REPORT 1350

METHODS FOR OBTAINING DESIRED HELICOPTER STABILITY CHARACTERISTICS AND PROCEDURES FOR STABILITY PREDICTIONS [1]

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SUMMARY

In the first part of this report a brief review is presented of methods available to the helicopter designer for obtaining desired stability characteristics by modifications to the airframe design. The discussion is based on modifications made during the establishment of flying-qualities criteria and includes sample results of theoretical studies of additional methods.

The conclusion is reached that it is now feasible to utilize combinations of methods whereby stability-parameter values are realized which in turn provide the desired stability characteristics.

Part II reviews some of the methods of predicting rotor stability derivatives. The procedures by which these rotor derivatives are employed to estimate helicopter stability characteristics have been summarized. Although these methods are not always adequate for predicting absolute values of the stability of the helicopter, the effects on stability of changes in individua derivatives can generally be estimated satisfactorily.

INTRODUCTION

The problems relating to stability of helicopters have been the subject of numerous published works. Requirements established by the military services for satisfactory helicopter stability are specified in reference 1. Some of the pertinent work on this subject by the National Advisory Committee for Aeronautics is described in references 2 to 21. The purpose of part I of this report is to summarize some of the physical methods available to the designer for obtaining desired stability values by changing the airframe design. Although the direct application considered is that of meeting flying-qualities criteria, it may be worth pointing out that other reasons often arise for designing a configuration so that specific amounts of stability are provided; for example, the most efficient combination of autopilot and airframe design may be desired.

In order to predict helicopter stability, as for example to estimate theoretically whether a proposed helicopter will meet the flying-qualities requirements, both the applicable equations of motion and the necessary stability derivatives must be determined.

Processes for applying the equations of motion have been well established as a result of the extensive stability analysis made for airplanes, and the modification of these procedures for use with helicopters has been found a secondary problem in comparison with the provision of values of stability derivatives. In the past few years, sufficient information has been provided to permit the most pertinent rotor derivatives to be predicted with fairly good accuracy in applications where no stall is present. A summary of this work is presented in part II of this report.

I—METHODS FOR OBTAINING DESIRED HELICOPTER STABILITY CHARACTERISTICS

The scope of the discussion in part I is outlined in the following table:

Characteristic	Rotor type investigated
Damping in roll	Single and tandem
Maneuver stability	Single and tandem
Speed stability	Single and tandem
Lateral oscillation and turn characteristics	Tandem

Available results on static directional (weathercock) stability of tandem helicopters are not included in this discussion because of thorough coverage in reference 2. For helicopter types other than single-rotor or tandem-rotor configurations, no direct information is provided herein but much can be inferred from the more appropriate of the two types covered.

For each of the stability characteristics covered, modifications made to the design in the process of exploring a range of characteristics during the establishment of criteria for desirable flying qualities are discussed. The value of these examples lies in the fact that they are demonstrated cases for which stability parameters have been measured. Optimum design generally requires the choice of a method or combination of methods; therefore, sample results of theoretical studies of additional methods are included in each case.

SYMBOLS

L_δ rolling moment due to control deflection

L_p rolling moment due to rolling angular velocity

ΔW_b change in weight of blade

[1] Supersedes NACA Technical Note 3945 by F. B. Gustafson and Robert J. Tapscott, 1957.

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R blade radius
I_1 blade moment of inertia about flapping hinge
γ blade mass constant
M_α pitching moment per unit angle-of-attack change
M_q pitching moment due to unit pitching angular velocity
L rolling moment
v sideslip velocity
Ω rotor rotational speed
σ rotor solidity
B_1 longitudinal cyclic pitch
V forward velocity
c_m blade section pitching-moment coefficient
$\Delta\theta$ difference between collective-pitch angles of front and rear rotors
$\Delta\alpha_d$ difference between angles of attack of front and rear rotors due to longitudinal swashplate tilt

DAMPING AND CONTROL POWER IN ROLL

The ratio of control power to damping, which frequently tends to reach values that lead to a major problem in the hovering characteristics of small low-inertia helicopters, is treated in table I. The test vehicle was a two-place single-rotor helicopter. As used herein, "control power" is the rolling moment per unit stick deflection and "damping" is the resisting moment per unit angular velocity of the helicopter. Since changes in control power are restricted by trim requirements, this discussion is concerned solely with changes in damping.

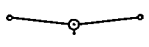
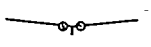
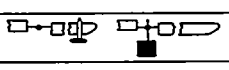
The requirement of the current military specification (ref. 1) is that the rate of roll per inch of stick displacement (often referred to as "sensitivity") be less than 20° per second. The test helicopter provided the opportunity to explore a fair-sized range of values, always on the satisfactory side, by adjusting a mechanical gyroscopic device. The value of 14 for L_δ/L_p is obtained with the device locked out, and the value of 5.2 is obtained with the device as far beyond the production setting as feasible. Both from theory and from flight measurements on other helicopters, it is known that, with lighter blades and no special device, the requirements of reference 1 would not have been met.

TABLE I

RATIO OF CONTROL POWER TO DAMPING

[Single rotor; 2,200-lb gross wt]

Parameter	Value to meet MIL-H-8501	Range tested	Method used
$\frac{\text{Control power}}{\text{Damping}}$, $\frac{L_\delta}{L_p}$, $\frac{\text{deg/sec}}{\text{in.}}$	<20	14 to 5.2	Available gyro device; L_δ fixed

Additional methods (L_δ fixed)		Amount for range tested
Increase in I_1 via tip wt.		$\Delta W_b = 0.6$
Hinge offset with $\gamma = 4.7$		2 % R
Aero servos		----------

In addition, and of perhaps even greater importance, studies made subsequent to the establishment of the existing specifications have demonstrated the direct value of increased damping irrespective of L_δ/L_p. (For example, see ref. 20.) Therefore, examination of several additional methods for changing the damping is advisable.

Increasing the blade inertia is one method of changing the damping. If use of a tip weight is considered, calculations indicate that 60 percent of the blade weight is needed to duplicate the full change in L_δ/L_p from 14 to 5.2.

Offsetting the flapping hinge will increase damping, but if this method is used the control linkage has to be changed to prevent increase in control power, or else little change in L_δ/L_p will result. With the heavy blades of the test helicopter, an offset of only 2-percent radius would be needed to change L_δ/L_p from 14 to 5.2; with light blades, an offset of 5-percent radius would be enough.

