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TECHNICAL NOTE 4207

EFFECT OF A STRINGER ON THE STRESS CONCENTRATION DUE TO A CRACK IN A THIN SHEET

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Langley Field, Va.

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EFFECT OF A STRINGER ON THE STRESS CONCENTRATION DUE TO A CRACK IN A THIN SHEET

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SUMMARY

A coefficient is obtained for determining the effect of a reinforcing stringer on the stress concentration factor at the tip of a crack in a thin sheet. The results are given for the case in which the stringer is intact and for the case in which the stringer is broken. In the first case the stress concentration factor for the stringer is also given.

INTRODUCTION

Some damage to aircraft structures due to fatigue or accident is statistically inevitable; thus, the fail-safe concept has entered into design considerations. One of the problems associated with this concept is the determination of the static strength of cracked parts. The mechanism of static failure of a structure weakened by the presence of a crack is by no means completely understood at the present time. However, an engineering theory which seems to hold some promise has recently become available (ref. 1). In this theory the significant quantity determining the strength of the cracked structure is the stress concentration factor at the end of the crack (corrected for plasticity and the so-called size effect). The fundamental information needed to apply the method is the stress concentration factor obtained from elasticity theory.

For many configurations an exact solution for the stress distribution from the theory of elasticity is very difficult to obtain. However, a considerable amount of information that is useful and adequate for practical applications has been obtained by making various idealizations and simplifications of the problems. As a further contribution, the results contained in the present paper were obtained.

The problem considered in the present paper is the determination of the relieving effect of a reinforcing stringer on the stress concentration at the tip of a crack in a thin sheet. The crack runs perpendicular to the stringer and extends an equal distance on either side of it. The state of stress in the sheet far away from the crack is a tensile stress

parallel to the stringer. The stress concentration factor for a crack in a thin sheet may be determined from a known formula. (See ref. 1.) The factor by which this known result can be multiplied in order to correct for the presence of the reinforcing stringer is determined in the present paper. The stress concentration factor in the stringer due to the crack and the correction factor for the crack in the case in which the stringer is broken are also found in the analysis. The results are presented graphically and in tabular form.

SYMBOLS

A	cross-sectional area of stringer
B	function defined in equation (30)
b	length of crack on one side of stringer
C	ratio between stress concentration factors for a cracked sheet with and without a stringer
C^*	corresponds to C in case where stringer is broken
E	Young's modulus for sheet material
E_{st}	Young's modulus for stringer material
F	analytic function defined in equation (11)
G	shear modulus for sheet material
H	stress function (see eqs. (2))
I_0, I_1	modified Bessel functions of first kind
K_0, K_1	modified Bessel functions of second kind
L_0, L_1	Struve functions of imaginary argument
P	load in stringer at its intersection with crack
$\bar{P}$	load concentration factor for stringer, $PE/\sigma AE_{st}$
$R(\ )$, $I(\ )$	real part of and imaginary part of

s	dummy variable of integration
t	sheet thickness
u	displacement in x_1-direction
x, y	dimensionless coordinates (see eqs. (4))
x_1, y_1	physical coordinates
z	complex variable, $x + iy$
γ	Euler's constant, 0.57722
ζ, η	complex variables
θ	dummy variable
λ	similarity parameter, $2btE/AE_{st}$
σ	direct stress in sheet at infinity
σ_x, τ	direct and shear stresses in sheet
$\sigma_{x,o}$	direct stress in sheet with a crack but without a stringer
Φ	analytic function, $\phi + i\psi$
Φ_o	corresponds to Φ for a sheet without a stringer
ϕ	dimensionless stress function (see eqs. (4))
ψ	dimensionless displacement (see eqs. (4))

Primes indicate differentiation and the notation $\sim$ indicates an asymptotic relationship.

ANALYSIS

Two simplifications of the problem are made in the present analysis. One simplification is that the sheet is assumed inextensional in the direction parallel to the crack. This orthotropic sheet was introduced by Hildebrand (ref. 2) and greatly simplifies the equations of plane stress. The other simplification is to treat the crack as a straight-line segment and assume that the strength of the stress singularity at

the end of the idealized crack is a measure of the stress concentration due to a thin crack with a small, but nonzero, radius of curvature at its end. The effect of the stringer on the strength of the stress singularity is found by solving the two similar problems of the cracked sheet with and without the stringer. The desired correction factor previously defined is taken to be the ratio of the two strengths thus found.

