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THEORETICAL INVESTIGATION OF VELOCITY DIAGRAMS OF A SINGLE-STAGE TURBINE FOR A TURBOJET ENGINE AT MAXIMUM THRUST PER SQUARE FOOT TURBINE FRONTAL AREA

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Cleveland, Ohio

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THEORETICAL INVESTIGATION OF VELOCITY DIAGRAMS OF A SINGLE-STAGE TURBINE FOR A TURBOJET ENGINE AT MAXIMUM THRUST PER SQUARE FOOT TURBINE FRONTAL AREA

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SUMMARY

An exploratory velocity-diagram analysis was made in order to establish the aerodynamic requirements of a single-stage, axial-flow turbine which satisfies the conditions of maximum thrust per square foot of turbine frontal area in a turbojet engine. The investigation was made for a range of turbine mean blade speeds and inlet temperatures, fixed flight conditions, and a constant centrifugal stress at the root of the rotor blading.

The results of this analysis indicate that, if this ultimate performance is to be realized, supersonic velocities relative to the leading and trailing edges of the turbine rotor are required for practically all cases investigated. The axial component of velocity leaving the turbine is sonic for all cases, and the velocity diagrams at the mean radius are of the impulse type.

A comparison was made employing conventional turbine aerodynamic design limits for one set of conditions from the analysis. For the conditions selected, a maximum-thrust turbine design with the same internal efficiency provided an engine thrust increase of approximately 17 percent.

INTRODUCTION

Emphasis on the development of high-speed aircraft has resulted in a demand for turbojet engines of high thrust, low frontal area, and light weight. It is essential that the turbine components of these

engines conform to this specification of small size and weight. Therefore, an exploratory velocity-diagram analysis was made at the NACA Lewis laboratory to establish the aerodynamic specifications of a single-stage turbine which provides these characteristics. Since turbine frontal area may be considered a general index of turbine weight, this analysis was made on the basis of obtaining maximum thrust per square foot of turbine frontal area. Conventional aerodynamic design limits were not considered as a limitation in developing the turbine velocity diagrams. Calculations were made for a range of compressor pressure ratios sufficient to include the maximum value of engine thrust for each turbine-inlet temperature and blade speed investigated. The same centrifugal stress at the root of the rotor blade was assumed for all blade speeds. 2423

This analysis, therefore, establishes the potential level of performance for a single-stage turbine, determines the gains potentially available over current designs, and indicates the manner and extent to which current limits must be exceeded to obtain these gains.

SYMBOLS

The following symbols are used in this report:

A turbine annulus area (sq ft/sq ft turbine frontal area)

c_p specific heat of air at constant pressure (0.24 Btu/(lb)($^{\circ}$R))

F engine thrust (lb/sq ft turbine frontal area)

g acceleration due to gravity (32.2 ft/sec^2)

J mechanical equivalent of heat (778.3 ft-lb/Btu)

P horsepower per square foot turbine frontal area

p absolute pressure (lb/sq ft)

R gas constant for air (53.3 ft/$^{\circ}$R)

r hub-tip radius ratio

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T absolute temperature ($^{\circ}R$)

U blade speed (ft/sec)

V absolute velocity (ft/sec)

W relative velocity (ft/sec)

w air weight flow (lb/(sec)(sq ft turbine frontal area))

α absolute flow angle (deg)

β relative flow angle (deg)

γ ratio of specific heats of air, 1.40

δ pressure reduction ratio, p'/p_0'

η internal efficiency

θ temperature reduction ratio, T'/T_0'

ρ density of gas (lb/cu ft)

ρ_b density of blade material (lb/cu ft)

σ blade centrifugal stress (lb/sq in.)

ψ blade stress taper factor

Subscripts:

0 NACA sea level air

1 compressor inlet

2 compressor discharge

3 upstream of turbine stator

4 upstream of turbine rotor

5 rotor exit

6 exhaust-cone exit

c compressor

m mean radius

T tip radius

t turbine

Superscript:

' stagnation or total conditions

GENERAL ANALYSIS

Conditions

An investigation was undertaken to determine the velocity diagram for a single-stage, axial-flow turbine component of a turbojet engine at maximum thrust per square foot turbine frontal area for a given turbine-inlet temperature and mean blade speed. Calculations were made for turbine-inlet temperatures of 2000°, 2750°, and 3500° R at mean blade speeds of 1000, 1200, and 1400 feet per second.

The hub-tip radius ratio and tip speed for each mean blade speed were calculated for a centrifugal stress at the blade root of 30,000 pounds per square inch, which is an assumed value based on current design practice. The calculation procedure used in determining these hub-tip ratios is outlined in appendix A.

The flight condition used in the analysis was 600 miles per hour at sea level, corresponding to a flight Mach number of 0.79.

Assumptions

Engine. - Sea-level ambient air was assumed to enter the engine at a velocity of 880 feet per second (600 mph) with a ram efficiency of 100 percent. The adiabatic efficiency of the compressor was taken as 85 percent, with no loss in total pressure in the combustor. The weight of the fuel added during combustion was neglected. The efficiency of the exhaust nozzle was taken as 100 percent for the range of exhaust-cone pressure ratios encountered in this analysis. The values of c_p and γ were assumed constant through the engine.

Turbine. - The turbine was assumed to have a cylindrical blade annulus. The internal efficiency was taken as 100 percent in calculating the turbine velocity diagrams and was considered as a variable (80 to 100 percent) in correcting the engine thrust for turbine efficiency. The tangential component of the absolute velocity immediately downstream of the turbine was assumed to be dissipated without energy recovery; the axial component was assumed to be completely recovered.

Method of Analysis

In order to determine the turbine conditions and the requirements for maximum thrust per square foot turbine frontal area, the compressor pressure ratio for this engine condition must first be evaluated. The range of compressor pressure ratios for which calculations were made was

sufficient to include the maximum value of thrust for each turbine-inlet temperature and mean blade speed investigated. The compressor pressure ratio at maximum thrust was determined graphically from a curve of thrust against compressor pressure ratio. The aerodynamic requirements of the turbine at the maximum thrust condition were determined indirectly from this plot since a turbine velocity diagram was calculated for each compressor pressure ratio considered.

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The turbine velocity diagram which was calculated for each compressor pressure ratio is unique in that the maximum air weight flow is being passed through the turbine at the specific power output required by the compressor. Therefore, a turbojet engine utilizing a turbine design based on the derived velocity diagram would develop the maximum engine thrust per square foot turbine frontal area for the assumed conditions.

The effect of turbine efficiency on engine thrust was determined by considering that the compressor work was reduced from its value at 100-percent turbine efficiency by the inefficiency of the turbine, with corresponding reductions in both engine air weight flow and pressure ratio across the exhaust cone. The total-static pressure ratio across the turbine was maintained constant for each velocity diagram over the range of turbine efficiencies investigated; the axial component of the velocity leaving the turbine was assumed to be independent of turbine efficiency.

Calculation Methods and Procedure

The velocity diagrams were determined for the mean radius since performance at this section closely approximates the over-all performance of the turbine stage.

The following procedure was used to relate the thrust of a turbojet engine to the turbine velocity diagram and ambient conditions:

The thrust for a turbojet engine installed in a plane moving with a velocity V_1 is given by

$$F = \frac{w}{g}(V_6 - V_1) \tag{1}$$

From the turbine velocity diagram (fig. 1), the air weight flow per square foot turbine frontal area may be expressed as

$$w = \rho_4 V_4 \sin \alpha_4 A \tag{2}$$

and the velocity of the air leaving the engine as

$$V_6 = \left[\overline{W_5 \sin \beta_5}^{\,2} + 2gJc_p \left(T_5 - T_6\right)\right]^{\frac{1}{2}} \tag{3}$$

The annulus area A per square foot turbine frontal area is related to the hub-tip radius ratio as follows: 2423

$$A = (1-r^2) \tag{4}$$

The compressor power and the turbine power are equal and may be expressed for a unit air weight flow as follows:

$$\frac{P_c}{w} = \frac{Jc_pT_1'}{550\eta_c}\left[\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{5}$$

$$\frac{P_t}{w} = \frac{U_m}{550g}\left(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m\right) \tag{6}$$

Equations (2) to (6) relate the turbine air weight flow, the velocity of the air leaving the exhaust cone, and the compressor power requirement to the turbine dimensions and the ideal turbine velocity diagram.

The next step in the analysis is to assume a sufficient number of compressor pressure ratios to define the one which results in the maximum thrust for each turbine-inlet temperature and blade speed. Each pressure ratio will specify a turbine power requirement as indicated in equations (5) and (6). The problem then resolves itself into determining a turbine velocity diagram for each compressor pressure ratio such that the maximum weight flow is being passed by the turbine at the specific power required by the compressor. This will result in the maximum engine thrust for the assumed conditions.

The maximum air weight flow immediately downstream of a turbine rotor of given annular area will occur when the axial component of the velocity at this station is sonic, which may be expressed as

$$W_5 \sin \beta_5 = \sqrt{\gamma g R T_5} \tag{7}$$

Since the air density in the turbine is usually lowest immediately downstream of the turbine rotor, it is essential that the absolute exit whirl $(W_5 \cos \beta_5 - U_m)$ be kept as small as possible if the maximum air-handling capacity of the turbine annulus is to be obtained.

Thus the solution to the problem of obtaining maximum weight flow at a specified work output depends on two points: (1) a sonic axial velocity component leaving the turbine rotor, and (2) minimum absolute whirl leaving the rotor.

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For any specific power output P/w and mean blade speed U_m the net change in the tangential components of the absolute velocities entering and leaving the rotor at the mean radius in the ideal case can be classified into three categories: (1) less than, (2) equal to, or (3) greater than twice the mean blade speed.

