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TECHNICAL NOTE 4376

ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF TEMPERATURE RECOVERY FACTORS FOR FULLY DEVELOPED FLOW OF AIR IN A TUBE

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SUMMARY

An analysis was made for predicting temperature recovery factors for fully developed flow in a tube. Most of the attention was confined to turbulent flow. Some qualitative results were obtained for laminar flow by setting the eddy diffusivity in the equation for turbulent flow equal to zero and using the incompressible parabolic velocity profile for laminar flow. For zero Mach number the laminar flow results were exact. Radial variation of properties was neglected in most of the calculations. The effect of wall temperature gradient along the tube was negligible for turbulent flow below Mach numbers of 0.9 and 0.98 for Reynolds numbers of 20,000 and 390,000, respectively. For laminar flow the effect became important at much lower Mach numbers.

Recovery factors were obtained experimentally for a range of Reynolds number from 630 to 30,000. Additional previously unpublished data are presented for Reynolds numbers up to 650,000. The results indicate that in the turbulent flow region the recovery factor is approximately independent of Reynolds number. In the transition region for Reynolds numbers between 2000 and 3000 the recovery factor is reduced abruptly to a value lower than that obtained for the turbulent flow region.

Comparison of the predicted recovery factor with experimental data indicated that the effective value of ratio of eddy diffusivities for heat transfer to momentum transfer varies from 1 at a Reynolds number of 5000 to 1.09 at a Reynolds number of 400,000. The ratio of eddy diffusivities was also obtained from measurements of heat-transfer coefficients and an analysis. Comparison of predicted with measured heat-transfer coefficients indicates effective eddy diffusivity ratios slightly less than those calculated from recovery factors.

The analysis substantiated the experimental result that correlations for heat transfer without frictional heating can be applied to the case with frictional heating by basing the heat-transfer coefficient on the difference between the wall temperature and the adiabatic wall temperature rather than on the difference between the wall and the fluid bulk temperature.

INTRODUCTION

Most of the work on temperature recovery factors has been confined to supersonic flow over external surfaces (ref. 1). Frictional heating at high supersonic Mach numbers, together with material limitations, necessitates accurate calculations of surface temperatures in this region.

A knowledge of recovery factors is also necessary for calculating convective heat transfer for flow in tubes at high subsonic Mach numbers and small temperature differences. This case was studied experimentally and reported in reference 2. The work was extended to supersonic flow in the study of reference 3. It is shown in reference 2 and by the analysis herein that, for turbulent flow, low-velocity heat-transfer correlations can be used for higher velocities if the heat-transfer coefficient is based on the difference between the wall and the adiabatic wall temperatures rather than on the difference between the wall and the fluid bulk temperatures. Calculation of the adiabatic wall temperature, or the temperature the wall would assume with no heat transfer, depends on a knowledge of the recovery factor.

The present experimental work covers a range of Reynolds numbers from 640 to 30,000. The values reported in reference 2 were obtained for a Reynolds number of approximately 30,000. Previously unpublished values obtained in connection with the work in reference 4 were measured for Reynolds numbers from 250,000 to 650,000 and are reported herein.

Very little analytical work has been done on fully developed recovery factors, although some analysis appears in reference 5. (Fully developed is herein taken to mean that the recovery factor is independent of distance from the tube inlet and that the velocity and total-temperature profiles are similar at various distances from the tube inlet.) In the present report, most of the attention is confined to fully developed subsonic turbulent flow. Some qualitative results are given for laminar flow by dropping the eddy diffusivity from the equations for turbulent flow.

In the turbulent case the recovery factor is quite sensitive to the ratio of eddy diffusivities for heat transfer to momentum transfer. The mean effective eddy diffusivity ratio at a cross section of the tube can be estimated by comparing the present analysis with the measured values of recovery factors reported herein.

SYMBOLS

C constant of integration

c_p specific heat of fluid at constant pressure

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D	inside tube diameter
h	heat-transfer coefficient for no frictional heating, $q_0/(t_0 - t_b)$
$h_{0,a}$	heat-transfer coefficient with frictional heating, $q_0/(t_0 - t_{0,a})$
k	thermal conductivity of fluid
n	constant, 0.124
p	static pressure
q	rate of heat transfer per unit area toward tube center
q_0	rate of heat transfer per unit area from inside wall toward center
R	gas constant
r	radius
r_0	inside tube radius
T	total temperature, $t + \frac{u^2}{2c_p}$, deg abs
T_b	bulk or average total temperature of fluid at cross section of tube, deg abs
$T_{b,a}$	bulk or average total temperature of fluid at cross section of tube with no heat transfer, deg abs
T_0,t_0	wall temperature, deg abs
$t_{0,a}$	adiabatic wall temperature, deg abs
t	static temperature, deg abs
t_a	static temperature with no heat transfer, deg abs
t_b	bulk or average static temperature of fluid at cross section of tube, deg abs
$t_{b,a}$	bulk or average static temperature of fluid at cross section of tube with no heat transfer, deg abs
u	velocity parallel to axis of tube
u_b	bulk or average velocity at cross section of tube

u_r velocity in radial direction

v velocity in negative radial direction

x distance along tube

y distance from tube wall

γ ratio of specific heats, taken as 1.40 for air

ϵ eddy diffusivity for momentum

ϵ_h eddy diffusivity for heat

$\varkappa$ Kármán constant, 0.36

μ viscosity

ν kinematic viscosity, μ/ρ

ρ density

ρ_b bulk or average density

τ shear stress in fluid

τ_0 shear stress at wall

Dimensionless Parameters:

α eddy diffusivity ratio, ϵ_h/ϵ

f friction factor, $-\dfrac{D(dp/dx)_{fr}}{2\rho u_b^2} = \dfrac{2\tau_0}{\rho u_b^2}$

M Mach number, $\dfrac{u_b}{\sqrt{\gamma R t_b}}$

Nu Nusselt number for no frictional heating, hD/k

$Nu_{0,a}$ Nusselt number with frictional heating, $h_{0,a}D/k$

Pr Prandtl number, $c_p\mu/k$

Re Reynolds number, $\rho u_b D/\mu$

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r_0^+ tube radius parameter, $\frac{\sqrt{\tau_0/\rho}\, r_0}{\nu}$

$T^{+\prime}$ total-temperature parameter for adiabatic case, $\frac{2(T_0 - T)c_p\rho}{\tau_0}$

$T_b^{+\prime}$ total bulk temperature parameter for adiabatic case, $\frac{2(T_0 - T_b)c_p\rho}{\tau_0}$

t_b^+ bulk temperature parameter, $\frac{(t_0 - t_b)c_p\tau_0}{q_0\sqrt{\tau_0/\rho}}$

$t_{0,a}^+$ adiabatic wall temperature parameter, $\frac{(t_0 - t_{0,a})c_p\tau_0}{q_0\sqrt{\tau_0/\rho}}$

u^+ velocity parameter, $u/\sqrt{\tau_0/\rho}$

u_b^+ bulk velocity parameter, $u_b/\sqrt{\tau_0/\rho}$

u_1^+ value of u^+ at y_1^+

y^+ wall distance parameter, $(\sqrt{\tau_0/\rho}\, y)/\nu$

y_1^+ value of y^+ at intersection of curves for flow close to wall and flow at distance from wall

φ recovery factor, $1 - \frac{T_b - T_0}{u_b^2/2c_p}$

Subscript:

fr on friction-pressure gradient

ANALYSIS

In order to calculate the recovery factors for flow through a tube, the temperature distribution in the tube will first be obtained from the energy equation. The energy equation for fully developed turbulent flow in a tube is derived in appendix A and can be written as

$$(r_0 - y)c_p\rho u \frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left[(r_0 - y)\left(k\frac{\partial t}{\partial y} - \rho c_p \overline{v't'} + \mu u \frac{\partial u}{\partial y} - \rho u \overline{u'v'}\right)\right] \quad (1)$$

where T is the total temperature and t the static temperature. The bars denote time averages, and the primes indicate fluctuating components. Equation (1) is the same as equation (A7) in appendix A if the bars over the time-averaged velocities, temperatures, and properties are dropped.

By introducing the eddy diffusivities for momentum and heat transfer, defined by

$$\overline{u'v'} \equiv -\epsilon \frac{\partial u}{\partial y}$$

$$\overline{t'v'} \equiv -\epsilon_h \frac{\partial t}{\partial y}$$

and setting

$$\tau = (\mu + \rho\epsilon) \frac{du}{dy} \tag{2}$$

equation (1) becomes

$$(r_0 - y)c_p\rho u \frac{\partial T}{\partial x} = \frac{\partial}{\partial y} \left\{(r_0 - y) \left[(k + \rho c_p a\epsilon) \frac{\partial t}{\partial y} + u\tau\right]\right\} \tag{3}$$

where a is the ratio of eddy diffusivities for heat transfer to momentum transfer.

The present section concerns the case where the wall is insulated, and hence

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 0 \tag{4}$$

If equation (3) is multiplied through by dy and integrated between the wall and the center of the tube, there results

$$c_p \frac{\partial}{\partial x} \left\{\int_0^{r_0} \rho u T(r_0 - y)dy\right\} = \int_0^{r_0} d\left\{(r_0 - y)\left[(k + \rho c_p a\epsilon) \frac{\partial t}{\partial y} + u\tau\right]\right\} \tag{5}$$

where ρu was regarded as independent of x from continuity. The right side of the equation is zero because the expression in brackets is zero at the insulated wall and at the center of the passage. The bulk temperature is defined

$$T_b \equiv \frac{\int_0^{r_0} \rho u T(r_0 - y)dy}{\int_0^{r_0} \rho u(r_0 - y)dy} \tag{6}$$

for constant specific heat. The numerator and denominator in equation (6) are independent of x by equation (5) and continuity. Therefore

$$\frac{dT_b}{dx} = 0 \tag{7}$$

This result is used in the next paragraph. Equation (7) can, of course, be obtained by other methods, but it is of interest that it follows from equation (3). Although $dT_b/dx = 0$ by equation (7), $dT/dx \neq 0$ at some radii, so that the left side of equation (1) will be retained and an estimate made of its importance.