Formulation of Boundary-Value Problem

The thin sheet with a crack and attached stringer is represented in figure 1. According to the orthotropic plane-stress theory of reference 2, the stress-displacement relations are

$$\left.\begin{aligned} \sigma_x &= E\frac{\partial u}{\partial x_1} \\ \tau &= G\frac{\partial u}{\partial y_1} \end{aligned}\right\} \tag{1}$$

where σ_x and τ are the direct and shear stresses, respectively, E is Young's modulus, G is the shear modulus, and u is the displacement in the x_1-direction. The displacement in the y_1-direction is zero from symmetry. Equilibrium is satisfied if the stresses are given in terms of a stress function H as follows:

$$\left.\begin{aligned} \sigma_x &= \frac{\partial H}{\partial y_1} \\ \tau &= -\frac{\partial H}{\partial x_1} \end{aligned}\right\} \tag{2}$$

An equation of equilibrium for the stringer may be obtained by considering the portion of the stringer from the origin to x_1 as a free body. (See fig. 2.) The displacements in the stringer must be the same as those in the sheet along the x_1-axis; hence, from equation (1) the stress in the stringer must be $\sigma_x \frac{E_{st}}{E}$, where E_{st} is Young's modulus for the stringer material. The required equilibrium equation is thus

$$
\begin{aligned}
P &= \sigma_x A \frac{E_{st}}{E} + 2 \int_0^{x_1} t\tau \, dx_1 \\
&= A \frac{E_{st}}{E} \frac{\partial H}{\partial y_1} - 2t \int_0^{x_1} \frac{\partial H}{\partial x_1} \, dx_1 \\
&= A \frac{E_{st}}{E} \frac{\partial H}{\partial y_1} - 2tH
\end{aligned} \tag{3}
$$

where P is the load in the stringer at $x_1 = 0$, A is the cross-sectional area of the stringer, t is the thickness of the sheet, and $H(0,0)$ has arbitrarily been chosen as zero.

Introduce dimensionless variables and parameters as follows:

$$
\left.
\begin{aligned}
x_1 &= \sqrt{\frac{E}{G}}\, bx & \sigma_x &= \sigma \frac{\partial \phi}{\partial y} \\
y_1 &= by & \tau &= -\sigma \sqrt{\frac{G}{E}} \frac{\partial \phi}{\partial x} \\
H &= \sigma b \phi & \overline{P} &= \frac{PE}{\sigma A E_{st}} \\
u &= -\frac{\sigma b}{\sqrt{EG}} \psi & \lambda &= \frac{2btE}{AE_{st}}
\end{aligned}
\right\} \tag{4}
$$

where σ is the direct stress in the sheet at infinity.

The following equations may now be obtained by eliminating σ_x and τ from equations (1) and (2):

$$
\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{5}
$$

which are the Cauchy-Riemann equations. It follows that

$$\phi + i\psi = \Phi(x + iy) = \Phi(z) \tag{6}$$

where Φ is an analytic function of the complex variable z.

Because of symmetry, a boundary-value problem may be formulated for the upper half-plane only. From equation (3)

$$\frac{\partial\phi}{\partial y} - \lambda\phi = \overline{P} \qquad (y = 0) \tag{7}$$

Since $\sigma_x = 0$ on the crack

$$\phi = 0 \qquad (x = 0,\ 0 \leqq y \leqq 1) \tag{8}$$

From symmetry, $u = 0$ on the line $(x = 0,\ y \geqq 1)$ and at the point $(x = 0,\ y = 0)$; therefore,

$$\psi = 0 \qquad \begin{pmatrix} x = 0,\ y \geqq 1); \\ x = 0,\ y = 0) \end{pmatrix} \tag{9}$$

Since $\sigma_x \longrightarrow \sigma$ and $\tau \longrightarrow 0$ at infinity,

$$\Phi \sim -iz \qquad (z \longrightarrow \infty) \tag{10}$$

Solution of Boundary-Value Problem

The boundary-value problem just formulated for Φ is of the mixed type. However, the problem may be reduced to the more familiar Dirichlet type by introducing a new unknown function F defined as follows:

$$\begin{aligned} F(z) &= \Phi' + i\lambda\Phi + i\overline{P} \\ &= \left(\frac{\partial\psi}{\partial y} - \lambda\psi\right) - i\left(\frac{\partial\phi}{\partial y} - \lambda\phi - \overline{P}\right) \end{aligned} \tag{11}$$

