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LIFT ON A BENT, FLAT PLATE

By F. Keune

Translation of "Auftrieb einer geknickten ebenen Platte." (Bericht der Aerodynamischer Versuchsanstalt Göttingen.) Luftfahrtforschung, March 20, 1936, Annual Volume

Washington
February 1955

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LIFT ON A BENT, FLAT PLATE*

By F. Keune

For the theoretical treatment of fin and rudder, or of a wing with flap, the profile is simplified in such a way that a bent flat plate may be considered. The aerodynamic forces on it can be calculated by means of known approximations due to Munk (ref. 1) or Glauert (ref. 2). The problem was recently treated exactly by the Russian work of Chapligin and Arjanikov (ref. 3). The calculations below will be carried out following the latter work, and the difference between them and the usual approximations will be determined.[1]

OUTLINE

1. Theory
2. Results
3. Summary
4. References

1. THEORY

The lift coefficient of a profile at angle of attack α can be written in the form

$$c_a = c \sin (\alpha - \alpha_o) \tag{1}$$

The problem is to determine the lift-curve slope c and the angle of zero lift α_o. For the flat plate, the theoretical lift-curve slope is $c = 2\pi$; for other profiles, it deviates only a little from this value. The angle of zero lift for the flat plate is, moreover, $\alpha_o = 0^o$. With the deflection of the flap, both values vary as a function of the chord ratio t_R/t and the deflection angle β.

*"Auftrieb einer geknickten ebenen Platte." (Bericht der Aerodynamischer Versuchsanstalt Göttingen.) Luftfahrtforschung, March 20, 1936, Annual Volume, pp. 85-87.

[1]I am especially grateful to Dr. I. Lotz for many suggestions in carrying out the analysis.

In order to calculate c and α_0 we must determine the flow around the bent plate and in particular the circulation needed for smooth flow at the trailing edge B (fig. 1).

With the aid of conformal mapping, the region outside of the profile (fig. 1) in the z plane is mapped, according to the Schwarz-Christoffel procedure (ref. 4), onto the lower half of the λ plane (fig. 2b), so that the border of the profile (see fig. 2a) on the μ axis and the point at infinity in the z plane transform to the point

$$\lambda = -ik$$

The mapping function has for this case the form

$$z = c \int_0^{\lambda} \frac{(\lambda - \mu_{1.6})(\lambda - \mu_{3.4})}{(\lambda^2 + k^2)^2} \frac{(\lambda - \mu_5)^n}{(\lambda - \mu_2)^n} d\lambda \tag{2}$$

where the exponent n is related to the deflection angle β of the flap (see fig. 1) by

$$\beta = n\pi \tag{3}$$

In order to be able to evaluate the integral (see eq. (2)) one carries out an appropriate linear transformation, by which the λ plane goes into the ζ plane. The μ axis is thereby mapped onto the ξ axis. If we consider now the point-by-point matching of the z plane with the ζ plane (fig. 3), there results from this transformation: the leading edge A (fig. 1) and the point A^* ($\zeta = \zeta_1$); the location of the bend on the suction side and the zero point O^*; the trailing edge B and the point B^* ($\zeta = \zeta_2$); the location of the bend on the pressure side of the profile and the infinitely distant point on the ξ axis. The point at infinity in the z plane falls at the point $\zeta_3 = 1 - i\delta$. For these positions of the designated points the integral of the Schwarz-Christoffel formula (2) can be calculated. If we introduce the abbreviation $\kappa^2 = 1 + \delta^2 (1 - n^2)$, we obtain the mapping function

$$z = 2(t - t_R) \frac{\kappa - 1}{1 - n^2} \left(\frac{\kappa + n}{1 + n}\right)^n \frac{\zeta^{1-n}}{(\zeta - 1)^2 + \delta^2} \tag{4}$$

