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TECHNICAL NOTE 2550

DETERMINATION OF SHAPES OF BOATTAIL BODIES OF REVOLUTION FOR MINIMUM WAVE DRAG

By Mac C. Adams

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Langley Field, Va.

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SUMMARY

By use of an approximate equation for the wave drag of slender bodies of revolution in a supersonic flow field, the optimum shapes of certain boattail bodies are determined for minimum wave drag. The properties of three specific families of bodies are determined, the first family consisting of bodies having a given length and base area and a contour passing through a prescribed point between the nose and base, the second family having fixed length, base area, and maximum area, and the third family having given length, volume, and base area. The method presented is easily generalized to determine minimum-wave-drag profile shapes which have contours that must pass through any prescribed number of points.

According to linearized theory, the optimum profiles are found to have infinite slope at the nose but zero radius of curvature so that the bodies appear to have pointed noses, a zero slope at the body base, and no variation of wave drag with Mach number. For those bodies having a specified intermediate diameter (that is, location and magnitude given), the maximum body diameter is shown to be larger, in general, than the specified diameter. It is also shown that, for bodies having a specified maximum diameter, the location of the maximum diameter is not arbitrary but is determined from the ratio of base diameter to maximum diameter.

INTRODUCTION

The wave drag of slender bodies of revolution having cross-sectional areas with a zero slope at the base was shown by Von Kármán (reference 1) to be given approximately by a double integral dependent only on the body shape and independent of Mach number. By use of Von Kármán's integral and the calculus of variations, several authors (references 1 to 4) have treated the problem of determining optimum body shapes to give minimum wave drag. All these investigations have been concerned with either closed bodies or a body, such as a shell, having its maximum thickness at the base; however, none have treated bodies having boattails, and this problem is considered herein.

PERMANENT RECORD

Ward (reference 5) has shown that bodies having a finite slope at the base give rise to drag terms in addition to Von Kármán's integral and these additional terms include Mach number effects. The present paper shows, however, that the optimum bodies must have a zero slope at the base and consequently the additional terms vanish. The determination of the minimum-wave-drag bodies of revolution with boattailing then resolves itself to minimizing the same integral as used by the reference papers but with a more general treatment of the body profile.

Although the analysis to follow is concerned with wave drag only, it should be remembered that the additional drag resulting from base pressure and skin friction is also of importance. A brief discussion of this additional drag is given in the concluding remarks.

SYMBOLS

D	wave drag
q	dynamic pressure of free stream
M	free-stream Mach number
$\beta = \sqrt{M^2 - 1}$	
l	body length
x, ξ	distances made nondimensional with respect to $l/2$ and measured along body axis from midpoint of body
$r(x)$	body radius
$S(x)$	nondimensional body cross-sectional area $\left(\pi\frac{r(x)^2}{(l/2)^2}\right)$
A	specified cross-sectional area of body divided by $(l/2)^2$
a	body diameter corresponding to area A
S_{max}	maximum body cross-sectional area divided by $(l/2)^2$
B	body-base area divided by $(l/2)^2$

d	maximum body diameter
C_D	drag coefficient $\left(\frac{D}{q(l/2)^2 S_{max}}\right)$
V	body volume divided by $(l/2)^3$
c	distance, divided by $l/2$, from midpoint of body to location of specified area A
e	distance, divided by $l/2$, from midpoint of body to location of S_{max}
λ	Lagrange multiplier

ANALYSIS

By means of linearized theory, Ward (reference 5) has shown that the wave drag of a body of revolution in a supersonic flow field is given approximately by

$$\frac{D}{q\left(\frac{l}{2}\right)^2} = -\frac{1}{2\pi}\int_{-1}^{1}\int_{-1}^{1} S''(x)S''(\xi)\log_e|x - \xi|\,dx\,d\xi +$$

$$\frac{S'(1)}{\pi}\int_{-1}^{1} S''(x)\log_e(1 - x)dx - \frac{1}{2\pi}\left[S'(1)\right]^2\log_e \frac{\beta}{2}\sqrt{\frac{S(1)}{\pi}} \tag{1}$$

where the body cross-sectional area and the coordinate distances have been written in nondimensional form. The requirements in the derivation of equation (1) are that the body profile have no corners (a continuous slope) and that the rate of change of the cross-sectional area at the nose be zero $(S'(-1) = 0)$.

Optimum body having given length and base area and a contour passing through prescribed point between nose and base.- In certain practical problems it may be desired to have a minimum-wave-drag body of revolution which has a given length and base area and which has a contour that must pass through a specified diameter at a particular point between the nose

and base of the body. Such a situation would arise, for example, if a body of revolution were desired which would enclose some given rocket or jet engine.

Although the analysis to follow determines the optimum body shape which is required to pass through three given points, that is, the nose, the base, and some in-between point, the manner in which the analysis could be generalized to determine optimum body shapes which must pass through any prescribed number of points will become apparent.