The boundary-value problem for F is as follows:

$$R(F) = 0 \qquad (x = 0,\ y > 1) \tag{12}$$

$$\left.\begin{aligned} I(F) &= 0 \qquad (y = 0) \\ I(F) &= \overline{P} \qquad (x = 0,\ y < 1) \end{aligned}\right\} \tag{13}$$

$$F \sim \lambda z \qquad (z \longrightarrow \infty) \tag{14}$$

The solution for F is found to be

$$F = \frac{2\overline{P}}{\pi} \log \frac{\sqrt{z^2 + 1} - 1}{z} + \lambda\sqrt{z^2 + 1} + \frac{C}{\sqrt{z^2 + 1}} \tag{15}$$

where the arbitrary constant C is real and the radical is positive on the positive real axis. From equation (11), it follows that

$$\Phi' + i\lambda\Phi = F - i\overline{P} \tag{16}$$

Solving for Φ with the use of the relationship $\Phi \sim -iz$ at infinity gives

$$\Phi = -\frac{\overline{P}}{\lambda} - i\sqrt{z^2 + 1} + e^{-i\lambda z} \int_{i\infty}^{z} \left(\frac{2\overline{P}}{\pi} \log \frac{\sqrt{\zeta^2 + 1} - 1}{\zeta} + \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}}\right) e^{i\lambda\zeta} d\zeta \tag{17}$$

Interpretation of C.- The stress concentration factor $\overline{P}$ for the stringer and the coefficient C are as yet unknown. Before proceeding to determine them as functions of λ, it is convenient to show that C itself is the ratio between the stress concentration factors for a cracked sheet with and without a stringer.

The solution to the problem of determining the complex stress function for the cracked sheet without a stringer may be obtained from equations (15) and (16) by letting $\lambda \longrightarrow \infty$, which is equivalent to letting $E_{st}A \longrightarrow 0$ since $\lambda = \frac{2btE}{E_{st}A}$. The result is

$$\Phi_o = -i\sqrt{z^2 + 1} \tag{18}$$

The stress field is determined from the derivative of the stress function, which in the neighborhood of $z = i$ (the tip of the crack) behaves as follows:

$$\Phi_o' \sim \frac{1}{\sqrt{2i(z - i)}} \qquad (z \longrightarrow i) \tag{19}$$

There is evidently a singularity at $z = i$. For the cracked sheet with a stringer,

$$\Phi' \sim \frac{C}{\sqrt{2i(z - i)}} \qquad (z \longrightarrow i) \tag{20}$$

as is evident from equation (16) since $\Phi(i) = 0$ from equations (8) and (9). On the line $(x = 0,\ y > 1)$, $\tau = 0$ and $\sigma_x = i\sigma\Phi'$. Thus, at the tip of the crack,

$$\frac{\sigma_x}{\sigma_{x,o}} = \frac{\Phi'}{\Phi_o'} \sim C \tag{21}$$

and C is evidently the required ratio.

Determination of C and $\overline{P}$.- The two conditions available for determining the two unknowns $\overline{P}$ and C as functions of λ are

$$\left.\begin{aligned} \Phi(0) &= 0 \\ \Phi(i) &= 0 \end{aligned}\right\} \tag{22}$$

which follow from equations (8) and (9). When applied to equation (17), these conditions yield

$$\Phi(0) = -\frac{\overline{P}}{\lambda} - i + \int_{i\infty}^{0}\left(\frac{2\overline{P}}{\pi}\log\frac{\sqrt{\zeta^2 + 1} - 1}{\zeta} + \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}}\right)e^{i\lambda\zeta}d\zeta = 0 \tag{23}$$

$$\Phi(i) = -\frac{\overline{P}}{\lambda} + e^{\lambda} \int_{i\infty}^{i} \left(\frac{2\overline{P}}{\pi} \log \frac{\sqrt{\zeta^2 + 1} - 1}{\zeta} + \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}} \right) e^{i\lambda\zeta} d\zeta = 0 \tag{24}$$

By use of equation (24), equation (23) becomes

$$-i + \frac{\overline{P}}{\lambda}\left(e^{-\lambda} - 1\right) + \int_{i}^{0} \left(\frac{2\overline{P}}{\pi} \log \frac{\sqrt{\zeta^2 + 1} - 1}{\zeta} + \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}} \right) e^{i\lambda\zeta} d\zeta = 0 \tag{25}$$

When the log term is integrated by parts, equations (25) and (24) become, respectively,

$$-i + \frac{2\overline{P}}{i\pi\lambda} \int_{i}^{0} \frac{1 - e^{i\lambda\zeta}}{\zeta\sqrt{\zeta^2 + 1}} d\zeta + \int_{i}^{0} \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}} e^{i\lambda\zeta} d\zeta = 0 \tag{26}$$

$$\frac{\overline{P}}{\lambda} + \frac{2\overline{P}}{i\pi\lambda} \int_{i}^{i\infty} \frac{1 - e^{i\lambda\zeta}}{\zeta\sqrt{\zeta^2 + 1}} d\zeta + \int_{i}^{i\infty} \frac{i\zeta + C}{\sqrt{\zeta^2 + 1}} e^{i\lambda\zeta} d\zeta = 0 \tag{27}$$