A third design approach which makes possible increased damping is the use of aerodynamic servocontrols. Sketches of two types, as viewed from above the rotor, are shown in table I. One type uses a surface behind the outer part of the blade and the other uses a surface attached at right angles at the blade root. Such devices apparently can be designed to provide increases in damping sufficient to cover the test range. Some methods permit increase in both control power and damping, a combination which is sometimes desirable; examples are flapping-hinge offset and aerodynamic servocontrols as just discussed.

Damping and control power in roll for the tandem helicopter require no separate discussion since, in roll, it is possible simply to consider two single rotors instead of one.

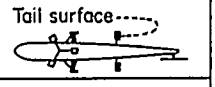
MANEUVER STABILITY OF SINGLE-ROTOR HELICOPTERS

Two separate maneuver-stability (or divergence) problems with single-rotor helicopters at cruising speed are treated in table II. With one helicopter, the variations were made by way of angle-of-attack stability, and with the other, by way of damping in pitch. For the first case, a change to a value of angle-of-attack stability of 300 pound-feet per radian was enough to cause the requirement to be met. The range covered went from the unstable value of 7,000 pound-feet

TABLE II

MANEUVER STABILITY

[Single rotor; gross wts.: (1) 5,000 lb and (2) 2,200 lb; cruise speed]

Major parameter varied	Value to meet MIL-H-8501	Range tested	Method used
(1) M_α, $\frac{\text{lb-ft}}{\text{radian}}$	<300	7,000 to −2,100	Tail surface 
(2) M_q, $\frac{\text{lb-ft}}{\text{radian/sec}}$	<−1,200 (rotor)	−710 to −1,930	Available gyro device

Parameter	Additional methods	Amount for 1/2 range tested
M_α	Increased rpm	33 %
	Forward c.g. with offset hinges	Δc.g. = 2.5% R with 5% offset
M_q	Similar to roll sensitivity	

per radian to the stable value of −2,100 pound-feet per radian. (Minus is stable in accepted stability theory.) The test conditions actually extended somewhat farther than the value of −2,100 pound-feet per radian on the stable side but the M_α values were not recorded.

The tail assembly used has been discussed in published papers, particularly reference 3. It may bear repeating, however, that a total tail area of 0.5 percent of the rotor area could make the difference between the helicopter's diverging excessively in a few seconds and being able to fly through rough air without the longitudinal control being moved. This result was obtained after linking the tail to the cyclic controls, which so reduced the design compromises as to permit use of a somewhat more effective tail.

Tests were made with a different helicopter (labeled (2) in table II), wherein the damping in pitch was varied in such a way as to bracket the condition for which the requirement was met. The value of −1,200 pound-feet per radian needed to cause the specification to be met falls about midway in the range of −700 to −1,900 pound-feet per radian covered. Damping is another quantity where the minus sign is indicative of a stable condition. As to the method used, the investigation was the same as that which provided the damping-in-roll values given in table I.

As to alternate methods, for large changes in angle-of-attack stability M_α, there do not seem to be many which will cover the range singlehanded. Increase in rotor speed is listed because the effect of such an increase is calculated to be sufficient to warrant thought of some compromise with design for optimum power. It will be understood that rotational speed is appropriate because in all other respects the design is fixed; more fundamentally, what is implied is lower values of pitch and tip-speed ratio. A 33-percent increase in rotor rotational speed is estimated to give half the range covered and would not have been enough, in itself, to meet the specification. The effects are not linear and much greater increase in stability by this method would cause extreme compromise with performance.

If offset flapping hinges are used, then the aircraft center-of-gravity position affects angle-of-attack stability. An estimate for this helicopter is that, with the hinges at 5 percent blade radius, moving the center of gravity 7 inches (2.5 percent of the rotor radius) forward from the rotor shaft would produce as much change as the 33-percent increase in rotor rotational speed. With the same offset, a center-of-gravity shift of 14 inches, if such is tolerable, would produce the change to the −2,100 value and thus would permit the requirements to be exceeded.

For alternate methods of varying the rotor damping in pitch M_q, the methods given in the roll-sensitivity discussion again apply.

The discussion of this section is in terms of existing criteria which in turn relate most specifically to normal-acceleration characteristics in maneuvers. It might be remarked that in certain instances, when the normal-acceleration criteria for maneuver stability are just met, the pitching velocity still shows appreciable divergent tendencies. Consideration of the problem suggests that when such situations arise, by carrying somewhat farther the methods that have just been described, logical development with time should be achieved for pitching velocity as well as normal acceleration, so that no fundamentally new problems seem to be represented.

MANEUVER STABILITY OF TANDEM-ROTOR HELICOPTERS

Maneuver stability of the tandem helicopter, again at cruising speed, is treated in table III. The parameter considered is again pitching moment per unit angle-of-attack change, and with the test helicopter the unstable value of 57,000 pound-feet per radian corresponded to a value that would just barely meet the requirements set forth in the specification. While a change of only 15,000 pound-feet per radian would have sufficed to just reach the marginal value of 57,000, a change of 88,000 in the direction of increased stability was explored. It is desirable to be able to make these larger changes. The test method used was not altogether appropriate for this discussion because so much of the range was obtained by choice of power setting. The available center-of-gravity range produced a useful but subordinate change, a forward center of gravity being the more stable.

Because the test method was not altogether appropriate for large changes, calculations are again used to illustrate other methods. Changes which affect the relative lift-curve slopes of the front and rear rotors seem especially effective. It is estimated that, if the front-rotor radius is decreased and the rear-rotor radius is increased to provide a total difference of 10 percent of the mean radius, the test value of stability increase would be realized. If, instead, a differential in rotor solidities were effected by putting wider blades on the rear rotor and narrower ones on the front rotor, this same stability increase could be obtained with a solidity differential of 45 percent of the mean value.

