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TECHNICAL NOTE 2623

COMPARISON OF SUPERSONIC MINIMUM-DRAG AIRFOILS DETERMINED BY LINEAR AND NONLINEAR THEORY

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SUMMARY

Supersonic profiles of minimum pressure drag for a given thickness ratio and for a given area have been determined with the use of a nonlinear pressure relation and are compared with minimum-drag profiles found by linearized theory. The results show that the profiles are determined with sufficient accuracy by linear theory over the entire supersonic Mach number range since the drag coefficients for these profiles are only slightly higher than those for optimum profiles determined by nonlinear theory. Linear theory appears to be adequate for determining profiles of minimum drag for other auxiliary structural conditions since moderate deviations from the optimum shape have only a small influence on the pressure drag.

The parameters determining the airfoil shape for a given thickness ratio found by both the linear and nonlinear theory are presented in graphs as a function of the base pressure coefficient. With the use of these results, the optimum profiles for any stream Mach number and thickness coefficient are readily determined. A comparison of the pressure drag coefficients for optimum profiles determined by linear and nonlinear theory is presented for the Mach number range from 1.5 to 10.0. In addition, several optimum profiles for a given area have been calculated by both the linear and nonlinear theory.

INTRODUCTION

Drougge (reference 1) has determined the airfoil section shape for minimum pressure drag at supersonic speeds subject to such auxiliary conditions as given bending and torsional stiffness. These calculations were made by using the linearized expression for the pressure coefficient; the effect of a base was not considered. Recently, Chapman (reference 2) has shown that the section shape for minimum pressure drag as determined by linearized theory may have a blunt trailing edge. The use of linearized theory for determining optimum profiles facilitates the mathematical

development; however, the results are subject to question particularly at high-supersonic Mach numbers.

The purpose of the present paper is to compare the section shapes for minimum pressure drag (subject to certain auxiliary conditions) determined by linear and nonlinear theory in order to estimate the errors introduced by the linearized form of the pressure coefficient and to determine its range of validity for calculations of this nature. For this purpose, it was considered sufficient to examine two problems. The problems chosen were the determination of the profile for minimum drag for a given thickness ratio and the determination of the profile for minimum drag for a given area.

The nonlinear form of the expression for the pressure coefficient used in the present analysis is derived in reference 3 where it is shown to be in excellent agreement with the exact expression for stream Mach numbers greater than 1.5. The variation of base pressure coefficient with stream Mach number was assumed in order to determine actual airfoil shapes. This base-pressure curve was based in part upon experimental data and the known variation of vacuum pressure coefficient with Mach number.

NOTATION

X chordwise distance from leading edge

Y airfoil ordinate

c airfoil chord; also, $\frac{4}{\gamma + 1}$

γ ratio of specific heat at constant pressure to specific heat at constant volume

$x = \frac{X}{c}$

$y = \frac{Y}{c}$

M_∞ stream Mach number

$m = \sqrt{M_\infty^2 - 1}$

P pressure coefficient

t	thickness coefficient
$h = 1 - \frac{y_b}{t}$	
ϑ	flow deflection angle
$\theta = m\vartheta$	
c_d	drag coefficient
A	airfoil area

Subscripts:

1	front surface
2	rear surface
b	base
v	vacuum
w	refers to double-wedge profile
opt	optimum
TE	trailing edge

ANALYSIS AND DISCUSSION

The pressure drag of an airfoil is the drag due to the normal forces acting over the airfoil surface. For a profile with a blunt trailing edge the base drag must be added to the drag due to the normal forces over the forward surfaces to give the total pressure drag. With the surface pressure coefficient denoted by P, the airfoil shape by $Y(X)$, the chord by c, the base height by Y_b, and the base pressure coefficient by P_b, the pressure drag coefficient c_d of a thin symmetric two-dimensional airfoil at zero angle of attack is given approximately by

$$c_d = \frac{2}{c}\int_0^c \left(P - P_b\right)\frac{dY}{dX}\,dX = \frac{2}{c}\int_0^c P\,\frac{dY}{dX}\,dX - P_b\,\frac{Y_b}{c}$$

or with $x = \frac{X}{c}$, $y = \frac{Y}{c}$, and $y_b = \frac{Y_b}{c}$, the pressure drag may be written as

$$c_d = 2\int_0^1 P\,\frac{dy}{dx}\,dx - P_b y_b \tag{1}$$

Profile for minimum drag for a given thickness ratio.- In order to determine the profile for minimum drag for a given thickness ratio, it is required to determine the function $y(x)$ and the base height y_b which make c_d a minimum. For supersonic flow, to a high order of approximation, the pressure coefficient is a function only of the local slope $\frac{dy}{dx}$ and the stream Mach number. By use of the calculus of variations, any integral of the form $\int_{x_1}^{x_2} F\left(\frac{dy}{dx}\right)dx$ can be shown to have a stationary value for $\frac{dy}{dx}$ = Constant (reference 4, for example). Thus, since the integrand of equation (1) is a function only of the slope $\frac{dy}{dx}$, the profile of minimum drag has plane surfaces. Then, with $\vartheta = \frac{dy}{dx}$ and the notation of figure 1, the drag coefficient may be written as

$$c_d = 2x_1\vartheta_1 P_1 + 2\left(1 - x_1\right)\vartheta_2 P_2 - y_b P_b \tag{2}$$

where x_1 is the location of maximum thickness and

$$P_1 \equiv P\left(\vartheta_1\right) \qquad P_2 \equiv P\left(\vartheta_2\right)$$