Next make the substitution $\zeta = i\eta$ to obtain

$$\frac{2\overline{P}}{\pi\lambda} \int_{0}^{1} \frac{1 - e^{-\lambda\eta}}{\eta\sqrt{1 - \eta^2}} d\eta - \int_{0}^{1} \frac{C - \eta}{\sqrt{1 - \eta^2}} e^{-\lambda\eta} d\eta = 1 \tag{28}$$

$$\frac{\overline{P}}{\lambda} - \frac{2\overline{P}}{\pi\lambda} \int_{1}^{\infty} \frac{1 - e^{-\lambda\eta}}{\eta\sqrt{\eta^2 - 1}} d\eta + \int_{1}^{\infty} \frac{C - \eta}{\sqrt{\eta^2 - 1}} e^{-\lambda\eta} d\eta = 0 \tag{29}$$

The definite integrals occurring in equations (28) and (29) are expressible in terms of known functions as follows:

$$\int_{1}^{\infty} \frac{1 - e^{-\lambda\eta}}{\eta\sqrt{\eta^2 - 1}} d\eta = \int_{0}^{\lambda} K_0(s) ds$$

$$\int_1^\infty \frac{e^{-\lambda\eta}}{\sqrt{\eta^2 - 1}}\, d\eta = K_0(\lambda)$$

$$\int_1^\infty \frac{\eta e^{-\lambda\eta}}{\sqrt{\eta^2 - 1}}\, d\eta = K_1(\lambda)$$

where K_0 and K_1 are modified Bessel functions of the second kind. Let

$$B(\lambda) = \int_0^1 \frac{1 - e^{-\lambda\eta}}{\eta\sqrt{1 - \eta^2}}\, d\eta \tag{30}$$

The function $B(\lambda)$ and its first two derivatives are expressible in terms of the Struve functions of imaginary argument L_0 and L_1 and modified Bessel functions of the first kind I_0 and I_1 as follows (see ref. 3):

$$\left.\begin{aligned}
B(\lambda) &= \frac{\pi}{2}\int_0^\lambda \left[I_0(s) - L_0(s)\right]ds \\
B'(\lambda) &= \int_0^1 \frac{e^{-\lambda\eta}}{\sqrt{1 - \eta^2}}\, d\eta = \frac{\pi}{2}\left[I_0(\lambda) - L_0(\lambda)\right] \\
B''(\lambda) &= -\int_0^1 \frac{\eta e^{-\lambda\eta}}{\sqrt{1 - \eta^2}}\, d\eta = -1 + \frac{\pi}{2}\left[I_1(\lambda) - L_1(\lambda)\right]
\end{aligned}\right\} \tag{31}$$

The first integrals of the modified Bessel functions can be expressed as follows:

$$\left.\begin{aligned}
\int_0^\lambda I_0(s)ds &= \lambda I_0(\lambda) + \frac{\pi\lambda}{2}\left[I_0(\lambda)L_1(\lambda) - I_1(\lambda)L_0(\lambda)\right] \\
\int_0^\lambda K_0(s)ds &= \lambda K_0(\lambda) + \frac{\pi\lambda}{2}\left[K_0(\lambda)L_1(\lambda) + K_1(\lambda)L_0(\lambda)\right]
\end{aligned}\right\} \tag{32}$$

Equations (28) and (29) may now be written

$$\frac{2\overline{P}}{\pi\lambda}B - B'C = 1 + B'' \tag{33}$$

$$\frac{2\overline{P}}{\pi\lambda}\int_{\lambda}^{\infty} K_0(s)ds + K_0C = K_1 \tag{34}$$

where, in equation (34), use has been made of the formula $\int_0^{\infty} K_0(s)ds = \frac{\pi}{2}$. Equations (33) and (34) may be solved for C and $\overline{P}$ to give the following results

$$C = \frac{BK_1 - (1 + B'')\int_{\lambda}^{\infty} K_0(s)ds}{BK_0 + B'\int_{\lambda}^{\infty} K_0(s)ds} \tag{35}$$

$$\overline{P} = \frac{\pi\lambda}{2}\,\frac{B'K_1 + (1 + B'')K_0}{BK_0 + B'\int_{\lambda}^{\infty} K_0(s)ds} \tag{36}$$