It is assumed herein that the total-temperature profile is fully developed, or

$$\frac{T_0 - T}{T_0 - T_b} = f(y) \tag{8}$$

Differentiation of equation (8) with respect to x gives

$$\frac{dT}{dx} = \frac{dT_0}{dx}\left(1 - \frac{T_0 - T}{T_0 - T_b}\right) \tag{9}$$

where equation (7) was used.

Dimensionless Form of Energy Equation

In the remainder of the analysis the effect of radial variation of properties is neglected. It is shown in reference 6 that the effect of radial variation of properties on the turbulent velocity profile is negligible for adiabatic flow at subsonic Mach numbers. It is reasonable to assume that the same result holds for turbulent temperature profiles and recovery factors.

If equation (9) is substituted into equation (3) and dimensionless quantities are introduced, the following equation results

$$(r_0^+ - y^+)u^+\lambda\left(1 - \frac{T^{+\prime}}{T_b^{+\prime}}\right) = \frac{\partial}{\partial y^+}\left\{(r_0^+ - y^+)\left[-\frac{1}{2}\left(\frac{1}{Pr} + a\frac{\epsilon}{\nu}\right)\frac{dT^{+\prime}}{dy^+} - \frac{\frac{1}{Pr} + a\frac{\epsilon}{\nu}}{1 + \frac{\epsilon}{\nu}}u^+\frac{\tau}{\tau_0} + u^+\frac{\tau}{\tau_0}\right]\right\} \tag{10}$$

where du/dy was eliminated by equation (2) and

$$\lambda \equiv \frac{\frac{\partial T_0}{\partial x}c_p\mu}{\tau_0\sqrt{\tau_0/\rho}} \tag{11}$$

In obtaining equation (10), the static temperature was eliminated by using the relation

$$T = t + \frac{u^2}{2c_p} \tag{12}$$

An estimate of λ is made in appendix B by using one-dimensional flow theory. One-dimensional flow is a reasonable assumption for turbulent flow because the velocity and temperature profiles for that case tend to be uniform over most of the cross section. The resulting expression for λ is

$$\lambda = \frac{2T_b^{+\prime}}{r_0^+u_b^{+2}}\gamma\frac{M^2}{1 - M^2} \tag{13}$$

For the quantity τ/τ_0 in equation (10), the incompressible relation

$$\frac{\tau}{\tau_0} = 1 - \frac{y}{r_0} \tag{14}$$

will be used. It is shown in reference 6 that the velocity profile in turbulent flow is insensitive to the way in which the shear stress varies with radius. It would be expected that the same result would hold for recovery factors.

The use of equation (14) for compressible turbulent flow can also be justified by consideration of the momentum equation (eq. (A1)). Equation (A1) can be used directly for fully developed turbulent flow by replacing μ by $\mu + \rho\epsilon$ and letting $u_r = 0$. For turbulent flow, where the velocity profile tends to be flat, $u\,\partial u/\partial x \approx u_b\,\partial u_b/\partial x$. If the

pressure gradient is separated into the momentum pressure gradient $-\rho u_b \, \partial u_b/\partial x$ and the friction pressure gradient $-2\tau_0/r_0$, and equation (2) is introduced into equation (A1), equation (14) results on integration.

Substituting equations (13) and (14) into (10) and integrating twice gives

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$$T^{+\prime} = \int_0^{y^+} \frac{\frac{u^+}{r_0^+}(r_0^+ - y^+)^2 \left(1 - \frac{\frac{1}{Pr} + a\frac{\epsilon}{\nu}}{1 + \frac{\epsilon}{\nu}}\right) - \frac{2\gamma}{r_0^+ u_b^{+2}} \frac{M^2}{1 - M^2} \int_0^{y^+} u^+(T_b^{+\prime} - T^{+\prime})(r_0^+ - y^+)dy^+}{\frac{1}{2}(r_0^+ - y^+)\left(\frac{1}{Pr} + a\frac{\epsilon}{\nu}\right)} dy^+ \quad (15)$$

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where

$$T_b^{+\prime} = \frac{\int_0^{r_0^+} T^{+\prime} u^+(r_0^+ - y^+)dy^+}{\int_0^{r_0^+} u^+(r_0^+ - y^+)dy^+} \quad (16)$$

and

$$u_b^+ = \frac{2}{r_0^{+2}} \int_0^{r_0^+} u^+(r_0^+ - y^+)dy^+ \quad (17)$$

Expressions for Eddy Diffusivity

The eddy diffusivity ϵ in equation (15) is obtained from the following expressions, which are experimentally verified in references 4 and 7: For flow close to the wall $(y^+ < 26)$

$$\epsilon = n^2 uy\left(1 - e^{-n^2 uy/\nu}\right) \quad (18)$$

For flow in the region away from the wall $(y^+ > 26)$ the Kármán relation

$$\epsilon = \frac{\varkappa^2 (du/dy)^3}{(d^2u/dy^2)^2} \quad (19)$$

is used, where n and $\varkappa$ are experimental constants having the values 0.124 and 0.36, respectively.

By integration of equation (2) for the region away from the wall with viscous shear stress neglected, and using equations (14) and (19), von Kármán obtained the following dimensionless equation (ref. 8):

$$u^+ = \frac{1}{\varkappa}\left[\sqrt{1 - \frac{y^+}{r_0^+}} + \log_e\left(1 - \sqrt{1 - \frac{y^+}{r_0^+}}\right)\right] + C \tag{20}$$

Equation (19) becomes, on substitution of the first and second derivatives from equation (20),

$$\frac{\epsilon}{\nu} = 2\varkappa r_0^+\left(1 - \frac{y^+}{r_0^+}\right)\left(1 - \sqrt{1 - \frac{y^+}{r_0^+}}\right) \tag{21}$$

for flow in the region away from the wall.

For flow close to the wall, equation (18) can be written in dimensionless form as

$$\frac{\epsilon}{\nu} = n^2u^+y^+\left(1 - e^{-n^2u^+y^+}\right) \tag{22}$$

where the values of u^+ are obtained from the generalized velocity distribution in figure 1.

Values of $T^{+'}$ as a function of y^+ for various values of r_0^+ and Mach number can now be calculated from equation (15).

Temperature Recovery Factors

The definition of recovery factor usually used in experimental work (ref. 2), and which is used here, can be written

$$\varphi \equiv 1 - \frac{T_b - T_0}{u_b^2/2c_p} \tag{23}$$

An alternate definition which is sometimes used is

$$\varphi \equiv \frac{t_0 - t_b}{u_b^2/2c_p} \tag{24}$$

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However, equations (23) and (24) are strictly equivalent only for a uniform velocity distribution, inasmuch as

$$T_b = t_b + \frac{\alpha u_b^2}{2c_p} \tag{25}$$

where α is a factor accounting for the nonuniformity of the velocity profile, having a value of 1 for a uniform profile. Inasmuch as total, rather than static, bulk temperatures are measured in the present experiments, the use of equation (23) is to be preferred.

Equation (23) can be written in terms of readily calculated quantities as

$$\varphi = 1 + \frac{T_b^{+\prime}}{u_b^{+2}} \tag{26}$$

where $T_b^{+\prime}$ and u_b^+, for a given r_0^+ and M, are obtained from equations (15), (16), and (17) and the u^+ against y^+ plot in figure 1, obtained from reference 7. If it is desired to obtain φ as a function of the usual Reynolds number rather than of r_0^+, the Reynolds number can be calculated from

$$Re = 2u_b^+ r_0^+ \tag{27}$$

External Heat Transfer

For calculating the external heat transfer for flow through a tube at high velocities, the energy equation (3) can be written as

$$(r_0 - y)c_p\rho u \frac{\partial T}{\partial x} = -\frac{\partial}{\partial y}\left[(r_0 - y)q\right] \tag{28}$$

where

$$q \equiv -(k + \rho c_p \mathcal{a}\epsilon)\frac{dt}{dy} - u\tau \tag{29}$$

For the case of no external heat transfer from the wall, it is shown in the section Effect of Wall Temperature Gradient on Recovery Factor that the left side of equation (3) can be neglected except at very high subsonic Mach numbers. Equation (28) then shows that $q \approx 0$ at all radii. For no external heat transfer equation (29) therefore becomes

$$0 = -(k + \rho c_p \mathcal{a}\epsilon)\frac{dt_a}{dy} - u\tau \tag{30}$$

where $u\tau$ is assumed to be unaffected by heat transfer (small heat transfer). Subtracting equation (30) from equation (29) gives

$$q = -(k + \rho c_p \mathcal{A}\epsilon) \frac{d(t - t_a)}{dy} \tag{31}$$

Integrating equation (31) from the wall to y gives

$$t_a - t + t_0 - t_{0,a} = \int_0^y \frac{q}{(k + \rho c_p \mathcal{A}\epsilon)} dy \tag{32}$$

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Multiplying equation (32) by $(r_0 - y)u\,dy$, integrating from the wall to the tube center, and dividing by $\int_0^{r_0} u(r_0 - y)dy$ give

$$t_{b,a} - t_b + t_0 - t_{0,a} = \frac{\int_0^{r_0} u(r_0 - y)\left[\int_0^y \frac{q}{(k + \rho c_p \mathcal{A}\epsilon)} dy\right] dy}{\int_0^{r_0} u(r_0 - y)dy} \tag{33}$$

Setting $t_b = t_{b,a}$, or $T_b = T_{b,a}$, since the heat transfer is assumed to be small and thus the effect of heat transfer on the velocity profile can be neglected, results in

$$t_0 - t_{0,a} = \frac{\int_0^{r_0} u(r_0 - y)\left[\int_0^y \frac{q}{(k + \rho c_p \mathcal{A}\epsilon)} dy\right] dy}{\int_0^{r_0} u(r_0 - y)dy} \tag{34}$$

The quantity t_b was set equal to $t_{b,a}$ because the wall temperature is compared to the adiabatic wall temperature for the same bulk temperature.