Then with

$$\vartheta_1 = \frac{t}{2x_1} \qquad \vartheta_2 = -\frac{t}{2}\,\frac{h}{1 - x_1} \qquad h = 1 - \frac{y_b}{t}$$

where t is the thickness coefficient, equation (2) may be written as

$$c_d = t\left[P_1 - hP_2 + (h - 1)P_b\right] \tag{3}$$

Reference 3 shows that

$$P = \vartheta\left[\frac{\gamma + 1}{2}\vartheta + \sqrt{\left(\frac{\gamma + 1}{2}\right)^2\vartheta^2 + \frac{4}{M_\infty{}^2 - 1}}\right] \tag{4}$$

where M_∞ is the stream Mach number and γ is the ratio of specific heats at constant pressure and constant volume, is in excellent agreement with the pressure coefficients given by the exact shock or expansion relations for $1.5 \leqq M_\infty < \infty$. It may be noted that this expression approaches both the linearized value and the hypersonic value in the appropriate Mach number ranges. With $\theta = m\vartheta$ as the independent variable, equation (4) becomes

$$P = \frac{2}{m^2c} K(\theta)$$

where

$$m = \sqrt{M_\infty{}^2 - 1} \qquad c = \frac{4}{\gamma + 1}$$

$$K(\theta) = \theta\left(\theta + \sqrt{\theta^2 + c^2}\right)$$

Then, with these relations substituted into equation (3), the drag coefficient becomes

$$c_d = \frac{2t}{m^2c}\left[K_1 - hK_2 + (h - 1)\frac{c}{2} m^2P_b\right] \tag{5}$$

where

$$\theta_1 = \frac{mt}{2x_1} \qquad \theta_2 = -\frac{mt}{2}\frac{h}{1 - x_1}$$

$$K_1 \equiv K\left(\theta_1\right) \qquad K_2 \equiv K\left(\theta_2\right)$$

Values of x_1 and h are required which make the drag a minimum. Thus, solutions are required for the equations

$$\frac{\partial c_d}{\partial x_1} = 0 \qquad \frac{\partial c_d}{\partial h} = 0$$

for x_1 and h.

The base pressure coefficient is dependent upon Mach number, airfoil shape, Reynolds number, the nature of the boundary layer, and so forth. Of these the Mach number is by far the most important parameter. Since the base pressure depends to some extent on the base height, a term involving $\frac{\partial P_b}{\partial h}$ should be included in the second equation used to determine the airfoil of minimum drag. The omission of this term however is of little consequence. Its effect is to modify slightly the effective value of the base pressure. For turbulent boundary layers at high Reynolds numbers this change is small and the change in airfoil shape due to the neglect of this term is of little consequence. If the term $\frac{\partial P_b}{\partial h}$ is omitted, the equations defining the airfoil of minimum drag are

$$\frac{\partial c_d}{\partial x_1} = \frac{4}{m^3 c}\left(-\theta_1{}^2 K_1{}' + \theta_2{}^2 K_2{}'\right) = 0 \tag{6}$$

$$\frac{\partial c_d}{\partial h} = \frac{2t}{m^2 c}\left(-K_2 + \theta_2 K_2{}' + \frac{c}{2} m^2 P_b\right) = 0 \tag{7}$$

where

$$K' = \frac{\partial K}{\partial \theta} = \frac{K^2}{\theta^2\sqrt{\theta^2 + c^2}}$$

From the solutions of equations (6) and (7) for θ_1 and θ_2 as functions of $\left(M_\infty{}^2 - 1\right)P_b$, the location of maximum thickness and the trailing-edge thickness are determined from

$$\left.\begin{aligned} x_1 &= \frac{mt}{2\theta_1} \\ \frac{y_b}{t} &= 1 - h = 1 + \theta_2\left(\frac{2}{mt} - \frac{1}{\theta_1}\right) \qquad \left(-2\,\frac{\theta_1\theta_2}{\theta_1 - \theta_2} \leqq mt \leqq 2\theta_1\right) \end{aligned}\right\} \tag{8}$$

The lower limit for mt insures that the rear upper and lower surfaces can intersect only at the trailing edge. The upper limit for mt insures that the location of maximum thickness will lie between the leading and trailing edges of the airfoil.

It is important to note that θ_1 and θ_2 are functions only of $\left(M_\infty^2 - 1\right)P_b$. Therefore, x_1 and y_b/t are functions only of $t\sqrt{M_\infty^2 - 1}$ and $\left(M_\infty^2 - 1\right)P_b$. The number of parameters has thus been reduced from three to two. From equation (8) the trailing-edge thickness is zero for $mt = -2\frac{\theta_1\theta_2}{\theta_1 - \theta_2}$; thus, the profile of minimum drag has a sharp trailing edge for this condition. For profiles with a sharp trailing edge, only equation (6) need be solved and the location of maximum thickness then is a function only of $t\sqrt{M_\infty^2 - 1}$. For $mt = 2\theta_1$, the the position of maximum thickness for minimum drag is at the trailing edge. For values of $mt > 2\theta_1$, no mathematical minimum for the drag exists.

The solution of the equations defining the profile of minimum pressure drag is especially simple for linearized flow (reference 2, for example). The approximate expression for the pressure coefficient given by linearized theory is

$$P = \frac{2}{\sqrt{M_\infty^2 - 1}}\,\vartheta$$

and the equations which determine the profile of minimum drag are

$$\frac{\partial c_d}{\partial x_1} = \frac{4}{m^3}\left(-\theta_1^2 + \theta_2^2\right) = 0 \quad \text{or} \quad \theta_1 = -\theta_2 \tag{9}$$

$$\frac{\partial c_d}{\partial h} = \frac{t}{m^2}\left(-4\theta_2 + m^2 P_b\right) = 0 \quad \text{or} \quad \theta_2 = \frac{m^2 P_b}{4} \tag{10}$$

Then, the location of maximum thickness and the trailing-edge thickness are found to be

$$\left.\begin{aligned} x_1 &= -\frac{2t}{mP_b} \\ \frac{y_b}{t} &= \frac{mP_b}{2t} + 2 = 2 - \frac{1}{x} \qquad \left(-\frac{m^2 P_b}{4} \leqq mt \leqq -\frac{m^2 P_b}{2}\right) \end{aligned}\right\} \tag{11}$$

For a profile with a sharp trailing edge $\left(mt \leqq -\frac{m^2 P_b}{4}\right)$, the familiar result that, for minimum drag, the location of maximum thickness is at the midchord is found as the solution of equation (9) for $y_b = 0$.