Solution for broken stringer.- For the case in which the stringer is broken at $x = 0$, the factor C must be replaced by C^* obtained from the solution to the boundary-value problem appropriate for the broken stringer. The solution to this problem is easily obtained from the one already given. It is only necessary to set $\overline{P} = 0$ and drop the requirement $\Phi(0) = 0$. The requirement $\Phi(i) = 0$ is retained and leads to equation (34) as before, except $\overline{P} = 0$. Thus,

$$C^* = \frac{K_1}{K_0} \tag{37}$$

Computation of results.- Tables of the modified Bessel functions are readily available. Values of the Struve functions L_0 and L_1 may be obtained from tables given in reference 4 for the range $0 \leqq \lambda \leqq 10$ at intervals of 0.1. No tables seemed to be available for $\int_0^{\lambda} L_0(s)ds$;

therefore, values of this function for values of $\lambda \leqq 2$ were computed from a power series. For values of λ between 2 and 6, it was more convenient to compute the function B by numerical integration from the formula

$$B(\lambda) = \int_0^{\pi/2} \left(1 - e^{-\lambda \sin \theta}\right) \frac{d\theta}{\sin \theta} \tag{38}$$

which was obtained from equation (30) by an obvious substitution. For values of $\lambda > 6$, the function B was computed from the asymptotic series

$$B(\lambda) \sim \gamma + \log 2\lambda - 2 \sum_1^{\infty} \frac{1}{n} \left[\frac{(2n - 1)!}{(n - 1)!}\right]^2 \frac{1}{(2\lambda)^{2n}} \tag{39}$$

which also was obtained from equation (30) by well-known methods ($\gamma = 0.57722$ is Euler's constant). For the same values of λ, the function B' and B" were computed from the derivatives of equation (39).

Results of the computations for C, $\overline{P}$, and C* for $\lambda \leqq 100$ are given in table I within slide-rule accuracy. The results are also plotted in figures 3 to 5. For values of $\lambda > 100$, the following asymptotic formulas give C, $\overline{P}$, and C* within slide-rule accuracy:

$$\left.\begin{aligned} C &\sim 1 - \frac{1}{\gamma + \log 2\lambda} \\ \overline{P} &\sim \frac{\pi}{2} \frac{\lambda + 0.875}{\gamma + \log 2\lambda} \\ C^* &\sim 1 + \frac{1}{2\lambda} \end{aligned}\right\} \tag{40}$$

NUMERICAL EXAMPLE

Consider a sheet 0.1 inch thick reinforced by a stringer made of the same material with an area of 0.5 square inch. A crack 6 inches long extends 3 inches on either side of the stringer. The effective radius of curvature ρ_e at the tip of the crack is taken to be 0.002 inch. According to a well-known formula, the theoretical stress concentration factor K_T at the tip of the crack is given by

$$K_T = 1 + 2\sqrt{\frac{b}{\rho_e}} = 1 + 2\sqrt{\frac{3}{0.002}} = 78.6$$

In the present example,

$$\lambda = \frac{2bt}{A} = \frac{2 \times 3 \times 0.1}{0.5} = 1.2$$

From table I,

$$C = 0.688$$

$$\overline{P} = 1.593$$

The corrected stress concentration factor K_T' at the tip of the crack is thus

$$K_T' = CK_T = 0.688 \times 78.6 = 54.1$$

The stress concentration factor in the stringer is $\overline{P} = 1.593$. The Neuber stress concentration factor K_N for the crack, taking size effect into account, is (see ref. 1)

$$K_N = \frac{1}{2}\left(1 + K_T'\right) = 27.5$$

In practical applications, of course, this large stress concentration factor is considerably reduced when corrected for the effect of plasticity. (See ref. 1 for details.)

DISCUSSION

Examination of figures 3 to 5 reveals at least two qualitative features of the results which are of practical interest. One is the appreciable stress concentration in the stringer and the other is the detrimental influence of the stringer once it has broken. These results

confirm intuition. The stringer is expected to carry part of the load refused by the sheet because of the crack. If then the stringer breaks, the two intact halves of the stringer carry load into the region of the sheet around the middle of the crack which tends to spread the crack more than if there were no stringer.

Because of the idealizations made in obtaining the theoretical solution, some caution should be observed in applying the results. In the analysis the stringer is assumed to be continuously attached to the sheet along a line. In reality the stringer has some finite width and may be attached to the sheet by means of rivets. Thus the theoretical results cannot be expected to be accurate for crack lengths shorter than two or three times the rivet spacing, or two or three times the width of an integral stiffener.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., December 23, 1957.