In dimensionless form equation (34) becomes

$$t_{0,a}^{+} = \frac{\int_0^{r_0^+} u^+(r_0^+ - y^+)\left[\int_0^{y^+} \frac{q/q_0 \, dy^+}{\frac{1}{Pr} + \frac{a\epsilon}{\nu}}\right] dy^+}{\int_0^{r_0^+} u^+(r_0^+ - y^+)dy^+} \tag{35}$$

The exact variation of q/q_0 in equation (35) is not important for turbulent flow (ref. 4), except possibly at very low Reynolds numbers. For $\partial T/\partial x$ independent of y, equation (28) can be integrated first, from the wall to a point in the fluid, and then from the wall to the center of the tube. Eliminating $\partial T/\partial x$ between the two equations and converting the result to dimensionless form give

$$\frac{q}{q_0} = \frac{r_0^+}{r_0^+ - y^+} - \frac{r_0^+}{r_0^+ - y^+} \frac{\int_0^{y^+} u^+(r_0^+ - y^+)dy^+}{\int_0^{r_0^+} u^+(r_0^+ - y^+)dy^+} \tag{36}$$

Equation (36) is the same as the equation for no frictional heating obtained in reference 8. Note that q as used herein contains a contribution from dissipation (eq. (29)).

The Nusselt number is given by

$$Nu_{0,a} = \frac{2r_0^+ Pr}{t_{0,a}^+} \tag{37}$$

Equation (37) follows from the definitions of the various quantities involved.

If in equation (29) the term $u\tau$ had been neglected, that is, if frictional heating had been neglected in the analysis, $t_{0,a}^+$ in equation (35) would have been replaced by t_b^+, and the equation for Nusselt number would become

$$Nu = \frac{2r_0^+ Pr}{t_b^+} \tag{38}$$

Thus, the result for no frictional heating is identical to that for frictional heating if the bulk temperature in the definition of the heat-transfer coefficient is replaced by the adiabatic wall temperature. This is the result which was found experimentally in reference 2.

APPARATUS AND PROCEDURE

The present apparatus was designed to determine the effect on recovery factor of Reynolds numbers less than 30,000. Air at subatmospheric pressure was passed through a thin-walled tube test section.

Arrangement of Apparatus

The general arrangement of the apparatus is shown in figure 2. Air at room temperature and pressure first enters a temperature stabilizing box with a volume of 2 cubic feet packed with steel wool. Then it flows in turn through a throttling valve, a rotameter, an inlet mixing tank in which the total temperature and pressure of the air are measured, the test section, an outlet mixing tank in which the outlet total temperature is measured, a second throttling valve, an exhaust tank, and the laboratory altitude exhaust system. The density and velocity in the test section were controlled by the throttling valves.

The mixing tanks were thermally insulated by a layer of glass wool enclosed in aluminum foil. The test section was thermally insulated by a layer of glass wool enclosed in a thin steel cylinder 3 inches in diameter. The entire apparatus was enclosed in a tent to minimize the effects of fluctuating room air temperature and currents produced by the room air conditioner.

Test Section

A diagram of the test section is shown in figure 2. The test section was the middle portion of an Inconel tube 51 inches long with an inside diameter of 0.222 inch and a wall thickness of 0.014 inch. The distance between the first and last pressure tap was 44.4 inches and comprised the test section proper; thus the approach and exit sections were short (3.3 in.). The test-section tubing was fastened to the mixing tanks by means of 1-inch cubical Lucite blocks which minimized the conduction of heat at each end of the test section.

Static pressure was measured at the entrance and exit of the test section by means of pressure taps located in Lucite blocks.

The thermocouples consisted of 32-gage iron-constantan wire, first butt-welded to form the junction and then spot-welded to the test section. Location of the thermocouples is shown in figure 2. The wall temperature thermocouples were connected differentially with the inlet total-air temperature thermocouples, and the differential temperature was measured by means of a self-balancing potentiometer modified to give full-scale deflection at 1 millivolt.

Procedure

Before each run the reading on the potentiometer for no flow through the test section was checked to obtain the zero reading. Then the airflow rate and pressure level in the test section were set at the desired values. For high flow rates temperature equilibrium was usually reached in a few minutes, and all readings were recorded. For low flow rates temperature equilibrium was difficult to discern, and readings were recorded shortly after the difference between the inlet total temperature and the wall temperature had reached a maximum value. Readings were thereafter recorded whenever the temperature difference reached a maximum or minimum value for a 2- to 3-hour period.

Measured values of static pressure at the inlet of the test section, static-pressure drop along the test section, airflow rate, inlet total temperature of the air measured in the inlet mixing tank, and temperature difference between the inlet total temperature and the adiabatic wall temperature at the exit of the test section are given in table I.

RESULTS AND DISCUSSION

Experimental Results

Values of temperature recovery factor as defined by equation (23), average friction factor, and Reynolds number obtained in the present investigation are given in table I. The recovery factor is based on the measured temperature difference between the total air temperature in the inlet tank and the wall temperature at the exit of the test section. For airflow rates corresponding to Reynolds numbers above 3000, the measured temperature difference was affected slightly by fluctuations in the room air temperature of 1° or 2° F. The total air temperature thermocouple responded faster to changes in the air temperature than the wall temperature of the test section; hence, an increase in room air temperature resulted in a maximum difference between the total air temperature and the wall temperature, whereas a decrease in the room air temperature resulted in a minimum temperature difference. The room air temperature variations were caused by the cyclic operation of the room air-conditioning unit and were usually repetitive at half-hour intervals. The time required to reach equilibrium was only a few minutes at the higher flow rates, and hence the minimum and maximum values of temperature difference deviated only slightly from the time-averaged value, and the recovery factor can be indicated by the numerical average of the minimum and maximum values (e.g., the last six runs).

A strictly adiabatic flow condition was difficult to obtain in the experimental work, since a temperature difference existed between the surrounding air in the room and the wall of the test section. Therefore,

calculations were made to determine the effect on recovery factor of heat transferred to the air in the test section from the surroundings, even though there was no indication by measurements that the total air temperature increased between the inlet and exit of the test section. The calculations indicated that this effect on recovery factor was negligible.

The estimated maximum possible error in calculating the recovery factor is as follows:

Reynolds number	Error, percent
Above 16,000	±2
4,000 to 16,000	±6

Below a Reynolds number of 4,000 the maximum possible error ranges from ±8 to ±26 percent, and 70 percent of these data have an error of less than ±20 percent. Approximately two-thirds of the total error can be attributed to the combined errors of measuring $T_b - T_0$ and converting millivolts to degrees Fahrenheit.

Values of the recovery factors given in table I are plotted against Reynolds number in figure 3. Included are values of recovery factor obtained in reference 2 and some previously unpublished data obtained at the NACA Lewis laboratory in conjunction with the heat-transfer investigation presented in reference 4.

In the turbulent flow region (Reynolds numbers above 3000) the temperature recovery factor is nearly independent of Reynolds number. The average value is 0.88, which agrees with values for flow parallel to flat plates.

For Reynolds numbers below 3000 in the transition region the recovery factor is abruptly reduced to values less than 0.8. The minimum and maximum values of recovery factor showed the greatest deviation for a Reynolds number of 2900. During this run the static-pressure drop along the test section increased slightly, the flow rate was maintained constant, and there was an attendant increase in the maximum and minimum values of the recovery factor, which indicated the possibility that the flow conditions changed from a laminar to a turbulent velocity profile near the exit of the test section.

In the laminar flow region the recovery factors increased with decreasing Reynolds numbers. Several check runs were made in this region, and the results were not as reproducible as those obtained in the turbulent flow region.

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In figure 4 the average friction coefficients based on the static-pressure drop along the test section are plotted against Reynolds number. The results indicate that the transition from laminar to turbulent flow occurred principally at Reynolds numbers between 2000 and 3000, although some deviation from the predicted curve for fully turbulent flow occurred at Reynolds numbers up to 6000.

Analytical Results

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The equations required for calculating recovery factors and heat-transfer coefficients were obtained in the ANALYSIS section.

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Effect of wall temperature gradient on recovery factor. - The presence of Mach number in equation (15) is due to the fact that the wall temperature gradient is not zero. The wall temperature gradient was written in terms of Mach number (see eqs. (10), (11), and (13)) by using one-dimensional flow theory, inasmuch as Mach number is a more familiar quantity and more readily interpreted. Equations (11) and (13) or equation (B6) indicates that the wall temperature gradient becomes indefinitely large as the Mach number approaches 1.

Turbulent recovery factor as a function of Mach number for several Reynolds numbers, a Prandtl number of 0.73, and a ratio of eddy diffusivities for heat transfer to momentum transfer of 1 are plotted in figure 5. These curves were calculated from equations (15), (16), (17), (21), (22), (26), and (27) and the u^+ against y^+ plot in figure 1. Except for the case $M = 0$, equation (15) must be solved by iteration for a given value of $T_b^{+'}$, inasmuch as $T^{+'}$ occurs on both sides of the equation. The correct value of $T_b^{+'}$ was obtained by trial and error. That is, it was necessary to try several values of $T_b^{+'}$ on the right side of equation (15) in order to find a value which would agree with that given by equation (16). The curves indicate that the effect of wall temperature gradient is negligible at Mach numbers below 0.9 for $Re = 20,000$ and below 0.98 for $Re = 390,000$. The curve for the higher Reynolds number is cut off at the point shown because computational difficulties were encountered at higher Mach numbers.