In the case in which the tangential component change is less than twice the mean blade speed, the requirement of minimum whirl leaving the turbine can be satisfied by specifying zero exit whirl because the proportion of blade speed to work output makes this possible. Since the absolute velocity leaving the turbine is now axial and sonic, the maximum weight flow will be passed at the required work output. The velocity diagram is then calculated as shown in appendix B.

The second case, in which the change in tangential components of the absolute velocities V_4 and V_5 is equal to twice the mean blade speed, also permits the specification of zero exit whirl for the same reason as in the case outlined in the preceding paragraph. For this case, however, the tangential components of the velocities relative to the rotor leading and trailing edges will be equal and no change in state is required in the rotor to produce the required work. Since the absolute velocity leaving the rotor is axial and sonic, the axial component of the velocity upstream of the rotor is also sonic, so that both the rotor inlet and rotor outlet annuli are passing the maximum weight flow. The procedure used in calculating this type of diagram is given in appendix B.

In the consideration of the case in which the required change in tangential velocities exceeds twice the mean blade speed, there is a clearly defined limit below which the absolute exit whirl cannot be decreased, since the compensating increase in the absolute whirl entering the rotor will result in a reduction in weight flow at this point below the maximum obtainable at the trailing edge of the rotor. The lower limit in absolute exit whirl corresponds to the condition at which the maximum weight flow is being passed both upstream and downstream of the rotor. At this point, the absolute whirl entering the rotor will be at its highest value consistent with maximum weight flow and therefore will result in the minimum exit whirl leaving the turbine. Since the

flow is assumed to be isentropic in the ideal case, there will be no change in state in the rotor and therefore no change in relative velocity from the leading to the trailing edges of the rotor. The calculation procedure for this case is given in appendix B.

The static temperature downstream of the turbine rotor T_5 in all cases was calculated from the turbine velocity diagram using equation (7). The static temperature of the gas leaving the engine tail pipe was determined using the following equation, which assumes isentropic flow from the combustor to the tail pipe: 2423

$$T_6 = T_3' \left(\frac{p_6}{p_3'}\right)^{\frac{\gamma-1}{\gamma}} \tag{8}$$

where p_6 is ambient atmospheric pressure.

With these quantities determined, the velocity of the exhaust jet was calculated from equation (3)

$$V_6 = \left[\overline{W_5 \sin \beta_5}^{\,2} + 2gJc_p\,(T_5 - T_6)\right]^{\frac{1}{2}} \tag{3}$$

The turbine air weight flow per square foot turbine frontal area was determined from equations (2) and (4), which were combined as follows:

$$w = \frac{p_3'}{RT_3'}\left(1 - \frac{V_4^{\,2}}{2gJc_pT_3'}\right)^{\frac{1}{\gamma-1}} (1-r^2)\,V_4 \sin \alpha_4 \tag{9}$$

The engine thrust was then calculated by substituting the results of equations (3) and (9) into equation (1).

In correcting the engine thrust output derived from a given ideal turbine velocity diagram for turbine efficiency, the turbine power output was assumed to vary directly as the turbine efficiency for an assigned turbine total-static pressure ratio. The resulting reduction in compressor pressure ratio decreased both the turbine weight flow and the static-pressure ratio across the exhaust cone. These reductions were in direct proportion to the reduced compressor pressure ratio. The newly determined values of air weight flow and exhaust-cone pressure ratio together with the assumption that the axial velocity leaving the turbine was independent of turbine efficiency were sufficient to evaluate the engine thrust at the assumed turbine efficiency. The procedure is outlined in appendix C.

RESULTS AND DISCUSSION

Velocity Diagrams at Mean Radius

Mach numbers. - The changes in the Mach numbers of the velocities leaving the stator at maximum thrust are slight over the range of blade speeds and turbine-inlet temperatures investigated, as indicated in table I(a). An exception was noted in the Mach number relative to the rotor leading edge at a blade speed of 1400 feet per second and a turbine-inlet temperature of 2000° R because of the comparatively low inlet whirl to the rotor as determined from the turbine work and the blade speed.

For a given turbine-inlet temperature the Mach numbers relative to the rotor generally decrease slightly with increasing mean blade speed but become practically independent of blade speed at 3500° R.

The Mach numbers of the absolute velocities leaving the turbine decrease with rising blade speed but increase with turbine-inlet temperature. The Mach number of the axial component of the velocity leaving the rotor is 1.00 in all cases; this applies to the rotor inlet as well, with the one exception noted earlier.

The Mach numbers in the turbine blading at maximum thrust per square foot turbine frontal area indicate that supersonic velocities relative to the leading and trailing edges of the rotor are required at the mean radius in order for a single-stage turbine to meet the engine requirements at this condition for the range of inlet temperatures and blade speeds investigated. In general, the Mach numbers relative to the leading and trailing edges of the rotor are equal so that the impulse condition exists at the mean radius. This indicates that a rise in static pressure across one-half of the blade height will be required if simple radial equilibrium is to be satisfied downstream of the turbine rotor.

From consideration of simple radial equilibrium, there will be an increase in the stator exit Mach number toward the hub radius and a reduction in Mach number toward the tip radius, the degree of radial variation of Mach number being dependent on the hub-tip radius ratio.

Flow angles and turning angles. - The variations in the stator and rotor blade angles at maximum thrust per square foot turbine frontal area are small over the range of blade speeds and turbine-inlet temperatures investigated, as indicated in table I(b).

The turning in the stator, with the one exception noted earlier, varies from 56° to 60° approximately, with from 78° to 94° of turning being required in the rotor for all cases considered.

The greatest variation in flow angle is in the absolute angle leaving the rotor α_5, which includes values from 58° to 90°. These angles increase with mean blade speed for a given inlet temperature and are reduced as inlet temperature is increased.

The velocity diagrams at maximum thrust are given in figure 2 for the full range of conditions investigated.

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Effect of Blade Speed on Thrust per Square Foot Turbine Frontal Area

The values of thrust indicated in figures 3 to 5 are approximate since several simplifying assumptions were made in calculating them (see Assumptions). The values are given here as an index of comparison to be used in evaluating the results of the analysis and should not be interpreted as representing the performance of an actual engine.

The thrust increased with decreasing blade speed at all turbine-inlet temperatures for the range of blade speeds investigated. However, the percentage gain in engine thrust due to low blade speed was appreciably less as turbine-inlet temperature was increased. For a turbine internal efficiency of 0.90 at the maximum thrust condition, the gains in thrust at 1000 feet per second over those obtainable at 1200 feet per second and 1400 feet per second were 18 percent and 48 percent, respectively, at 2000° R; at 2750° R these increases were 9 percent and 25 percent, respectively; at 3500° R they were 3 percent and 12 percent, respectively.

Variation of Turbine Air Capacity with Blade Speed

The air-handling capacity of the gas turbine in the turbojet engine is a topic of general interest and is given here to indicate the extent of its variation with turbine-inlet temperature, mean blade speed, and turbine efficiency as established from this investigation. The results of the analysis are given in figure 6 as equivalent weight flow per square foot turbine frontal area referred to the compressor inlet conditions for the range of inlet temperatures and blade speeds covered in this analysis.

The equivalent weight flow decreased continually with increasing rotational speed for a given turbine-inlet temperature. The decrease in air-handling capacity with increasing rotational speed is, however, reduced on a percentage basis as turbine-inlet temperature increases.

A compressor air-handling capacity of 37 pounds per second per square foot of frontal area probably represents the maximum value that can be achieved in the next several years. The air-handling capacity of the maximum-thrust turbine of this analysis closely approximates this value at the following design conditions: internal efficiency, 0.90; inlet temperature, 2190° R; and mean blade speed, 1000 feet per second. At mean blade speeds of 1200 and 1400 feet per second, turbine-inlet temperatures of 2540° and 2940° R, respectively, would be required for the turbine to pass this air flow.

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Effect of Conservative Mach Number Limits on Thrust

In order to determine the effect on engine thrust of conventional turbine aerodynamic design limits, a turbine velocity diagram was calculated for a set of conditions used in this analysis with the assumption that the relative Mach number entering the rotor and the absolute Mach number leaving the rotor were 0.70 at the mean radius.

Results of these calculations indicate that for the same component efficiencies, turbine-inlet temperature, and blade speed, the engine thrust was approximately 17 percent less than the maximum value arrived at with no restrictions on turbine Mach numbers.

Effect of Blade Speed and Inlet Temperature on Compressor Pressure Ratio for Maximum Thrust

The compressor pressure ratio required for maximum engine thrust increased with both turbine-inlet temperature and mean blade speed for a given turbine efficiency. The effect of inlet temperature is more significant than blade speed, as indicated in figures 3 to 5.

The maximum thrust per square foot of turbine frontal area decreased with increasing blade speed for a given inlet temperature, as did the turbine air-handling capacity. This drop in thrust occurred despite the increases in compressor ratio which were required to obtain maximum thrust at the increased blade speeds. This indicates that the thrust gain potentially available from increasing pressure ratio were not realized because of the decrease in turbine air-handling capacity resulting from the reduced turbine annulus area.

For the range of blade speeds considered, the variation of engine thrust with compressor pressure ratio for a given turbine-inlet temperature and turbine efficiency is greatest at a blade speed of 1000 feet per second and diminishes steadily with increasing blade speed, as indicated in figures 3 to 5. The contours of these curves also indicate that the maximum engine thrust can be approximated over a limited range of compressor pressure ratios in the vicinity of maximum thrust at only a slight penalty in performance.

Effect of Centrifugal Stress on Thrust per Square Foot Turbine Frontal Area

For a given mean blade speed, the change in engine thrust per square foot of turbine frontal area with centrifugal stress at the root of the turbine rotor is proportional to the change in turbine annulus area. The results of calculations made for centrifugal rotor stresses of 20,000 to 60,000 pounds per square inch are given in figure 7 in the form of correction curves to be applied to engine thrust and equivalent air weight flow, as given in figures 3 to 6.