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2. Hildebrand, Francis B.: The Exact Solution of Shear-Lag Problems in Flat Panels and Box Beams Assumed Rigid in the Transverse Direction. NACA TN 894, 1943.

3. Watson, G. N.: A Treatise on the Theory of Bessel Functions. Second ed., The Macmillan Co., 1944, p. 329.

4. Anon.: Table of the Struve Functions $L_\nu(x)$ and $H_\nu(x)$. Jour. Math. and Phys., vol. XXV, no. 3, Oct. 1946, pp. 252-259.

TABLE I.- COMPUTED VALUES OF C, $\overline{P}$, AND C^*

λ	C	$\overline{P}$	C^*	λ	C	$\overline{P}$	C^*
0	0.637	1.000	∞	1.7	0.699	1.806	1.265
.1	.645	1.060	4.06	1.8	.701	1.848	1.251
.2	.652	1.115	2.73	1.9	.703	1.889	1.239
.3	.658	1.168	2.23	2.0	.704	1.930	1.228
.4	.662	1.219	1.960	3.0	.718	2.32	1.156
.5	.667	1.269	1.791	4.0	.729	2.69	1.119
.6	.670	1.318	1.676	5.0	.737	3.05	1.096
.7	.674	1.366	1.590	6.0	.744	3.40	1.080
.8	.677	1.413	1.524	7.0	.750	3.73	1.069
.9	.680	1.459	1.472	8.0	.755	4.06	1.061
1.0	.683	1.504	1.430	9.0	.759	4.38	1.054
1.1	.686	1.549	1.394	10.0	.763	4.70	1.049
1.2	.688	1.593	1.365	15.0	.777	6.21	1.033
1.3	.691	1.636	1.339	20.0	.787	7.64	1.025
1.4	.693	1.679	1.317	30.0	.800	10.35	1.017
1.5	.695	1.722	1.297	50.0	.816	15.40	1.010
1.6	.697	1.764	1.280	100.0	.834	27.0	1.005

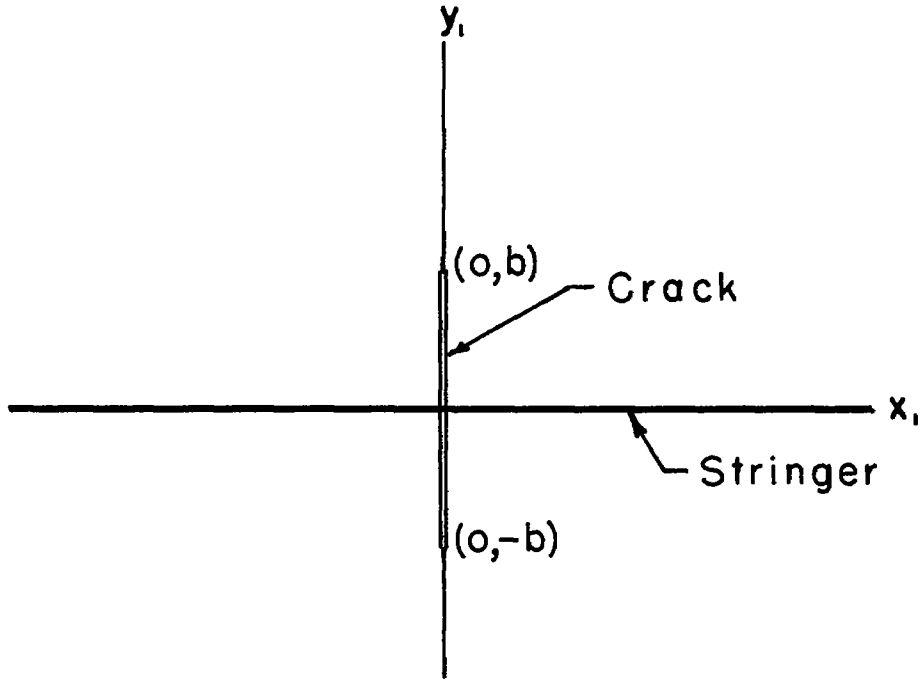

Figure 1.- Cracked sheet with a reinforcing stringer.

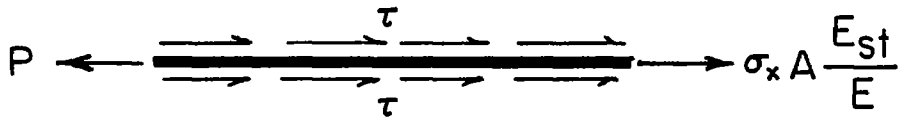

Figure 2.- Free-body diagram of a segment of the stringer.