The solution of equations (6) and (7) is shown in figure 2 where θ_1 and θ_2 are given as functions of $\left(M_\infty^2 - 1\right)P_b$. The dashed vertical line at $-\left(M_\infty^2 - 1\right)P_b$ equal to 1.428 is the limiting value of the vacuum pressure coefficient for infinite Mach number. The solutions for linearized flow are also shown in figure 2 for comparison. It may be noted that the values of θ_1 for linearized flow are in good agreement with the results of this analysis, whereas θ_2 shows appreciable deviation. Curves of $t\sqrt{M_\infty^2 - 1}$ are also shown as functions of $\left(M_\infty^2 - 1\right)P_b$ in figure 2. The upper set of curves gives the value of $t\sqrt{M_\infty^2 - 1}$ for which the optimum profile has the maximum-thickness location at the trailing edge and the lower set of curves gives the value of $t\sqrt{M_\infty^2 - 1}$ for which the optimum profile has a sharp trailing edge. At a given value of $\left(M_\infty^2 - 1\right)P_b$, true minimum-drag profiles do not exist for values of $t\sqrt{M_\infty^2 - 1}$ greater than the values given by the upper curve. The profile of least drag, however, under those conditions is one which has the maximum-thickness location at the trailing edge. For values of $t\sqrt{M_\infty^2 - 1}$ less than those of the lower curve, the profile of minimum drag has a sharp trailing edge and the position of maximum thickness for minimum drag may be determined from figure 3. Profiles for minimum pressure drag are readily determined with the use of figures 2 and 3. For a given base pressure coefficient and Mach number, values of θ_1 and θ_2 are found

from figure 2. Then for $-2\frac{\theta_1\theta_2}{\theta_1 - \theta_2} \leqq t\sqrt{M_\infty^2 - 1} \leqq 2\theta_1$, the position of maximum thickness and the trailing-edge thickness are found from equations (8) for a given thickness coefficient. For values of $t\sqrt{M_\infty^2 - 1} \leqq -2\frac{\theta_1\theta_2}{\theta_1 - \theta_2}$, the profile has a sharp trailing edge and the position of maximum thickness for minimum drag is determined directly from figure 3.

In order to show more clearly the variation of the shape of the airfoil for minimum drag with Mach number, a number of profiles have been calculated for the base pressures shown in figure 4. The base pressure coefficients in this figure are, in part, based upon some knowledge of the base pressures for turbulent boundary layers and, in part, upon the variation of the vacuum pressure coefficient with Mach number. For comparison the vacuum pressure coefficient P_V is also shown.

In figure 5 the position of maximum thickness and the trailing-edge thickness for minimum drag are presented as functions of Mach number for the base pressure coefficients of figure 4. Here it may be noted that, for a given thickness ratio, the minimum-drag airfoil has a sharp trailing edge for the lower Mach numbers and at higher Mach numbers the trailing edge is blunt. Further, the Mach number at which the minimum-drag profile first has a blunt trailing edge is higher for the thinner airfoils. For a given thickness ratio, the position of maximum thickness moves rearward with increasing Mach number until it is located at the trailing edge.

Also shown in figure 5 are the optimum profiles for the 6- and 10-percent-thick airfoils calculated from the linearized equations (11). For the 6-percent-thick airfoil the position of maximum thickness remains fixed at the midchord up to a stream Mach number of approximately 5. At Mach numbers greater than 5, the position of maximum thickness moves rearward and the trailing-edge thickness increases in a manner similar to that determined from the nonlinear equations. For the 10-percent-thick profile the position of maximum thickness and the trailing-edge thickness show an erratic behavior at the low Mach numbers, that is, at low values of the base pressure coefficient. At a stream Mach number of 1.5 the optimum position of maximum thickness is at the 0.56 chord. At Mach numbers from 1.5 to 2.1 the maximum-thickness location moves forward, and at higher Mach numbers it moves rearward in a manner similar to that determined from the nonlinear equations. In general, the linearized theory predicts the position of maximum thickness fairly accurately over a wide Mach number range but does not predict the optimum trailing-edge thickness so well.

In order to illustrate the effect of the airfoil geometry on drag, the drag coefficients of the optimum profiles have been calculated for the 6- and 10-percent-thick airfoils. In figure 6 the ratio of the optimum drag coefficient $c_{d_{opt}}$ to the drag coefficient of a double-wedge airfoil c_{d_w} is shown as a function of the stream Mach number for the profile determined by both the nonlinear and linear theory. For the 10-percent-thick airfoil the drag presented is not a minimum above a Mach number of approximately 6. The drag curve above this Mach number is that for an airfoil with the maximum thickness located at the trailing edge. Here it may be noted that the optimum profiles have substantially lower drag than the double-wedge profiles, particularly at high Mach numbers. Further, the drag reduction is considerably greater for the 10-percent-thick airfoil than for the 6-percent-thick airfoil. The dashed curves of figure 6 correspond to the drag coefficients for the profiles found by linear theory. In calculating the drag, however, the nonlinear expression for the pressure coefficient was used; that is, only the geometry of the airfoils was calculated by linear theory. Here it may be noted that the drag for the profiles determined by linear equations is only slightly higher than the drag of the optimum profiles found with the nonlinear relations. The largest difference in drag between the 6-percent-thick profiles determined from the nonlinear equations and the profiles determined from the linear equations is less than 4 percent; for the 10-percent-thick profiles the differences in drag are even less.