The position of the point $\zeta_3 = 1 - i\delta$, the image of the point at infinity of the z plane, depends on the ratio of the flap chord to the forward part of the wing and on the deflection of the flap. The position is determined by the equation

$$\frac{t}{t - t_R} = \sigma = \frac{\kappa - 1}{\kappa + 1} \left(\frac{\kappa + n}{\kappa - n}\right)^n \tag{5}$$

In the z plane a parallel stream prevails at infinity; on this stream (on account of the circulation around the plate) is superposed a vortex whose strength depends on the angle of attack. To the parallel flow of the z plane there corresponds a flow in the ζ plane which is produced by a dipole at the point $\zeta_3 = 1 - i\delta$; and to the circulation flow there corresponds a vortex flow around the same point. The strength of the circulation is given by the condition that in the z plane the flow is smooth at B; thus at B^* the velocity is zero. If we give the position of the stagnation point $\zeta_4 = \zeta_4(\sigma)$, then the velocity must have the form

$$w = \frac{d\varphi}{d\zeta} = 2(t - t_R)v_\infty N \frac{\kappa - 1}{1 - n^2}\left(\frac{\kappa + n}{1 + n}\right)^n \frac{(\zeta - \zeta_2)(\zeta + \zeta_4)}{[(\zeta - 1)^2 + \delta^2]^2} \tag{6}$$

In this equation the quantity N is a constant which is essentially dependent on the angle of attack α.

We determine the circulation from the residue of the velocity function w (see eq. (6)). One obtains this by resolution into partial fractions. According to lengthier calculation, which can be seen in Chapligin and Arjanikov (pp. 5-7, ref. 4), one has for the lift

$$c_a = \frac{4\pi}{(1 + \sigma)(\kappa + 1)}\left(\frac{\kappa + n}{1 + n}\right)^{(n+1)/2}\left(\frac{\kappa - n}{1 - n}\right)^{(1-n)/2} \sin(\alpha - \alpha_o) \tag{7}$$

where the angle of zero lift (pp. 7 and 8, ref. 4) is calculated as

$$+\alpha_o = -\left(\arctan \frac{n\sqrt{\kappa^2 - 1}}{\kappa\sqrt{1 - n^2}} + n \arctan \sqrt{\frac{\kappa^2 - 1}{1 - n^2}}\right) \tag{8}$$

By equating formulas (1) and (7) there is obtained for the lift-curve slope

$$c = \frac{4\pi}{(1 + \sigma)(\kappa + 1)}\left(\frac{\kappa + n}{1 - n}\right)^{(1+n)/2}\left(\frac{\kappa - n}{1 - n}\right)^{(1-n)/2} \tag{9}$$

In the evaluation one next determines, for a prescribed deflection angle β and ratio $\sigma = t_R/(t - t_R)$, the value κ according to

equation (5).[2] With this equation the calculation of the angle of zero lift α_o (eq. (8)), of the lift-curve slope c (eq. (9)) and of the lift c_a (eq. (1)) are feasible.[3]

2. RESULTS

The dependence of α_o, the angle of zero lift, on the ratio t_R/t for fixed-deflection angle β is seen in figure 5 (eq. (8)). Since for values $n = \beta/\pi > 0.1$ the curves for $t_R/t > 0.5$ become approximately proportional to one another, the curves for $\beta = 45^\circ$ and 60° are therefore not plotted any further.

If one plots the angle α_o of zero lift for different chord ratios against the deflection angle β (fig. 6), one obtains almost straight lines. The dashed lines representing Glauert's approximation deviate from these curves only for small chord ratio and there one has somewhat smaller values.

Figure 7 gives the ratio of the lift-curve slope c of the plate with flap deflection to the flat plate value of 2π. The lift coefficient c_a is referred to the original wing chord t. For the actual chord of the wing t', which becomes smaller with increasing flap deflection,[4] we obtain somewhat larger deviations from $c/2\pi = 1$ in the reversed sense ($c/2\pi > 1$). For all curves β, they are the largest with chord ratio $t_R/t = 0.5$ in both cases (eq. (9)). Glauert assumed that the quantity $c/2\pi$, referred to the original chord t, varies as the ratio t'/t. This assumption results in values that are too small.