Let it be supposed that the optimum body shape is given by the expressions

$$\left.\begin{aligned} S(x) &= f(x) \qquad (-1 \leqq x \leqq c) \\ S(x) &= g(x) \qquad (c \leqq x \leqq 1) \end{aligned}\right\} \tag{2}$$

Since the body must pass through three given points, the following conditions apply:

$$\left.\begin{aligned} f(-1) &= 0 \\ f(c) &= g(c) = A \\ g(1) &= B \end{aligned}\right\} \tag{3}$$

The limitation of equation (1) allows no corners on the body profile; hence,

$$f'(c) = g'(c) \tag{4}$$

and a further condition requires that $f'(-1) = 0$.

Next consider another body shape such that

$$\left.\begin{aligned} S(x) &= f(x) + \delta h(x) \qquad (-1 \leqq x \leqq c) \\ S(x) &= g(x) + \gamma k(x) \qquad (c \leqq x \leqq 1) \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} &h(-1) = h(c) = k(c) = k(1) = h'(-1) = 0 \\ &\delta h'(c) = \gamma k'(c) \end{aligned}\right\} \tag{6}$$

This second body profile passes through the same three prescribed points as the optimum body profile and also satisfies the other conditions placed on the optimum body. However, since $f(x)$ and $g(x)$ define the optimum profile, the variations in the profile, $\delta h(x)$ and $\gamma k(x)$, must necessarily produce an increased drag. It follows that $h(x)$ and $k(x)$ are completely arbitrary in form and that δ and γ are independent parameters. Substitution of equations (5) into equation (1) gives the drag as a function of the two independent parameters δ and γ. If the rear portion of the body is held fixed in its minimum-drag configuration, that is, $\gamma = 0$, then

$$\left[\frac{\partial D(\delta,0)}{\partial \delta}\right]_{\delta=0} = 0$$

$$= -\int_{-1}^{c} h''(\xi)\, d\xi \left[\int_{-1}^{c} f''(x)\log_e|x - \xi|\, dx + \int_{c}^{1} g''(x)\log_e|x - \xi|\, dx\right] + g'(1)\int_{-1}^{c} h''(x)\log_e(1 - x)\, dx \tag{7}$$

where

$$h(-1) = h(c) = h'(-1) = h'(c) = 0$$

In a similar manner, the forward portion of the body can be held fixed in its optimum configuration, that is, $\delta = 0$, and

$$\left[\frac{\partial D(0,\gamma)}{\partial \gamma}\right]_{\gamma=0} = 0$$

$$= -\int_c^1 k''(\xi)\, d\xi \left[\int_{-1}^c f''(x)\log_e|x - \xi|\, dx + \right.$$

$$\left.\int_c^1 g''(x)\log_e|x - \xi|\, dx\right] + k'(1)\left[\int_{-1}^c f''(x)\log_e(1 - x)\, dx + \right.$$

$$\left.\int_c^1 g''(x)\log_e(1 - x)\, dx\right] + g'(1)\int_c^1 k''(x)\log_e(1 - x)\, dx -$$

$$g'(1)k'(1)\log_e \frac{\beta}{2}\sqrt{\frac{g(1)}{\pi}} \tag{8}$$

where

$$k(c) = k(1) = k'(c) = 0$$

Partial integration of equations (7) and (8) yields

$$\int_{-1}^c h(\xi)\, d\xi \frac{d^2}{d\xi^2}\left[\int_{-1}^c \frac{f'(x)}{x - \xi}\, dx + \int_c^1 \frac{g'(x)}{x - \xi}\, dx\right] = 0 \tag{9}$$

and

$$\int_c^1 k(\xi)\, d\xi \frac{d^2}{d\xi^2}\left[\int_{-1}^c \frac{f'(x)}{x - \xi}\, dx + \int_c^1 \frac{g'(x)}{x - \xi}\, dx\right] +$$

$$g'(1) \lim_{x \to 1}\left[k'(x)\log_e(1 - x)\right] - g'(1)k'(1)\log_e \frac{\beta}{2}\sqrt{\frac{g(1)}{\pi}} = 0 \tag{10}$$

Inasmuch as $h(\xi)$ is an arbitrary function, equation (9) yields directly

$$\int_{-1}^{c} \frac{f'(x)}{x - \xi} dx + \int_{c}^{1} \frac{g'(x)}{x - \xi} dx = A_1\xi + A_2 \qquad (-1 \leqq \xi \leqq c) \qquad (11)$$