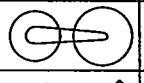
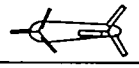
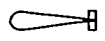
The size of horizontal-tail surface necessary to contribute the same range was computed, in order to illustrate the indication in reference 9 that a horizontal tail is relatively less effective for tandems than for single rotors. A value of 4 percent of rotor area is indicated in distinction to the 0.5

TABLE III

MANEUVER STABILITY

[Tandem rotor; 7,000-lb gross wt.; cruise speed]

Major parameter	Value to meet MIL-H-8501	Range tested	Method used
M_α, $\frac{\text{lb-ft}}{\text{radian}}$	<57,000	72,000 to −16,000	c.g. and power setting

Additional methods	Amount for range tested
ΔR (same Ω) 	10%
$\Delta\sigma$ 	45%
Tail surface 	100 sq ft or 4% of total rotor area

percent indicated for the single-rotor helicopters. Although the two cases are not strictly comparable, the relative order of magnitude is considered to be reasonable.

SPEED STABILITY OF SINGLE-ROTOR HELICOPTERS

Table IV treats the speed stability of a single-rotor helicopter. For a helicopter to be stable with speed, it is required that, with fixed pitch and throttle, the stick be moved farther forward for trim with increase in speed. The parameter used to measure this change is the longitudinal cyclic pitch B_1 per unit velocity V (in knots). The range tested (B_1/V from 0 to 0.06 degrees per knot) was obtained by changes in horizontal-tail setting. When sufficient upload was provided on the tail surface, the stable contribution of the rotor could be cancelled; when download was provided, it could be increased. Incidentally, most of this exploration of near-zero speed stability was carried out with the tail linked to the cyclic stick, the reason being that a high, fixed, nose-up tail setting can be dangerous in the event of inadvertent speed increase.

The rotor tends to be stable, but counteracting factors must still be considered. Some fuselage shapes can be as destabilizing as a nose-up tail setting, although a range of 30 foot-pounds per knot would be rather unusual. A large rotor-blade-section diving moment could more than cancel the stable tendency of the rotor. The increment Δc_m of 0.06 actually was suggested by the dangerous characteristics which arose with autogiros having $c_m = -0.06$. A rough estimate indicated the value of 0.06 to be the right order of magnitude to produce the change under discussion as well.

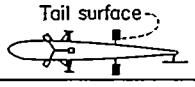
SPEED STABILITY OF TANDEM-ROTOR HELICOPTERS

Table V relates to speed stability of the tandem-rotor type of helicopter. The longitudinal control of this configuration is obtained primarily by changing the pitch-setting difference between front and rear rotors, and the parameter is written as $\Delta\theta/V$ to correspond. This quantity was varied from -0.01 to 0.003; thus, the requirement of at least zero is bracketed. This change was accomplished by changing the tilt angle between the front and rear rotor systems. The unstable setting was 0°; the stable one, 1.8° "toe-in."

TABLE IV

SPEED STABILITY

[Single rotor; 5,000-lb gross wt.; cruise speed]

Parameter	Value to meet MIL-H-8501	Range tested	Method used
$\frac{B_1}{V}$, $\frac{\text{deg}}{\text{knot}}$	>0	0 to 0.06	Tail surface 

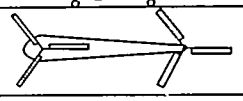
Additional factors	Amount for range tested
Moments, fuselage less tail	$\Delta\frac{\text{ft-lb}}{\text{knot}}=30$
Blade section pitching-moment coefficient	$\Delta c_m \approx 0.06$

TABLE V

SPEED STABILITY

[Tandem rotor; 7,000-lb gross wt.; cruise speed]

Parameter	Value to meet MIL-H-8501	Range tested	Method used
$\frac{\Delta\theta}{V}$, $\frac{\text{deg}}{\text{knot}}$	>0	−0.01 to 0.003	$-1.8°\ \Delta\alpha_d$

Additional factors	Amount for range tested
c.g. shift	22%
$\Delta\sigma$ 	34%

Calculations indicate that a center-of-gravity shift from midway between the shafts to 22 percent ahead of the midpoint should produce the equivalent range. The 22-percent value is based on total distance between shafts. Similarly, giving the rear rotor higher solidity and the front less, with a total difference $\Delta\sigma$ of 34 percent of the mean, should achieve this same result. These values are for cruising speed; at low speeds the effect of center of gravity can even reverse, whereas the solidity change becomes more effective. The use of these methods in combination thus holds special interest. It is also worth noting that these effects act in the same direction for speed stability as they do for maneuver stability.

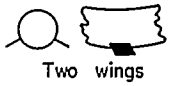
LATERAL-OSCILLATION AND TURN CHARACTERISTICS OF TANDEM-ROTOR HELICOPTERS

Tandem-rotor lateral-oscillation and turn characteristics at cruise speed are treated in table VI. One of the most effective parameters for both is rolling moment per unit sideslip velocity, or L/v, measured as foot-pounds of rolling moment per foot per second of sideslip velocity. Fortunately, this quantity, which is the effective dihedral, is not critical in its own right, provided it stays on the stable side. (By convention, stable values are negative.) Most of the required change was obtained by attaching small wings (surface area,

TABLE VI

LATERAL OSCILLATION AND TURN CHARACTERISTICS

[Tandem rotor; 7,000-lb gross wt.; cruise speed]

Major parameter	Value to meet MIL-H-8501	Range tested	Method used
Roll due to sideslip, $\frac{L}{V}$, $\frac{\text{ft-lb}}{\text{ft/sec}}$	>−26	−75 to −37	 Two wings 7 sq ft each

Additional method	Amount for range tested
Vertical c.g.– c.p. relationship	9 in.

7 square feet) to the landing gear. The front view and side view are shown. It was most convenient in these flying-qualities trials to extend the test conditions further by changing the flight conditions; but with so many designs beginning to appear with moderate-sized wings (much bigger than the panels used in these tests), making the full change needed may often require only the determination of the correct geometric dihedral.

One additional method is to change the vertical location of the center of gravity relative to the side-view area. Only a major change in power-plant location, such as a shift to overhead turbines, seems likely to make a change of the necessary magnitude (9-inch shift in center of gravity). Contributing changes, though, including some ventral-fin area, may often prove feasible.

Increasing the damping in roll is another effective approach that can be considered. This method is discussed, along with still other approaches, in reference 4.

CONCLUDING REMARKS FOR PART I

Examples have been presented to illustrate methods for improving helicopter stability. It will be realized that many of the numerical values which have been given apply only for the case studied. For example, a tandem helicopter might already have lower effective dihedral or might somehow be designed with less nose-down inclination of the principal inertia axis and, in either event, might meet the lateral requirements without reduction in effective dihedral.