Plotted for comparison are laminar recovery factors obtained by letting the eddy diffusivity in the equation for turbulent recovery factor equal zero and using the incompressible parabolic velocity profile for laminar flow. These results for laminar flow are probably only qualitative inasmuch as the one-dimensional approximations made in the analysis are not accurate for laminar flow. It is of interest that the effect of wall temperature gradient becomes important at much lower Mach numbers than for turbulent flow. For $M = 0$ equation (15) can be integrated, and the expression for recovery factor becomes

$$\varphi = 2Pr - 1 \tag{39}$$

Equation (39) is exact because the one-dimensional approximations are not involved when $M = 0$.

Reference 5 states that $\varphi = 2Pr$ rather than $2Pr - 1$. The difference is apparently due to the fact that the recovery factor is defined in terms of static temperatures (eq. (24)) rather than in terms of total temperatures (eq. (23)) in reference 5. As mentioned previously, the two definitions would be equivalent only for $\alpha = 1$ (uniform velocity profile).

It should be mentioned that the results for the range of Mach numbers where the recovery factors deviate appreciably from their incompressible values may be open to question even for the turbulent case, because the recovery factor was assumed to be independent of distance along the tube. At high Mach numbers the Mach number varies along the tube, so that if the recovery factor also varies appreciably with Mach number, it will vary along the tube. The analysis may therefore be chiefly useful for indicating the range of Mach numbers for which the effect of wall temperature gradient is negligible. Fortunately, the results indicate a considerable range of subsonic Mach numbers over which this is true. In the remainder of the calculations the effect of wall temperature gradient or of Mach number on the recovery factor is neglected.

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Ratio of eddy diffusivities for heat transfer to momentum transfer from recovery factors. - The ratio of eddy diffusivities for heat transfer to momentum transfer a occurs in equation (15). The ANALYSIS, together with experimental data on recovery factors, may therefore provide a method for estimating the ratio of the eddy diffusivities.

In figure 6 the predicted recovery factors are plotted against Reynolds number for various values of a. The curves indicate that a has a considerable effect on the recovery factor, the latter decreasing as a increases. This can be explained by the fact that an increase in a, or in the eddy diffusivity for heat transfer, tends to make the static temperature more uniform and thus the total temperature less uniform, since the two temperatures vary in opposite directions (the static temperature decreases with distance from the wall, whereas the total temperature increases for recovery factors less than 1).

In figure 7 the predicted recovery factor curve for an a of 1.07 is compared with the measured values of recovery factors shown in figure 3. The predicted curve is in reasonable agreement with the data but indicates a greater variation with Reynolds number than the measured values of recovery factor.

Comparison of the curves for various values of a in figure 6 with the measured values indicates that the best agreement would be obtained by using a value of a that varies from about 1 at a Reynolds number of 5000 to 1.09 at a Reynolds number of 400,000. This is the trend

that is predicted in reference 8 from a consideration of the effect of conduction to an eddy as it moves transversely. It is predicted there that a increases with Peclet number (Pe = RePr).

It should be mentioned that the values of a obtained from recovery factors are mean effective values. The local values of a may vary with radius. More is said about this in the next section.

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Eddy diffusivity ratio from heat-transfer measurements. - The ratio of eddy diffusivities can also be obtained from measurements of heat-transfer coefficients and from the ANALYSIS, since equation (35) contains a. Nusselt number as a function of Reynolds number can be calculated from equations (35), (36), and (37), or (38), (17), and (25), and figures 1 and 8. The variation of q/q_0 with y^+ and r_0^+ shown in figure 8 was obtained in reference 4. For obtaining Nusselt number as a function of Reynolds number, the parameter r_0^+ is assigned arbitrary values.

Calculated Nusselt numbers for values of a of 1 and 1.07 are plotted in figure 9. Included for comparison are experimental data from reference 4. The data are in slightly better agreement with the curve for an a of 1, although the difference between the two curves may be within the experimental error of the data. The curves and the data indicate that, as in the case of recovery factors, the values of a at the low Reynolds numbers are lower than at the high Reynolds numbers.

The slightly lower effective values of a for heat transfer than for recovery factors might be explained by assuming that a increases with distance from the wall. In the case of heat transfer the temperature gradient at the wall is very large compared with gradients at other positions, and hence the effective value of a should be close to the value at the wall.

In the case of recovery factors, since the temperature gradient is zero at the wall, the effective value of a should correspond to the local value at some point away from the wall; that is, the effective value may be higher. The assumption that a increases with distance from the wall is supported by experiments reported in reference 9.

SUMMARY OF RESULTS

Measured values of temperature recovery factors for air flowing in a tube indicated that:

1. In the turbulent flow region for Reynolds numbers greater than 3000, the recovery factor is approximately independent of Reynolds number and has an average value of 0.88.

2. In the transition region for Reynolds numbers between 2000 and 3000 the recovery factor is reduced abruptly to a value lower than that obtained for the turbulent flow region.

The following results were obtained from the analysis of recovery factors for fully developed flow in a tube:

1. Wall temperature gradient along the tube has a negligible effect on the turbulent recovery factor for Mach numbers less than about 0.9. The effect of wall temperature gradient on laminar recovery factors becomes important at much lower Mach numbers.

2. Comparison of the predicted recovery factors with experimental data indicated that the effective value of ratio of eddy diffusivities for heat transfer to momentum transfer varies from about 1 at a Reynolds number of 5000 to about 1.09 at a Reynolds number of 400,000. The effective values of eddy diffusivity ratio appeared to be slightly less for heat-transfer coefficients.

3. The analysis substantiated the experimental result that correlations for heat transfer without frictional heating can be applied to frictional heating by basing the heat-transfer coefficient on the difference between the wall temperature and the adiabatic wall temperature (for the same bulk temperature), rather than on the difference between the wall and the fluid bulk temperatures.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, July 22, 1958

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APPENDIX A

DERIVATION OF TURBULENT ENERGY EQUATION

The method of derivation given here is essentially the same as that used in reference 10 for flow over a flat plate. The equations of momentum, continuity, and energy used herein for symmetrical flow in a tube can be written, respectively, as

$$\rho u \frac{\partial u}{\partial x} + \rho u_r \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\mu r \frac{\partial u}{\partial r}\right) - \frac{\partial p}{\partial x} \tag{A1}$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{1}{r} \frac{\partial(r\rho u_r)}{\partial r} = 0 \tag{A2}$$

$$\rho u c_p \frac{\partial t}{\partial x} + \mu u_r c_p \frac{\partial t}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial t}{\partial r}\right) + \mu\left(\frac{\partial u}{\partial r}\right)^2 + u \frac{\partial p}{\partial x} \tag{A3}$$

Time derivatives are neglected in these equations, as flow is assumed statistically steady. Multiplying equation (A1) through by u and adding the result to equation (A3) give

$$\rho u \frac{\partial}{\partial x} \left(c_p t + \frac{u^2}{2}\right) + \rho u_r \frac{\partial}{\partial r} \left(c_p t + \frac{u^2}{2}\right) = \frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial t}{\partial r} + r\mu u \frac{\partial u}{\partial r}\right) \tag{A4}$$

where c_p is considered constant. If the instantaneous quantities in equation (A4) are replaced by the sum of their time average and fluctuating components, and the equation of continuity is used, the following is obtained:

$$\bar{\rho}\bar{u} \frac{\partial}{\partial x} \left(c_p \bar{t} + \frac{\bar{u}^2}{2}\right) + \left(\bar{\rho}\bar{u}_r + \overline{\rho' u_r'}\right) \frac{\partial}{\partial r} \left(c_p \bar{t} + \frac{\bar{u}^2}{2}\right) = \frac{1}{r} \frac{\partial}{\partial r} \left[r\left(\bar{k} \frac{\partial \bar{t}}{\partial r} + \bar{\mu}\bar{u} \frac{\partial \bar{u}}{\partial r} - \bar{\rho} c_p \overline{u_r' t'} - \bar{\rho}\bar{u}\, \overline{u' u_r'}\right)\right] \tag{A5}$$

where the boundary-layer assumptions have been used and the fluctuations of the physical properties are assumed small compared with their mean values. But because

$$\bar{\rho}\bar{u}_r + \overline{\rho' u_r'} = \overline{\rho u_r}$$

equation (A5) becomes

$$\bar{\rho}\bar{u}\frac{\partial}{\partial x}\left(c_p\bar{t} + \frac{\bar{u}^2}{2}\right) + \overline{\rho u_r}\frac{\partial}{\partial r}\left(c_p\bar{t} + \frac{\bar{u}^2}{2}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\bar{k}\frac{\partial\bar{t}}{\partial r} + \bar{\mu}\bar{u}\frac{\partial\bar{u}}{\partial r} - \overline{\rho c_p}\,\overline{u_r't'} - \bar{\rho}\bar{u}\,\overline{u'u_r'}\right)\right] \tag{A6}$$

For fully developed flow there is no mean radial mass flow, or, $\overline{\rho u_r}$ is zero. If, in addition, r is replaced by $(r_0 - y)$, u_r by $-v$, and the left side of equation (A6) is written in terms of total temperature, it becomes

$$(r_0 - y)c_p\bar{\rho}\bar{u}\frac{\partial\bar{T}}{\partial x} = \frac{\partial}{\partial y}\left[(r_0 - y)\left(\bar{k}\frac{\partial\bar{t}}{\partial y} - \rho c_p\,\overline{v't'} + \bar{\mu}\bar{u}\frac{\partial\bar{u}}{\partial y} - \bar{\rho}\bar{u}\,\overline{u'v'}\right)\right] \tag{A7}$$