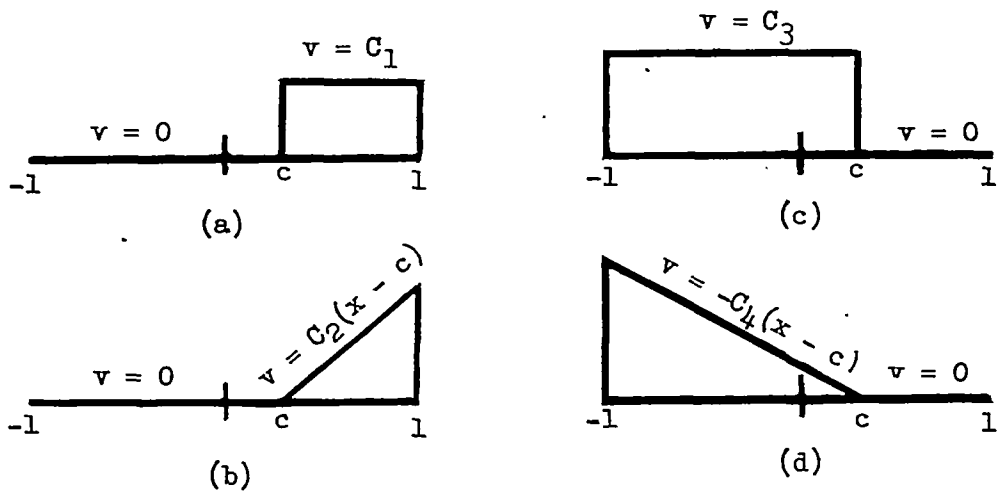
Equation (10) cannot be reduced so readily but requires more consideration. Since $k(\xi)$ is also arbitrary, it can be chosen such that $k'(1) = 0$ of sufficient order and then

$$\int_{-1}^{c} \frac{f'(x)}{x - \xi} dx + \int_{c}^{1} \frac{g'(x)}{x - \xi} dx = A_3\xi + A_4 \qquad (c \leqq \xi \leqq 1) \qquad (12)$$

Now suppose that $k'(1)$ is not zero but instead is some finite value. It then follows from equations (10) and (12) that $g'(1)$ must be zero; that is, the optimum body profile must have a zero slope at its base. It is interesting to observe that the Mach number effect on the drag disappears for the minimum-drag bodies.

The simultaneous solution of integral equations (11) and (12) will now give the optimum body profiles sought. These two equations are identical to those found in reference 2 in which minimum-drag closed bodies were considered.

It is recognized that equations (11) and (12) are analogous to the integral equations found in lifting-line theory. In the analogy $f'(x)$ and $g'(x)$ correspond to the vortex distribution, and the right-hand sides of the equations represent the induced-normal-velocity distributions on the two-dimensional wings with no thickness. The cross-sectional area of the optimum body is therefore a linear combination of the surface potential distributions which give rise to the following normal-velocity distributions on two-dimensional wings:



where

$$C_1 = A_3c + A_4 \qquad C_3 = A_1c + A_2$$

$$C_2 = A_3 \qquad C_4 = -A_1$$

In addition, a circulatory solution is required in order that the body have a finite base area; that is, the analogous wing has a specified lift.

Sketches (a) and (c) correspond to two-dimensional flat-plate wings with deflected flaps, and sketches (b) and (d) represent flaps rotating with a constant angular velocity. The surface-potential distributions for the four wings, as well as the circulatory solution, are all found in reference 6 and may be written

$$\varphi_{(a)} = \frac{C_1}{\pi}\left[\sqrt{1 - x^2}\cos^{-1}c - (x - c)\log_e N\right] \tag{13a}$$

$$\varphi_{(b)} = \frac{C_2}{2\pi}\left[\sqrt{1 - c^2}\sqrt{1 - x^2} + (x - 2c)\sqrt{1 - x^2}\cos^{-1}c - (x - c)^2\log_e N\right] \tag{13b}$$

$$\varphi_{(c)} = \frac{C_3}{\pi}\left[\sqrt{1 - x^2}\cos^{-1}(-c) + (x - c)\log_e N\right] \tag{13c}$$

$$\varphi_{(d)} = \frac{C_4}{2\pi}\left[\sqrt{1 - c^2}\sqrt{1 - x^2} - (x - 2c)\sqrt{1 - x^2}\cos^{-1}(-c) - (x - c)^2\log_e N\right] \tag{13d}$$

$$\varphi_{(\Gamma)} = \frac{\Gamma}{2\pi}\cos^{-1}(-x) \tag{13e}$$

where

$$N = \frac{1 - cx - \sqrt{1 - c^2}\sqrt{1 - x^2}}{|x - c|}$$

and $\varphi_{(\Gamma)}$ represents the circulatory solution. The constants in equations (13) are determined by applying the following conditions:

$$\left.\begin{aligned} S'(-1) &= S'(1) = 0 \\ S(c) &= A \\ S(1) &= B \end{aligned}\right\} \tag{14}$$

A further condition is needed to insure that the body have finite and continuous slope at the point c; that is, $C_1 = C_3$.