The cases presented are believed to illustrate that it is now feasible to choose one or more important stability parameters from theoretical studies and, in turn, to find a combination of methods whereby values of the parameters are realized which provide the desired stability characteristics.

The obtainment of desired stability characteristics with a minimum of design compromise requires first of all a high degree of fundamental understanding. To achieve this end a workable theory must be available. It is believed that the existence of such theory has been demonstrated. In addition to such use of theory, the outstanding key to the obtainment of desired characteristics with minimum design compromise seems to be the availability and use of a variety of physical methods in combination with one another. It is hoped that the sample methods presented may be helpful in suggesting still more approaches to the helicopter designer.

II—PROCEDURES FOR PREDICTING HELICOPTER STABILITY

The prediction of stability derivatives requires a knowledge of the contributions of both the rotor and the fuselage. Fuselage characteristics are not open to as specific an analysis as rotor characteristics, and preliminary estimates can be handled on the basis of data from previous designs and from wind-tunnel model tests. The fuselage seems to be subject to greater modification up to the time production starts; consequently, this discussion is confined primarily to the rotor contributions to the derivatives.

SYMBOLS

W	gross weight of helicopter, lb
m	mass of helicopter, slugs
g	acceleration due to gravity, 32.2 ft/sec^2
R	blade radius, ft
c	blade-section chord, ft
c_e	equivalent blade chord (on thrust basis), $\dfrac{\int_0^R cr^2dr}{\int_0^R r^2dr}$, ft
b	number of blades per rotor
σ	rotor solidity, $bc_e/\pi R$
Θ	instantaneous blade-section pitch angle (angle between line of zero lift of blade section and plane perpendicular to rotor shaft), $\theta - A_1 \cos\psi - B_1 \sin\psi$, radians
θ	collective-pitch angle (average value of Θ around azimuth), radians
ρ	mass density of air, slugs/cu ft
γ	mass constant of rotor blade (ratio of air forces to mass forces), $\rho caR^4/I_1$; also, flight-path angle, radians
I_X	helicopter rolling moment of inertia about center of gravity, slug-ft^2
I_Y	helicopter pitching moment of inertia about center of gravity, slug-ft^2
I_Z	helicopter yawing moment of inertia about center of gravity, slug-ft^2
$\Delta\theta$	difference between collective-pitch angles of front and rear rotors, positive when collective-pitch angle of rear rotor is greater, radians; also, increment of θ
I_1	mass moment of inertia of blade about flapping hinge, slug-ft^2
ΔT	difference between thrusts of front and rear rotors, positive when thrust of rear rotor is greater, lb
$\Delta R, \Delta(\Omega R), \Delta\sigma$	definitions analogous to that for ΔT
$\Delta\alpha_d$	difference between angles of attack of front and rear rotors due to longitudinal swashplate tilt, positive when rear-rotor angle of attack is greater
A_1, B_1	coefficients of $-\cos\psi$ and $-\sin\psi$, respectively, in expression for Θ; therefore, lateral and longitudinal cyclic pitch, respectively, radians
δ	control motion, inches from trim
t	time, sec
V	true airspeed of helicopter along flight path, ft/sec
v	sideslip velocity of helicopter in Y-direction, ft/sec
α	rotor angle of attack (angle between flight path and plane perpendicular to axis of no feathering), positive when axis is inclined rearward, radians
Ω	rotor angular velocity, radians/sec
μ	tip-speed ratio, $\dfrac{V\cos\alpha}{\Omega R}\left(\text{assumed equal to } \dfrac{V}{\Omega R}\right)$
U_T	component at blade element of resultant velocity perpendicular to blade-span axis and to axis of no feathering, ft/sec

$u_T = U_T/\Omega R$

ψ blade azimuth angle (measured in direction of rotation from downwind position of blade if wind axes are used; measured in direction of rotation from position of blade when it is pointing rearward along longitudinal axis of helicopter if body axes are used), radians

a slope of curve of blade section lift coefficient plotted against section angle of attack, per radian

L rotor lift, lb; also, rolling moment, lb-ft

T rotor thrust (component of rotor resultant force parallel to axis of no feathering), lb

Q rotor-shaft torque, lb-ft

C_T rotor thrust coefficient, $\frac{T}{\pi R^2 \rho (\Omega R)^2}$

C_Q rotor-shaft torque coefficient, $\frac{Q}{\pi R^2 \rho (\Omega R)^2 R}$

β blade flapping angle at particular azimuth position, $a_0 - a_1 \cos\psi - b_1 \sin\psi \ldots$, radians; also, sideslip angle, radians

a_0 constant term in Fourier series that expresses β; therefore, rotor coning angle

a_1 coefficient of $-\cos\psi$ in expression for β; therefore, longitudinal tilt of rotor cone

b_1 coefficient of $-\sin\psi$ in expression for β; therefore, lateral tilt of rotor cone

a' projection of angle between rotor resultant force vector and axis of no feathering in plane containing flight path and axis of no feathering

b' projection of angle between rotor resultant force vector and axis of no feathering in plane containing axis of no feathering and perpendicular to plane containing flight path and axis of no feathering

p helicopter rolling velocity, radians/sec

q helicopter pitching velocity, radians/sec

r helicopter yawing velocity, radians/sec; also, radial distance to blade element, ft

ϕ angle of roll, radians

N yawing moment, lb-ft

M pitching moment, lb-ft

h height of rotor hub above helicopter center of gravity, ft

η angle between principal longitudinal axis of inertia of helicopter and flight path, positive when nose is up, radians

X, Y, Z stability axes

Subscript:

AV average

Dots over symbols indicate derivatives with respect to time.