Profile for minimum drag for a given area.- The problem of determining the profile of minimum drag and satisfying a given structural condition is an isoperimetric problem of the calculus of variations. The equations for determining the supersonic profile of minimum drag for a given structural condition have been developed in reference 2. The equations for determining the airfoil of minimum drag for a given area (or torsional strength) are

$$\left.\begin{aligned}
&y'^2 \frac{\partial P}{\partial y'} = \lambda\left(\frac{t}{2} - y\right) \\
&A = 2\int_0^1 y\,dx \\
&P_{TE} - P_b + \left(y' \frac{\partial P}{\partial y'}\right)_{TE} = 0 \\
&y(0) = 0
\end{aligned}\right\} \tag{12}$$

where $y' = \frac{dy}{dx}$, λ is a constant to be determined from the solution, A is the area, and the subscript TE refers to the values at the trailing edge of the airfoil. The solution of these equations for the linearized form of the pressure coefficient gives the profile shape as

$$y = \frac{t}{2}\frac{x}{x_1}\left(2 - \frac{x}{x_1}\right)$$

where x_1 is the position of maximum thickness and is given by

$$x_1 = \frac{1 - \frac{mP_b}{12A}}{1 - \frac{mP_b}{4A}}$$

and the thickness coefficient is given by

$$t = \frac{3}{2}Ax_1\left(1 - \frac{mP_b}{12A}\right)$$

The solution of equations (12) for the nonlinear form of the pressure coefficient was obtained by an iterative procedure.

Figure 7 presents a comparison of the profiles determined by linear and nonlinear theory for $A = 0.05$ for several Mach numbers and for the base pressure coefficients given in figure 4. The linearized theory gives a location of maximum thickness farther forward and a smaller trailing-edge thickness and thickness ratio than given by nonlinear theory. These differences in geometry of the profiles determined from linear and nonlinear theory follow the same trend as for the airfoils of a given thickness ratio; this trend may be expected for other auxiliary structural conditions as well. The drag of the profiles found by linear and nonlinear theory (equation (4) was used for computing the pressures for the evaluation of the drag in each case) differed by less than 2 percent for Mach numbers from 2 to 10.

CONCLUDING REMARKS

The results show that the profile shape for minimum drag for a given thickness ratio or for a given area is determined with sufficient accuracy by linear theory over the entire supersonic Mach number range since the drag coefficients for these profiles are only slightly higher than those for

optimum profiles determined by nonlinear theory. It would appear that linear theory should also be adequate for determining profiles for minimum drag for other auxiliary structural conditions since moderate deviations from the optimum shape have only a small influence on the pressure drag coefficient.

It appears that, when airfoils with finite trailing-edge thicknesses are considered, the linearized theory may be used for determining profiles of minimum drag (although not the drag itself) at least up to Mach numbers of 10.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., November 14, 1951

REFERENCES

1. Drougge, Georg: Wing Sections with Minimum Drag at Supersonic Speeds. Rep. No. 26, Aero. Res. Inst. of Sweden (Stockholm), 1949.

2. Chapman, Dean R.: Airfoil Profiles for Minimum Pressure Drag at Supersonic Velocities - General Analysis with Application to Linearized Supersonic Flow. NACA TN 2264, 1951.

3. Ivey, H. Reese, and Cline, Charles W.: Effect of Heat-Capacity Lag on the Flow through Oblique Shock Waves. NACA TN 2196, 1950.

4. Jeffreys, Harold, and Jeffreys, Bertha Swirles: Methods of Mathematical Physics. Cambridge Univ. Press, 1946.

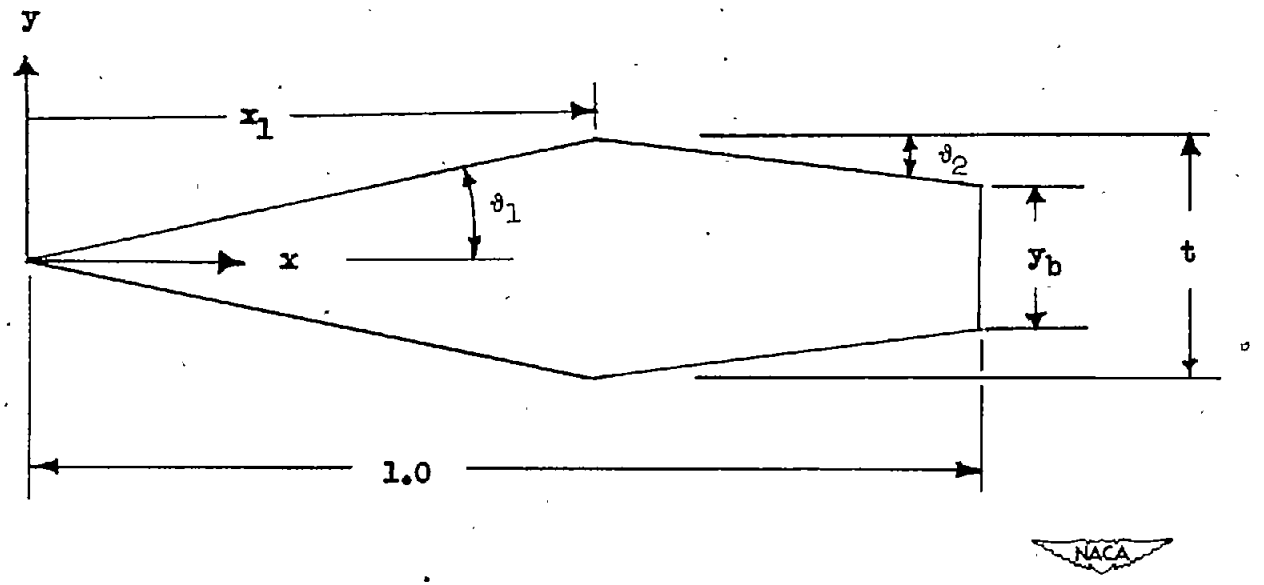

Figure 1.- Optimum profile for a given thickness ratio.