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APPENDIX B

CALCULATION OF dT_0/dx

The temperature recovery factor is given in terms of total bulk temperature as

$$\varphi = 1 - \frac{T_b - T_0}{u_b^2/2c_p} \tag{B1}$$

For fully developed flow the recovery factor is independent of x. Differentiating equation (B1) and using equation (7) give

$$\frac{dT_0}{dx} = -\frac{(T_b - T_0)}{u_b/2}\frac{du_b}{dx} \tag{B2}$$

In the present development one-dimensional flow is assumed; that is, the fact that the velocity and temperature vary with radial position is neglected. This is a reasonable assumption for turbulent flow. By using the continuity relation $w = \rho_b u_b A$, equation (B2) can be written as

$$\frac{dT_0}{dx} = \frac{2(T_b - T_0)}{\rho_b}\frac{d\rho_b}{dx} \tag{B3}$$

In order to obtain $d\rho_b/dx$ in terms of known quantities, the total-pressure drop is written as the sum of the momentum and the friction pressure drops, or

$$\frac{dp}{dx} = u_b^2\frac{d\rho_b}{dx} - \frac{2f}{D}\rho_b u_b^2 \tag{B4}$$

Differentiating the perfect gas law $p = \rho_b R t_b$ and the definition of total bulk temperature for one-dimensional flow ($\alpha = 1$)

$$T_b \equiv t_b + \frac{u_b^2}{2c_p}$$

and using equation (7) and continuity give on substitution into (B4)

$$\frac{d\rho_b}{dx} = \frac{f\rho_b u_b^2/r_0}{u_b^2 - Ru_b^2/c_p - Rt_b} \tag{B5}$$

Substituting equation (B5) into (B3) and using the perfect gas relation $c_p = R\gamma/(\gamma - 1)$ and the definition of Mach number give

$$\frac{dT_0}{dx} = -\frac{2(T_b - T_0)f}{r_0}\frac{\gamma M^2}{1 - M^2} \tag{B6}$$

or, using the definitions of λ, $T_b^{+'}$, and f, the resulting equation is

$$\lambda = \frac{2T_b^{+'}}{r_0^+ u_b^{+2}}\gamma\frac{M^2}{1 - M^2} \tag{B7}$$

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TABLE I. - EXPERIMENTAL DATA

Inlet static pressure, lb/sq ft abs	Static-pressure drop along test section, lb/sq ft abs	Total inlet air temperature, °R	Air-flow, lb/hr	Fluid bulk total temperature minus wall temperature, °R		Recovery factor		Average half friction factor	Reynolds number evaluated at bulk total temperature of fluid	Reynolds number evaluated at exit bulk static temperature of fluid
				Maximum	Minimum	Maximum	Minimum			
681	286	531	6.59	2.70	2.58	0.86	0.85	4.20×10^{-3}	10,300	10,600
553	193	531	4.94	1.73	1.57	.88	.86	4.43	7,720	7,860
460	122	531	3.57	1.02	.87	.89	.87	4.83	5,580	5,630
384	69.2	530	2.37	.50	.43	.89	.87	5.59	3,700	3,730
188	28.9	530	.81	.55	.55	.88	.88	6.5	1,300	1,300
221	33.2	530	1.45	1.12	1.02	.75	.73	4.17	2,270	2,290
264	44.6	530	1.64	.80	.72	.81	.79	5.18	2,560	2,580
326	59.4	531	2.06	.62	.60	.86	.85	5.35	3,230	3,250
138	28.9	531	.85	.73	.68	.84	.82	6.5	1,300	1,300
132	28.9	532	.83	.73	.68	.85	.84	6.4	1,300	1,300
168	30.6	531	1.05	.75	.72	.82	.82	5.45	1,650	1,660
163	30.6	530	1.04	.90	.88	.80	.79	5.31	1,630	1,650
207	32.4	530	1.36	1.17	.88	.79	.72	4.35	2,120	2,140
300	59.0	529	1.85	1.25	.86	.79	.90	6.13	2,900	2,920
216	55.9	529	1.85	.68	.68	.88	.83	5.69	2,890	2,910
225	33.4	530	1.47	1.15	1.15	.72	.72	4.16	2,300	2,310
214	32.7	530	1.43	.98	.88	.79	.77	4.06	2,230	2,250
216	32.6	530	1.47	1.28	1.23	.71	.70	3.06	2,310	2,320
162	30.6	530	1.07	1.17	.65	.86	.74	5.04	1,670	1,680
165	30.4	528	1.08	1.01	.95	.79	.77	5.00	1,700	1,710
185	28.5	529	.83	.92	.82	.81	.78	6.4	1,300	1,300
70.5	24.3	530	.40	.35	.22	.96	.93	11	630	630
297	54.4	530	1.83	.48	.48	.88	.88	5.67	2,870	2,890
1668	753	530	18.8	3.72	3.53	.87	.86	3.10	29,500	30,700
1657	736	531	18.9	3.67	3.62	.86	.8	3.00	29,600	30,700
1416	584	530	15.4	3.15	3.11	.86	.86	3.20	24,100	24,900
1247	465	530	12.9	2.68	2.64	.87	.87	3.18	20,200	20,800
1052	325	530	9.86	1.83	1.78	.86	.85	3.62	15,400	15,700

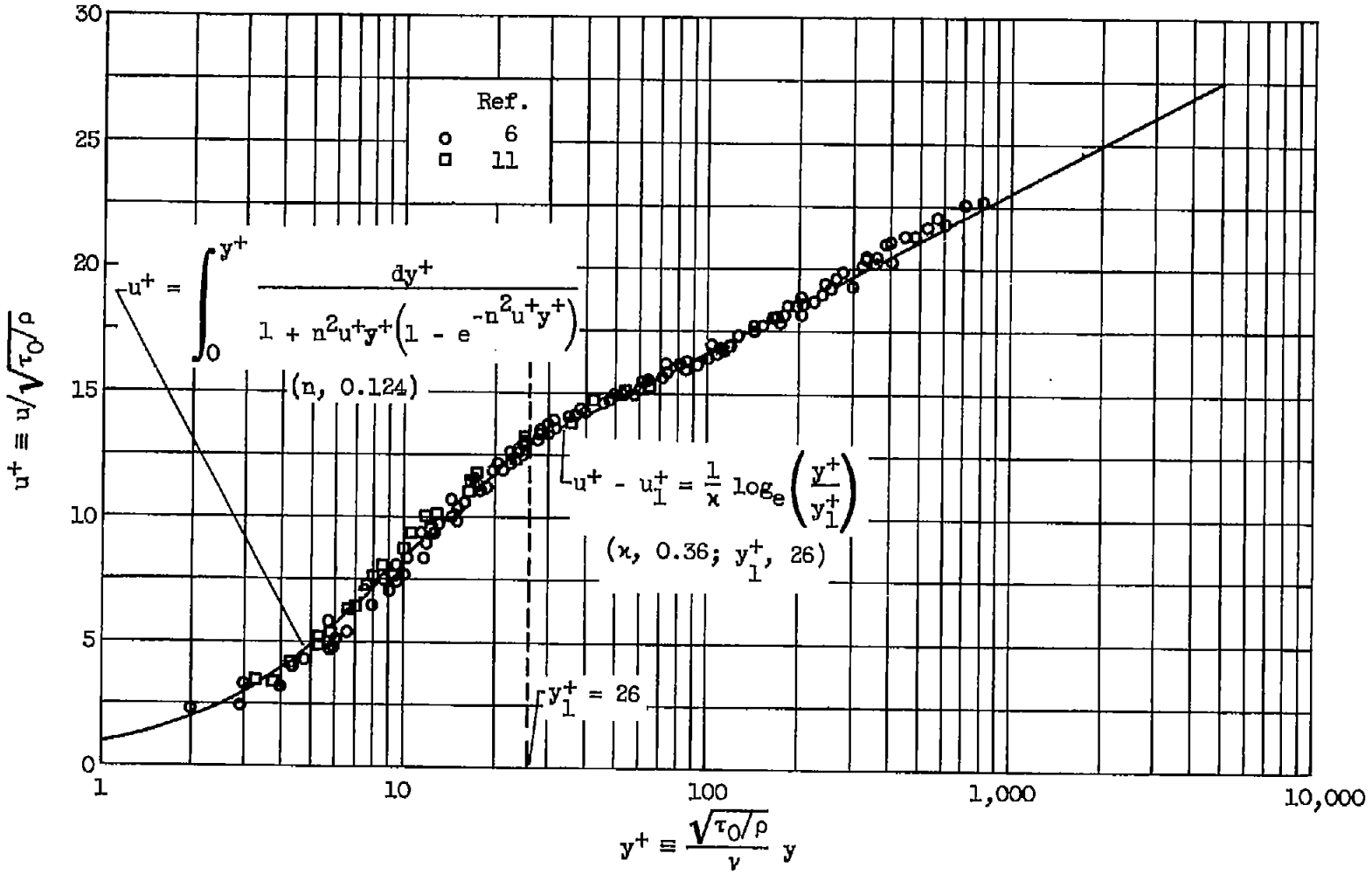
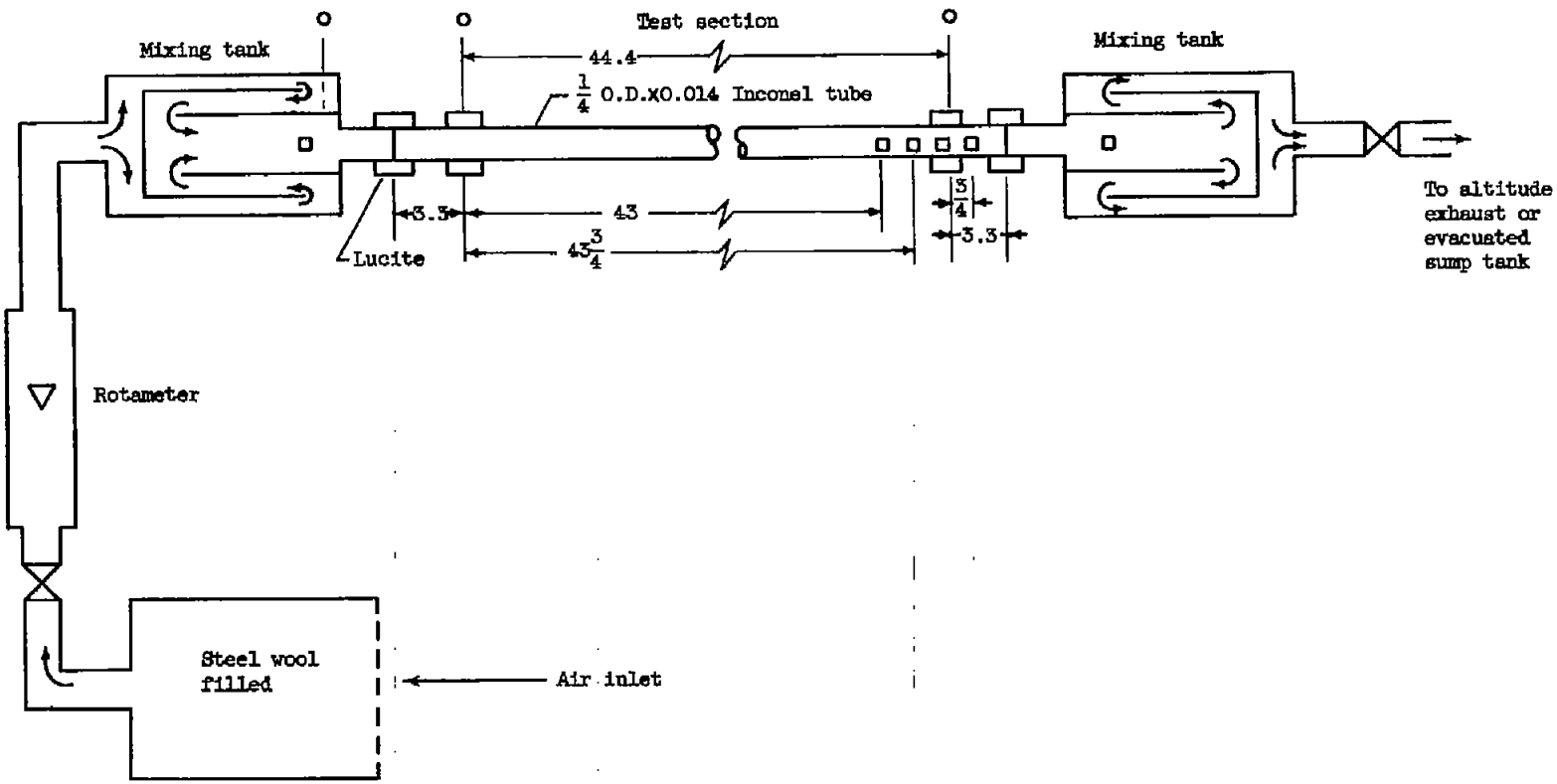