Figures 8 and 9 show the dependence of the lift coefficient c_a on the ratio t_R/t for fixed deflection β with an angle of attack of the profile of $\alpha = 5^\circ$ or $\alpha = 10^\circ$ (eq. (1)).

Figure 10 is a cross plot of figure 8 and points out that the dependence of lift on the deflection angle β for values $\beta > 15^\circ$ does not remain linear any longer. The linear dependence assumed by Glauert thus holds only for deflections to 15°. Glauert has replaced the sine of the angle of attack, referred to the zero-lift direction, with $\alpha - \alpha_o$ itself. The lift becomes too large, therefore, for large $\alpha - \alpha_o$. Up to deflection of $\beta = 15^\circ$ this error is counterbalanced in practice by the too-small value of the lift-curve slope.

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[2]For calculation of the quantities c and α_o for chord ratios $t_R/t > 0.5$, one employs the values $0 \leq t_R/t \leq 0.5$, while imagining the wing surface and flap exchanged, as shown in figures 4a and b, and also the potential correspondingly changed.

[3]The specified formulas hold for positive and negative deflections; only the sign of α_o and therefore the lift c_a are changed.

[4]$t'^2 = (t - t_R)^2 + t_R^2 + 2(t - t_R)t_R \cos\beta$, (see fig. 1 and the sketch in fig. 11).

Figure 11 gives the circulation distribution for the chord ratio $t_R/t = 0.5$ and the deflection angle $\beta = 30^\circ$ with the angle of attack $\alpha = 7^\circ$; Glauert's approximations are shown by dashed lines. For this case the lift, referred to the original chord, is $c_a = 3.25$ and according to Glauert $c_a = 3.33$, referred to the same chord.

3. SUMMARY

The lift on a bent, flat plate is calculated exactly by use of conformal mapping, and the results for the angle α_o of zero lift, the lift-curve slope c, the lift coefficient c_a and the circulation distribution $\frac{1}{v_\infty}\frac{d\Gamma}{dx}$ are compared with those obtained by Glauert's approximation. This approximation suffices for deflection angles of the flap $\beta < 15^\circ$, when the angle of attack α is so large that the small inexactitudes of the value of α_o of Glauert's approximation in the term $(\alpha - \alpha_o)$ can be neglected; otherwise, one obtains values somewhat too small in this region. The results for the lift-curve slope are practically the same for $\beta < 9^\circ$.

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4. REFERENCES

1. Munk, M.: Elements of the Wing Section Theory and of the Wing Theory. NACA Rep. 191, 1924.

2. Glauert, H.: Theoretical Relationship for an Aerofoil with Hinged Flap. ARC Rep. and Mem. No. 1095, 1927.

3. Chapligin, S. A., and Arjanikov, N. S.: On the Forward Aerofoil and Flap Slot Theory. Trans. Centr. Aero-Hydrodyn. Inst., No. 105, 1931.