ROTOR STABILITY DERIVATIVES

In general, calculation of the stability derivatives needed for a study of helicopter characteristics depends on the knowledge of the individual rotor derivatives shown in the following table:

$$\begin{array}{ccccc}
\frac{\partial(C_T/\sigma)}{\partial\alpha} & \frac{\partial(C_T/\sigma)}{\partial V} & \frac{\partial(C_T/\sigma)}{\partial\Omega} & \frac{\partial(C_T/\sigma)}{\partial\theta} & \frac{\partial(C_T/\sigma)}{\partial q} \\
\frac{\partial a'}{\partial\alpha} & \frac{\partial a'}{\partial V} & \frac{\partial a'}{\partial\Omega} & \frac{\partial a'}{\partial\theta} & \frac{\partial a'}{\partial q} = \frac{\partial b'}{\partial p} \\
\frac{\partial(C_Q/\sigma)}{\partial\alpha} & \frac{\partial(C_Q/\sigma)}{\partial V} & \frac{\partial(C_Q/\sigma)}{\partial\Omega} & \frac{\partial(C_Q/\sigma)}{\partial\theta} & \frac{\partial(C_Q/\sigma)}{\partial q} \\
\frac{\partial a_0}{\partial\alpha} & \frac{\partial a_0}{\partial V} & \frac{\partial a_0}{\partial\Omega} & \frac{\partial a_0}{\partial\theta} & \frac{\partial a_0}{\partial q} \\
\frac{\partial a_1}{\partial\alpha} & \frac{\partial a_1}{\partial V} & \frac{\partial a_1}{\partial\Omega} & \frac{\partial a_1}{\partial\theta} & \frac{\partial a_1}{\partial q} \\
\frac{\partial b_1}{\partial\alpha} & \frac{\partial b_1}{\partial V} & \frac{\partial b_1}{\partial\Omega} & \frac{\partial b_1}{\partial\theta} & \frac{\partial b_1}{\partial q}
\end{array}$$

As shown, the rotor parameters considered are C_T/σ and a', the magnitude and angle of the thrust vector, respectively; rotor torque C_Q/σ; blade coning angle a_0; and b_1 and a_1, the lateral and longitudinal flapping, respectively. These rotor parameters are functions of five independent variables: rotor angle of attack α, forward speed V, rotational speed Ω, collective pitch θ, and pitching velocity q. The values of the derivatives are determined by the variations from trim of the rotor parameters with changes in each of the five variables. The total number of derivatives shown here is large; however, for most applications it is generally possible to reduce the number considerably. For instance, if the rotor under consideration has no flapping-hinge offset, the flapping coefficients are not significant; thus, the number of derivatives to be considered is immediately reduced by about one-half. If necessary, however, these derivatives can be determined from the equations of references 10 and 14.

Of the remaining derivatives, the lift due to pitching and torque due to pitching generally can be neglected. For most cases, then, the derivatives needed have been reduced to those enclosed within the lines in the preceding table. All the C_T/σ, C_Q/σ, and a' derivatives can be obtained from the figures or equations of references 8 and 10, as is shown in the following discussion.

The thrust due to angle of attack and that due to collective pitch are presented as functions of tip-speed ratio in figure 1. If the solidity and tip-speed ratio for a given case are known, these quantities can be read directly from the curves. The equations on which this figure is based and the processes by which they were derived are discussed in reference 8.

The change in thrust coefficient with tip-speed ratio is presented in reference 8 in the form of an equation from which, when the flight condition is known, the rate of change

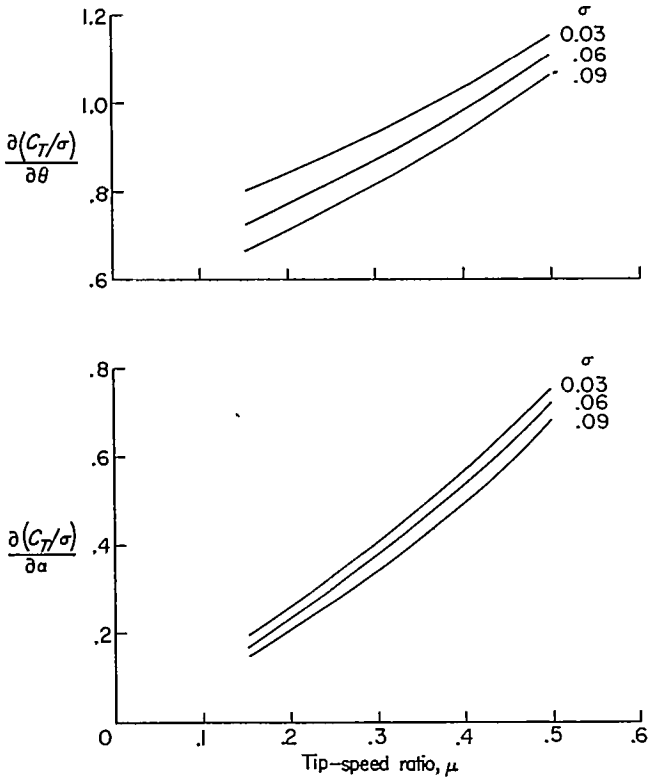


FIGURE 1.—Variation of C_T/σ derivatives with respect to μ.

of thrust coefficient with forward speed and rotational speed can be computed.

The change in inclination of the rotor force vector due to steady pitching or rolling velocity has been derived in reference 10. This derivative is shown in figure 2 as a function of the parameter $\frac{\theta}{C_T/\sigma}$. As the effect of tip-speed ratio μ is small, the formula shown in figure 2 is fairly accurate below a tip-speed ratio of 0.5 for both roll and pitch. The rotor damping moment is determined simply by multiplying the quantity obtained from the figure by $18/\gamma\Omega$ and by the rotor thrust and rotor height above the helicopter center of gravity; that is,

$$\frac{\partial M}{\partial q}=\frac{\partial a'/\partial q}{18/\gamma\Omega}\left(Th\frac{18}{\gamma\Omega}\right)$$

For the rotor-vector angle and rotor torque derivatives, charts such as those shown in figure 3 have been derived and are published in reference 8. These charts are given for a range of collective-pitch angles from 0° to 14° at 2° increments; the sample shown herein is for a collective pitch of 8°. In these charts, the longitudinal rotor-vector angle a' is plotted against the ratio of thrust coefficient to solidity for specified values of tip-speed ratio. Lines of constant power parameter $C_Q/C_T\mu$ are also plotted on the charts. Combinations of these parameters which result in angles of attack of 12° and 16° on the retreating blade are indicated by dashed lines; these lines, in effect, serve as limit lines above which account must be taken of stall. By using slopes or differences from these charts in conjunction with

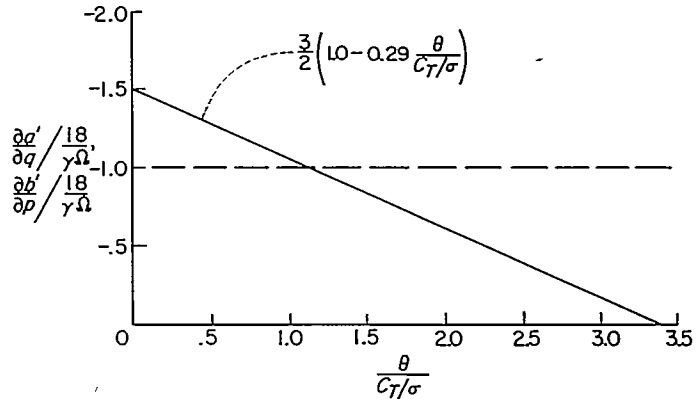


FIGURE 2.—Chart for determining rotor damping in pitch and roll.