The least-drag body, which has given length and base area and which has a contour that must pass through the point c where $S = A$, is then given by

$$S(x) = \left[A - \frac{B}{\pi}\cos^{-1}(-c)\right]\frac{\sqrt{1 - x^2}}{\left(1 - c^2\right)^{3/2}}(1 - cx) + \frac{B}{\pi}\frac{\sqrt{1 - x^2}}{1 - c^2}(x - c) + \left[\frac{A - \frac{B}{\pi}\cos^{-1}(-c)}{\left(1 - c^2\right)^2} - \frac{Bc}{\pi\left(1 - c^2\right)^{3/2}}\right](x - c)^2\log_e N + \frac{B}{\pi}\cos^{-1}(-x) \tag{15}$$

Some typical profiles defined by equation (15) are shown in figure 1, where an arbitrary thickness ratio of approximately 20 percent has been chosen. All the profiles defined by equation (15) have infinite slope at the nose. However, the radii of curvature approach zero at the noses so that the noses appear to have finite angles.

For vanishing base area, equation (15) reduces to the body profiles considered in reference 2. The optimum shell for given caliber and length is obtained for $A \rightarrow B$ and $c \rightarrow 1$. The result is

$$S(x) = \frac{B}{\pi} x \sqrt{1 - x^2} + \frac{B}{\pi} \cos^{-1}(-x) \tag{16}$$

and is in agreement with reference 1.

It should be pointed out that the maximum cross-sectional area is in general not coincident with, and may be larger than, either of the areas A or B (see fig. 1(a)). This result is contrary to two-dimensional profiles where the maximum thickness is coincident with the largest ordinate through which the profile is required to pass. The maximum area of the body of revolution coincides with the point A only when

$$\frac{\pi A}{B} = \frac{\sqrt{1 - c^2}}{c} + \cos^{-1}(-c) \tag{17}$$

The profiles in this case give minimum wave drag for given length, base area, and maximum area, and the area distribution is given by

$$S(x) = \frac{B}{\pi c} \sqrt{1 - x^2} + \frac{B}{\pi c} \frac{(x - c)^2}{\sqrt{1 - c^2}} \log_e N + \frac{B}{\pi} \cos^{-1}(-x) \tag{18}$$

The quantity c in this equation represents the location of the maximum diameter and is determined, for any given ratio B/A $(A = S_{max})$, from equation (17). A graph of equation (17) is shown in figure 2 and some typical profiles defined by equations (17) and (18) (which are special cases of equation (15)) are presented in figure 1(b).

The wave drag for the optimum bodies of equation (15) is easily found to be

$$\frac{D}{q\left(\frac{l}{2}\right)^2} = -\frac{A}{2}\left(C_2 + C_4\right) + \frac{BC_2}{2}$$

$$= \pi A\left[\frac{A - \frac{B}{\pi}\cos^{-1}(-c)}{\left(1 - c^2\right)^2} - \frac{Bc}{\pi\left(1 - c^2\right)^{3/2}}\right] +$$

$$B\left\{-\frac{A - \frac{B}{\pi}\cos^{-1}(-c)}{\left(1 - c^2\right)^2}\left[c\sqrt{1 - c^2} + \cos^{-1}(-c)\right] + \right.$$

$$\left.\frac{B}{\pi}\frac{1 - c^2 + c\sqrt{1 - c^2}\cos^{-1}(-c)}{\left(1 - c^2\right)^2}\right\} \tag{19}$$

The drag coefficient for optimum bodies of a given length, base area, and maximum area is then found to be

$$C_D = \frac{1}{\pi}\frac{B^2}{Ac^2} \tag{20}$$

where $A = S_{max}$ and c is defined by equation (17). This drag coefficient is shown in figure 3 plotted against B/A.

If the optimum profiles which must pass through several specified points were sought, it follows from the present section that integral

equations like equations (11) and (12) must be satisfied in each of the specified regions.

Of some practical interest may be the least-drag body with a given length and base area and which must have as a part of its profile a cylindrical collar of constant radius. The problem in this case reduces to the solution for a tandem biplane configuration having the same upwash distribution on the wing surface as in the previous problem.

Optimum body having given length, volume, and base area.- It follows from the previous section that again the optimum bodies having given length, volume, and base area will also have a zero slope at the base. The calculus of variations then gives the following integral to be made stationary:

$$-\frac{1}{2\pi}\int_{-1}^{1}\int_{-1}^{1} S''(x)S''(\xi)\log_e|x - \xi|\,dx\,d\xi + \lambda\int_{-1}^{1} S(x)\,dx \tag{21}$$

with the following conditions on the body shape:

$$\left.\begin{aligned} &S(-1) = S'(-1) = S'(1) = 0 \\ &S(1) = B \\ &\int_{-1}^{1} S(x)\,dx = V \end{aligned}\right\} \tag{22}$$

By following a procedure similar to that used in the previous section, the following integral equation is deduced:

$$\int_{-1}^{1} \frac{S'(x)}{x - \xi}\,dx = K_1\xi^2 + K_2\xi + K_3 \qquad (-1 \leqq \xi \leqq 1) \tag{23}$$

Again by use of the wing analogy, the solution to equation (23) is found in reference 6, and the constants are determined from equation (22). The final result is

$$S(x) = \frac{8}{3}\frac{V - B}{\pi}\left(1 - x^2\right)^{3/2} + \frac{B}{\pi}x\sqrt{1 - x^2} + \frac{B}{\pi}\cos^{-1}(-x) \tag{24}$$

For vanishing base area B this result reduces to the optimum body for given length and volume as found in reference 2.