4. Courant, R., and Hurwitz, A.: Funktionentheorie, Springer-Verlag, Berlin, 1922.

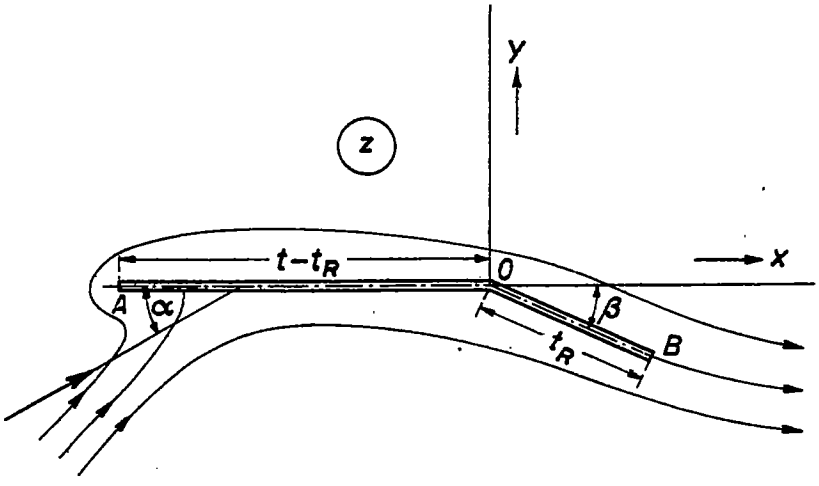

Figure 1.- The bent or hinged flat plate.

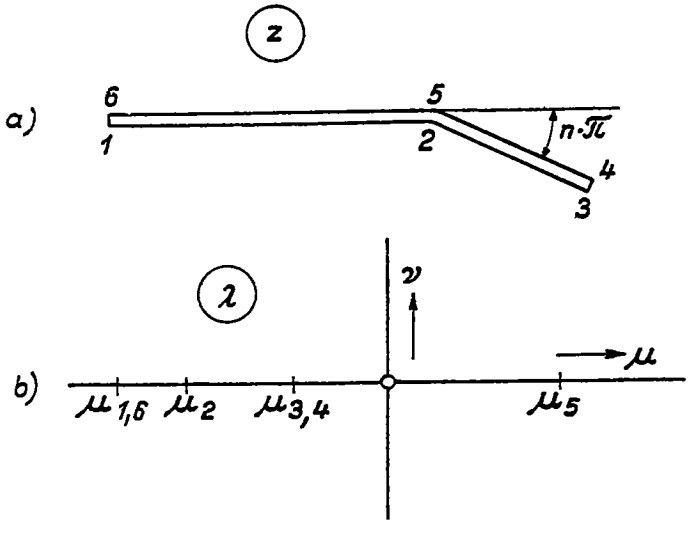

Figure 2.- Mapping of the outer region of the bent, flat plate onto a half plane.

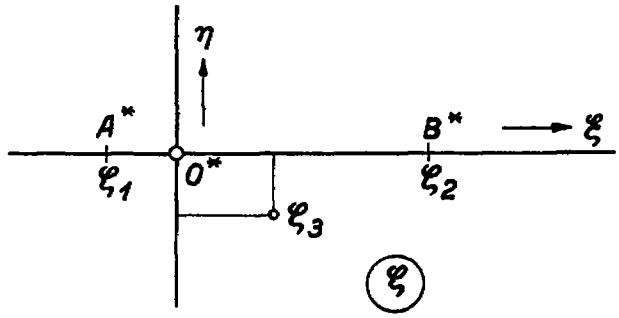

Figure 3.- Correspondence of the z plane (fig. 1) to the ζ plane.

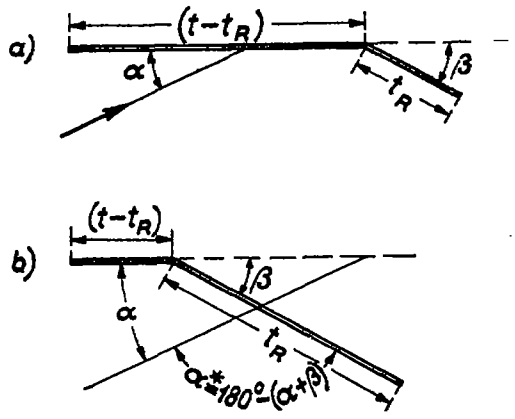

Figure 4.- Exchange of the wing surface and flap.