These calculations indicate that for the range of mean blade speeds investigated the greatest gain in engine thrust per square foot of turbine frontal area with increased stress is obtained at a mean blade speed of 1400 feet per second. This gain is diminished considerably at a mean blade speed of 1200 feet per second. When the mean blade speed is decreased to 1000 feet per second, the gain in thrust per square foot of turbine frontal area is reduced still further and reaches a maximum value at approximately 50,000 pounds per square inch, beyond which the increase in turbine frontal area required to obtain a larger annulus becomes prohibitive. This limit is reached when the ratio of turbine annulus area to turbine frontal area is a maximum for the fixed mean diameter and mean blade speed being considered.

The results of this analysis are applicable over the range of flight speeds and altitudes for which the ram temperature is constant, as indicated in figure 8.

SUMMARY OF RESULTS

An investigation was made to determine the aerodynamic requirements at the mean radius of a single-stage, stress-limited turbine for application to a turbojet engine at maximum thrust per square foot turbine frontal area. The analysis was made for a range of turbine mean blade speeds and inlet temperatures, with no assumptions as to aerodynamic design limits having been made. The following results were obtained:

1. Supersonic velocities relative to the leading and trailing edges of the turbine rotor blades were required if a single-stage, axial-flow turbine was to meet the requirements of a turbojet engine at maximum thrust per square foot turbine frontal area.

2. In general, the velocity diagram at the mean radius at maximum thrust was the impulse type; that is, the flow was accelerated only in the stator and no change in state occurred in the rotor.

3. The axial component of the velocity leaving the turbine was sonic in all cases. Where the impulse condition in the rotor existed, the axial component of the velocity upstream of the rotor was sonic also so that the annuli at the leading and trailing edges of the rotor were operating at their maximum air-handling capacities.

4. The application of conventional turbine aerodynamic design limits to a set of conditions investigated in this analysis resulted in a reduction of 17 percent in thrust from that obtained when no aerodynamic limits were imposed on the turbine.

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5. The variations in blade angles and turning angles at maximum thrust were small over the range of blade speeds and turbine-inlet temperatures investigated. The turning in the stator varied from 56° to 65°, with 78° to 94° of turning being required in the rotor for the range of conditions investigated.

6. The absolute Mach numbers at the stator trailing edge varied from 1.59 to 1.97 at maximum thrust. The Mach numbers relative to the leading edge of the rotor varied from 0.94 to 1.46; those relative to the trailing edge varied from 1.29 to 1.46.

7. The air-handling capacity of the maximum-thrust turbine of this analysis closely approximates 37 pounds per second per square foot of frontal area at the following design conditions: internal efficiency, 0.90; inlet temperature, 2190° R; mean blade speed, 1000 feet per second. At mean blade speeds of 1200 and 1400 feet per second, turbine-inlet temperatures of 2540° and 2940° R, respectively, would be required for the turbine to pass this air flow.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

CALCULATION OF HUB-TIP RATIOS FOR CONSTANT STRESS

The following procedure was used to calculate the hub-tip ratios and tip speeds for variable mean blade speed with a constant centrifugal stress at the blade root and a turbine frontal area of 1 square foot (reference 1): 2423

$$\sigma = \frac{\psi\rho_b U_T^2}{2g144}(1-r^2) \tag{A1}$$

A blade stress taper factor ψ of 0.69, which is consistent with current practice, was assumed. The density of the blade material was 490 pounds per cubic foot, and the assumed centrifugal stress at the blade root was 30,000 pounds per square inch.

The tip speed U_T may be expressed as

$$U_T = \frac{2U_m}{1+r} \tag{A2}$$

in terms of the mean blade speed and the hub-tip ratio. Rewriting equation (A1) as

$$\left(\frac{2U_m}{1+r}\right)^2 (1-r^2) = \frac{288g\sigma}{\psi\rho_b} \tag{A3}$$

yields

$$\frac{1-r^2}{(1+r)^2} = \frac{288g\sigma}{4\psi\rho_b U_m^2} = \frac{288\times32.2\times30,000}{4\times0.69\times490\times U_m^2} \tag{A4}$$

or

$$(1-r^2) = \frac{2.057\times10^5}{U_m^2}(1+r)^2$$

which may be solved directly for the hub-tip ratio r at each of the assumed mean blade speeds. Calculated results for the three mean blade speeds used in the analysis are given in the following table:

U_m (ft/sec)	r	U_T (ft/sec)	A (sq/ft)
1000	0.6588	1205.6	0.5660
1200	.7500	1371.4	.4375
1400	.8100	1546.9	.3439

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APPENDIX B

CALCULATION OF VELOCITY DIAGRAMS

Case I - $(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m) < 2U_m$

Given:

$\frac{P_c}{w}$	$W_5 \cos \beta_5 = U_m$
U_m	$V_5 = \left(\gamma g R T_5\right)^{\frac{1}{2}}$
p_3'	A
T_3'	

The change in tangential momentum per pound per second of air flow is given by

$$(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m) = \frac{550g\left(\frac{P_c}{w}\right)}{U_m}$$

or, since

$$W_5 \cos \beta_5 = U_m$$

$$V_4 \cos \alpha_4 = \frac{550g\left(\frac{P_c}{w}\right)}{U_m} \tag{B1}$$

The stagnation temperature downstream of the turbine may be calculated as follows:

$$T_5' = T_3' - \frac{550\left(\frac{P_c}{w}\right)}{778c_p} \tag{B2}$$

The static temperature T_5 can now be determined:

$$T_5 = T_5' - \frac{V_5^2}{2gJc_p} = T_5' - \frac{\gamma g R T_5}{2gJc_p}$$

or

$$T_5 = \frac{T_5'}{1 + \frac{\gamma g R}{2 g J c_p}} \tag{B3}$$

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so that

$$V_5 = \left[\gamma g R \left(\frac{2T_5'}{\gamma + 1}\right)\right]^{\frac{1}{2}} \tag{B4}$$

Both W_5 and β_5 can now be calculated since $W_5 \sin \beta_5$ is equal to V_5, and $W_5 \cos \beta_5$ is given as equal to U_m.

The following procedure was used to determine the air weight flow from conditions at the trailing edge of the rotor:

The static density ρ_5 is given by

$$\rho_5 = \rho_3' \left(\frac{T_5}{T_3'}\right)^{\frac{1}{\gamma-1}} = \frac{p_3'}{RT_3'} \left(\frac{T_5}{T_3'}\right)^{\frac{1}{\gamma-1}} \tag{B5}$$

The weight flow in pounds per second per square foot turbine frontal area is

$$w = \rho_5 V_5 A$$

or

$$w = \frac{p_3'}{RT_3'} \left(\frac{T_5}{T_3'}\right)^{\frac{1}{\gamma-1}} \left[\gamma g R \left(\frac{2T_5'}{\gamma + 1}\right)\right]^{\frac{1}{2}} A \tag{B6}$$

The equation of continuity was used to determine the stator diagram as follows:

$$\rho_4 V_4 \sin \alpha_4 \, A = \rho_5 V_5 A$$

The density downstream of the stator is given by

$$\rho_4 = \rho_3' \left(\frac{T_4}{T_3'}\right)^{\frac{1}{\gamma-1}}$$

From the energy equation across the stator,

$$\frac{T_4}{T_3'} = 1 - \left(\frac{\overline{V_4 \cos \alpha_4}^2 + \overline{V_4 \sin \alpha_4}^2}{2gJc_p\ T_3'}\right)$$

Since $V_4 \cos \alpha_4$ is given in equation (B1), the continuity equation may be written as

$$\rho_3' \left(1 - \frac{\overline{V_4 \cos \alpha_4}^2 + \overline{V_4 \sin \alpha_4}^2}{2gJc_p\ T_3'}\right)^{\frac{1}{\gamma-1}} V_4 \sin \alpha_4 = \rho_5 V_5 \qquad \text{(B7)}$$

Equation (B7) may be solved by trial and error for $V_4 \sin \alpha_4$, and V_4, α_4, W_4, and β_4 may then be determined using standard trigonometric relations.

The velocity of the exhaust jet is then calculated from the following equation:

$$V_6 = \left[V_5^2 + 2gJc_p\ (T_5 - T_6)\right]^{\frac{1}{2}} \qquad \text{(B8)}$$

From the assumption of isentropic flow from the turbine inlet to the exhaust tail cone,

$$T_6 = T_3' \left(\frac{p_6}{p_3'}\right)^{\frac{\gamma}{\gamma-1}} \qquad \text{(B9)}$$

The thrust per square foot turbine frontal area may then be calculated from equations (B4), (B6), (B8), and (B9) as follows:

$$F = \frac{w}{g}(V_6 - V_1) \qquad \text{(B10)}$$

Case II - $(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m) = 2U_m$

Given:

$\frac{P_c}{w}$	$W_5 \cos \beta_5 = U_m = W_4 \cos \beta_4$
U_m	$V_5 = \left(\gamma g R T_5\right)^{\frac{1}{2}}$
p_3'	A
T_3'	

The procedure used in this case is identical with that used in Case I up to and including equation (B6). However, since there is no change in state in the rotor for Case II, the static temperatures upstream and downstream of the rotor will be equal; that is, $T_5 = T_4$. Therefore, the stator exit velocity may be calculated directly from downstream static temperature.

$$V_4 = \left[2gJc_p \left(T_3' - T_5\right)\right]^{\frac{1}{2}} \tag{B11}$$

The velocity diagram may then be determined using standard trigonometric relations.

The engine thrust is determined as indicated in equations (B8) through (B10).