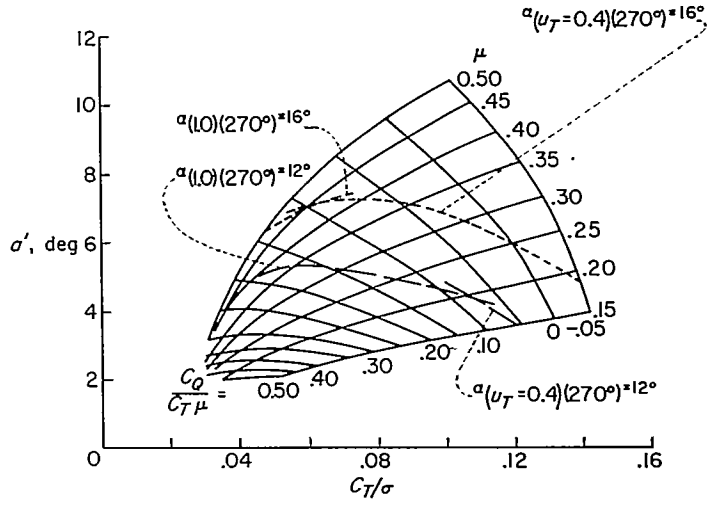


FIGURE 3.—Sample chart for determining derivatives of a' and C_Q/σ. $\theta=8°$.

other derivatives and some simple equations, the remaining derivatives of a' and of the torque coefficient can be obtained. As an example, consider the derivative of a' with respect to angle of attack. From charts such as figure 3, the rate of change of a' with thrust coefficient can be obtained at a given tip-speed ratio and thrust coefficient by scaling off the slope of the tip-speed-ratio line at the desired thrust coefficient. When this quantity is multiplied by the change in thrust coefficient with angle of attack, which has already been discussed, the result is the change in a' with angle of attack. The other a' and the torque coefficient derivatives can be obtained by similar procedures. These procedures are discussed in reference 8.

PREDICTION OF HELICOPTER STABILITY CHARACTERISTICS

It is believed that the rotor contribution to the essential helicopter derivatives can be predicted by using the rotor derivatives that have been discussed. These rotor derivatives are applicable, in most cases, to a study of the stability characteristics of either a single-rotor or a tandem-rotor configuration. Differences arise in using the rotor derivatives to determine the helicopter derivatives for use in the equations of motion and in accounting for the effects of flow

interference for a specific configuration. For most purposes, the lateral and longitudinal characteristics of the helicopter can be studied separately. Generally, equations of motion derived on the basis of constant forward speed are sufficient and are applicable to both single- and tandem-rotor configurations.

Longitudinal characteristics.—In a study of the longitudinal characteristics of the helicopter (refs. 5 and 11), an important criterion is that the time history of the normal acceleration shall be concave downward within 2 seconds after a step input to the longitudinal control; that is, the slope of the normal acceleration curve shall reach its maximum value and begin to decrease within 2 seconds. In order to assist in estimating theoretically whether a prospective helicopter will meet this criterion, the following equations of motion were devised:

$$\frac{\partial L}{\partial q}q+\frac{\partial L}{\partial \theta}\Delta\theta+\frac{\partial L}{\partial \alpha}\Delta\alpha-\frac{WV}{g}\dot{\gamma}=0$$

$$\frac{\partial M}{\partial q}q+\frac{\partial M}{\partial \theta}\Delta\theta+\frac{\partial M}{\partial \alpha}\Delta\alpha+\frac{\partial M}{\partial B_1}\Delta B_1-I_Y\dot{q}=0$$

$$\Delta\alpha=\int_0^t q\,dt-\Delta\gamma-\Delta B_1$$

These equations are based on stability axes and derived on the assumptions of constant forward speed, constant rotor speed, and constant stability derivatives. The assumption is also made that the dynamic maneuver can be represented by a series of static conditions; hence, the rotor parameters are always at their equilibrium values as determined by the instantaneous values of angle of attack, pitch angle, tip-speed ratio, and pitching velocity. The first and second equations shown represent, respectively, equilibrium normal to the flight path and equilibrium in pitch. The third equation simply relates the variables of the first two equations to permit a solution.

The form of these equations applies specifically to the single-rotor configuration. For the tandem configuration, the pitching moment due to control motion results primarily from differential collective pitch of the two rotors rather than from cyclic pitch; thus, the term in the equations expressing pitching moment due to control motion must be modified to account for this difference.

In reference 5, these equations of motion have been solved for the flight-path angle γ, which, in turn, permits an expression to be written for the time history of the normal acceleration. Values of combinations of derivatives have been determined which, when substituted into the expression for the time history of normal acceleration, will result in a time history that is concave downward at 2 seconds. These values have been presented in the form of a chart. (See fig. 4.) Thus the curve in figure 4 is a boundary line that separates combinations of significant longitudinal stability derivatives which result in satisfactory characteristics from those which result in unsatisfactory characteristics according to the criterion previously mentioned. Shown along