Typical body profiles as given by equation (24) are shown in figure 4. The profile shapes at the nose and base of the body can be seen to have essentially the same character as the profiles found in the previous section.

The wave drag for those bodies having a given volume and base area is found to be

$$\frac{D}{q\left(\frac{l}{2}\right)^2} = \frac{1}{\pi}\left[8V(V - 2B) + 9B^2\right] \tag{25}$$

The location of the maximum thickness for the bodies defined by equation (24) is given by

$$e = \frac{1}{4\left(\frac{V}{B} - 1\right)} \tag{26}$$

Therefore, for a given ratio V/B or a given location of maximum thickness, the ratio of the base area to the maximum area is readily found. A graph of the location of the maximum thickness against B/S_{max} is shown in figure 5. The wave-drag coefficients for the same values of B/S_{max} are shown in figure 6.

CONCLUDING REMARKS

By use of an approximate equation for the wave drag of slender bodies of revolution in a supersonic flow field, the minimum-wave-drag body shapes have been determined for three cases: (1) the body has given length and base area and a contour that passes through a prescribed point between the nose and base, (2) the body has given length, base area, and maximum area, and (3) the body has given length, base area, and volume.

The optimum body profiles are shown to have: (1) infinite slope at the nose but zero radius of curvature so that the bodies appear to have pointed noses, (2) a zero slope at the body base, and (3) no variation of wave drag with Mach number. For those bodies having a specified intermediate diameter (that is, location and magnitude given) the maximum body diameter is found to be, in general, larger than the specified diameter. It was also found that, for bodies having a specified maximum diameter, the location of the maximum diameter is not arbitrary but is determined from the ratio of base diameter to maximum diameter.

In order to find body profiles that give minimum total drag, consideration must be given to skin-friction and base-pressure drag. As for skin-friction drag, it can be said that the profile shape is of secondary importance and that the wetted area of the body and the Reynolds number are the primary factors.

In cases where the base pressure is independent of the body shape, for example, where a supersonic jet exits at the base, the optimum profiles sought are the same as those determined herein. If no jet exits at the body base, the base pressure is dependent upon the body shape. Just what the dependence is is not sufficiently clear at the present time. However, the profiles determined may require some slight changes and further investigation, in which account is taken for both the wave drag and base-pressure drag, would be necessary.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., August 21, 1951

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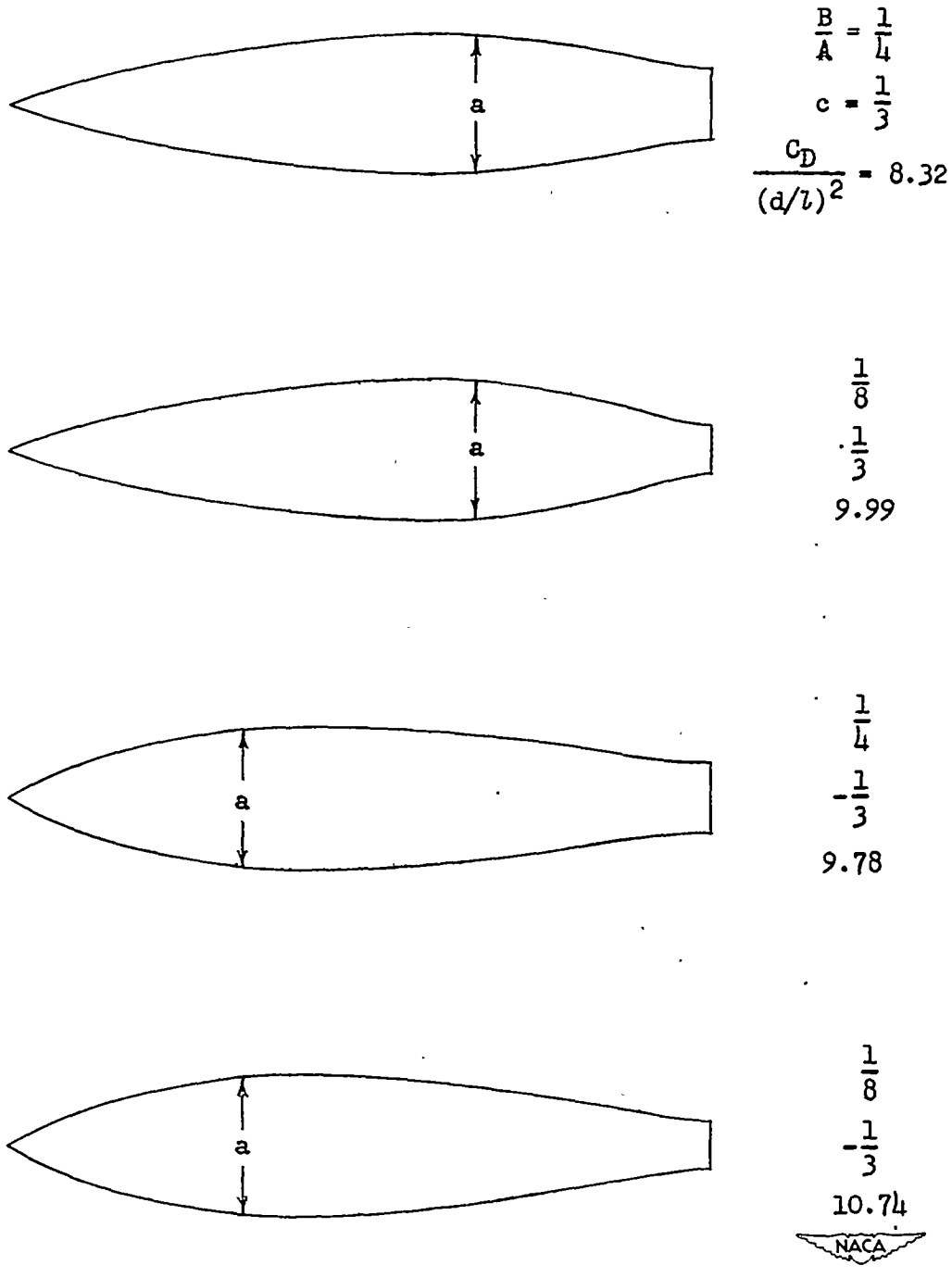
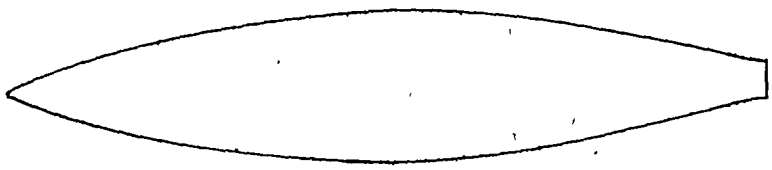