Case III - $(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m) > 2U_m$

Given:

$\frac{P_c}{w}$	$W_4 \cos \beta_4 = W_5 \cos \beta_5$
U_m	$V_5 \sin \alpha_5 = \left(\gamma g R T_5\right)^{\frac{1}{2}} = V_4 \sin \alpha_4$
p_3'	A
T_3'	

The change in tangential momentum per pound per second of air flow is given by

$$(V_4 \cos \alpha_4 + W_5 \cos \beta_5 - U_m) = \frac{550g\left(\frac{P_c}{w}\right)}{U_m}$$

From figure 1

$$W_4 \cos \beta_4 = V_4 \cos \alpha_4 - U_m$$

or

$$(W_4 \cos \beta_4 + W_5 \cos \beta_5) = \frac{550g\left(\frac{P_c}{w}\right)}{U_m} \tag{B12}$$

Since $W_4 \cos \beta_4$ and $W_5 \cos \beta_5$ are equal,

$$W_4 \cos \beta_4 = \frac{550g\left(\frac{P_c}{w}\right)}{2U_m} = W_5 \cos \beta_5$$

and

$$V_4 \cos \alpha_4 = U_m + \frac{550g\left(\frac{P_c}{w}\right)}{2U_m} \tag{B13}$$

The static temperature downstream of the turbine stator (or rotor) must be determined so that

$$V_4 \sin \alpha_4 = \left(\gamma g R T_4\right)^{\frac{1}{2}} = V_5 \sin \alpha_5$$

Since

$$T_4 = T_3' - \frac{\overline{V_4 \sin \alpha_4}^2 + \overline{V_4 \cos \alpha_4}^2}{2gJc_p} \tag{B14}$$

then

$$V_4 \sin \alpha_4 = \left[\left(\frac{\gamma-1}{\gamma+1}\right)\left(2gJc_pT_3' - \overline{V_4 \cos \alpha_4}^2\right)\right]^{\frac{1}{2}} \tag{B15}$$

The static temperature T_4 can then be calculated from equation (B14) and the static density can be determined from the isentropic relation

$$\rho_4 = \frac{p_3'}{RT_3'}\left(\frac{T_4}{T_3'}\right)^{\frac{1}{\gamma-1}} \tag{B16}$$

The air weight flow is then calculated using equations (B15) and (B16) as follows:

$$w = \rho_4 V_4 \sin \alpha_4 A$$

or

$$w = \frac{p_3'}{RT_3'}\left(\frac{T_4}{T_3'}\right)^{\frac{1}{\gamma-1}}\left[\left(\frac{\gamma-1}{\gamma+1}\right)\left(2gJc_pT_3' - \overline{V_4 \cos \alpha_4}^2\right)\right]^{\frac{1}{2}} A \tag{B17}$$

The static temperature of the air leaving the exhaust cone is calculated from equation (B9). The exhaust jet velocity is determined from equations (B9), (B14), and (B15) as substituted into the following equation:

$$V_6 = \left[\overline{V_5 \sin \alpha_5}^2 + 2gJc_p\left(T_5 - T_6\right)\right]^{\frac{1}{2}} \tag{B18}$$

The engine thrust is calculated from equations (B17) and (B18) as indicated in equation (B10).

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APPENDIX C

CORRECTION OF THRUST FOR TURBINE EFFICIENCY

When the compressor power at 100 percent turbine efficiency is reduced directly by the inefficiency of the turbine, the corrected compressor power per pound of air can be given as

$$\left(\frac{P_c}{w}\right)_{corr} = \eta_t \left(\frac{P_c}{w}\right)_{\eta_t = 1.00} \tag{C1}$$

for an assumed turbine efficiency η_t.

The corrected compressor power corresponds to a new compressor pressure ratio which may be determined from equation (5) in the text or more conveniently from a curve of compressor specific power against compressor pressure ratio.

Since the turbine configuration has not been altered, the turbine air weight flow may be considered to vary directly with turbine-inlet pressure (or compressor-discharge pressure) and may be determined from the corrected compressor pressure ratio.

The static pressure downstream of the turbine varies directly as the compressor pressure ratio since the pressure ratio across the turbine p_3'/p_5 and the ram pressure are fixed. Thus the static-pressure ratio across the exhaust cone p_6/p_5 is also dependent on the compressor pressure ratio. Because the axial component of the velocity leaving the turbine rotor has been assumed to be independent of turbine efficiency, the corrected thrust is now specified by the corrected compressor pressure ratio as determined from the turbine efficiency.

REFERENCES

1. LaValle, Vincent L., and Huppert, Merle C.: Effects of Several Design Variables on Turbine-Wheel Weight. NACA TN 1814, 1949.

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TABLE I - IDEAL TURBINE VELOCITY DIAGRAMS AT MAXIMUM THRUST

(a) Mach numbers at mean radius

Velocity	Inlet temperature, °R								
	2000			2750			3500		
	Mean blade speed, U_m, ft/sec								
	1000	1200	1400	1000	1200	1400	1000	1200	1400
	Mach number								
V_4	1.78	1.83	1.59	1.82	1.88	1.94	1.81	1.87	1.97
W_4	1.34	1.29	0.94	1.43	1.40	1.38	1.46	1.45	1.46
W_5	1.34	1.29	1.30	1.43	1.40	1.38	1.46	1.45	1.46
V_5	1.05	1.01	1.00	1.12	1.07	1.03	1.18	1.12	1.08

(b) Flow angles and turning angles at mean radius

Angle	Inlet temperature, °R								
	2000			2750			3500		
	Mean blade speed, U_m, ft/sec								
	1000	1200	1400	1000	1200	1400	1000	1200	1400
	Flow and turning angles								
α_4	34.1	33.2	25.2	33.4	32.2	31.0	33.5	32.3	30.4
β_4	48.3	50.7	46.0	44.6	45.6	46.6	43.1	43.8	43.4
β_5	48.3	50.7	50.2	44.6	45.6	46.6	43.1	43.8	43.4
α_5	72.8	83.5	90.0	62.8	69.6	77.4	58.1	63.2	67.5
Turning in rotor	83.4	78.6	83.8	90.8	88.8	86.8	93.8	92.4	93.2

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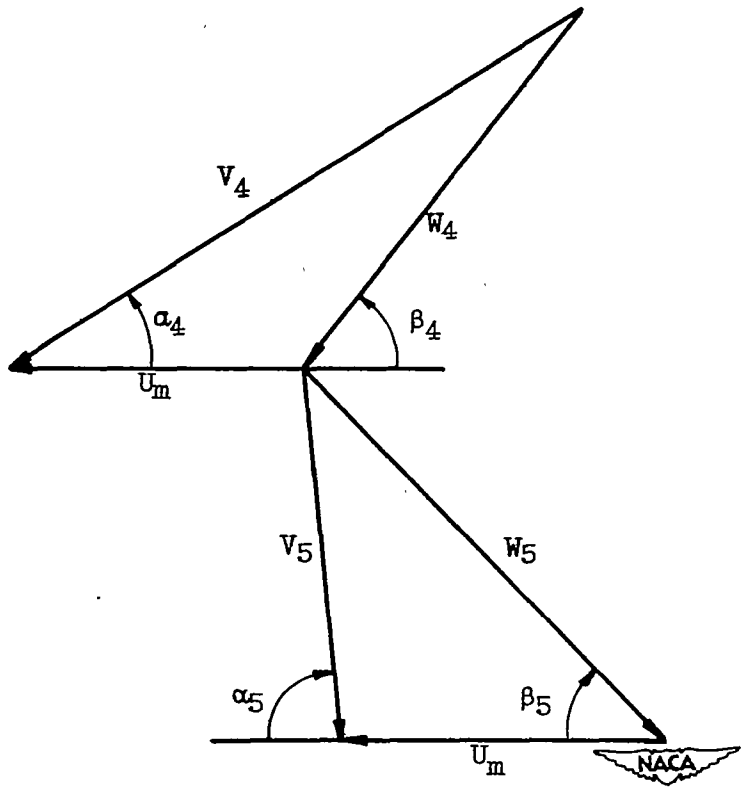

Figure 1. - Nomenclature used in turbine velocity diagrams.