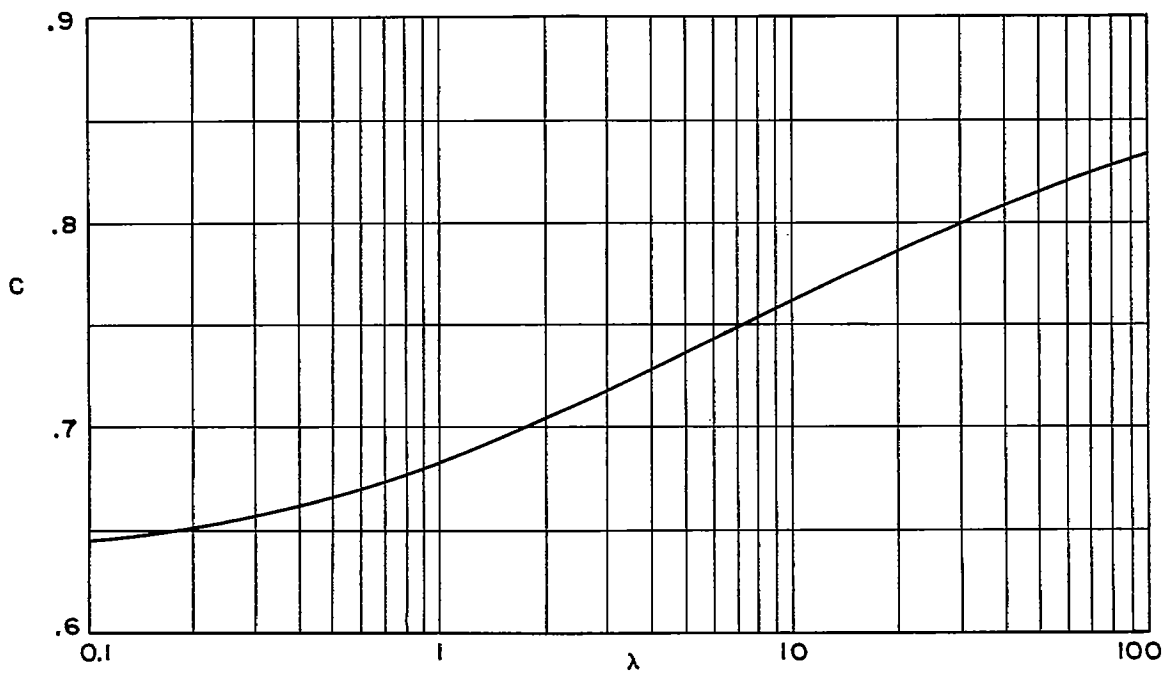

Figure 3.- Variation with λ of the ratio C of the stress concentration factors in a cracked sheet with and without a stringer for the case in which the stringer is intact. $\lambda = \frac{2btE}{AE_{st}}$.