Figure 1. - Generalized velocity distribution for adiabatic turbulent flow.



○ Pressure tap location
□ Thermocouple location

Figure 2. - Arrangement of apparatus and design of test section (dimensions in inches).

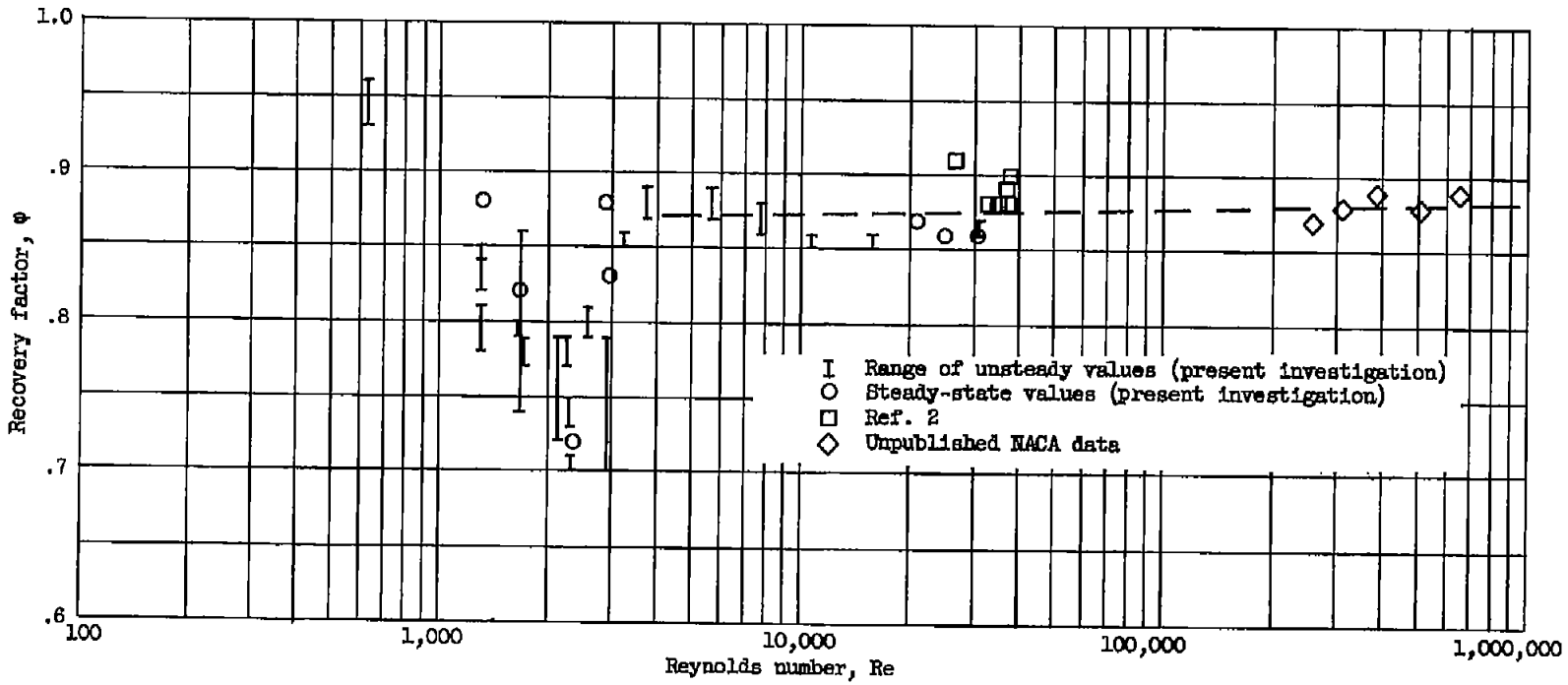


Figure 3. - Variation of recovery factor at tube exit with Reynolds number.

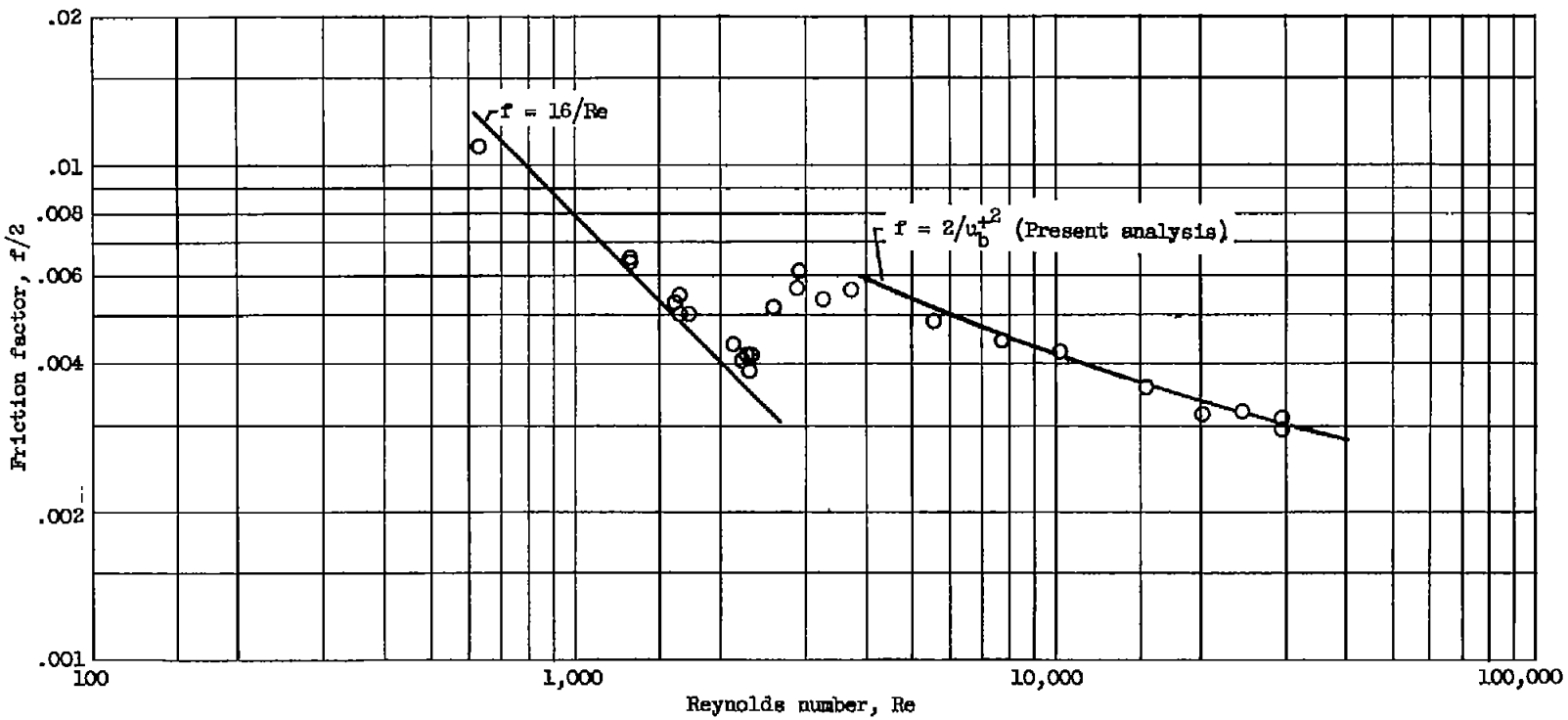


Figure 4. - Correlation of isothermal average friction factor with Reynolds number evaluated at bulk total temperature of fluid.