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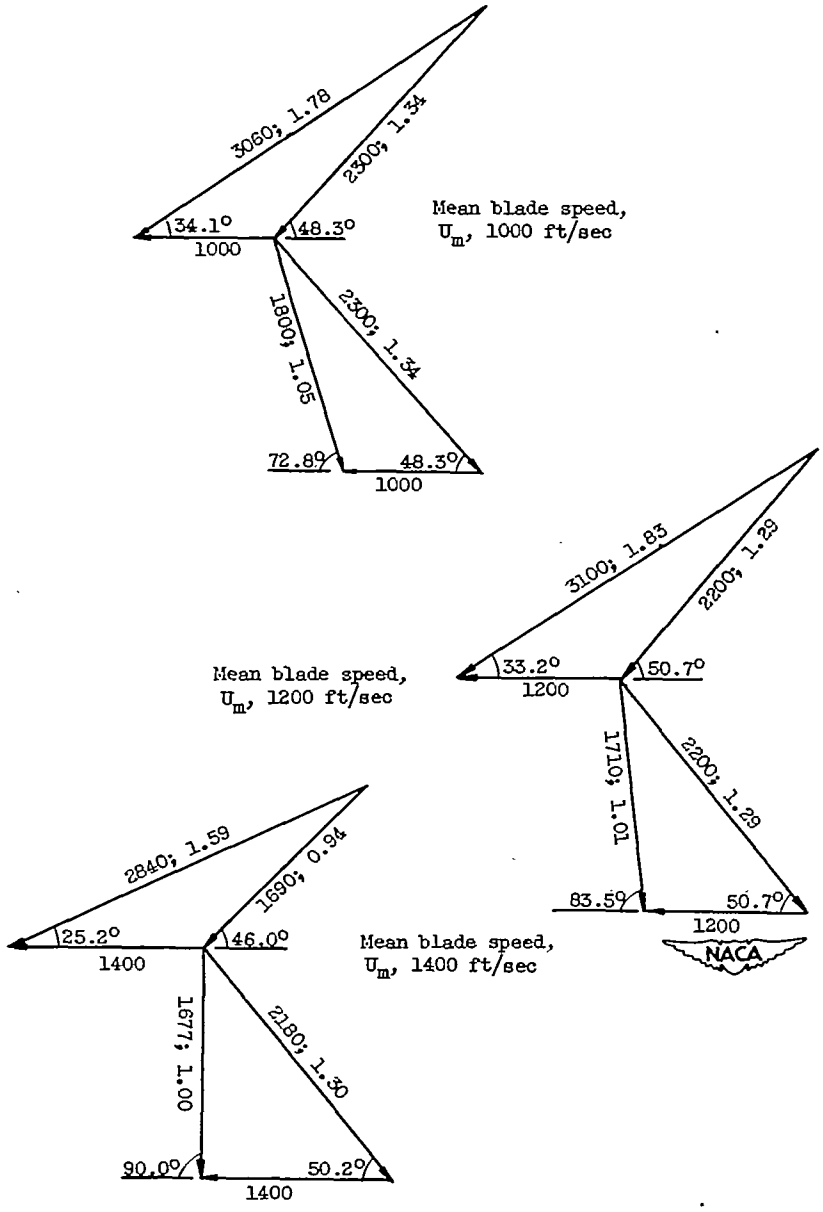
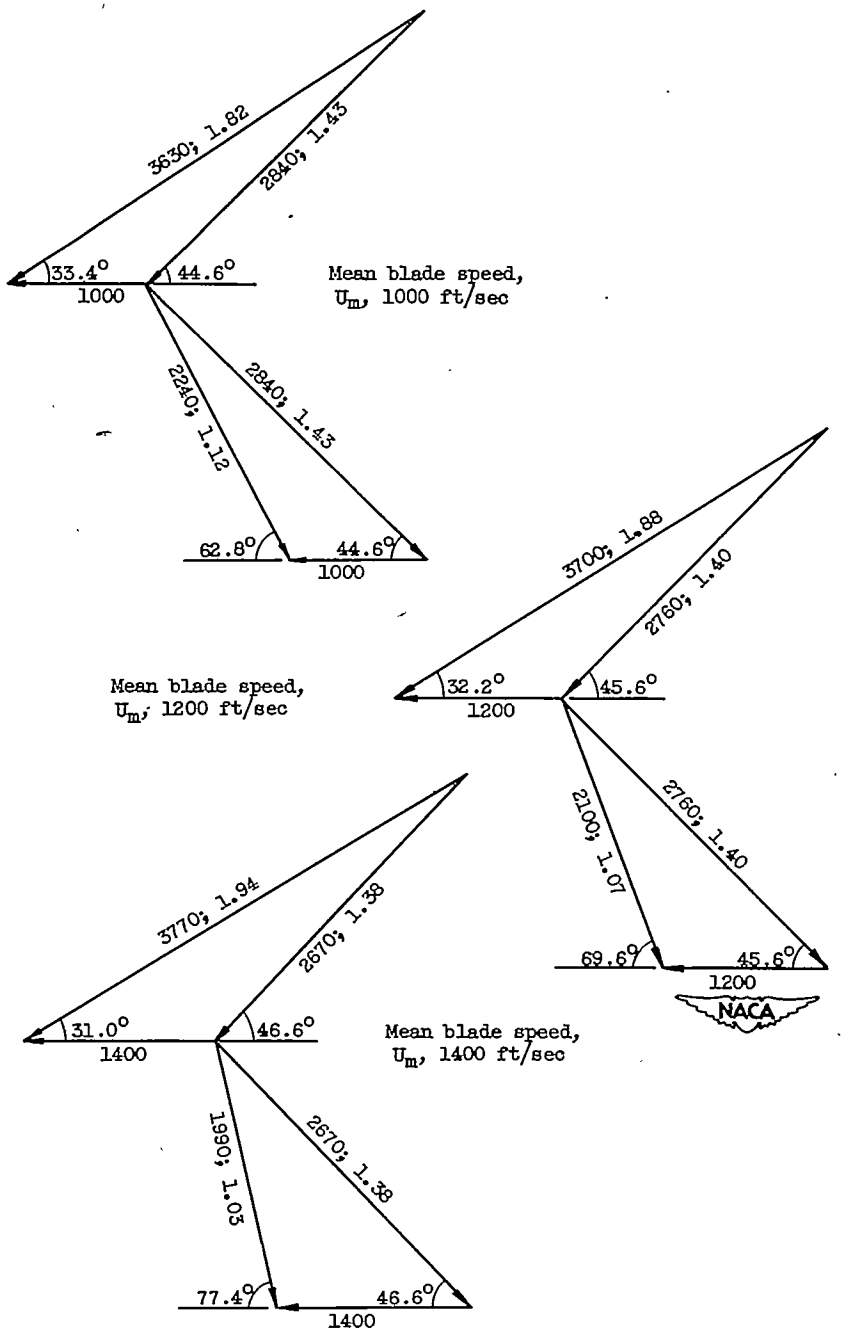


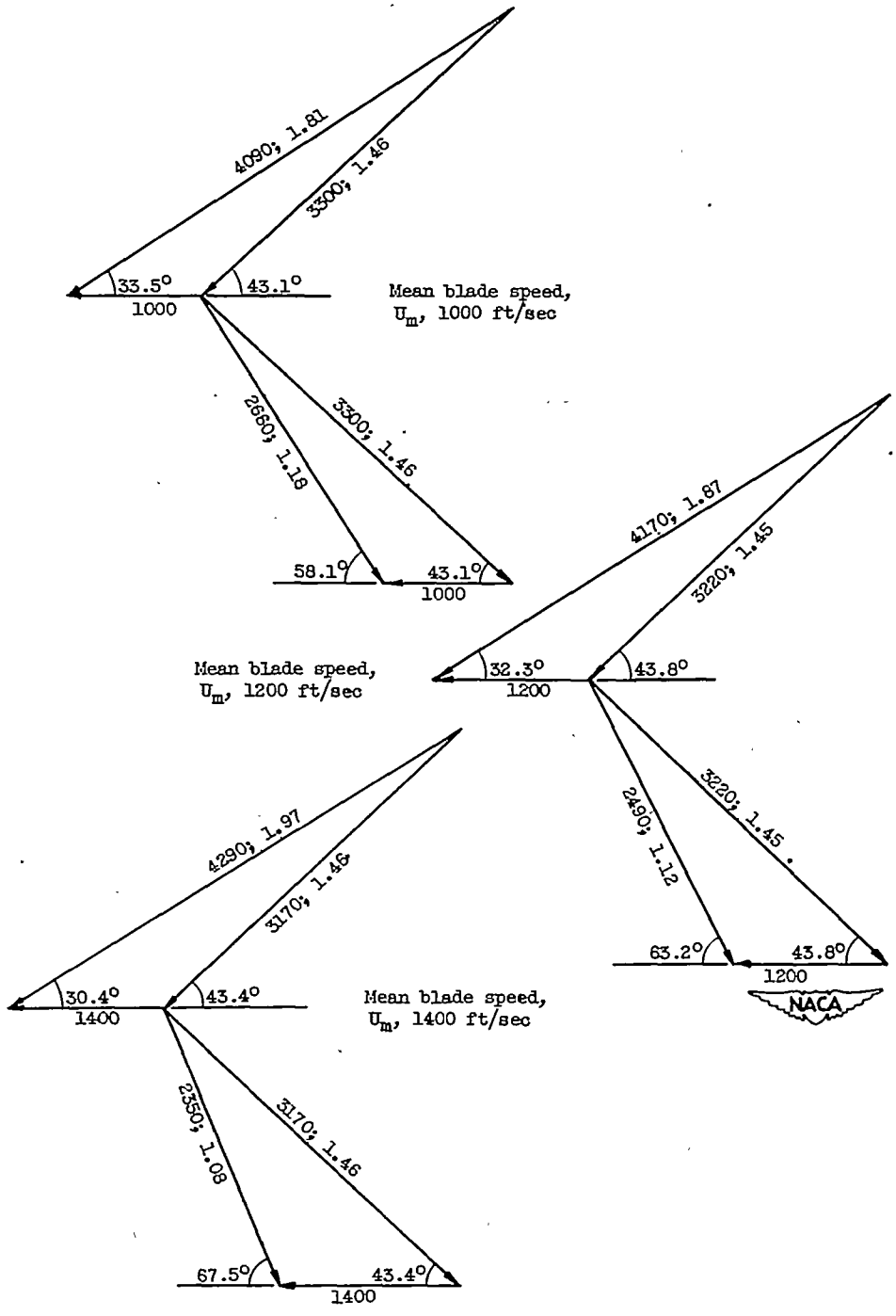
(a) Inlet temperature, 2000° R.

Figure 2. - Ideal velocity diagrams at mean radius for maximum thrust per square foot turbine frontal area. Velocities are in feet per second; numbers to right of velocities are local Mach numbers.