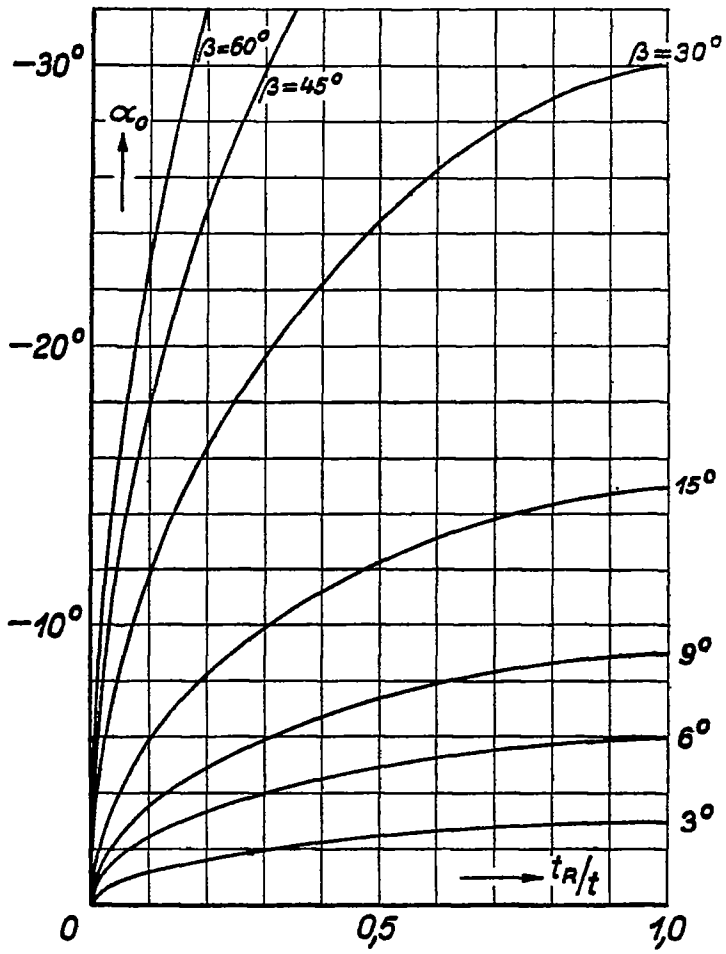

Figure 5.- Angle of zero lift α_0 as a function of the chord ratio for various flap deflections.

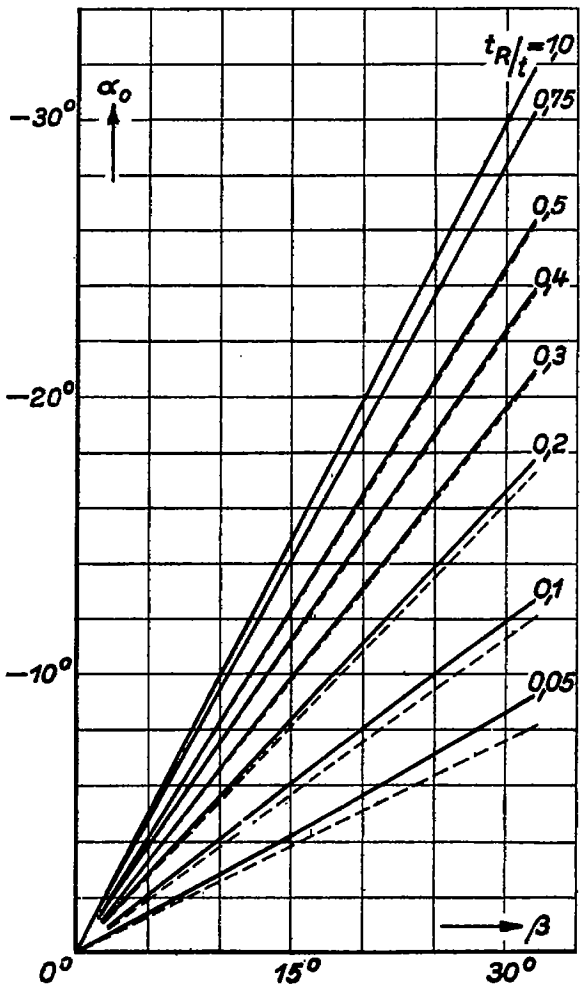

Figure 6.- Zero lift angle as a function of the flap deflection β for various chord ratios. Glauert's approximation is shown by dashed lines.

