.

CONCLUDING REMARKS

The axial-momentum theory for compressible flow predicts flow conditions through a propeller significantly different from the conditions predicted by the incompressible-flow theory. These differences are greatest at high Mach numbers and high power loadings, but, because the magnitudes of the effects of these differences cannot be evaluated, the possibility of encountering important effects under less extreme conditions should not be overlooked.

Such important parameters as rotation, profile drag, and number of blades should be investigated. These parameters may either amplify or counteract the effects anticipated from simple considerations. Tests designed to investigate propeller operation along the lines indicated by this compressible-flow theory would be valuable.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., April 26, 1951

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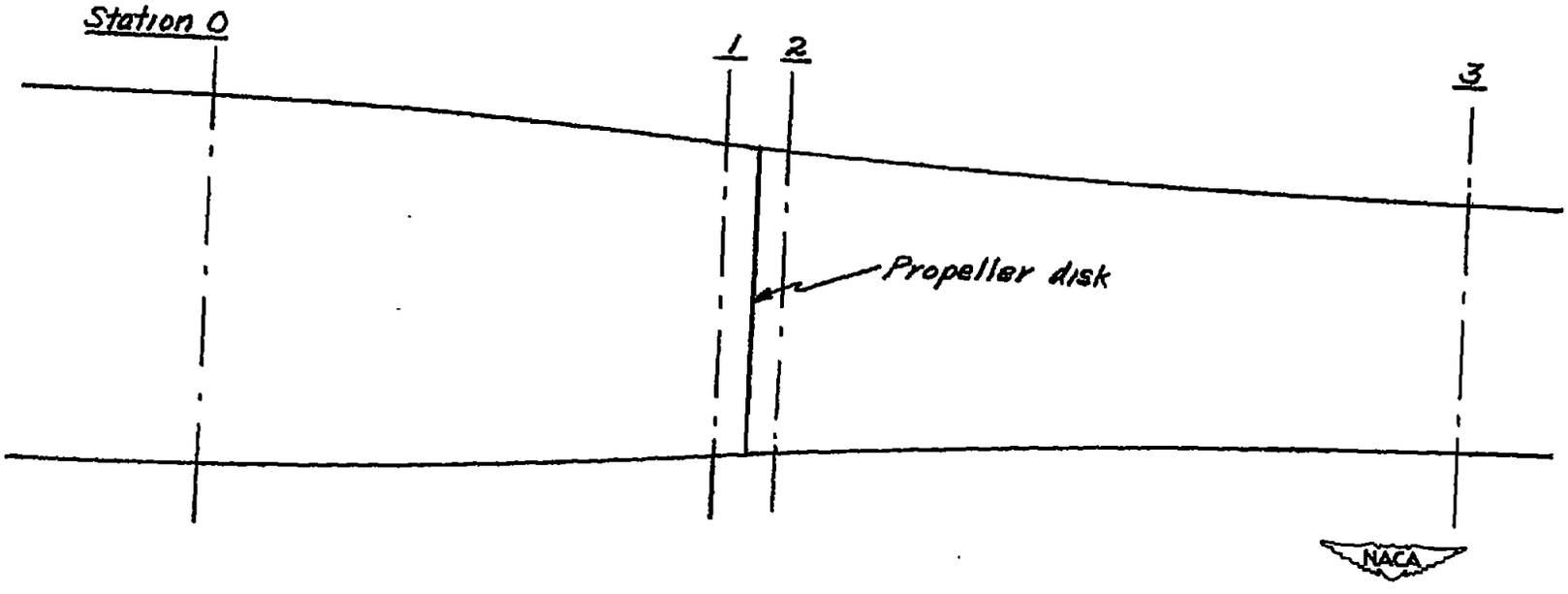

Figure 1.- Flow pattern assumed in development of theory.