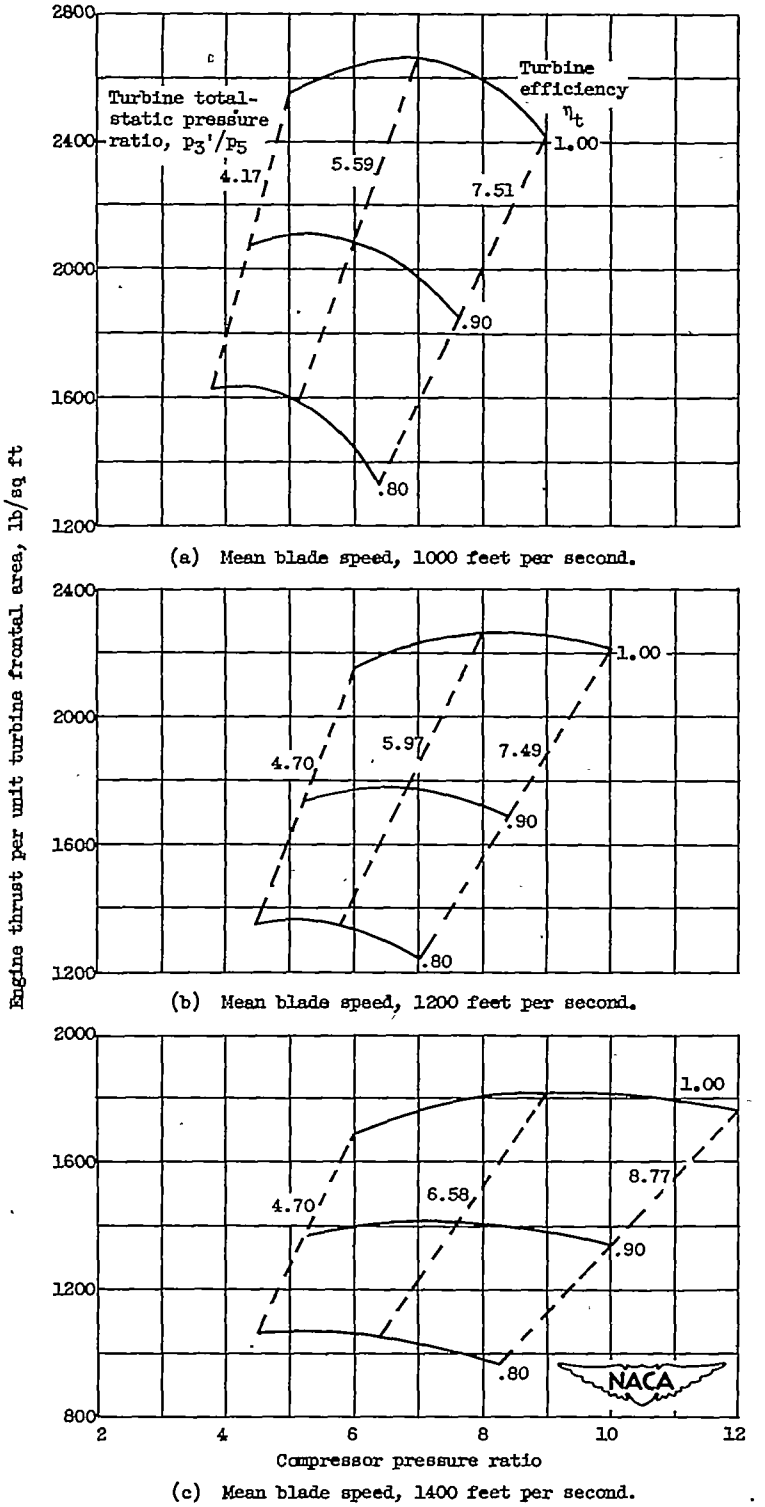
(b) Inlet temperature, 2750° R.

Figure 2. - Continued. Ideal velocity diagrams at mean radius for maximum thrust per square foot turbine frontal area. Velocities are in feet per second; numbers to right of velocities are local Mach numbers.



(c) Inlet temperature, 3500° R.

Figure 2. - Concluded. Ideal velocity diagrams at mean radius for maximum thrust per square foot turbine frontal area. Velocities are in feet per second; numbers to right of velocities are local Mach numbers.

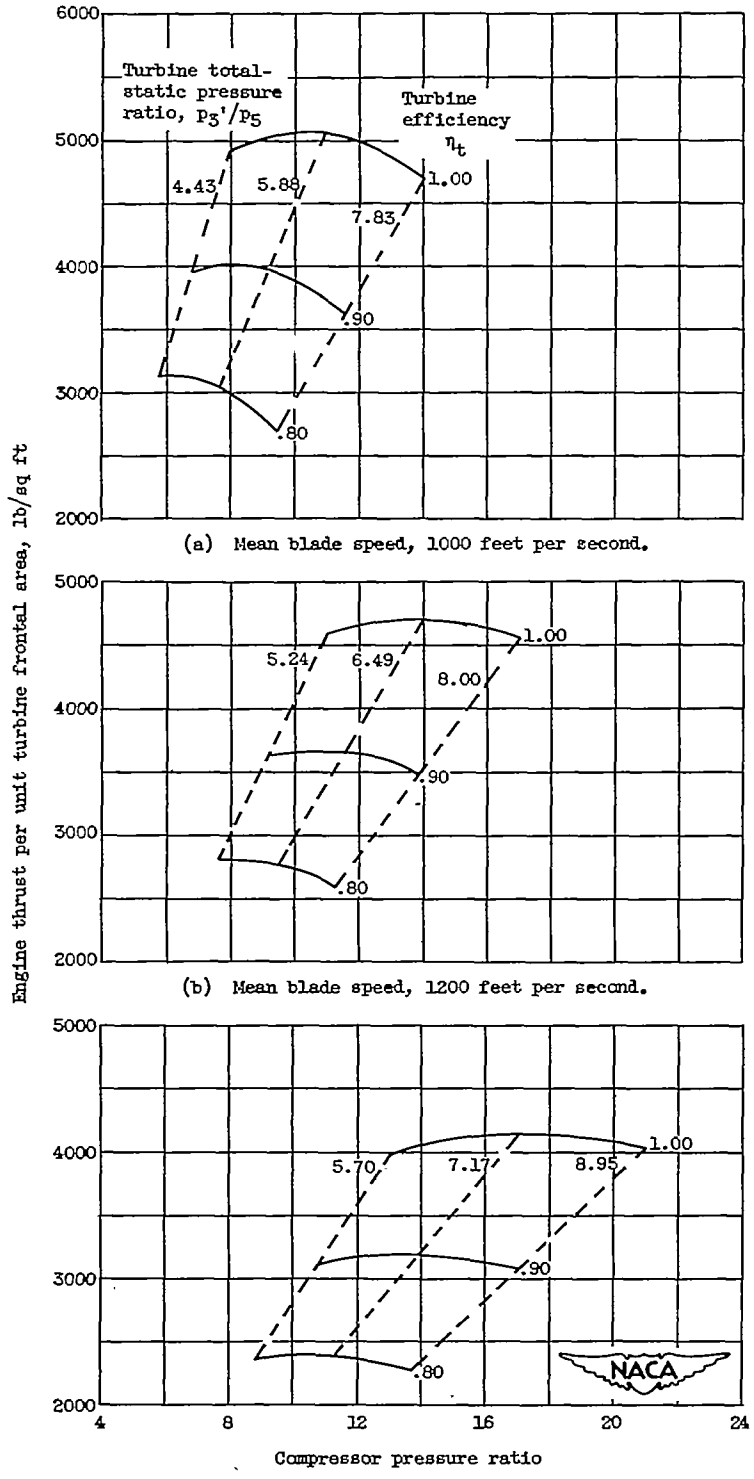


(a) Mean blade speed, 1000 feet per second.

(b) Mean blade speed, 1200 feet per second.

(c) Mean blade speed, 1400 feet per second.

Figure 3. - Effect of turbine parameters on engine thrust at 600 miles per hour and turbine-inlet temperature of 2000° R.



(a) Mean blade speed, 1000 feet per second.

(b) Mean blade speed, 1200 feet per second.

(c) Mean blade speed, 1400 feet per second.

Figure 4. - Effect of turbine parameters on engine thrust at 600 miles per hour and turbine-inlet temperature of 2750° R.

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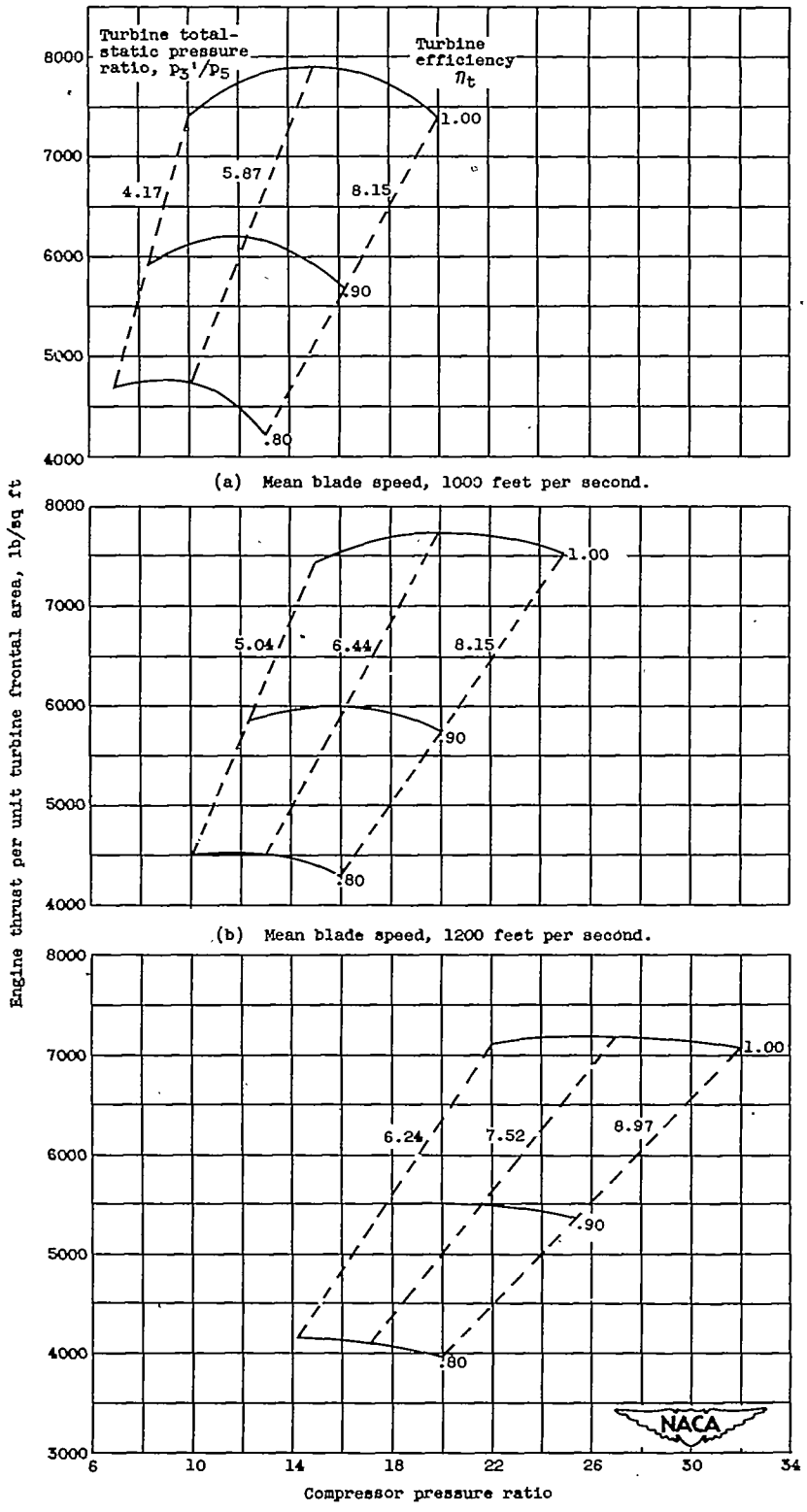

(a) Mean blade speed, 1000 feet per second.