(a) Bodies having a given length and base diameter and a contour passing through another specifically located diameter.

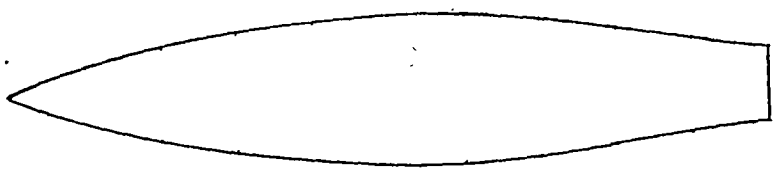
Figure 1.- Typical optimum body profiles defined by equation (15). Thickness, 20 percent.



$\frac{B}{A} = \frac{1}{16}$

$c = 0.02$

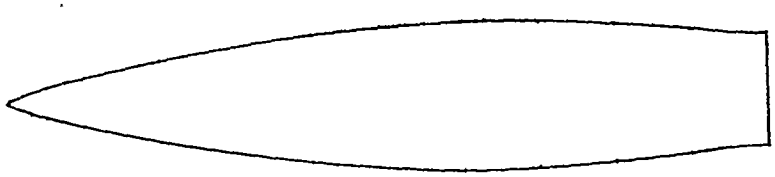
$\frac{C_D}{(d/l)^2} = 9.29$



$\frac{1}{4}$

0.09

7.53



$\frac{9}{16}$

0.26

4.75

(b) Bodies having given length, base area, and maximum area ($A = S_{max}$).

Figure 1.- Concluded.

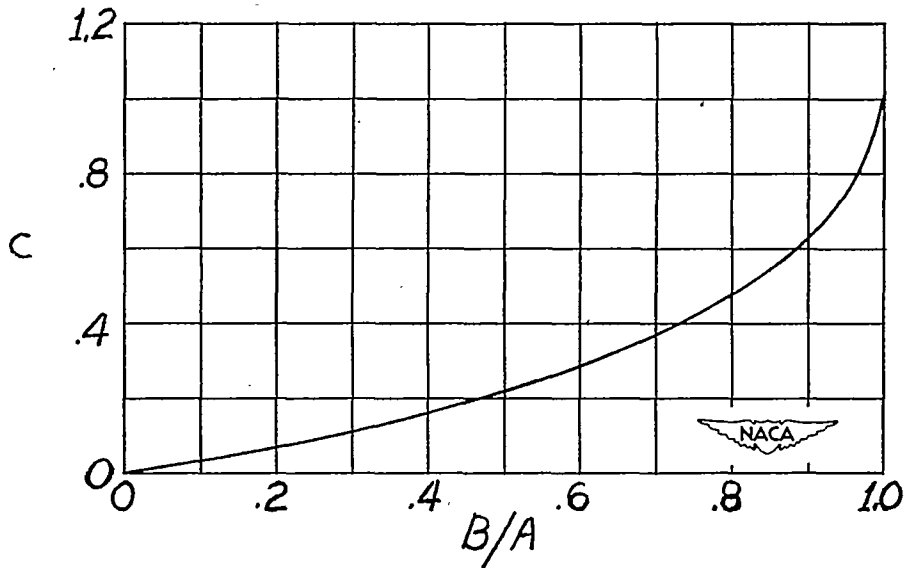


Figure 2.- Location of maximum diameter for optimum bodies of given length, base area, and maximum area ($A = S_{max}$).