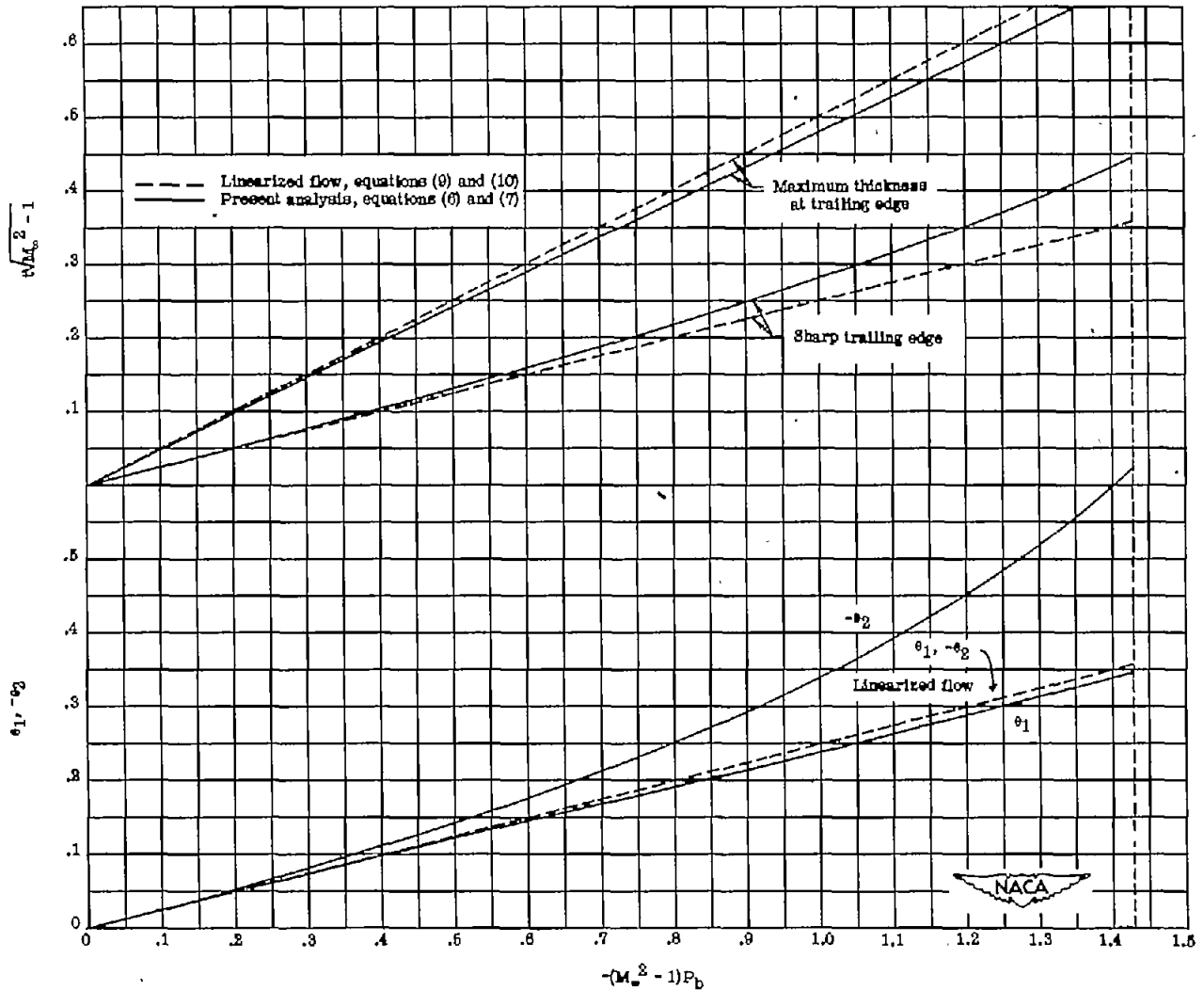

Figure 2.- The parameters θ_1 and θ_2 for profiles of minimum drag as functions of base pressure and the limits on $t\sqrt{M_\infty^2 - 1}$ for which the solutions are valid.