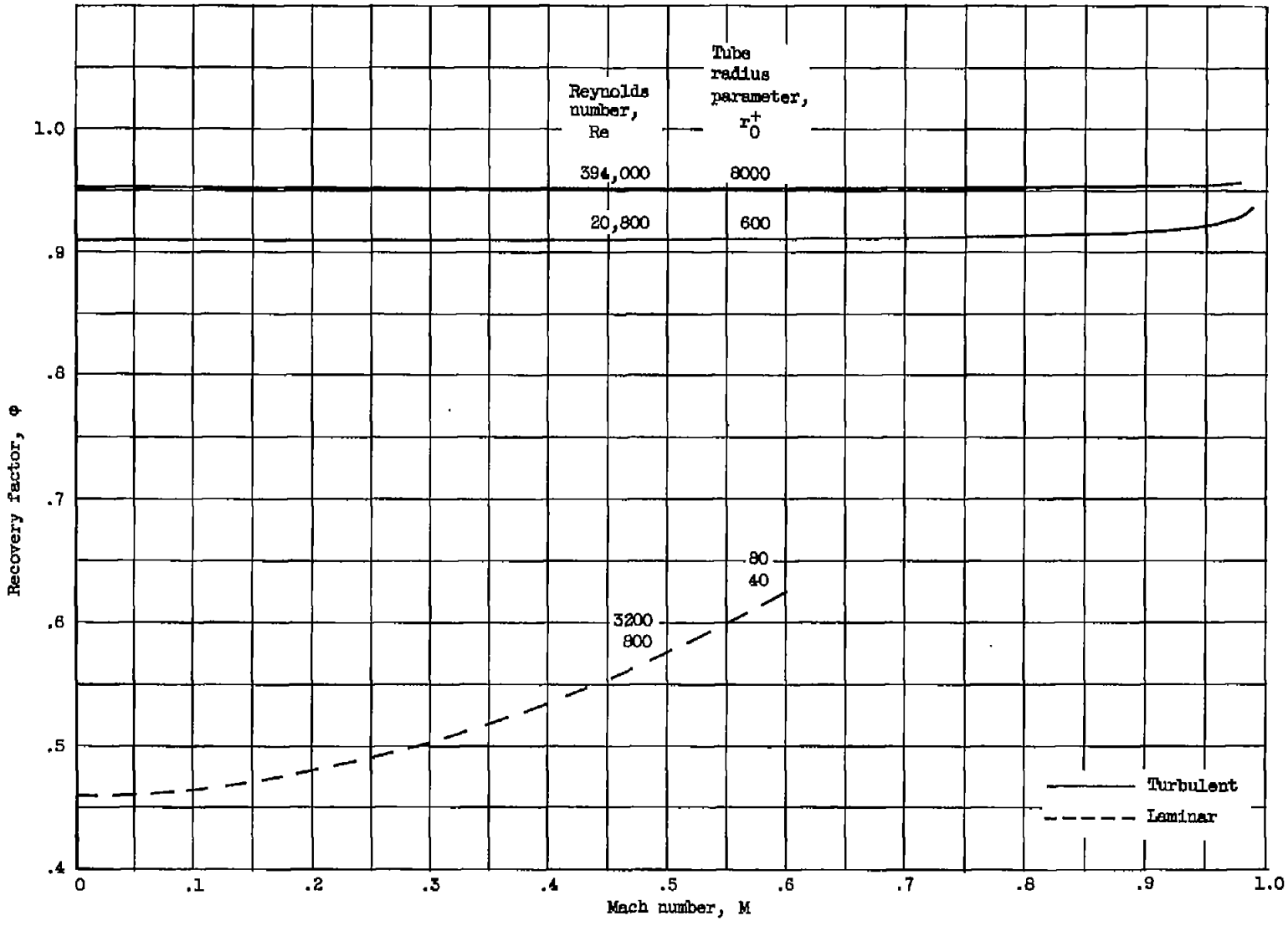


Figure 5. - Variation of recovery factor with Mach number at Prandtl number of 0.73 and several Reynolds numbers. Eddy diffusivity ratio, 1.0.

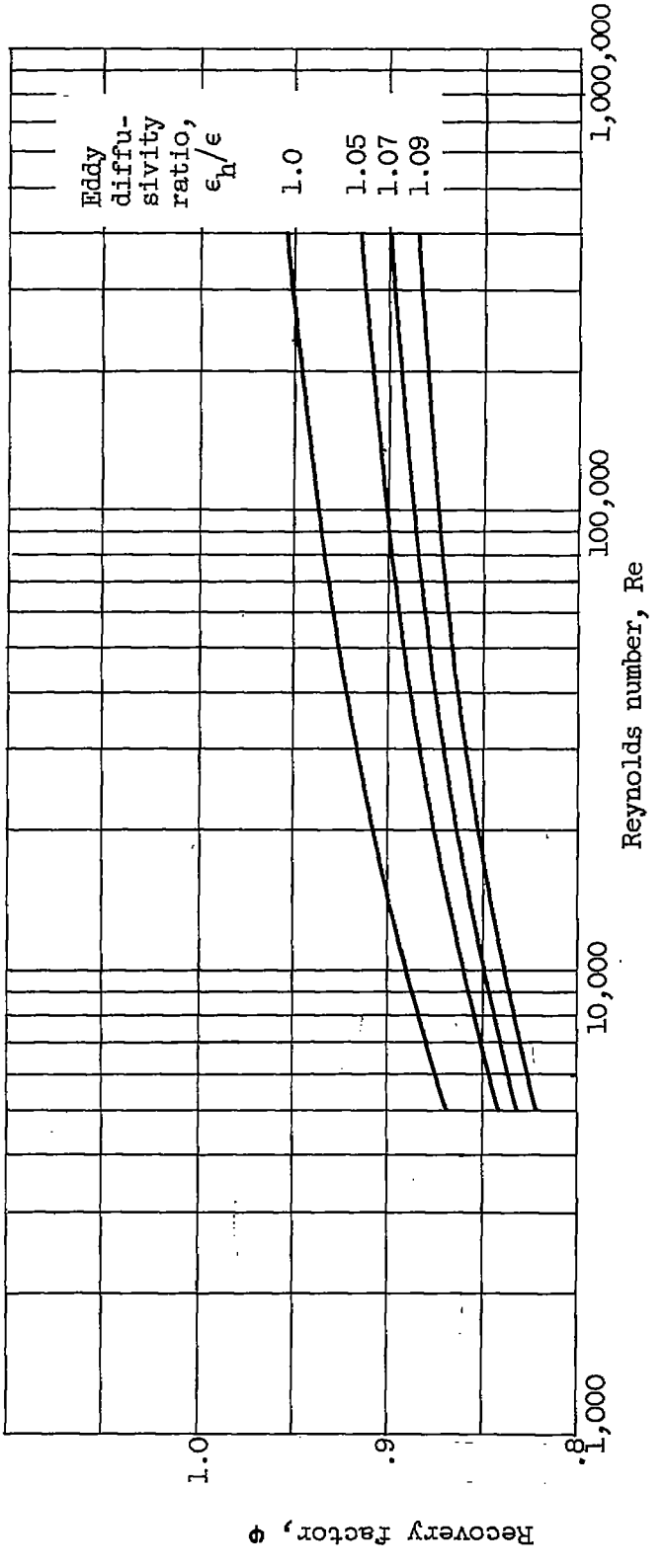


Figure 6. - Variation of recovery factor with Reynolds number for various eddy diffusivity ratios.