(b) Mean blade speed, 1200 feet per second.

(c) Mean blade speed, 1400 feet per second.

Figure 5. - Effect of turbine parameters on engine thrust at 600 miles per hour and turbine-inlet temperature of 3500° R.

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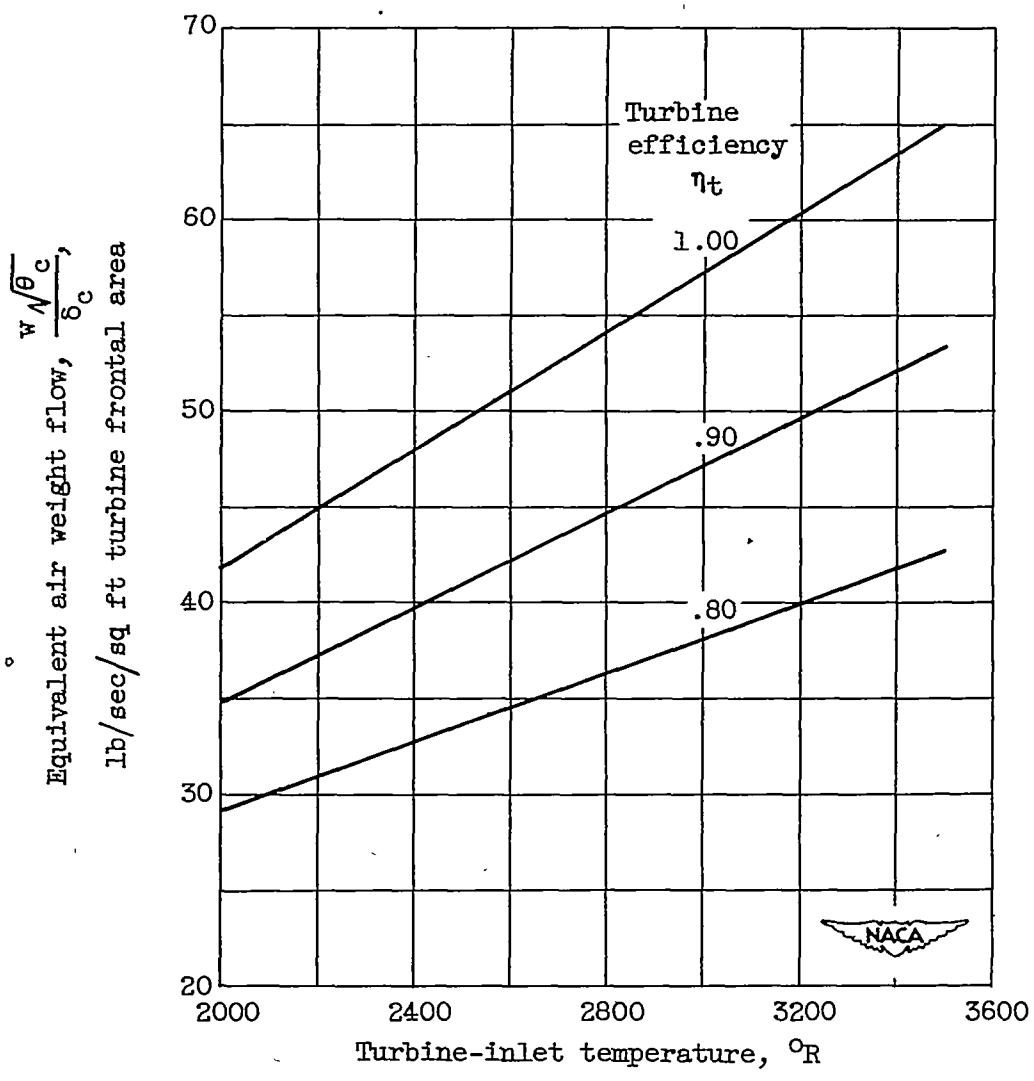

(a) Mean blade speed, 1000 feet per second.

Figure 6. - Variation of equivalent air weight flow at maximum thrust with turbine-inlet temperature and turbine efficiency.

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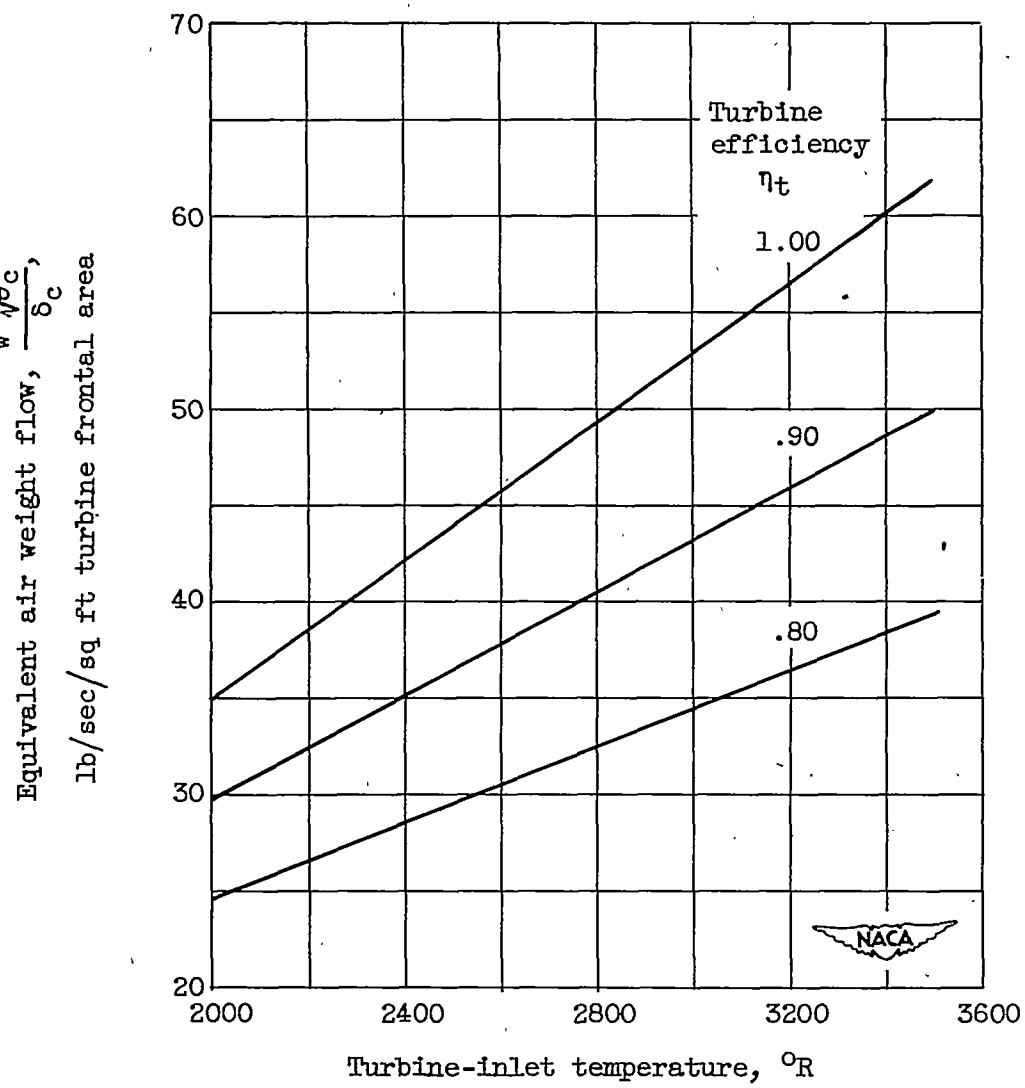

(b) Mean blade speed, 1200 feet per second.

Figure 6. - Continued. Variation of equivalent air weight flow at maximum thrust with turbine-inlet temperature and turbine efficiency.