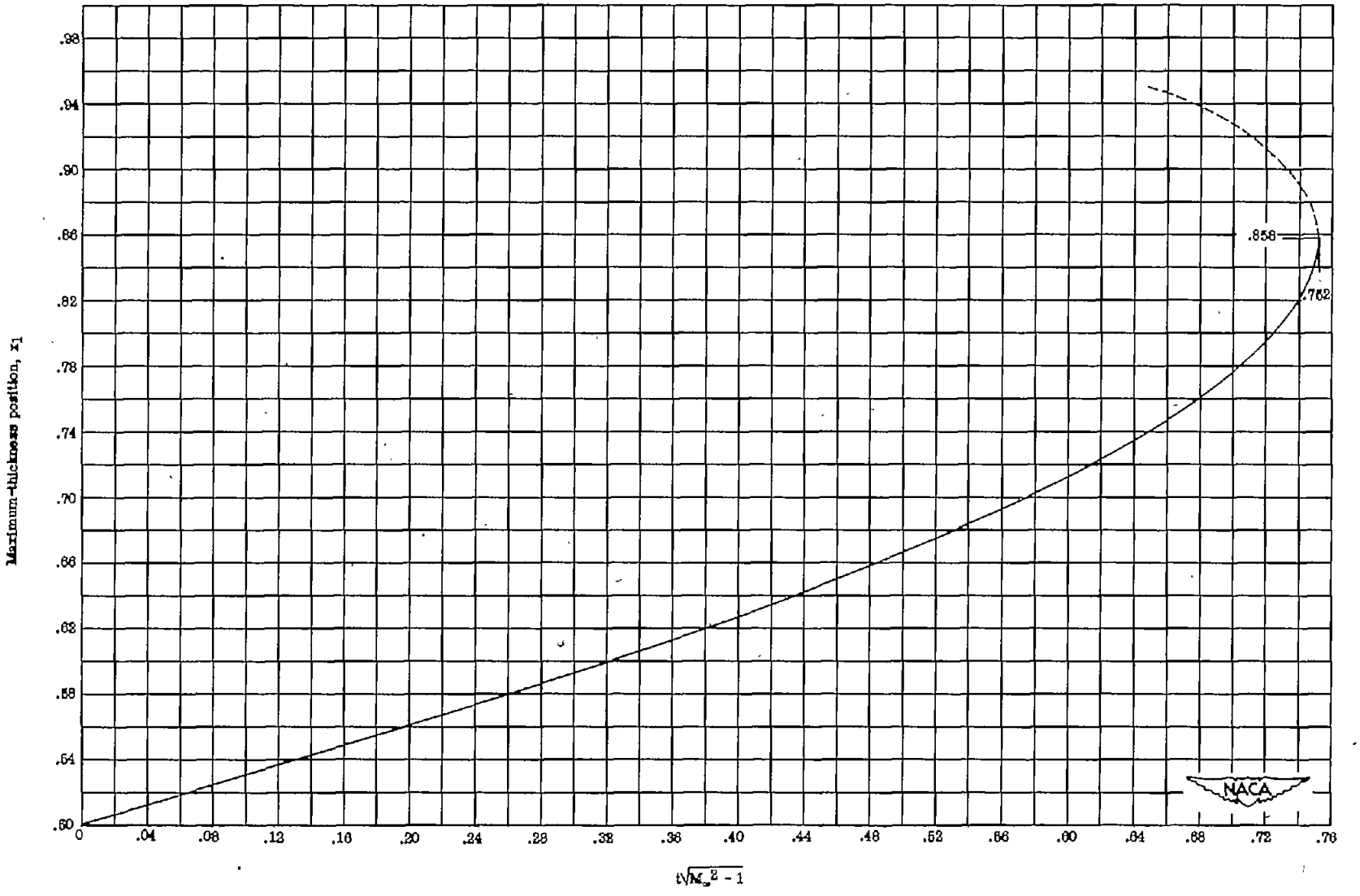

Figure 3.- Position of maximum thickness for sharp-trailing-edge airfoils of minimum drag.

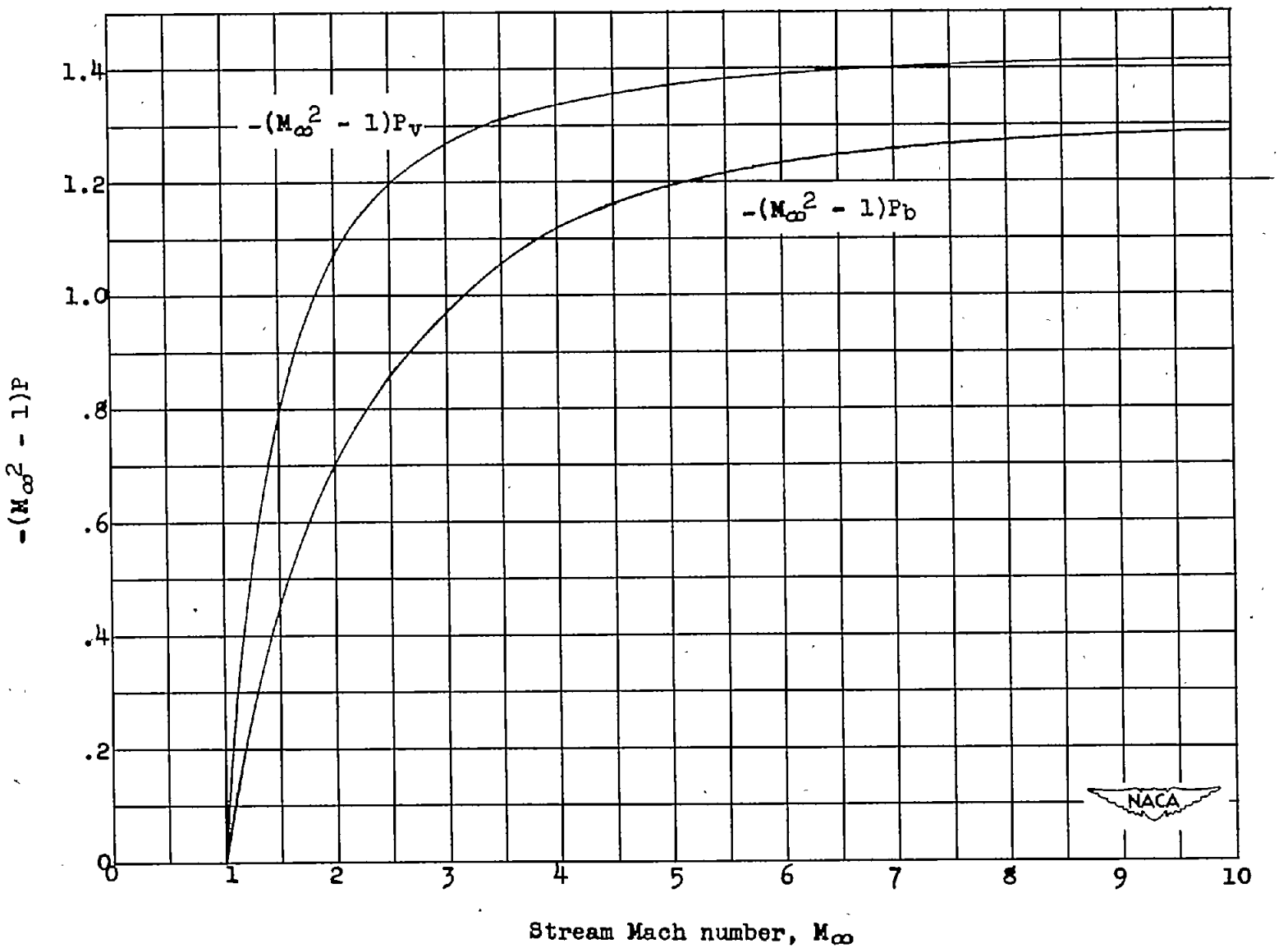

Figure 4.- Vacuum pressure coefficient and base pressure coefficient as functions of stream Mach number.