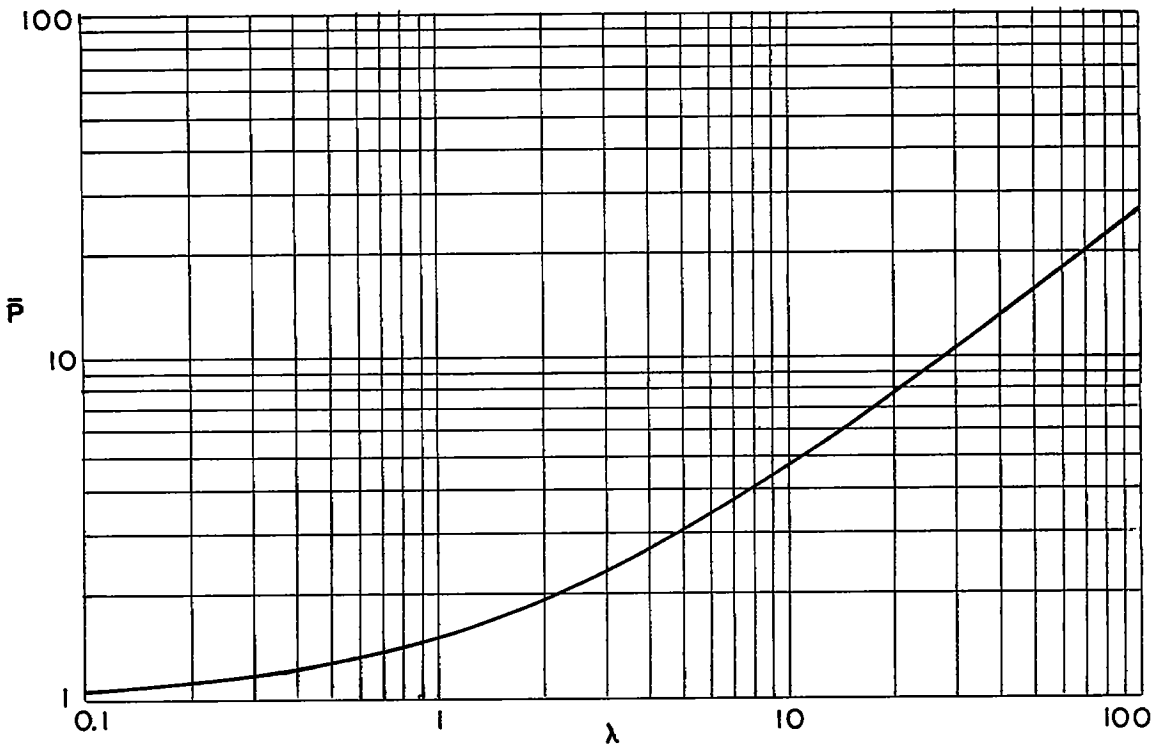

Figure 4.- Variation of the stringer stress concentration factor $\bar{P}$ with $\lambda = \frac{2btE}{AE_{st}}$.

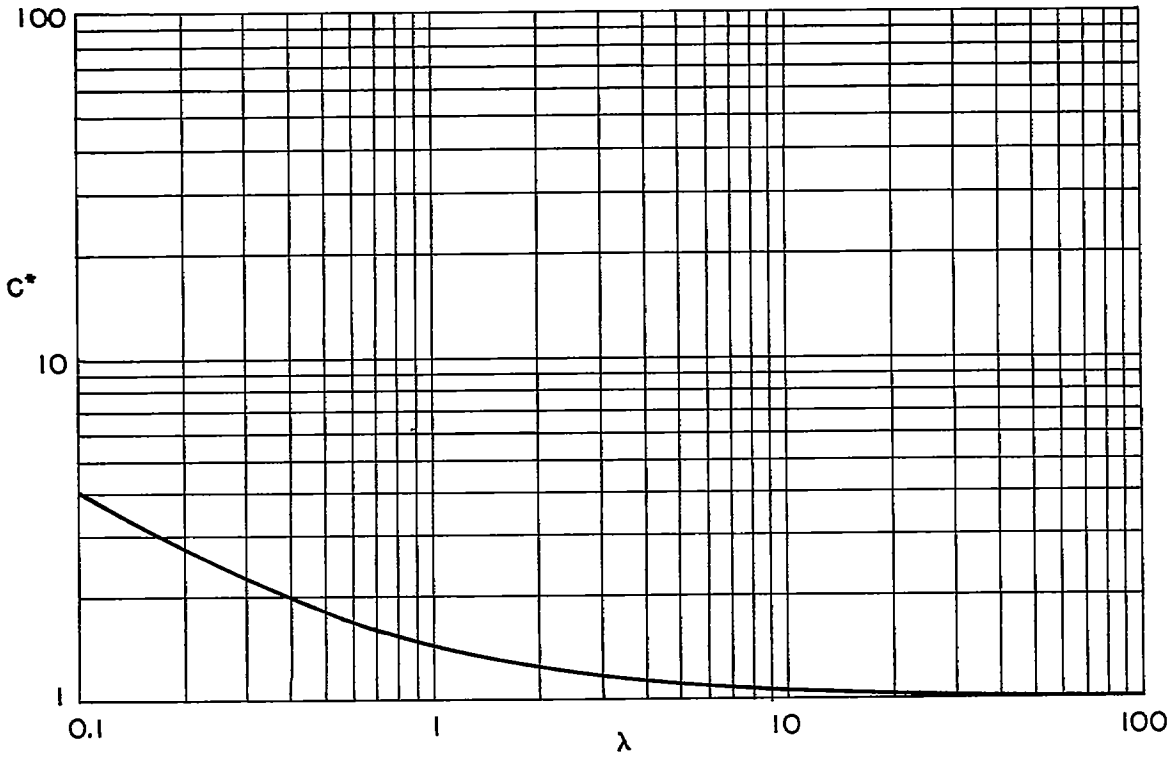

Figure 5.- Variation with λ of the ratio C^* of the stress concentration factors in a cracked sheet with and without a stringer for the case in which the stringer is broken. $\lambda = \frac{2btE}{AE_{st}}$.