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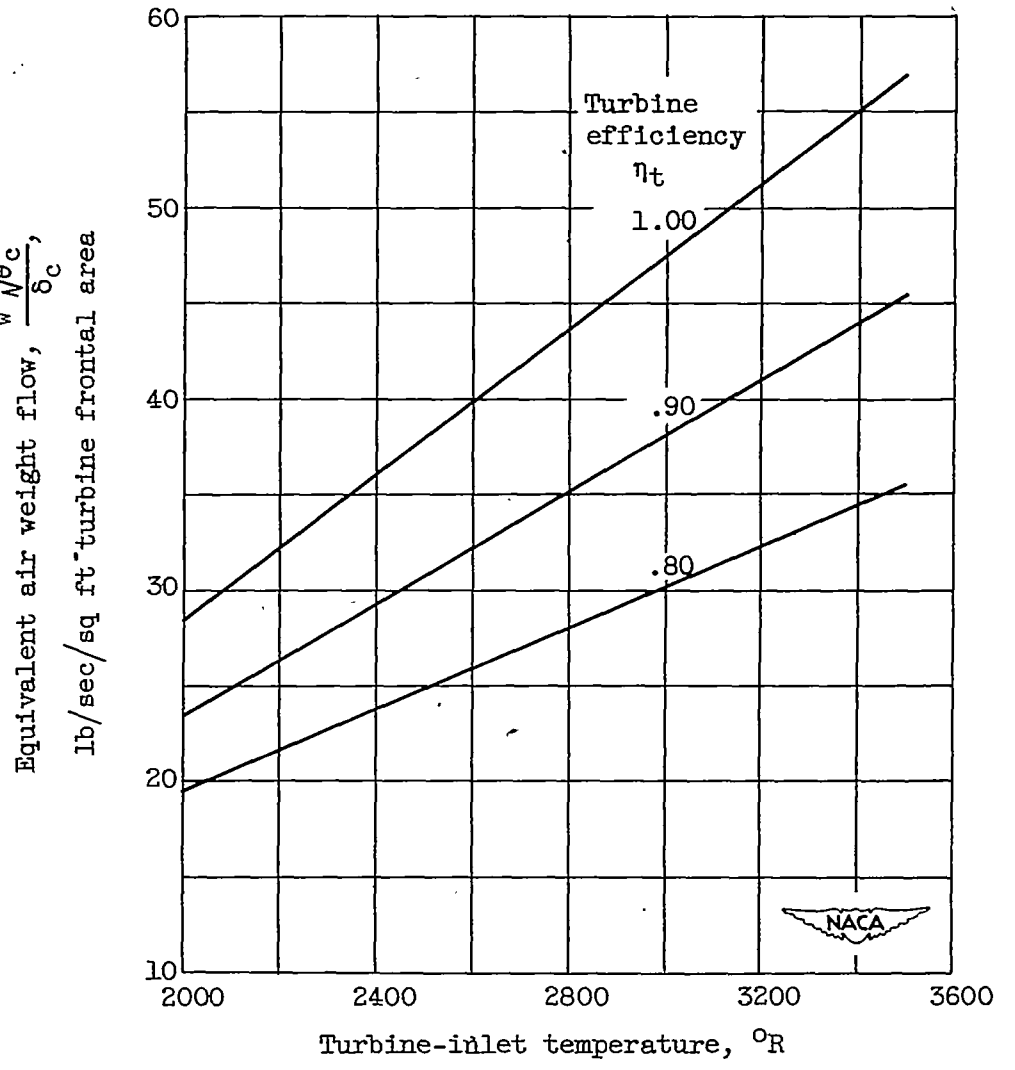

(c) Mean blade speed, 1400 feet per second.

Figure 6. - Concluded. Variation of equivalent air weight flow at maximum thrust with turbine-inlet temperature and turbine efficiency.

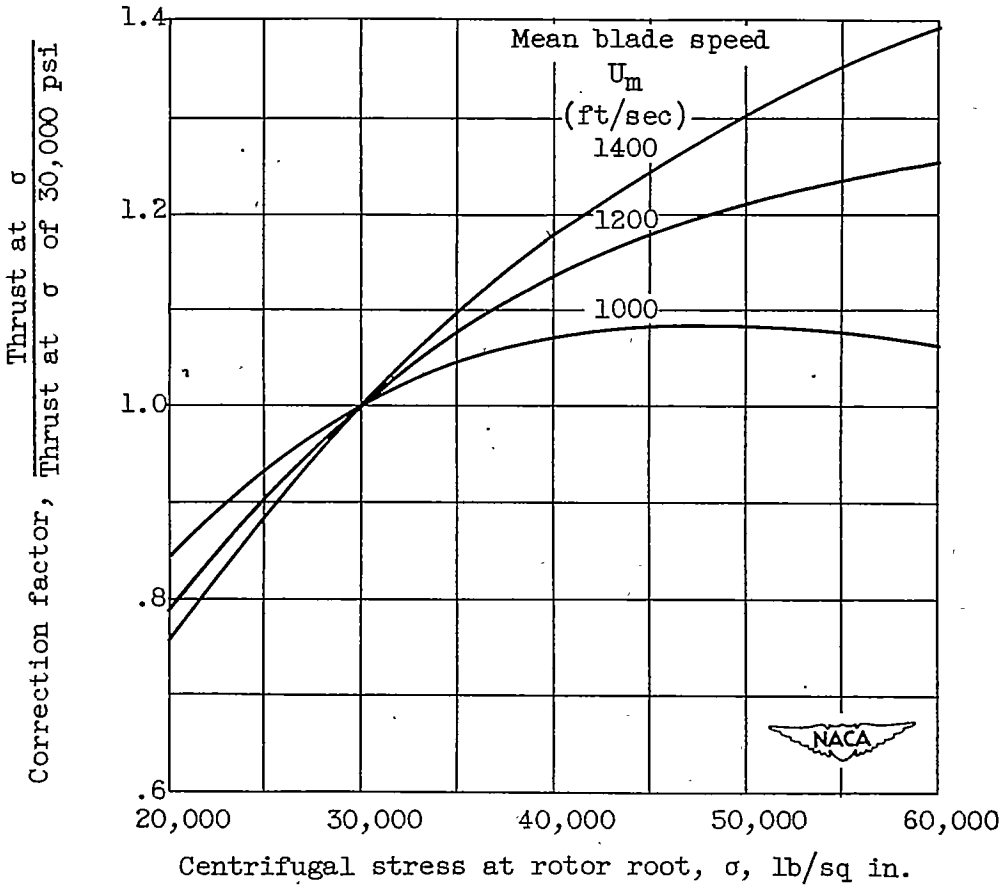

Figure 7. - Effect of centrifugal root stress σ on engine thrust per square foot turbine frontal area.

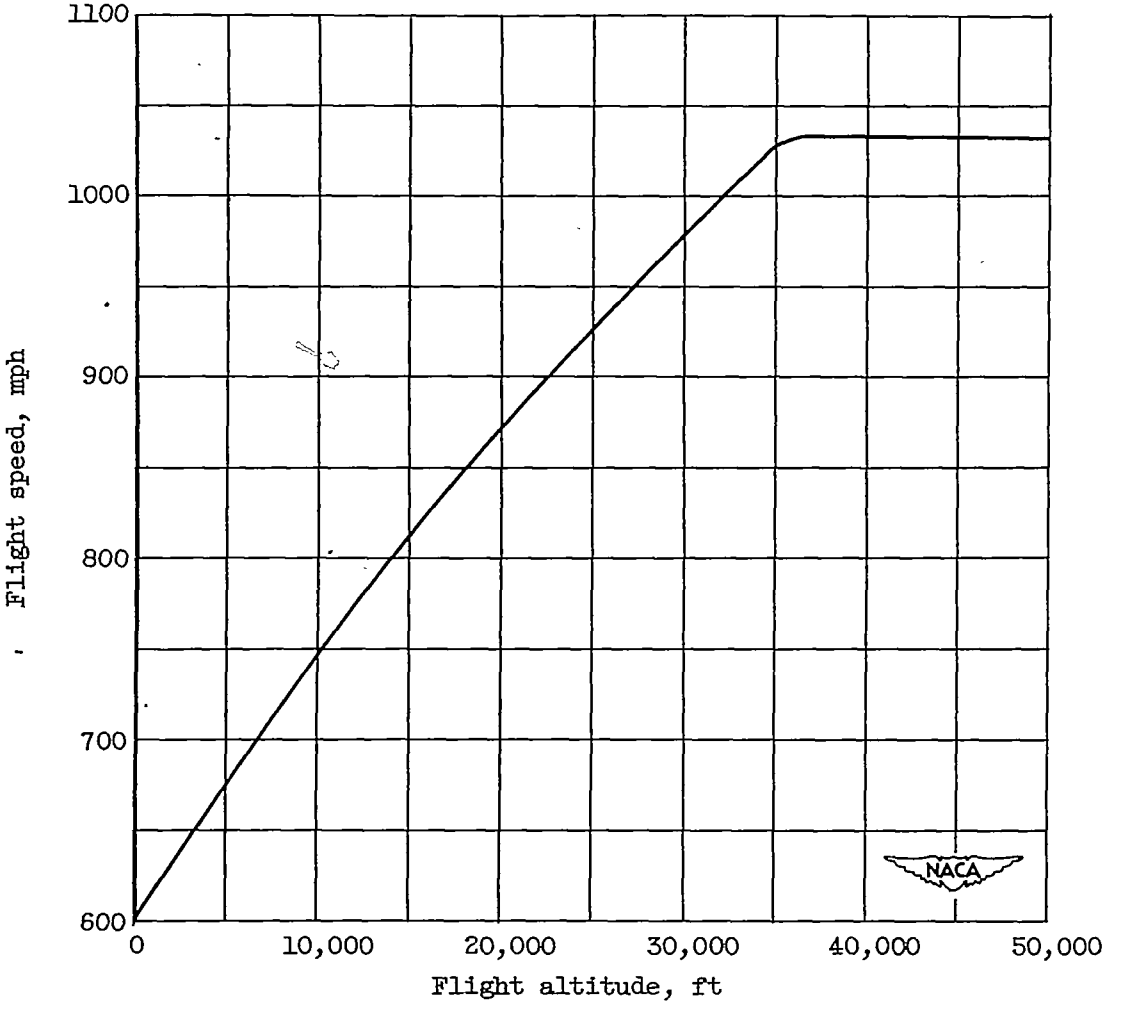

Figure 8. - Range of applicability of turbine velocity diagrams. Variation of flight speed with altitude at constant ram temperature, 600 miles per hour at sea level.

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