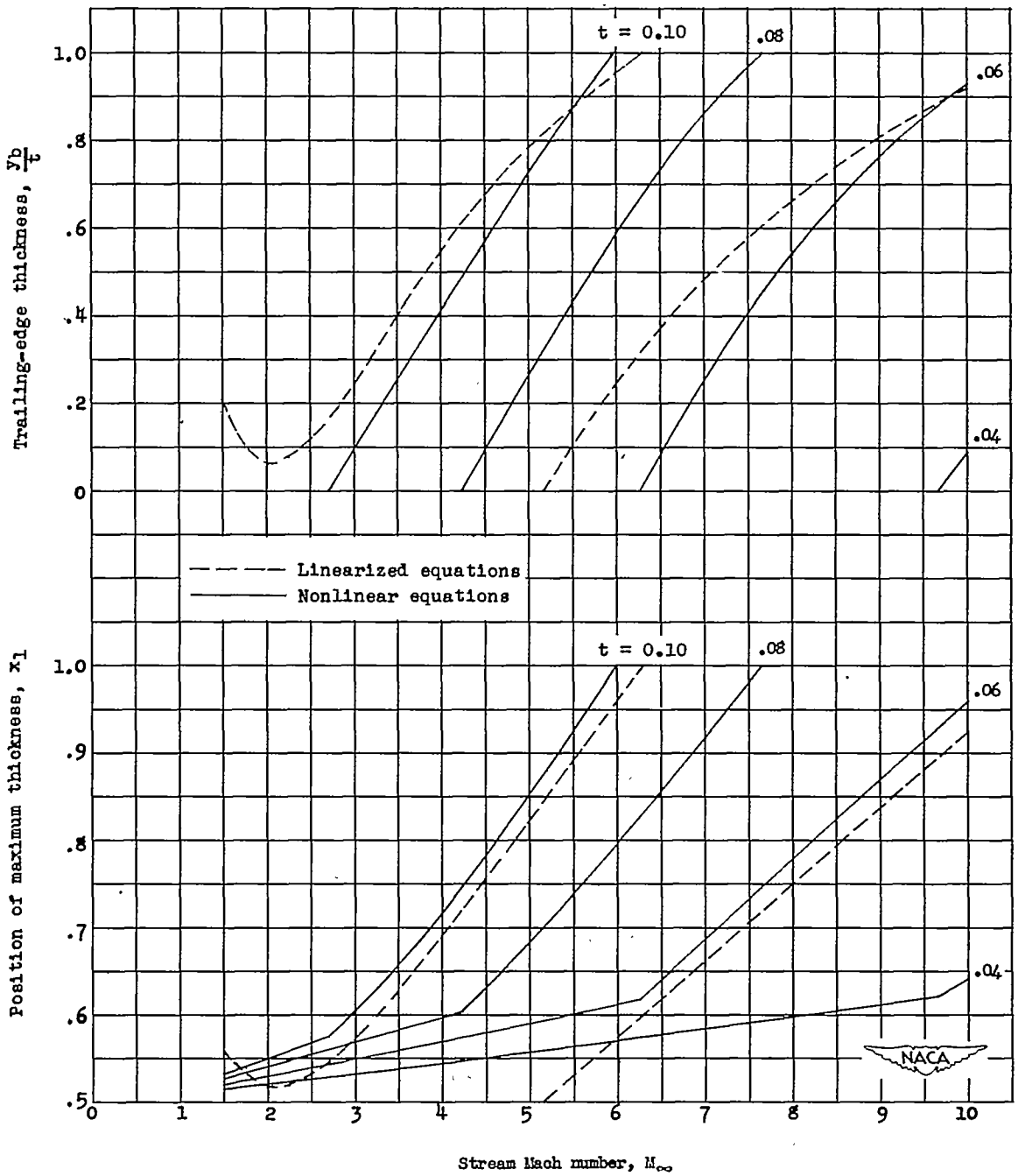

Figure 5.- Position of maximum thickness and trailing-edge thickness for airfoils of minimum drag for base pressures presented in figure 4.