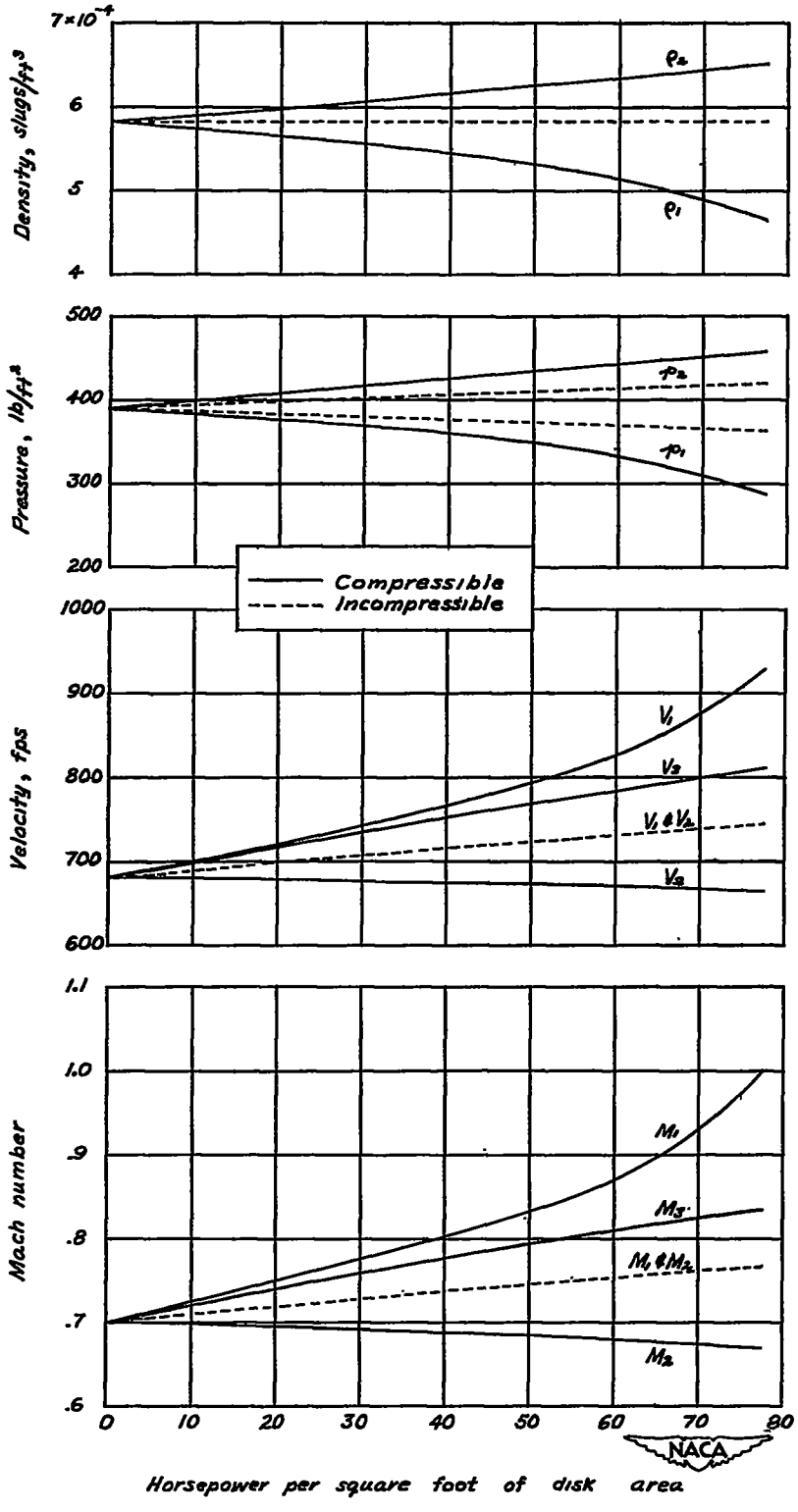

Figure 2.- Axial-flow conditions for an ideal propeller at a flight Mach number of 0.7 and an altitude of 40,000 feet.

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