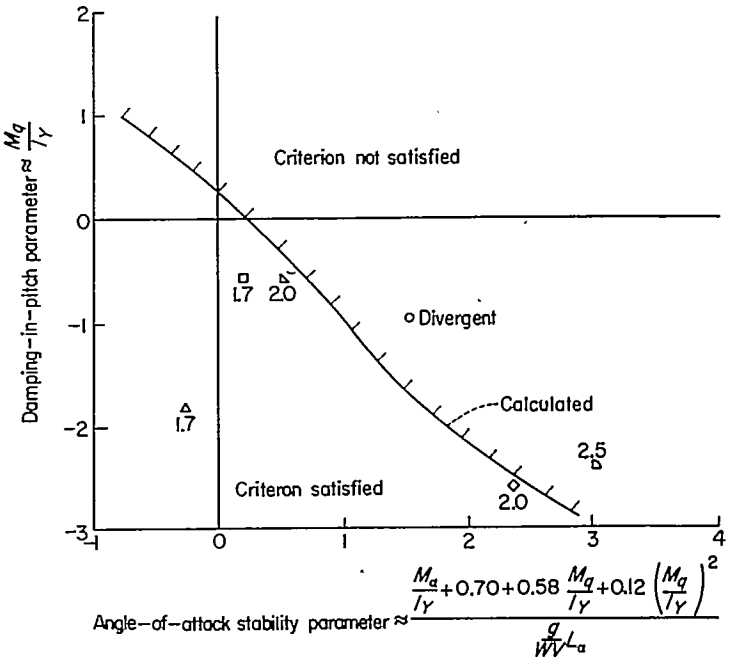


FIGURE 4.—Maneuver-stability chart for 2-second criterion.

with the theoretical curve are data points corresponding to five helicopter configurations. The derivatives for the configurations corresponding to the points shown were measured in flight and the adjacent number is the approximate time for the corresponding normal-acceleration time history to become concave downward. For both single- and tandem-rotor helicopters, the theoretical curve is indicated to be qualitatively correct for separating configurations which have satisfactory maneuver stability from those which have unsatisfactory maneuver stability according to the criterion; that is, the experimental points for which the normal-acceleration time history becomes concave downward in less than 2 seconds fall in the satisfactory region whereas, for times of more than 2 seconds, the points fall in the unsatisfactory region.

Figure 4 shows that an increase in the angle-of-attack stability or damping in pitch (that is, more negative M_α or M_q) or increases in the rotor lift-curve slope L_α are stabilizing. Also, it can be determined from this plot that an increase in the pitching moment of inertia I_Y can be destabilizing. Since the change in lift slope cannot be large and the lift slope is not expected to change sign, the primary changes in stability must be brought about by changes in the damping-in-pitch or angle-of-attack stability.

In the discussion of longitudinal stability so far, the forward speed has been assumed to be constant. For the tandem configuration, however, the downwash effects of the front rotor acting on the rear rotor cause an unstable variation in pitching moment with speed. A study of the speed stability of the tandem helicopter was made in reference 6. In that study, on the basis of the available derivatives discussed previously, an expression for the change of stick position with speed was derived. The equation, along with a plot of constants based on the rotor derivatives, is shown in

figure 5. These constants are presented for several values of rotor solidity over a range of tip-speed ratios. After a flight condition is selected and the tip-speed ratio and thrust coefficient are thereby established, the slope of the variation of stick position with speed can be determined. This equation takes into account the effects on speed stability of center-of-gravity position, differential rotor speed, differential rotor radius and solidity, and "longitudinal dihedral." Effects of these parameters can be studied either together or separately.

This analysis of tandem-helicopter speed stability is of particular value in studies, such as the study of the relative effects of changes in the various parameters, wherein it is not necessary to know accurately the value of downwash or the fuselage contribution. In order to predict the absolute value of speed stability, the downwash must be estimated for the configuration and flight condition under consideration, and the variation of the fuselage moments must be known. The results of flow-field measurements

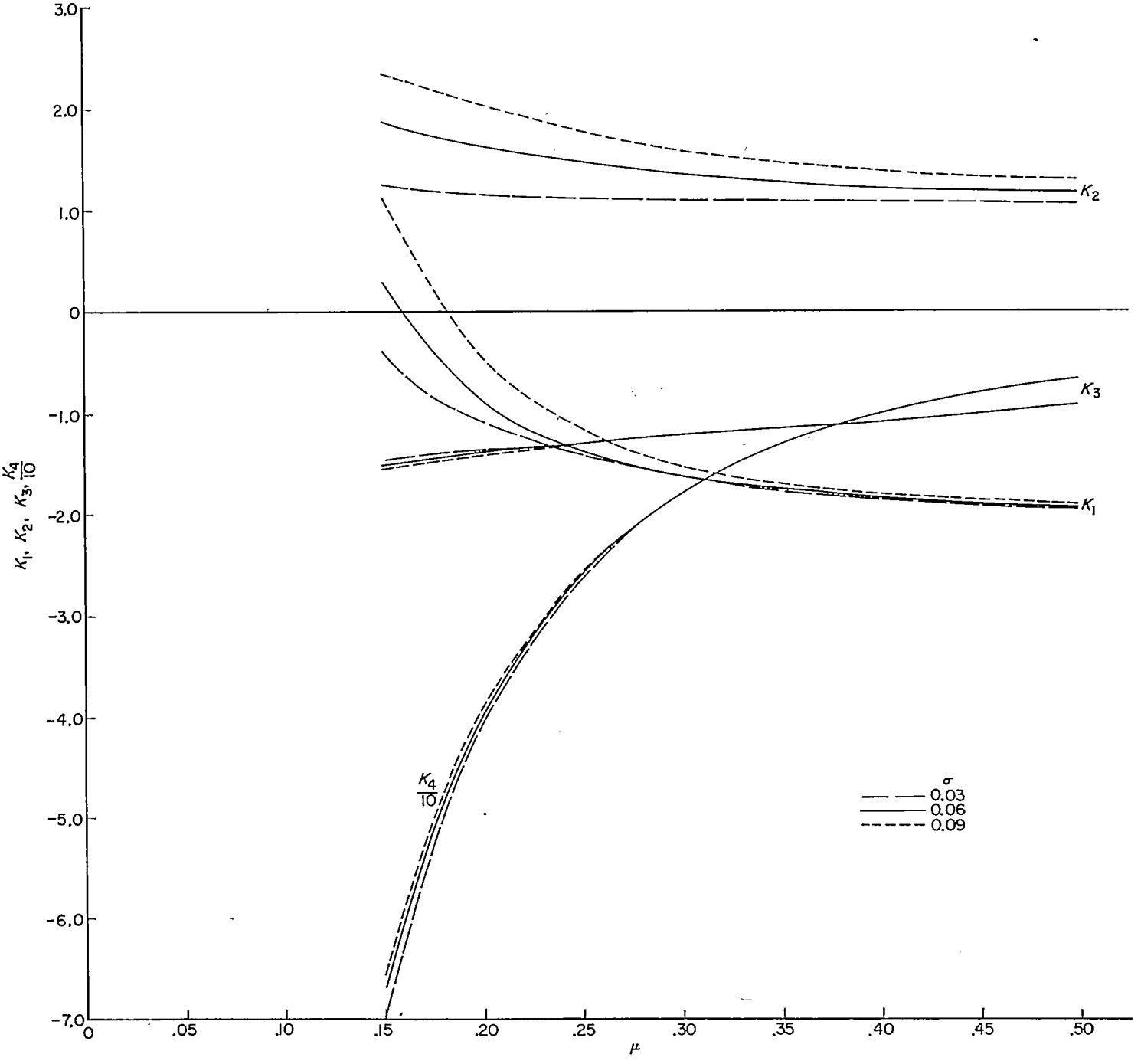


FIGURE 5.—Equation for speed stability of tandem-rotor helicopters.