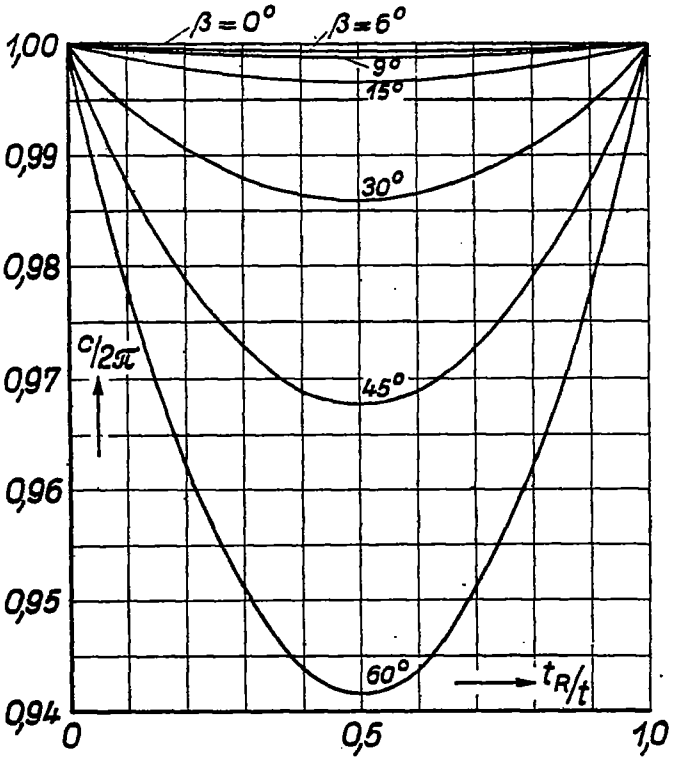

Figure 7.- Ratio of lift-curve slope c to flat plate as a function of the chord ratio for various flap deflections.

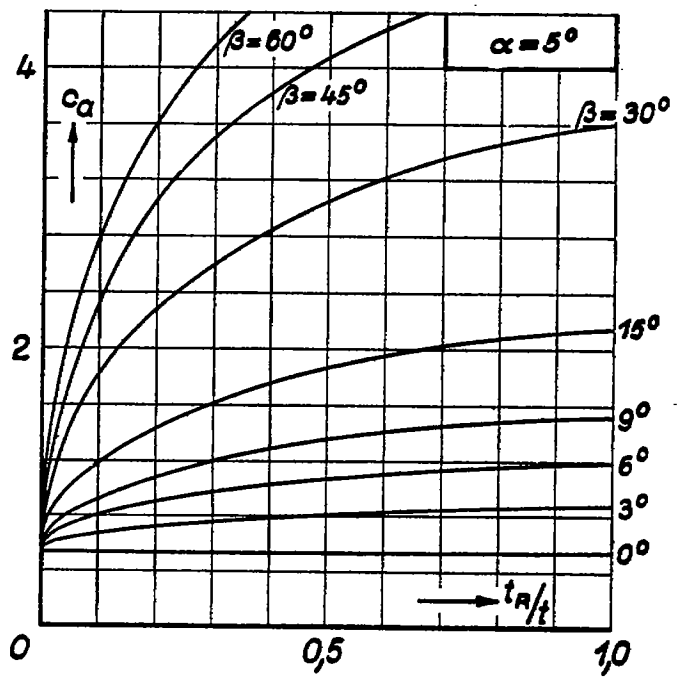

Figure 8.- Dependence of the lift coefficient c_a on chord ratio for various flap deflections at an angle of attack $\alpha = +5^\circ$.