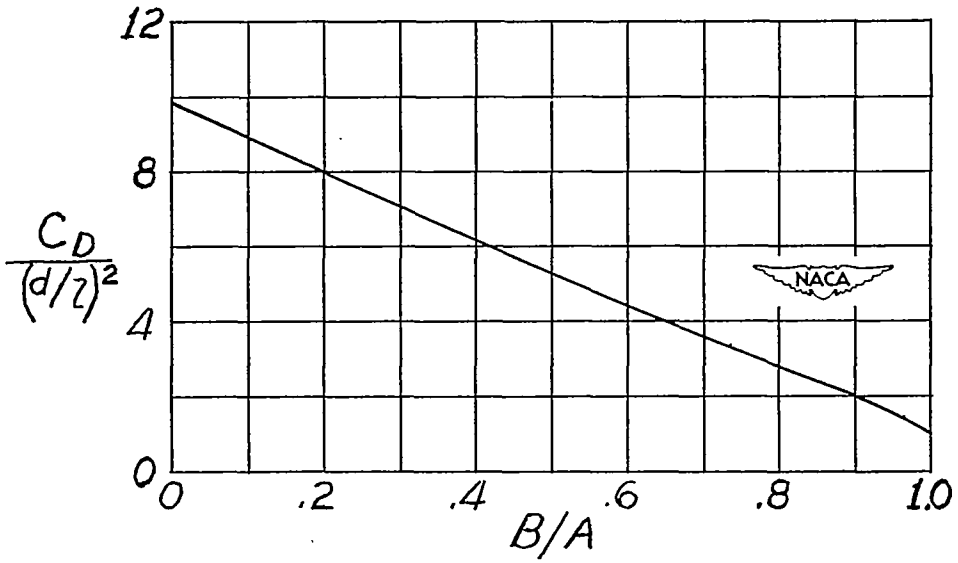


Figure 3.- Wave-drag coefficient for optimum bodies of given length, base area, and maximum area ($A = S_{max}$).