$$\frac{d(\Delta\theta)}{d\mu}=K_1\left(\frac{C_T}{\sigma}\right)_{AV}\left(\frac{\Delta T}{W}-\frac{\Delta R}{R_{AV}}\right)+K_2\left(\frac{C_T}{\sigma}\right)_{AV}\left[\frac{\Delta\sigma}{\sigma_{AV}}+2\frac{\Delta(\Omega R)}{(\Omega R)_{AV}}\right]+K_3\Delta\alpha_d+K_4C_T$$

presented in references 15 to 18 should be useful in estimating the magnitude of the downwash applicable to a specific condition.

Lateral-directional characteristics.—The next important item in helicopter stability is the lateral-directional characteristics. If roll, yaw, and sideslip are considered as degrees of freedom, the equations of motion are as follows:

$$I_{x'}\ddot{\phi}+\frac{\partial L}{\partial p}\dot{\phi}+\frac{\partial L}{\partial v}v-\frac{\partial L}{\partial \delta}\delta=0$$

$$I_{z'}\ddot{\psi}+\frac{\partial N}{\partial r}\dot{\psi}+\frac{\partial N}{\partial v}v-\frac{\partial N}{\partial \delta}\delta=0$$

$$\left(mV-\frac{\partial Y}{\partial r}\right)\dot{\psi}-\left(mV\sin\eta+\frac{\partial Y}{\partial p}\right)\dot{\phi}-$$

$$T\phi-T\psi\tan\gamma+m\dot{v}-\frac{\partial Y}{\partial v}v-\frac{\partial Y}{\partial \delta}\delta=0$$

For convenience in determining the derivatives, these equations are based on principal axes of inertia (primed symbols) rather than on the relative wind. In general, the rotor contribution to the derivatives needed in these equations can also be determined from rotor theory. Fuselage contributions can be predicted from wind-tunnel data and previous experience or from flight data where available.

These equations have been found useful particularly in estimating the effects on helicopter characteristics of changes in the individual stability derivatives. As an example, figure 6 shows the theoretically predicted time histories of rolling velocity and sideslip angle of a tandem-rotor helicopter. The curves on the left-hand side of figure 6 represent the predicted time histories for the original helicopter. On the right-hand side of figure 6 are time histories of the same quantities when the effective-dihedral derivative is reduced 50 percent. This figure indicates that a 50-percent reduction in the effective dihedral would substantially improve the oscillatory characteristics of the helicopter. In an attempt to improve the characteristics, this means was tried experimentally. The results are shown in figure 7, wherein the experimentally measured time histories of rolling velocity and sideslip angle before and after the derivative change are compared. The comparison shows that, as predicted by the theory, a reduction in the effective dihedral improved the

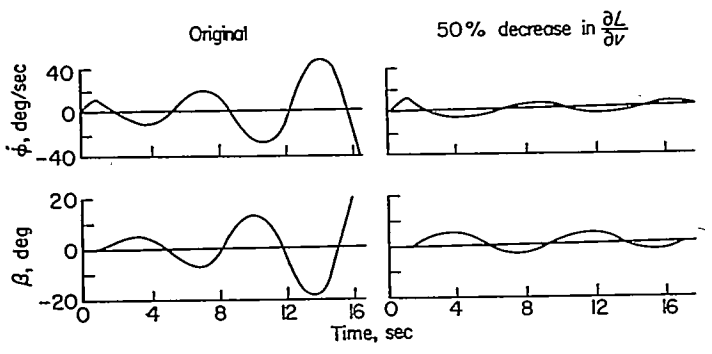


FIGURE 6.—Predicted effect of reduction in effective dihedral on lateral oscillation.

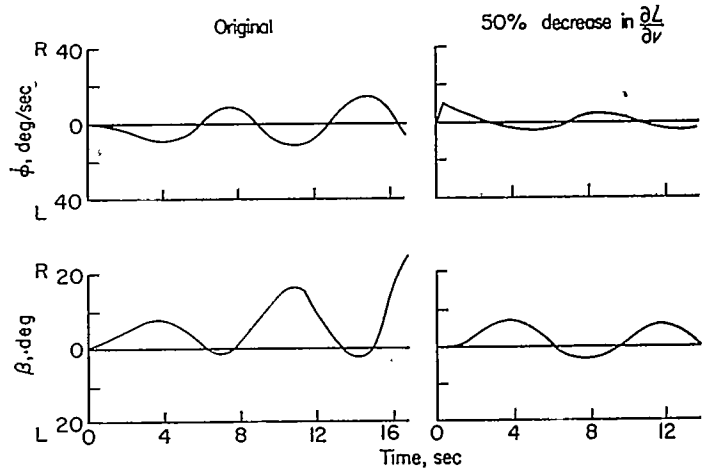


FIGURE 7.—Measured effect of reduction in effective dihedral on lateral oscillation.

oscillation. Thus, in this case, the theory was employed successfully to indicate the course to be followed in making an improvement.

CONCLUDING REMARKS FOR PART II

It will frequently not be feasible to predict accurately absolute magnitudes for the stability of the complete helicopter, particularly because of the difficulty of predetermining final, full-scale fuselage characteristics. However, on the basis of the studies discussed herein, a first approximation can be made, and by making some comparatively straightforward flight measurements of stability derivatives and reemploying the theory to show what modifications are needed, it appears feasible to handle at least those problems with which direct experience has been had. It appears likely that, in most cases, changes in several derivatives simultaneously would be necessary to achieve most efficiently the desirable stability characteristics, and the equations discussed herein should prove very useful in this type of study.

LANGLEY AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *November 20, 1956.*

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