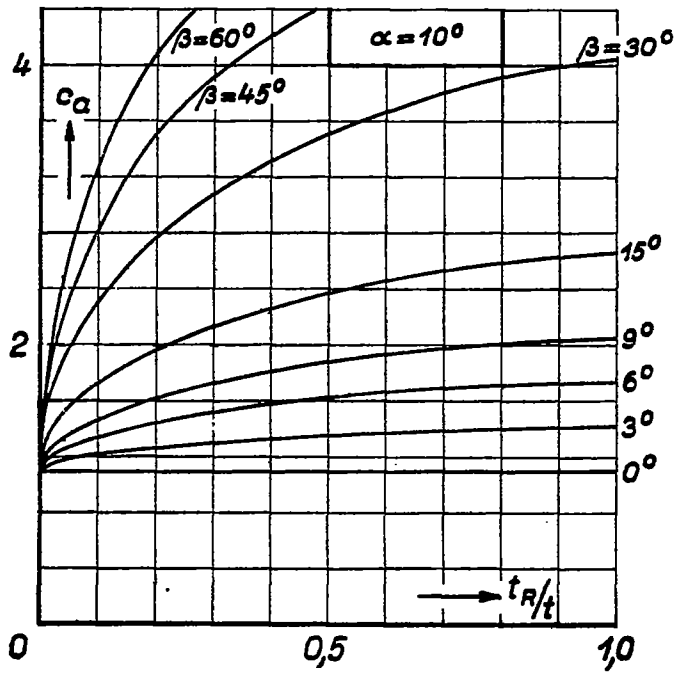

Figure 9.- Dependence of lift coefficient c_a on chord ratio for various flap deflections at angle of attack $\alpha = +10^{\circ}$.

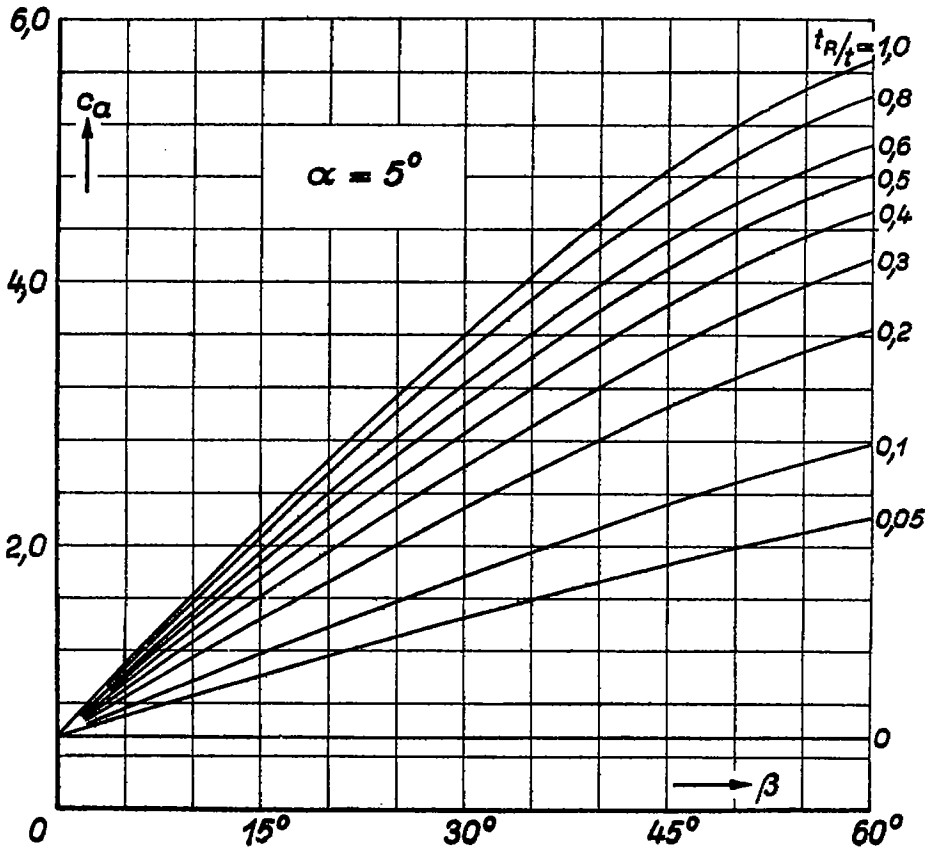

Figure 10.- Lift coefficient c_a as a function of flap deflection for various chord ratios at angle of attack $\alpha = +5^\circ$.