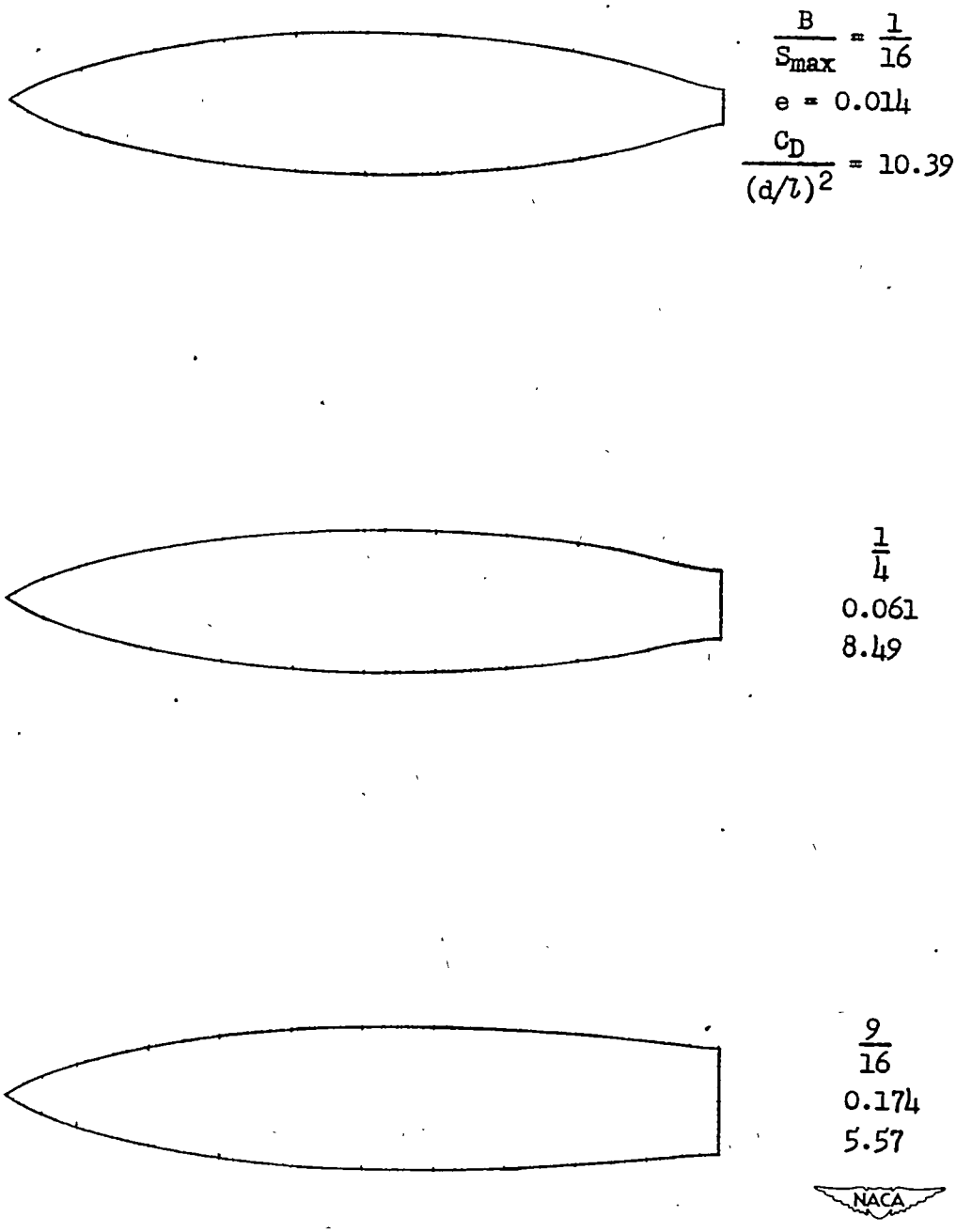


Figure 4.- Typical optimum body profiles which have given length, volume, and base diameter defined by equation (24). Thickness, 20 percent.

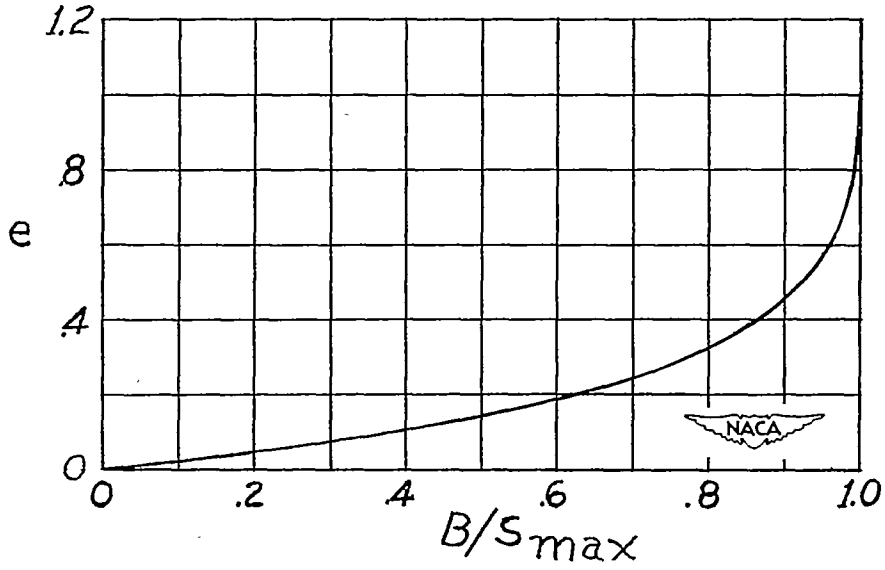


Figure 5.- Location of maximum body diameter for a given ratio of base area to maximum area for bodies of a given length, volume, and base area.

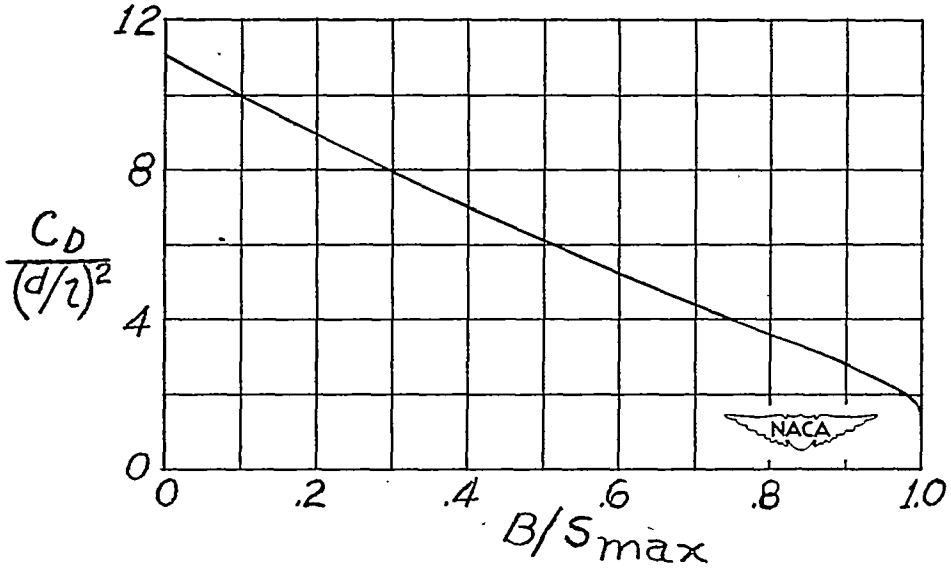


Figure 6.- Wave-drag coefficient for a given ratio of base area to maximum area for bodies of a given length, volume, and base area.

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