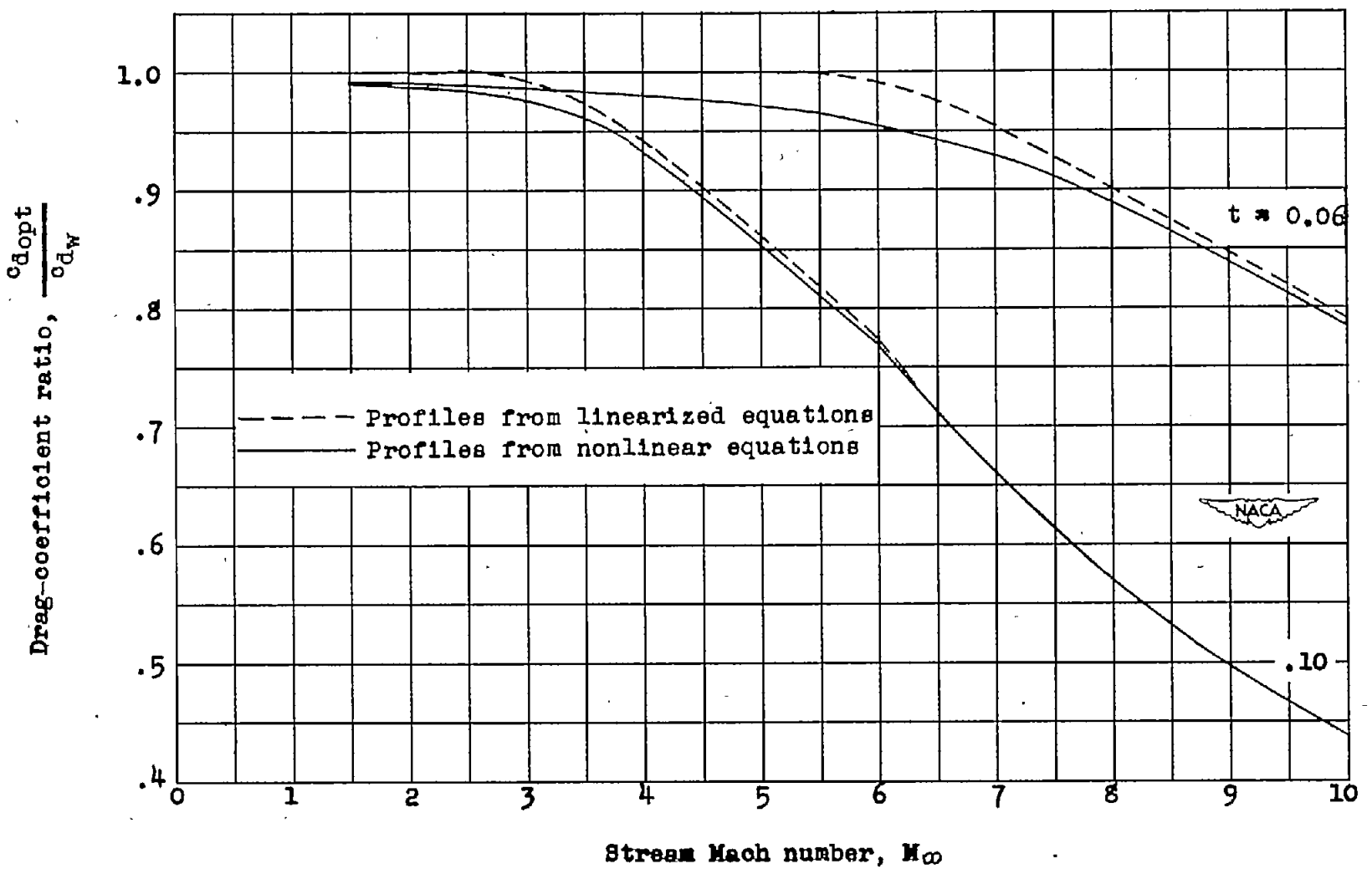

Figure 6.- Ratio of drag coefficient of optimum airfoil to drag coefficient of double-wedge airfoil.

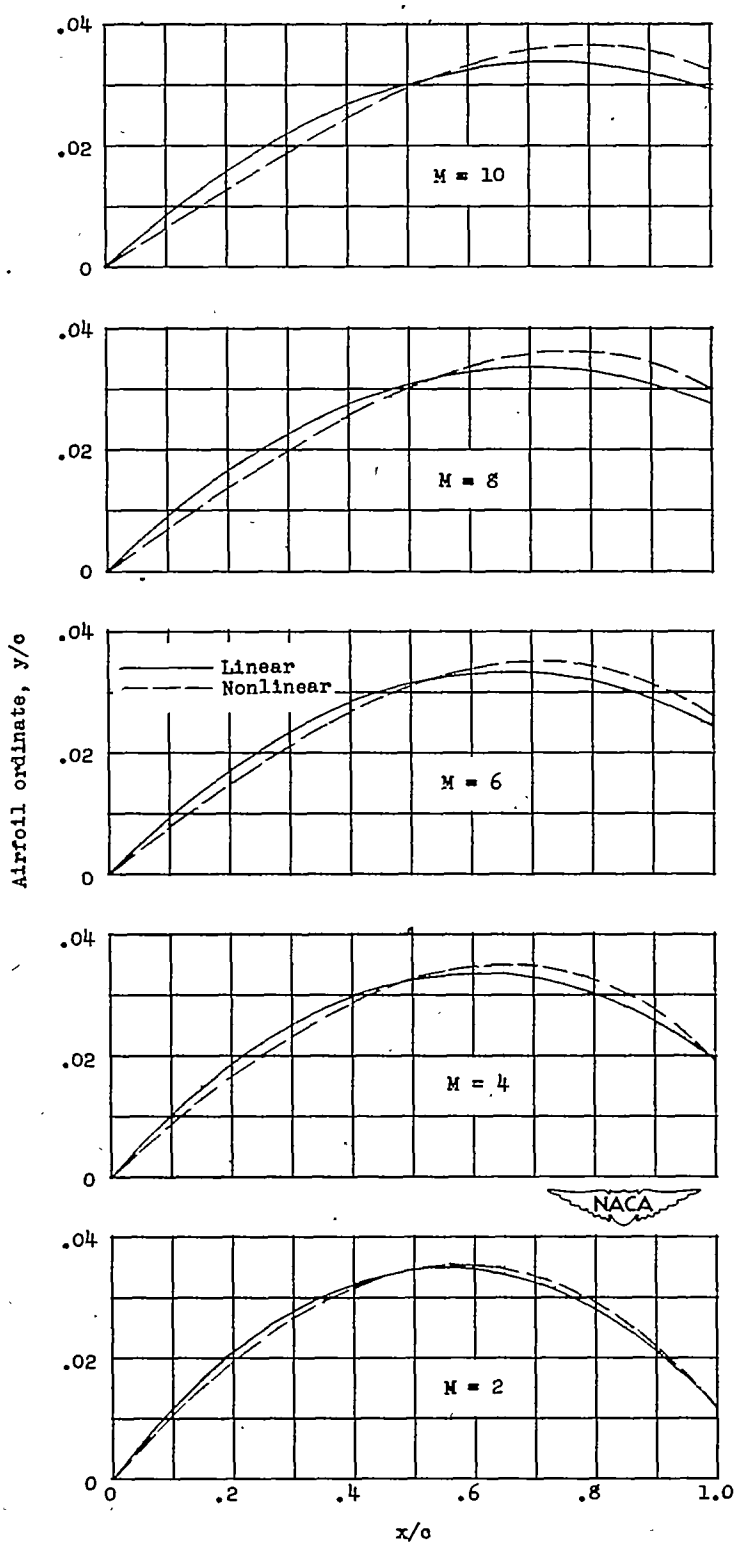

Figure 7.- Comparison of optimum profiles for a given area as determined by linear and nonlinear theory. A = 0.05.

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