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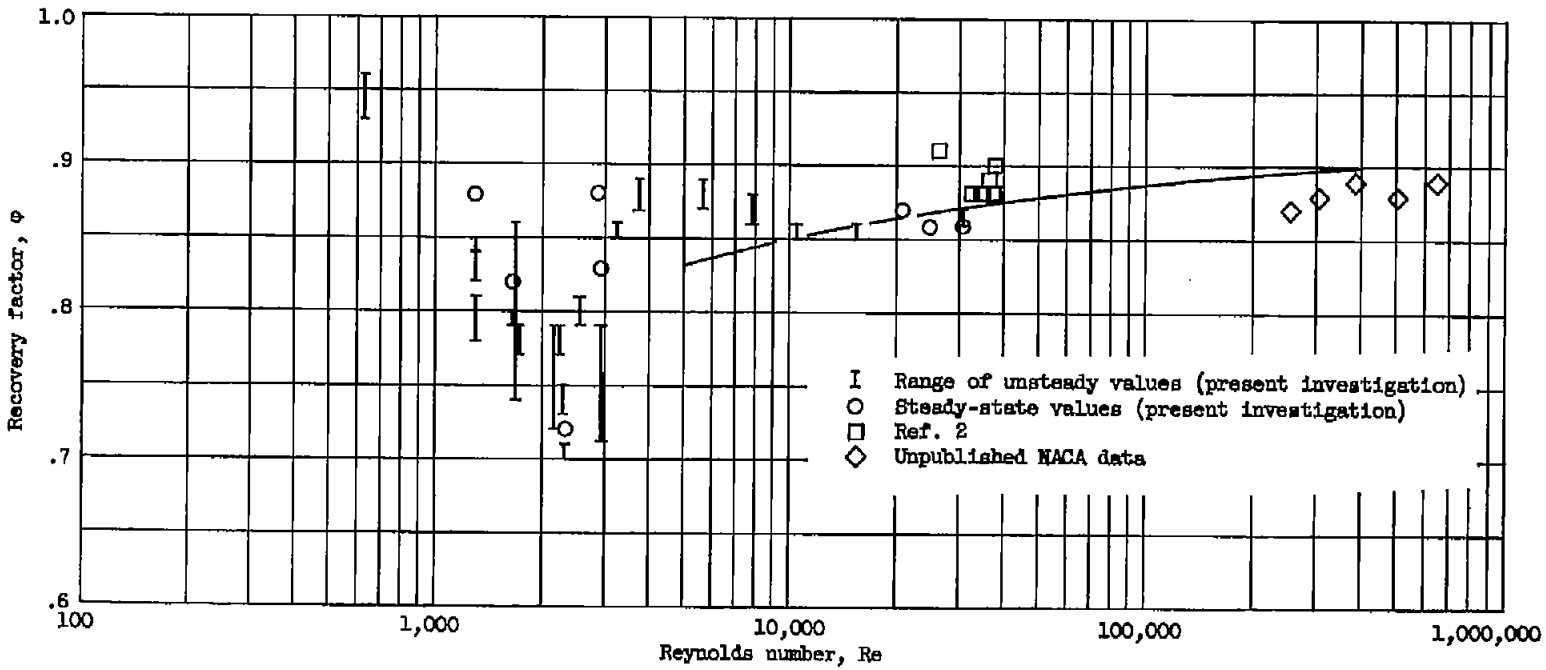

Figure 7. - Comparison of predicted recovery factor curve for eddy diffusivity ratio of 1.07 with measured recovery factor at tube exit.

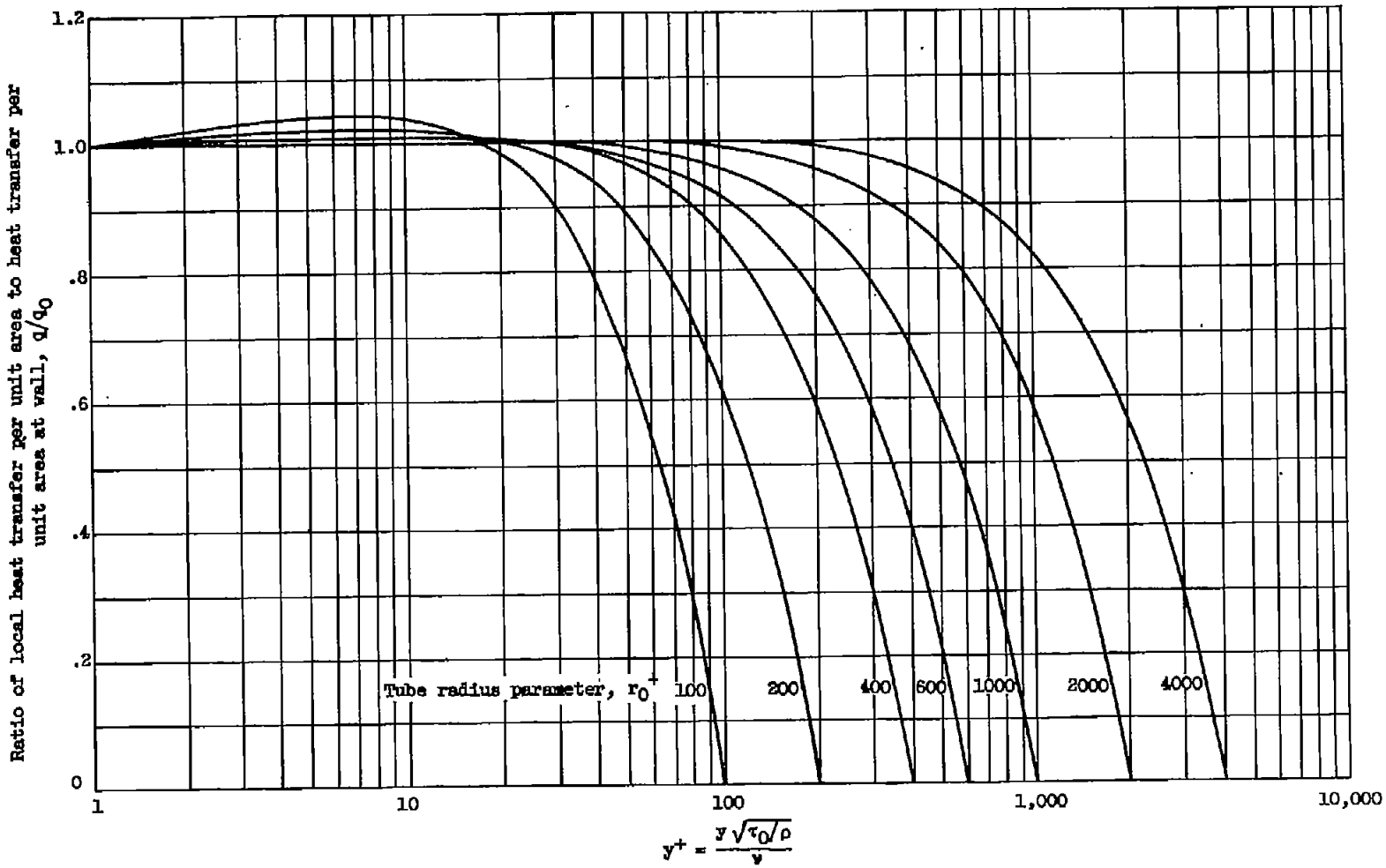

Figure 8. - Variation of heat transfer per unit area across tube.

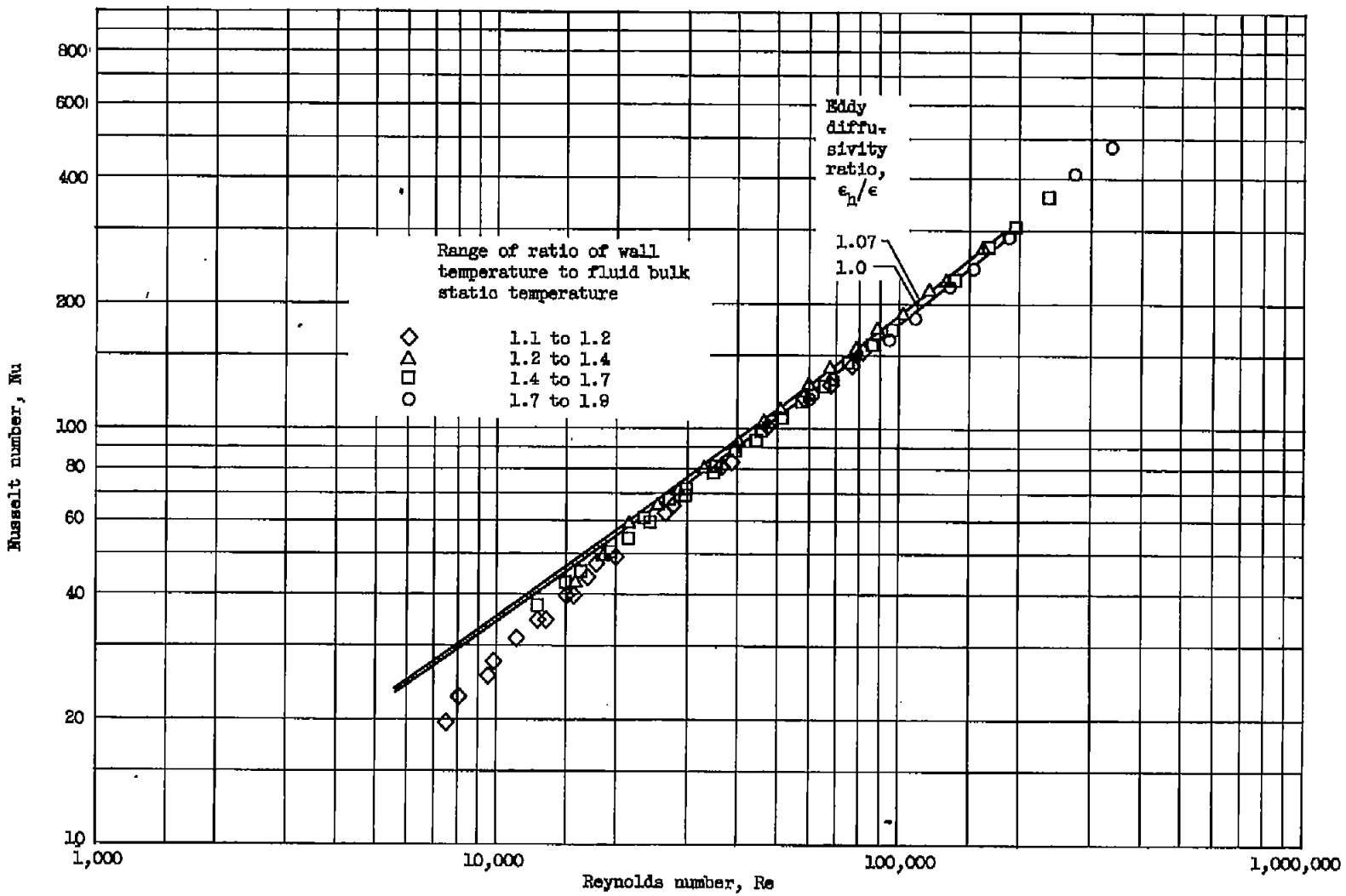

Figure 9. - Variation of Nusselt number with Reynolds number at various ratios of wall temperature to fluid bulk static temperature and several values of ϵ_h/ϵ. (Properties evaluated at film temperature.)

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