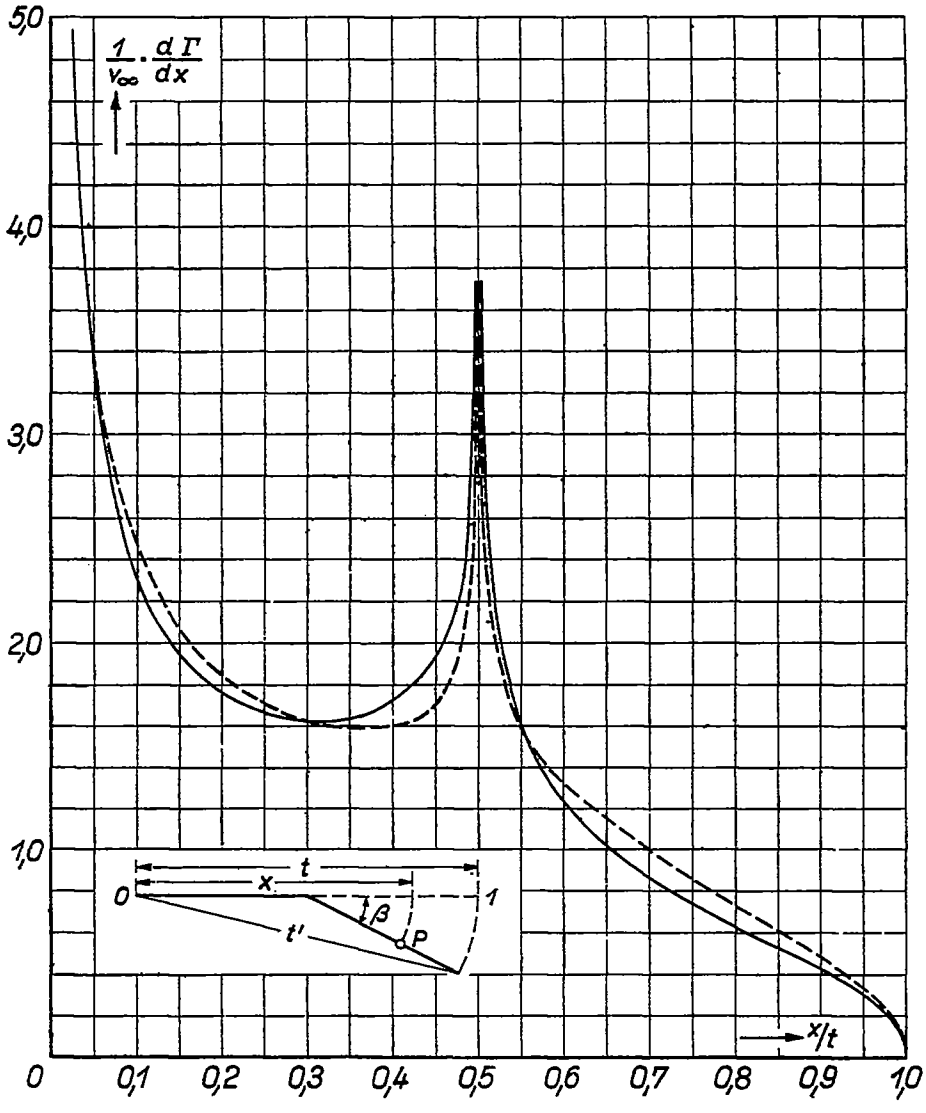

Figure 11.- Circulation distribution with $t_R/t = 0.5$, $\alpha = 7^\circ$, $\beta = 30^\circ$. The dashed lines give Glauert's approximation.

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